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Common fixed points of mappings satisfying implicit contractive conditions

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Abstract

In this article we obtain, in the setting of metric spaces or ordered metric spaces, coincidence point, and common fixed point theorems for self-mappings in a general class of contractions defined by an implicit relation. Our results unify, extend, generalize many related common fixed point theorems from the literature.

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Keywords: implicit relation, contraction, coincidence point, fixed point, common fixed point

Introduction and preliminaries

It is well known that the contraction mapping principle, formulated and proved in the Ph.D. dissertation of Banach in 1920, which was published in 1922 [1], is one of the most important theorems in classical functional analysis. The study of fixed and common fixed points of mappings satisfying a certain metrical contractive condition attracted many researchers, see for example [2,3] and for existence results for fixed points of contractive non-self-mappings, see [4-6]. Among these (common) fixed point theorems, only a few give a constructive method for finding the fixed points or the common fixed points of the mappings involved. Berinde in [7-15] obtained (common) fixed point theorems, which were called constructive (common) fixed point theorems, see [12]. These results have been obtained by considering self-mappings that satisfy an explicit contractive-type condition. On the other hand, several classical fixed point theorems and common fixed point theorems have been recently unified by considering general contractive conditions expressed by an implicit relation, see Popa [16,17] and Ali and Imdad [18]. Following Popa's approach, many results on fixed point, common fixed point and coincidence point has been obtained, in various ambient spaces, see [16-25] and references therein.

In [21], Berinde obtained some constructive fixed point theorems for almost contractions satisfying an implicit relation. These results unify, extend, generalize related results (see [2,3,7-16,21,25-38]).

In this article we obtain, in the setting of metric spaces or ordered metric spaces, coincidence point, and common fixed point results for self-mappings in a general class of contractions defined by an implicit relation. Our results unify, extend, generalize many of related common fixed point theorems from literature.

Let X be a non-empty set and $f, T: X \to X$. A point $x \in X$ is called a coincidence point of f and T if $Tx = fx$. The mappings f and T are said to be weakly compatible if they commute at their coincidence point (i.e., $Tfx = fTx$ whenever $Tx = fx$). Suppose $TX \subset fX$. For every $x_0 \in X$ we consider the sequence $\{x_n\} \subset X$ defined by $fx_n = Tx_{n-1}$ for all $n \in \mathbb{N}$, we say that $\{Tx_n\}$ is a T -f -sequence with initial point x_0.

Let X be a non-empty set. If (X, d) is a metric space and $(X, \preccurlyeq)$ is partially ordered, then $(X, d, \preccurlyeq)$ is called an ordered metric space. Then, $x, y \in X$ are called comparable if $x \preccurlyeq y$ or $y \preccurlyeq x$ holds. Let $f, T: X \to X$ be two mappings, T is said to be f -non-decreasing if $fx \preccurlyeq fy$ implies $Tx \preccurlyeq Ty$ for all $x, y \in X$. If f is the identity mapping on X, then T is non-decreasing.

Throughout this article the letters $\mathbb{R}_+$ and $\mathbb{N}$ will denote the set of all non-negative real numbers and the set of all positive integer numbers.

Fixed point theorems for mappings satisfying an implicit relation

A simple and natural way to unify and prove in a simple manner several metrical fixed point theorems is to consider an implicit contraction type condition instead of the usual explicit contractive conditions. Popa [16,17] initiated this direction of research which produced so far a consistent literature (that cannot be completely cited here) on fixed point, common fixed point, and coincidence point theorems, for both single-valued and multi-valued mappings, in various ambient spaces; see the recent nice paper [21] of Berinde, for a partial list of references.

In [21], Berinde considered the family $\mathcal{F}$ of all continuous real functions $F : \mathbb{R}_+^6 \to \mathbb{R}_+$ and the following conditions:

(F_{1a}) F is non-increasing in the fifth variable and $F(u, v, v, u, u + v, 0) \le 0$ for $u, v \ge 0$ implies that there exists $h \in [0, 1)$ such that $u \le hv$;

(F_{1b}) F is non-increasing in the fourth variable and $F(u, v, 0, u + v, u, v) \le 0$ for $u, v \ge 0$ implies that there exists $h \in [0, 1)$ such that $u \le hv$;

(F_{1c}) F is non-increasing in the third variable and $F(u, v, u+v, 0, v, u) \le 0$ for $u, v \ge 0$ implies that there exists $h \in [0, 1)$ such that $u \le hv$;

(F_2) $F(u, u, 0, 0, u, u) > 0$, for all $u > 0$.

He gave many examples of functions corresponding to well-known fixed point theorems and satisfying most of the conditions (F_{1a})-(F_2) above, see Examples 1-11 of [21].

Example **1**. The following functions $F \in \mathcal{F}$ satisfy properties F_2 and F_{1a}-F_{1c} (see Examples 1-6, 9, and 11 of [21]).

(i) $F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - at_2$, where $a \in [0, 1)$;

(ii) $F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - b(t_3 + t_4)$, where $b \in [0, 1/2)$;

(iii) $F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - c(t_5 + t_6)$, where $c \in [0, 1/2)$;

(iv) $F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - a\max\{t_2, \frac{t_3+t_4}{2}, \frac{t_5+t_6}{2}\}$, where $a \in [0, 1)$;

(v) $F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - at_2 - b(t_3 + t_4) - c(t_5 + t_6)$, where $a, b, c \in [0, 1)$ and $a + 2b + 2c < 1$;

(vi) $F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - a\max\{t_2, \frac{t_3+t_4}{2}, t_5, t_6\}$, where $a \in [0, 1)$;

(vii) $F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - at_2 - L\min\{t_3, t_4, t_5, t_6\}$, where $a \in [0, 1)$;

(viii) $F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - a\max\{t_2, t_3, t_4, \frac{t_5+t_6}{2}\} - L\min\{t_3, t_4, t_5, t_6\}$, where $a \in [0, 1)$ and $L \ge 0$.

Example **2**. The function $F \in \mathcal{F}$, given by

$$F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - a \max\{t_2, t_3, t_4, t_5, t_6\},$$

where $a \in [0, 1/2)$ satisfies properties F_2 and F_{1a}-F_{1c} with $h = \frac{a}{1-a} < 1$.

Motivated by [21], the following theorem is one of the main results in this article.

Theorem 1. *Let (X, d) be a metric space and $T, f: X \to X$ be self-mappings such that $TX \subseteq fX$. Assume that there exists $F \in \mathcal{F}$, satisfying (F_{1a}), such that for all $x, y \in X$*

$$F(d(Tx, Ty), d(fx, fy), d(fx, Tx), d(fy, Ty), d(fx, Ty), d(fy, Tx)) \leq 0. \tag{1}$$

If fX is a complete subspace of X, then T and f have a coincidence point. Moreover, if T and f are weakly compatible and F satisfies also F_2, then T and f have a unique common fixed point. Further, for any $x_0 \in X$, the T-f-sequence $\{Tx_n\}$ with initial point x_0 converges to the common fixed point.

Proof. Let $x_0 \in X$ be an arbitrary point. As $TX \subseteq fX$, one can choose a T-f-sequence $\{Tx_n\}$ with initial point x_0. If we take $x = x_n$ and $y = x_{n+1}$ in (1) and denote with $u = d(Tx_n, Tx_{n+1})$ and $v = d(Tx_{n-1}, Tx_n)$ we get that

$$F(u, v, v, u, d(Tx_{n-1}, Tx_{n+1}), 0) \leq 0.$$

By triangle inequality, $d(Tx_{n-1}, Tx_{n+1}) \leq d(Tx_{n-1}, Tx_n) + d(Tx_n, Tx_{n+1}) = u + v$ and, since F is non-increasing in the fifth variable, we have

$$F(u, v, v, u, u + v, 0) \leq 0$$

and hence, in view of assumption (F_{1a}), there exists $h \in [0, 1)$ such that $u \leq hv$, i.e.,

$$d(Tx_n, Tx_{n+1}) \leq hd(Tx_{n-1}, Tx_n) \quad \text{for all } n \in \mathbb{N}. \tag{2}$$

By (2), in a straightforward way, we deduce that $\{Tx_n\}$ is a Cauchy sequence. Since fX is complete, there exist $z, w \in X$ such that $z = fw$ and

$$\lim_{n\to+\infty} Tx_n = \lim_{n\to+\infty} fx_n = fw = z. \tag{3}$$

By taking $x = x_n$ and $y = w$ in (1), we obtain that

$$F(d(Tx_n, Tw), d(fx_n, fw), d(fx_n, Tx_n), d(fw, Tw), d(fx_n, Tw), d(fw, Tx_n)) \leq 0. \tag{4}$$

As F is continuous, using (3) and letting $n \to +\infty$ in (4), we get

$$F(d(fw, Tw), d(fw, fw), d(fw, fw), d(fw, Tw), d(fw, Tw), d(fw, fw)) \leq 0$$

which, by assumption (F_{1a}), yields $d(fw, Tw) \leq 0$, i.e., $fw = Tw = z$. Thus, we have proved that T and f have a coincidence point.

Now, we assume that T and f are weakly compatible, then $fz = fTw = Tfw = Tz$.

We show that $Tz = z = Tw$.

Suppose $d(Tz, Tw) > 0$, by taking $x = z$ and $y = w$ in (1), we get

$$F(d(Tz, Tw), d(fz, fw), d(fz, Tz), d(fw, Tw), d(fz, Tw), d(fw, Tz)) \leq 0,$$

i.e.,

$$F(d(Tz, Tw), d(Tz, Tw), 0, 0, d(Tz, Tw), d(Tz, Tw)) \leq 0,$$

which is a contradiction by assumption (F_2). This implies that $d(Tz, Tw) = 0$ and hence $fz = Tz = Tw = z$. So T and f have a common fixed point.

The uniqueness of the common fixed point is a consequence of assumption (F_2). Clearly, for any $x_0 \in X$, the T-f-sequence $\{Tx_n\}$ with initial point x_0 converges to the unique common fixed point. □

Remark 1. From (2) we deduce the unifying error estimate

$$d(Tx_{n+i-1}, z) \le \frac{h^i}{1-h} d(Tx_{n-1}, Tx_n).$$

From this we get both the a priori estimate

$$d(Tx_n, z) \le \frac{h^n}{1-h} d(Tx_0, Tx_1), \quad n = 1, 2, \ldots$$

and the a posteriori estimate

$$d(Tx_n, z) \le \frac{h}{1-h} d(Tx_{n-1}, Tx_n), \quad n = 1, 2, \ldots$$

which are extremely important in applications, especially when approximating the solutions of nonlinear equations.

If $f = I_X$ from Theorem 1, we deduce the following result of fixed point for one self-mapping, see [21].

Corollary 1. *Let (X, d) be a complete metric space and $T: X \to X$. Assume that there exists, $F \in \mathcal{F}$ satisfying (F_{1a}), such that for all $x, y \in X$*

$$F(d(Tx, Ty), d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)) \le 0.$$

Then T has a fixed point. Moreover, if F satisfies also F_2, then T has a unique fixed point. Further, for any $x_0 \in X$, the Picard sequence $\{T^n x_0\}$ with initial point x_0 converges to the fixed point.

Common fixed point in ordered metric spaces

The existence of fixed points in ordered metric spaces was investigated by Turinici [39], Ran and Reurings [40], Nieto and Rodríguez-López [41]. See, also [42-45], and references therein. A common fixed point result in ordered metric spaces for mappings satisfying implicit contractive conditions is given by the next theorem.

Theorem 2. *Let $(X, d, \preccurlyeq)$ be a complete ordered metric space and $T, f: X \to X$ be self-mappings such that $TX \subseteq fX$. Assume that there exists $F \in \mathcal{F}$, satisfying (F_{1a}), such that for all $x, y \in X$ with $fx \preccurlyeq fy$*

$$F(d(Tx, Ty), d(fx, fy), d(fx, Tx), d(fy, Ty), d(fx, Ty), d(fy, Tx)) \le 0. \tag{5}$$

If the following conditions hold:

(i) *there exists $x_0 \in X$ such that $fx_0 \preccurlyeq Tx_0$;*

(ii) *T is f-non-decreasing;*

(iii) *for a non-decreasing sequence $\{fx_n\} \subseteq X$ converging to $fw \in X$, we have $fx_n \preccurlyeq fw$ for all $n \in \mathbb{N}$ and $fw \preccurlyeq f fw$;*

then T and f have a coincidence point in X. Moreover, if

(iv) *T and f are weakly compatible;*

(v) *F satisfies also F_2,*

then T and f have a common fixed point. Further, for any $x_0 \in X$, the T-f-sequence $\{Tx_n\}$ with initial point x_0 converges to a common fixed point.

Proof. Let $x_0 \in X$ such that $fx_0 \preccurlyeq Tx_0$ and let $\{Tx_n\}$ be a T-f-sequence with initial point x_0. Since $fx_0 \preccurlyeq Tx_0$ and $Tx_0 = fx_1$, we have $fx_0 \preccurlyeq fx_1$. As T is f-non-decreasing we get that $Tx_0 \preccurlyeq Tx_1$. Continuing this process we obtain

$$fx_0 \preccurlyeq Tx_0 = fx_1 \preccurlyeq Tx_1 = fx_2 \preccurlyeq \cdots \preccurlyeq Tx_n = fx_{n+1} \preccurlyeq \cdots .$$

In what follows we will suppose that $d(Tx_n, Tx_{n+1}) > 0$ for all $n \in \mathbb{N}$, since if $Tx_n = Tx_{n+1}$ for some n, then $fx_{n+1} = Tx_n = Tx_{n+1}$. This implies that x_{n+1} is a coincidence point for T and f and the result is proved. As $fx_n \preccurlyeq fx_{n+1}$ for all $n \in \mathbb{N}$, if we take $x = x_n$ and $y = x_{n+1}$ in (5) and denote $u = d(Tx_n, Tx_{n+1})$ and $v = d(Tx_{n-1}, Tx_n)$ we get that

$$F(u,\ v,\ v,\ u,\ d(Tx_{n-1},\ Tx_{n+1}),\ 0) \le 0.$$

By triangle inequality, $d(Tx_{n-1}, Tx_{n+1}) \le d(Tx_{n-1}, Tx_n) + d(Tx_n, Tx_{n+1}) = u + v$ and, since F is non-increasing in the fifth variable, we have

$$F(u,\ v,\ v,\ u,\ u+v,\ 0) \le 0$$

and hence, in view of assumption (F_{1a}), there exists $h \in [0, 1)$ such that $u \le hv$, i.e.,

$$d(Tx_n,\ Tx_{n+1}) \le hd(Tx_{n-1},\ Tx_n). \tag{6}$$

By (6), we deduce that $\{Tx_n\}$ is a Cauchy sequence. Since (X, d) is complete, there exist $z, w \in X$ such that $z = fw$ and

$$\lim_{n\to+\infty} Tx_n = \lim_{n\to+\infty} fx_n = fw = z. \tag{7}$$

By condition (iii), $fx_n \preccurlyeq fw$ for all $n \in \mathbb{N}$, if we take $x = x_n$ and $y = w$ in (5) we get

$$F(d(Tx_n, Tw),\ d(fx_n,\ fw),\ d(fx_n, Tx_n),\ d(fw,\ Tw),\ d(fx_n, Tw),\ d(fw,\ Tx_n) \le 0.$$

As F is continuous, using (7) and letting $n \to +\infty$ we obtain

$$F(d(fw,\ Tw),\ d(fw,\ fw),\ d(fw,\ fw),\ d(fw,\ Tw),\ d(fw,\ Tw),\ d(fw,\ fw)) \le 0$$

which, by assumption (F_{1a}), yields $d(fw, Tw) \le 0$, i.e., $fw = Tw$. Thus we have proved that T and f have a coincidence point.

If T and f are weakly compatible we show that z is a common fixed point for T and f. As $fz = fTw = Tfw = Tz$, by condition (iii), we have that $fw \preccurlyeq f fw = fz$.

Now, by taking $x = w$ and $y = z$ in (5) we get

$$F(d(Tw,\ Tz),\ d(fw,\ fz),\ d(fw,\ Tw),\ d(fz,\ Tz),\ d(fw,\ Tz),\ d(fz,\ Tw)) \le 0.$$

Assumption (F_2) implies $d(Tz, Tw) = 0$ and hence $fz = Tz = Tw = z$. So T and f have a common fixed point. From the proof it follows that, for any $x_0 \in X$, the T-f-sequence $\{Tx_n\}$ with initial point x_0 converges to a common fixed point. □

We shall give a sufficient condition for the uniqueness of the common fixed point in Theorem 2.

Theorem 3. *Let all the conditions of Theorem 2 be satisfied. If the following conditions hold*

(vi) *for all $x, y \in fX$ there exists $v_0 \in X$ such that $fv_0 \preccurlyeq x$, $fv_0 \preccurlyeq y$;*

(vii) *F satisfies F_{1c},*

then T and f have a unique common fixed point.

Proof. Let z, w be two common fixed points of T and f with $z \neq w$. If z and w are comparable, say $z \preccurlyeq y$. Then taking $x = z$ and $y = w$ in (5), we obtain

$$F(d(Tz, Tw), d(fz, fw), d(fz, Tz), d(fw, Tw), d(fz, Tw), d(fw, Tz)) \leq 0,$$

which is a contradiction by assumption (F_2) and so $z = w$.

If z and w are not comparable, then there exists $v_0 \in X$ such that $fv_0 \preccurlyeq fz = z$ and $fv_0 \preccurlyeq fw = w$.

As T is f-non-decreasing from $fv_0 \preccurlyeq fz$ we get that

$$fv_1 = Tv_0 \preccurlyeq Tz = fz.$$

Continuing we obtain

$$fv_{n+1} = Tv_n \preccurlyeq Tz = fz \quad \text{for all} \quad n \in \mathbb{N}.$$

Then, taking $x = v_n$ and $y = z$ in (5) we obtain

$$F(d(Tv_n, Tz), d(fv_n, fz), d(fv_n, Tv_n), d(fz, Tz), d(fv_n, Tz), d(fz, Tv_n)) \leq 0,$$

i.e.,

$$F(d(Tv_n, Tz), d(Tv_{n-1}, Tz), d(Tv_{n-1}, Tv_n), d(fz, Tz), d(Tv_{n-1}, Tz), d(Tz, Tv_n)) \leq 0.$$

Denote $u = d(Tv_n, Tz)$ and $v = d(Tv_{n-1}, Tz)$. As F is non-increasing in the third variable, we get

$$F(u, v, u + v, 0, v, u) \leq 0.$$

By assumption F_{1c}, there exists $h \in [0, 1)$ such that $u \leq hv$, i.e.,

$$d(Tv_n, Tz) \leq hd(Tv_{n-1}, Tz), \quad \text{for all } n \in \mathbb{N}.$$

This implies that $d(Tv_n, Tz) = d(Tv_n, z) \to 0$ as $n \to +\infty$.

With similar arguments, we deduce that $d(Tv_n, w) \to 0$ as $n \to +\infty$. Hence

$$0 < d(w, z) \leq d(w, Tv_n) + d(Tv_n, z) \to 0$$

as $n \to +\infty$, which is a contradiction. Thus T and f have a unique common fixed point. □

If $f = I_X$ from Theorems 2 and 3, we deduce the following results of fixed point for one self-mapping.

Corollary 2. *Let $(X, d, \preccurlyeq)$ be a complete ordered metric space and T: $X \to X$. Assume that there exists $F \in \mathcal{F}$, satisfying (F_{1a}), such that for all $x, y \in X$ with $x \preccurlyeq y$*

$$F(d(Tx, Ty), d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)) \leq 0. \tag{8}$$

If the following conditions hold:

(i) *there exists $x_0 \in X$ such that $x_0 \preccurlyeq Tx_0$;*

(ii) *T is non-decreasing;*

(iii) *for a non-decreasing sequence $\{x_n\} \subseteq X$ converging to $w \in X$, we have $x_n \preccurlyeq w$ for all $n \in \mathbb{N}$,*

then T has a fixed point in X. Further, for any $x_0 \in X$, the Picard sequence $\{T^n x_0\}$ with initial point x_0 converges to a fixed point.

Corollary 3. *Let all the conditions of Corollary 2 be satisfied. If the following conditions hold*

(v) *F satisfies F_2;*

(vi) *for all $x, y \in X$ there exists $v_0 \in X$ such that $v_0 \preccurlyeq x$, $v_0 \preccurlyeq y$;*

(vii) *F satisfies F_{1c},*

then T has a unique fixed point.

If F is the function in Example 2, then by Theorem 3 we obtain a fixed point theorem that extends the result of Theorem 3 of [44].

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Authors' Contributions
All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

Competing interests
The author declares that they have no competing interests.

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