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Dynamical analysis of a biological resource management model with impulsive releasing and harvesting

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Abstract

In this study, we consider a biological resource management predator-prey model with impulsive releasing and harvesting at different moments. First, we prove that all solutions of the investigated system are uniformly ultimately bounded. Second, the conditions of the globally asymptotic stability predator-extinction boundary periodic solution are obtained. Third, the permanence condition of the investigated system is also obtained. Finally, the numerical simulation verifies our results. These results provide reliable tactic basis for the biological resource management in practice.

Keywords: predator-prey model, impulsive releasing, impulsive harvesting, extinction, permanence

1 Introduction

Biological resources are renewable resources. Economic and biological aspects of renewable resources management have been considered by Clark [1]. In recent years, the optimal management of renewable resources, which has direct relationship to sustainable development, has been studied extensively by many authors [2-4]. Especially, the predator-prey models with harvesting (or dispersal and competition) are investigated by many articles [5-8]. In general, the exploitation of population should be determined by the economic and biological value of the population. It is the purpose of this article to analyze the exploitation of the predator-prey model with impulsive releasing and harvesting at different moments.

Impulsive delay differential equations are suitable for the mathematical simulation of the evolutionary process. The application of impulsive delay differential equations to population dynamics is an interesting topic since it is reasonable and correct in modelling the evolution of population, such as pest management [9]. Moreover, impulsive delay differential equations are used in various fields of applied sciences too, for example physics, ecology, pest control and so on. According to the nature of biological resource management, Jiao et al. [10] introduced the stocking on prey at fixed moments, and considering the following impulsive delay differential equation

$$\left. \begin{cases} x'_1(t) = x_1(t)(a - bx_1(t)) - \frac{\beta x_1(t)}{1 + cx_1(t)}x_3(t), \\ x'_2(t) = rx_3(t) - re^{-w\tau_1}x_3(t - \tau_1) - wx_2(t), \\ x'_3(t) = re^{-w\tau_1}x_3(t - \tau_1) + \frac{k\beta x_1(t)}{1 + cx_1(t)}x_3(t) - d_3x_3(t) - Ex_3(t) - d_4x_3^2(t), \end{cases} \right\} t \neq n\tau, \tag{1.1}$$

$$\left. \begin{cases} \Delta x_1(t) = \mu, \\ \Delta x_2(t) = 0, \\ \Delta x_3(t) = 0, \end{cases} \right\} t = n\tau, \quad n = 1, 2, \dots$$

$$(\varphi_1(\zeta), \varphi_2(\zeta), \varphi_3(\zeta)) \in C_+ = C([-\tau_1, 0], \mathbb{R}_+^3), \quad \varphi_i(0) > 0, \quad i = 1, 2, 3.$$

The biological meanings of the parameters in (1.1) can be seen in [10]. Jiao and Chen [10] consider the mature predator population is harvested continuously. In fact, the population with economic value are harvested discontinuously. It will be arisen at fixed moments or state-dependent moments, that is to say, the releasing population and harvesting population should be occurred at differential moments in [10]. In this article, in order to model the fact of the biological resource management, we investigate a differential equation with two impulses for the biological resource management.

2 The model

It is well known that the basic Lotka-Volterra predator-prey model can be written as

$$\begin{cases} \frac{dx_1(t)}{dt} = x_1(t)(r - ax_1 - bx_2(t)), \\ \frac{dx_2(t)}{dt} = x_2(t)(-d + cx_1(t)), \end{cases} \tag{2.1}$$

where $x_1(t)$ and $x_2(t)$ are densities of the prey population and the predator population, respectively, $r > 0$ is the intrinsic growth rate of prey, $a > 0$ is the coefficient of intraspecific competition, $b > 0$ is the per-capita rate of predation of the predator, $d > 0$ is the death rate of predator, $c > 0$ denotes the product of the per-capita rate of predation and the rate of converting prey into predator. If $rc < ad$ is satisfied, the predator $x_2(t)$ will go extinct and the prey will tend to r/a , that is to say, system (2.1) has boundary equilibrium $(r/a, 0)$. If $rc > ad$ is satisfied, system (2.1) has globally asymptotically stable unique positive equilibrium $(d/c, rc - ad/cb)$.

System (2.1) is an organic growth model, that is to say, there is no intervention management on system (2.1). Obviously, the dynamical behaviors of system (2.1) is very simple. As a matter of fact, the mankind more and more devote themselves to investigate and empolder the ecosystem with the development of society. Bases on the ideology, we develop (2.1) by introducing releasing the prey and harvesting the predator and prey at different fixed moments, that is, we consider the following impulsive differential equation

$$\left. \begin{cases} \frac{dx(t)}{dt} = x(t)(a - bx(t)) - \beta x(t)y(t), \\ \frac{dy(t)}{dt} = k\beta x(t)y(t) - dy(t), \end{cases} \right\} t \neq (n+l)\tau, \quad t \neq (n+1)\tau, \tag{2.2}$$

$$\left. \begin{cases} \Delta x(t) = -\mu_1 x(t), \\ \Delta y(t) = -\mu_2 y(t), \end{cases} \right\} t = (n+l)\tau, \quad n = 1, 2, \dots$$

$$\left. \begin{cases} \Delta x(t) = \mu, \\ \Delta y(t) = 0, \end{cases} \right\} t = (n+1)\tau, \quad n = 1, 2, \dots$$

where $x(t)$ denotes the density of the predator population at time t . $y(t)$ denotes the density of the prey population Y at time t . $a > 0$ denotes the intrinsic growth rate of the prey population X . $b > 0$ denotes the coefficient of the intraspecific competition in prey population X . $\beta > 0$ denotes the per-capita rate predation of the predator population Y . $k > 0$ denotes product of the per-capita rate and the rate of conversing prey population X into predator population Y . $d > 0$ denotes the death rate of the predator population Y . $0 < \mu_1 < 1$ denotes the harvesting rate of prey population X at $t = (n + l)\tau$, $n \in \mathbb{Z}^+$. $0 < \mu_2 < 1$ denotes the harvesting rate of predator population Y at $t = (n + l)\tau$, $n \in \mathbb{Z}^+$. $\mu > 0$ denotes the released amount of prey population X at $t = (n + 1)\tau$, $n \in \mathbb{Z}^+$. $\Delta x(t) = x(t^+) - x(t)$, where $x(t^+)$ represents the density of prey population X immediately after the impulsive releasing (or harvesting) at time t , while $x(t)$ represents the density of prey population X before the impulsive releasing (or harvesting) at time t . $\Delta y(t) = y(t^+) - y(t)$, where $y(t^+)$ represents the density of predator population Y immediately after the impulsive harvesting at time t , while $y(t)$ represents the density of predator population Y before the impulsive harvesting at time t . $0 < l < 1$, and τ denotes the period of impulsive effect.

3 The lemmas

Before discussing main results, we will give some definitions, notations and lemmas. Let $R_+ = [0, \infty)$, $R_+^2 = \{z \in R^2 : z > 0\}$. Denote $f = (f_1, f_2)$ the map defined by the right hand of system (2.2). Let $V : R_+ \times R_+^2 \rightarrow R_+$, then V is said to belong to class V_0 , if

(i) V is continuous in $(n\tau, (n + l)\tau) \times R_+^2$ and $[(n + l)\tau, (n + 1)\tau] \times R_+^2$ for each $\lim_{(t,y) \rightarrow (n\tau^+,z)} V(t,z) = V(n\tau^+,z)$, $\lim_{(t,y) \rightarrow ((n+l)\tau^+,z)} V(t,z) = V((n+l)\tau^+,z)$ and $\lim_{(t,z) \rightarrow ((n+l)\tau^+,z)} V(t,z) = V((n+l)\tau^+,z)$ exists.

(ii) V is locally Lipschitzian in z .

Definition 3.1. $V \in V_0$, then for $(t,z) \in (n\tau, (n + l)\tau) \times R_+^2$ and $[(n + l)\tau, (n + 1)\tau] \times R_+^2$, the upper right derivative of $V(t, z)$ with respect to the impulsive differential system (2.2) is defined as

$$D^+V(t, z) = \limsup_{h \rightarrow 0} \frac{1}{h} [V(t + h, z + hf(t, z)) - V(t, z)].$$

The solution of system (2.2), denote by $z(t) = (x(t), y(t))^T$, is a piecewise continuous function $z : R_+ \rightarrow R_+^2$, $z(t)$ is continuous on $(n\tau, (n + l)\tau) \times R_+^2$ and $[(n + l)\tau, (n + 1)\tau] \times R_+^2$ ($n \in \mathbb{Z}_+, 0 \leq l \leq 1$). Obviously, the global existence and uniqueness of solutions of (2.2) is guaranteed by the smoothness properties of f , which denotes the mapping defined by right-side of system (2.2) (see Lakshmikantham [4]).

Before we have the main results. We need give some lemmas which will be used in the next. Since $(dx(t)/dt = 0)$ whenever $x(t) = 0$, $(dy(t)/dt = 0)$ whenever $y(t) = 0$, $t = n\tau$, $x(n\tau^+) = (1 - \mu_1)x(n\tau)$, $y(n\tau^+) = (1 - \mu_2)y(n\tau)$, and $t = (n + l)\tau$, $x((n + l)\tau^+) = x((n + l)\tau) + \mu$, $\mu \geq 0$. We can easily have

Lemma 3.2. Suppose $z(t)$ is a solution of system (2.2) with $z(0^+) \geq 0$, then $z(t) \geq 0$ for $t \geq 0$. and further $z(t) > 0$ $t \geq 0$ for $z(0^+) > 0$.

Now, we show that all solutions of (2.3) are uniformly ultimately bounded.

Lemma 3.3. There exists a constant $M > 0$ such that $x(t) \leq M$, $y(t) \leq M$ for each solution $(x(t), y(t))$ of (2.2) with all t large enough.

Proof. Define $V(t) = kx(t) + y(t)$. When $t \neq n\tau$ and $t \neq (n + l)\tau$, we have

$$\begin{aligned} D^+V(t) + dV(t) &= k[(a + d)x(t) - bx^2(t)] \\ &\leq -kb \left[x(t) - \frac{k(a + d)}{2b} \right]^2 + \frac{k(a + d)^2}{4b} \leq M_0, \end{aligned}$$

where $M_0 = k(a + d)^2/4b$. When $t = n\tau$, $V(n\tau^+) = kx(n\tau^+) + y(n\tau^+) = (1 - \mu_1)kx(n\tau) + (1 - \mu_2)y(n\tau) \leq kx(n\tau) + y(n\tau) = V(n\tau)$. When $t = (n + l)\tau$, $V((n + l)\tau^+) = kx((n + l)\tau^+) + y((n + l)\tau^+) = kx((n + l)\tau) + \mu + y((n + l)\tau) = V((n + l)\tau) + \mu$. From ([6], Lemma 2.2, p. 23]), for $t \in (n\tau, (n + l)\tau]$ and $((n + l)\tau, (n + 1)\tau]$, we have

$$\begin{aligned} V(t) &\leq V(0^+)e^{-dt} + \frac{M_0}{d}(1 - e^{-dt}) + \mu \frac{e^{-d(t-\tau)}}{1 - e^{-d\tau}} + \mu \frac{e^{-d\tau}}{e^{d\tau} - 1} \\ &\rightarrow \frac{M_0}{d} + \mu \frac{e^{d\tau}}{e^{d\tau} - 1}, \quad \text{as } t \rightarrow \infty. \end{aligned}$$

So $V(t)$ is uniformly ultimately bounded. Hence, by the definition of $V(t)$, there exists a constant $M > 0$ such that $x(t) \leq M$, $y(t) \leq M$ for t large enough. The proof is complete.

If $y(t) = 0$, we obtain the subsystem of system (2.2)

$$\begin{cases} \frac{dx(t)}{dt} = x(t)(a - bx(t)), & t \neq (n + l)\tau, \quad t \neq (n + 1)\tau, \\ \Delta x(t^+) = -\mu_1 x(t), & t = (n + l)\tau, \quad n \in Z^+, \\ \Delta x(t^+) = \mu, & t = (n + 1)\tau, \quad n \in Z^+, \\ x(0^+) = x(0) > 0. \end{cases} \quad (3.1)$$

It is easy to solve the first equation of system (3.1) between pulses

$$x(t) = \begin{cases} \frac{ae^{a(t-n\tau)}x(n\tau^+)}{a + b [e^{a(t-n\tau)} - 1] x(n\tau^+)}, & t \in (n\tau, (n + l)\tau], \\ \frac{ae^{a(t-(n+l)\tau)}x((n + l)\tau^+)}{a + b [e^{a(t-(n+l)\tau)} - 1] x((n + l)\tau^+)}, & t \in ((n + l)\tau, (n + 1)\tau]. \end{cases} \quad (3.2)$$

By considering the last two equations of system (3.1), we obtain the following stroboscopic map of system (3.1):

$$x((n + 1)\tau^+) = \frac{(1 - \mu_1)ae^{a\tau}x(n\tau^+)}{a + be^{a\tau} [1 + (1 - \mu_1)(e^{a(1-l)\tau} - 1)] x(n\tau^+)} + \mu. \quad (3.3)$$

Taking $A = (1 - \mu_1)ae^{a\tau} > 0$ and $B = be^{a\tau} [1 + (1 - \mu_1)(e^{a(1-l)\tau} - 1)] > 0$, we can rewrite (3.3) as

$$x((n + 1)\tau^+) = \frac{Ax(n\tau^+)}{a + Bx(n\tau^+)} + \mu. \quad (3.4)$$

Referring to [11], we can easily prove that (3.4) has unique positive fixed point

$$x^* = \frac{(A + \mu B - a) + \sqrt{(A + \mu B - a)^2 + 4\mu a B}}{2B}, \quad (3.5)$$

which can be easily proved to be globally asymptotically stable.

Then, we can derive the following lemma:

Lemma 3.4. System (3.1) has a positive periodic solution $\widetilde{x}(t)$. For every solution $x(t)$ of system (3.1), we have $x(t) \rightarrow \widetilde{x}(t)$ as $t \rightarrow \infty$, where

$$\widetilde{x}(t) = \begin{cases} \frac{ae^{a(t-n\tau)}x^*}{a+b[e^{a(t-n\tau)}-1]x^*}, & t \in (n\tau, (n+l)\tau], \\ \frac{(1-\mu_1)ae^{a(t-n\tau)}x^*}{a+b[(e^{al\tau}-1)+(1-\mu_1)(e^{a(t-n\tau)}-e^{al\tau})]x^*}, & t \in ((n+l)\tau, (n+1)\tau]. \end{cases} \quad (3.6)$$

4 The dynamics

In this article, we will prove that the predator-extinction periodic solution is globally asymptotically stable and system (2.2) is permanent.

4.1 The extinction

From above discussion, we know that (2.2) has a predator-extinction periodic solution $(\widetilde{x}(t), 0)$. Then we have following theorem.

Theorem 4.1. If

$$\ln \frac{1}{1-\mu_1} > a\tau - 2 \left[\ln \left(1 + \frac{b(e^{al\tau}-1)x^*}{a} \right) + \ln \left(1 + \frac{b(1-\mu_1)(e^{a\tau}-e^{al\tau})x^*}{a+b(e^{al\tau}-1)} \right) \right], \quad (4.1)$$

and

$$\ln \frac{1}{1-\mu_2} > \frac{k\beta}{b} \left[\ln \left(1 + \frac{b(e^{al\tau}-1)x^*}{a} \right) + \ln \left(1 + \frac{b(1-\mu_1)(e^{a\tau}-e^{al\tau})x^*}{a+b(e^{al\tau}-1)} \right) \right] - d\tau, \quad (4.2)$$

hold, then predator-extinction periodic solution $(\widetilde{x}(t), 0)$ of (2.2) is globally asymptotically stable. Where x^* is defined as (3.5):

Proof. First, we prove the local stability. Define $x_1(t) = x(t) - \widetilde{x}(t)$, $y(t) = y(t)$, we have the following linearly similar system of system (2.2):

$$\begin{pmatrix} \frac{dx_1(t)}{dt} \\ \frac{dy(t)}{dt} \end{pmatrix} = \begin{pmatrix} a - 2b\widetilde{x}(t) & -\beta\widetilde{x}(t) \\ 0 & -d \end{pmatrix} \begin{pmatrix} x_1(t) \\ y(t) \end{pmatrix}.$$

It is easy to obtain the fundamental solution matrix

$$\Phi(t) = \begin{pmatrix} \exp(\int_0^t (a - 2b\widetilde{x}(s)) ds) & * \\ 0 & \exp(-dt) \end{pmatrix}.$$

There is no need to calculate the exact form of (*) as it is not required in the following analysis. The linearization of the third and fourth equations of (2.2) is

$$\begin{pmatrix} x_1((n+l)\tau^+) \\ y((n+l)\tau^+) \end{pmatrix} = \begin{pmatrix} 1-\mu_1 & 0 \\ 0 & 1-\mu_2 \end{pmatrix} \begin{pmatrix} x_1((n+l)\tau) \\ y((n+l)\tau) \end{pmatrix}.$$

The linearization of the fifth and sixth equations of (2.2) is

$$\begin{pmatrix} x_1((n+1)\tau^+) \\ y((n+1)\tau^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1((n+1)\tau) \\ y((n+1)\tau) \end{pmatrix}.$$

The stability of the periodic solution $(\widetilde{x}(t), 0)$ is determined by the eigenvalues of

$$M = \begin{pmatrix} 1 - \mu_1 & 0 \\ 0 & 1 - \mu_2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Phi(\tau),$$

which are

$$\lambda_1 = (1 - \mu_2) \exp(-d\tau) < 1, \lambda_2 = (1 - \mu_1) \exp\left(\int_0^\tau (a - 2b\widetilde{x}(s)) ds\right),$$

According to the Floquet theory [6], if $|\lambda_2| < 1$, i.e. (4.1) holds, then $(\widetilde{x}(t), 0)$ is locally stable.

The following study is to prove the global attraction, choose $\varepsilon > 0$ such that

$$\rho = (1 - \mu_2) \exp\left(\int_0^\tau [k\beta(\widetilde{x}(t) + \varepsilon) - d] dt\right) < 1, \tag{4.3}$$

From the first equation of (2.2), we notice that $dx(t)/dt \leq x(t)(a - bx(t))$, so we consider following impulsive differential equation

$$\begin{cases} \frac{dz(t)}{dt} = z(t)(a - bz(t)), & t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \Delta z(t) = -\mu_1 z(t), & t = (n+l)\tau, \\ \Delta z(t) = \mu, & t = (n+1)\tau, \\ z(0^+) = z(0^+), \end{cases} \tag{4.4}$$

From Lemma 3.4 and comparison theorem of impulsive equation (see [[6], Theorem 3.1.1]), we have $x(t) \leq z(t)$ and $z(t) \rightarrow \widetilde{z}(t)$ as $t \rightarrow \infty$, that is

$$x(t) \leq z(t) \leq \widetilde{x}(t) + \varepsilon, \tag{4.5}$$

for all t large enough, for convenience, we may assume (4.2) hold for all $t \geq 0$. From (2.2) and (4.5), we get.

$$\begin{cases} \frac{dy(t)}{dt} \leq (k\beta(\widetilde{x}(t) + \varepsilon) - d)y(t), & t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \Delta y(t^+) = -\mu_2 y(t), & t = (n+l)\tau, n = 1, 2, \dots \\ \Delta y(t^+) = 0, & t = (n+1)\tau, n = 1, 2, \dots \end{cases} \tag{4.6}$$

So $y((n+l+1)\tau^+) \leq y((n+l)\tau^+)(1 - \mu_2) \exp\left[\int_{(n+l)\tau}^{(n+l+1)\tau} (k\beta(\widetilde{x}(s) + \varepsilon) - d) ds\right]$, hence $y((n+l)\tau^+) \leq y(l\tau^+) \rho^n$ and $y((n+l)\tau^+) \rightarrow 0$ as $n \rightarrow \infty$. Since $0 < y(t) \leq y((n+l)\tau^+)(1 - \mu_1) \frac{ae^{at}x^*}{e^{a+b(e^{at}\tau-1)x^*}t + \frac{a(1-\mu_2)e^{at}x^*}{a+b[(e^{at}\tau-1)+e^{at}\tau-e^{at}\tau]}x^*}(1-l)\tau$ for $(n+l)\tau < t \leq (n+l+1)\tau$, therefore $y(t) \rightarrow 0$ as $t \rightarrow \infty$.

Next we prove that $x(t) \rightarrow \widetilde{x}(t)$ as $t \rightarrow \infty$. For $\varepsilon > 0$, there must exist a $t_0 > 0$ such that $0 < y(t) < \varepsilon$ for all $t \geq t_0$. Without loss of generality, we assume that $0 < y(t) < \varepsilon$ for all $t \geq 0$, then, for the first equation of system (2.2), we have

$$x(t)[(a - \beta\varepsilon) - bx(t)] \leq \frac{dx(t)}{dt} \leq x(t)(a - bx(t)), \tag{4.7}$$

then, we have $z_1(t) \leq x(t) \leq z_2(t)$, and $z_1(t) \rightarrow \widetilde{x}(t)$, $z_2(t) \rightarrow \widetilde{x}(t)$, as $t \rightarrow \infty$. While $z_1(t)$ and $z_2(t)$ are the solutions of

$$\begin{cases} \frac{dz_1(t)}{dt} = z_1(t)[(a - \beta\varepsilon) - bz_1(t)], & t \neq (n+l)\tau, \quad t \neq (n+1)\tau, \\ \Delta z_1(t^+) = -\mu_1 z_1(t), & t = (n+l)\tau, \\ \Delta z_1(t^+) = \mu, & t = (n+1)\tau, \\ z_1(0^+) = x(0^+), \end{cases} \quad (4.8)$$

and

$$\begin{cases} \frac{dz_2(t)}{dt} = z_2(t)[a - bz_2(t)], & t \neq (n+l)\tau, \quad t \neq (n+1)\tau, \\ \Delta z_2(t^+) = -\mu_2 z_2(t), & t = (n+l)\tau, \\ \Delta z_2(t^+) = \mu, & t = (n+1)\tau, \\ z_2(0^+) = I(0^+), \end{cases} \quad (4.9)$$

respectively. And

$$\widetilde{z}_1(t) = \begin{cases} \frac{(a - \beta\varepsilon)e^{(a-\beta\varepsilon)(t-n\tau)} z_1^*}{(a - \beta\varepsilon) + b[e^{(a-\beta\varepsilon)(t-n\tau)} - 1] z_1^*}, & t \in (n\tau, (n+l)\tau], \\ \frac{(1 - \mu_1)(a - \beta\varepsilon)e^{(a-\beta\varepsilon)(t-n\tau)} z_1^*}{(a - \beta\varepsilon) + b[(e^{(a-\beta\varepsilon)\tau} - 1) + (1 - \mu_1)(e^{(a-\beta\varepsilon)(t-n\tau)} - e^{(a-\beta\varepsilon)\tau})] z_1^*}, & t \in ((n+l)\tau, (n+1)\tau]. \end{cases} \quad (4.10)$$

where

$$z_1^* = \frac{(A_1 + \mu B_1 - (a - \beta\varepsilon)) + \sqrt{(A_1 + \mu B_1 - (a - \beta\varepsilon))^2 + 4\mu(a - \beta\varepsilon)B_1}}{2B_1}, \quad (4.11)$$

and $A_1 = (1 - \mu_1)(a - \beta\varepsilon)e^{(a-\beta\varepsilon)\tau} > 0$ and $B_1 = be^{(a - \beta\varepsilon)\tau} [1 + (1 - \mu_1)(e^{(a - \beta\varepsilon)(1-l)\tau} - 1)] > 0$. Therefore, for any $\varepsilon_1 > 0$, there exists a t_1 , $t > t_1$ such that

$$\widetilde{z}_1(t) - \varepsilon_1 < x(t) < \widetilde{z}_2(t) + \varepsilon,$$

Let $\varepsilon \rightarrow 0$, so we have

$$\widetilde{x}(t) - \varepsilon_1 < x(t) < \widetilde{x}(t) + \varepsilon_1,$$

for t large enough, which implies $x(t) \rightarrow \widetilde{x}(t)$ as $t \rightarrow \infty$. This completes the proof.

4.2 The permanence

The following study is to investigate the permanence of system (2.2). Before starting this study, we should give the following definition.

Definition 4.2. System (2.2) is said to be permanent if there are constants $m, M > 0$ (independent of initial value) and a finite time T_0 such that for all solutions $(x(t), y(t))$ with all initial values $x(0^+) > 0, y(0^+) > 0, m \leq x(t) \leq M, m \leq y(t) \leq M$ holds for all $t \geq T_0$. Here T_0 may depends on the initial values $(x(0^+), (y(0^+))$.

Theorem 4.3. If

$$\begin{aligned} & \ln \frac{1}{1 - \mu_2} \\ & < \frac{k\beta}{b} \left[\ln \left(1 + \frac{b(e^{a\tau} - 1)x^*}{a} \right) + \ln \left(1 + \frac{b(1 - \mu_1)(e^{a\tau} - e^{a\tau})x^*}{a + b(e^{a\tau} - 1)} \right) \right] - d\tau, \end{aligned} \quad (4.12)$$

holds, then, system (2.2) is permanent. Where x^* is defined as (3.5).

Proof. Let $(x(t), y(t))$ be a solution of (2.2) with $x(0) > 0, y(0) > 0$. By Lemma 3.3, we have proved there exists a constant $M > 0$ ($\beta M < a$) such that $x(t) \leq M, y(t) \leq M$ for t large enough. We may assume $x(t) \leq M, y(t) \leq M$ for $t \geq 0$.

In this view, for the first equation of system (2.2), we have

$$\frac{dx(t)}{dt} > x(t)[(a - \beta M) - bx(t)], \quad (4.13)$$

then, we obtain the following comparative impulsive differential equation

$$\begin{cases} \frac{dn(t)}{dt} = n(t) [(a - \beta M) - bn(t)], & t \neq (n + l)\tau, \quad t \neq (n + 1)\tau, \\ \Delta n(t^+) = -\mu_1 n(t), & t = (n + l)\tau, \\ \Delta n(t^+) = \mu, & t = (n + 1)\tau, \\ n(0^+) = x(0^+). \end{cases} \quad (4.14)$$

Analyzing (4.14) with similarity as (4.8), we have $n(t) \rightarrow \widetilde{n}(t)$, and

$$\widetilde{n}(t) = \begin{cases} \frac{(a - \beta M)e^{(a - \beta M)(t - n\tau)} n^*}{(a - \beta M) + b[e^{(a - \beta M)(t - n\tau)} - 1] n^*}, & t \in (n\tau, (n + l)\tau), \\ \frac{(1 - \mu_1)(a - \beta M)e^{(a - \beta M)(t - n\tau)} n^*}{(a - \beta M) + b[e^{(a - \beta M)\tau} - 1] + (1 - \mu_1)[e^{(a - \beta M)(t - n\tau)} - e^{(a - \beta M)\tau}] n^*}, & t \in ((n + l)\tau, (n + 1)\tau), \end{cases} \quad (4.15)$$

where

$$n^* = \frac{(A_2 + \mu B_2 - (a - \beta M)) + \sqrt{(A_2 + \mu B_2 - (a - \beta M))^2 + 4\mu(a - \beta M)B_2}}{2B_2}, \quad (4.16)$$

and $A_2 = (1 - \mu_1)(a - \beta M)e^{(a - \beta M)\tau} > 0$ and $B_2 = be^{(a - \beta M)\tau} [1 + (1 - \mu_1)(e^{(a - \beta M)(1 - l)\tau} - 1)] > 0$. Following comparative theory of impulsive differential equation [6], we know there exists a ε_2 such that $x(t) > \widetilde{n}(t) - \varepsilon_2$ for all t large enough, and $\varepsilon_2 > 0$. So $x(t) > [n^* + (1 - \mu_1)(a - \beta M)e^{(a - \beta M)\tau} / (a - \beta M) + b(e^{(a - \beta M)\tau} - 1)] - \varepsilon_2 = m_2$ for t large enough. Thus, we only need to find $m_1 > 0$ such that $y(t) \geq m_1$ for t large enough. We will do it in the following two steps.

(1) By the condition of Theorem 2, we can select $m_3 > 0, \varepsilon_1 > 0$ small enough such that $0 < m_3 < \frac{a}{\beta}$, and

$$\sigma = \frac{k\beta}{b} \left[\ln \left(1 + \frac{b(e^{(a - \beta m_3)\tau} - 1)z^*}{a - \beta m_3} \right) + \ln \left(1 + \frac{b(1 - \mu_1)(e^{(a - \beta m_3)\tau} - e^{(a - \beta m_3)\tau})z^*}{a - \beta m_3 + b(e^{a\tau} - 1)} \right) \right] - k\beta\varepsilon_1\tau - d\tau > 0.$$

where z^* is defined as (4.20). We will prove that $y(t) < m_3$ cannot hold for $t \geq 0$.

Otherwise,

$$\begin{cases} \frac{dx(t)}{dt} \geq x(t) [(a - \beta m_3) - bx(t)], & t \neq (n+l)\tau, \quad t \neq (n+1)\tau, \\ \Delta x(t) = -\mu_1 x(t), & t = (n+l)\tau, \\ \Delta x(t) = x(t) + \mu, & t = (n+1)\tau, \end{cases} \quad (4.17)$$

By Lemma 3.4, we have $x(t) \geq z(t)$ and $z(t) \rightarrow \overline{z(t)}$, $t \rightarrow \infty$, where $z(t)$ is the solution of

$$\begin{cases} \frac{dz(t)}{dt} \geq z(t) [(a - \beta m_3) - bz(t)], & t \neq (n+l)\tau, \quad t \neq (n+1)\tau, \\ \Delta z(t) = -\mu_1 z(t), & t = (n+l)\tau, \\ \Delta z(t) = z(t) + \mu, & t = (n+1)\tau, \end{cases} \quad (4.18)$$

and

$$\overline{z(t)} = \begin{cases} \frac{(a - \beta m_3)e^{(a - \beta m_3)(t - n\tau)} n^*}{(a - \beta m_3) + b[e^{(a - \beta m_3)(t - n\tau)} - 1]z^*}, & t \in (n\tau, (n+l)\tau], \\ \frac{(1 - \mu_1)(a - \beta m_3)e^{(a - \beta m_3)(t - n\tau)} z^*}{(a - \beta m_3) + b[e^{(a - \beta m_3)t} - 1] + (1 - \mu_1)[e^{(a - \beta m_3)(t - n\tau)} - e^{(a - \beta m_3)t}]} z^*, & t \in ((n+l)\tau, (n+1)\tau], \end{cases} \quad (4.19)$$

where

$$z^* = \frac{(A_3 + \mu B_3 - (a - \beta m_3)) + \sqrt{(A_3 + \mu B_3 - (a - \beta m_3))^2 + 4\mu(a - \beta m_3)B_3}}{2B_3}, \quad (4.20)$$

and $A_3 = (1 - \mu_1)(a - \beta m_3)e^{(a - \beta m_3)\tau} > 0$ and $B_3 = be^{(a - \beta m_3)l\tau} [1 + (1 - \mu_1)(e^{(a - \beta m_3)(1-l)\tau} - 1)] > 0$.

Therefore, there exists a $T_1 > 0$ such that

$$x(t) \geq z(t) \geq \overline{z(t)} - \varepsilon_1,$$

and

$$\begin{cases} \frac{dy(t)}{dt} \geq y(t)[k\beta(\overline{z(t)}) - \varepsilon_1] - d, & t \neq (n+l)\tau, \\ \Delta y(t) = -\mu_2 y(t), & t = (n+l)\tau, \quad n = 1, 2, \dots \end{cases} \quad (4.21)$$

for $t \geq T_1$. Let $N_1 \in \mathbb{N}$ and $N_1\tau > T_1$, integrating (4.21) on $((n+l-1)\tau, (n+l)\tau)$, $n \geq N_1$, we have

$$y((n+l)\tau) \geq (1 - \mu_2)^k y((n+l-1)\tau) \exp \left(\int_{(n+l-1)\tau}^{(n+l)\tau} [k\beta(\overline{z(t)}) - \varepsilon_1] dt \right) = (1 - \mu_2)^k y((n+l-1)\tau) e^{\sigma},$$

then, $y((N_1 + k + l)\tau) \geq (1 - \mu_2)^k y((N_1 + l)\tau) e^{k\sigma} \rightarrow \infty$, as $k \rightarrow \infty$, which is a contradiction to the boundedness of $y(t)$. Hence there exists a $t_1 > 0$ such that $y(t) \geq m_3$.

(2) If $y(t) \geq m_3$ for $t \geq t_1$, then our aim is obtained. Hence, we only need to consider those solutions which leave region $R = \{(x(t), y(t)) \in R_+^2 : y(t) < m_3\}$ and reenter it again. Let $t^* = \inf_{t \geq t_1} \{y(t) < m_3\}$, there are two possible cases for t^* .

Case 1. $t^* = (n+l-1)\tau$, $n_1 \in \mathbb{Z}_+$, then $x(t) \geq m_3$ for $t \in [t_1, t^*)$ and $y(t^*) = m_3$, and $y(t^{*+}) = y(t^*) \leq m_3$. Select $n_2, n_3 \in \mathbb{N}$, such that

$$(n_2 - 1)\tau > T_2 = \frac{\ln\left(\frac{\varepsilon_1}{M}\right)}{k\beta m_3 - d},$$

$$(1 - \mu_2)^{n_2} e^{n_3\sigma} e^{n_2\sigma_1\tau} > (1 - \mu_2)^{n_2} e^{n_3\sigma} e^{(n_2+1)\sigma_1\tau} > 1,$$

where $\sigma_1 = k\beta m_2 - d < 0$. Let $T = n_1\tau + n_2\tau$. We claim that there must be a $t_2 \in [t^*, t^* + T]$ such that $y(t_2) > m_3$, otherwise, consider (4.21) with $z(t^{*+}) = y(t^{*+})$. We have

$$z(t) = z(n_1^+)e^{(k\beta m_3 - d)t} + \overline{z(t)}, \quad t \in ((n - 1)\tau, (n + l - 1)\tau], \quad (4.22)$$

and $n_1 + 1 \leq n \leq n_2 + n_3$, then

$$|z(t) - \overline{z(t)}| < Me^{(k\beta m_3 - d)(t - n_1\tau)} < \varepsilon_1,$$

and $y(t) \leq z(t) \leq \overline{z(t)} + \varepsilon_1$, $(n_1 + n_2 - 1)\tau \leq t \leq t^* + T$, which implies (4.22) holds for $t^* + n_2\tau \leq t \leq t^* + T$. As in step 1, we have

$$y(t^* + T) \geq y(t^* + n_2)\tau e^{n_3\sigma},$$

The second equation of system (2.1) gives

$$\begin{cases} \frac{dy(t)}{dt} \geq y(t)(k\beta m_2 - d) = \sigma_1 y(t), & t \neq (n + l - 1)\tau, \\ \Delta y(t) = -\mu_2 y(t), & t = (n + l - 1)\tau, \end{cases} \quad (4.23)$$

Integrating (4.23) on $[t^*, t^* + n_2\tau]$, we have

$$y(t^* + n_2\tau) \geq (1 - \mu_2)^{n_2} m_3 e^{\sigma_1 n_2 \tau},$$

thus we have

$$y(t^* + T) \geq (1 - \mu_2)^{n_2} m_3 e^{\sigma_1 n_2 \tau} e^{n_3\sigma} > m_3,$$

which is a contradiction. Let $\bar{t} = \inf_{t \geq t^*} \{y(t) \geq m_3\}$, thus $y(\bar{t}) \geq m_3$ for $t \in [t^*, \bar{t}]$, we have $y(t) \geq y(t^*)e^{\sigma_1(t - t^*)} \geq (1 - \mu_2)^{n_2 + n_3} m_3 e^{\sigma_1(n_2 + n_3)\tau} = m_1$ for $t \geq \bar{t}$. So we have $y(t) \geq m_1$. The same arguments can be continued since $y(\bar{t}) \geq m_3$. Hence $y(t) \geq m_1$ for all $t \geq \bar{t}$.

Case 2. $t \neq (n + l - 1)\tau$, $n \in Z_+$, then $y(t) \geq m_3$ for $t \in [t_1, t^*]$, and $y(t^*) = m_3$. Suppose $t^* \in ((n'_1 + l - 1)\tau, (n'_1 + l)\tau)$, ($t \in Z_+$), then there are two possible cases for $t \in (t^*, (n'_1 + l)\tau)$.

Case 2(a). $y(t) \leq m_3$ for all $t \in (t^*, (n'_1)\tau)$. Similar to case 1., we can prove that there must be a $t'_2 \in [(n'_1 + l)\tau, (n'_1 + l)\tau + T]$ such that $y(t'_2) > m_3$. Here we omit it.

Let $\bar{t} = \inf_{t > t^*} \{y(t) > m_3\}$, then $y(t) \leq m_3$ for $t \in (t^*, \bar{t})$ and $y(\bar{t}) = m_3$. For $t \in (t^*, \bar{t})$, we have

$$y(t) \geq (1 - p_1)^{n_2 + n_3} m_3 e^{(n_2 + n_3 + 1)\sigma_1\tau}.$$

Let $m_1 = (1 - \mu_2)^{n_2 + n_3} m_3 e^{(n_2 + n_3 + 1)\sigma_1\tau} < m'_1$, so $y(t) \geq m_1$ for $t \in (t^*, \bar{t})$. For $t > \bar{t}$, the same arguments can be continued since $y(t) \geq m_1$.

Case 2(b). There exists a $t \in (t^*, (n'_1)\tau)$ such that $y(t) > m_3$. Let $\hat{t} = \inf_{t > t^*} \{y(t) > m_3\}$, then $y(t) \leq m_3$ for $t \in (t^*, \hat{t})$ and $y(\hat{t}) = m_3$. For $t \in (t^*, \hat{t})$, (4.23) holds true, integrating (4.23) on (t^*, \hat{t}) , we derive

$$y(t) \geq y(t^*)e^{\sigma_1(\hat{t} - t^*)} \geq m_3 e^{\sigma_1\tau} > m_1,$$

Since $y(\hat{t}) \geq m_3$ for $t > \hat{t}$, the same arguments can be continued. Hence $y(t) \geq m_3$ for $t \geq t_1$. This completes the proof.

5 Discussion

In this article, according to the fact of biological resource management, we proposed and investigated a predator-prey model with impulsive releasing prey population and impulsive harvesting predator population and prey population at different fixed moment. We analyze that the predator-extinction periodic solution of this system is globally asymptotic stability. If it is assumed that $x(0) = 2, y(0) = 2, a = 2, b = 1, d = 1, \beta = 0.6, k = 0.9, \mu_1 = 0.2, \mu_2 = 0.6, \mu = 3, l = 0.25, \tau = 1$, obviously, the condition of predator-extinction are satisfied, then the predator-extinction periodic solution of system (2.1) is globally asymptotically stable (the numerical simulation can be seen in Figures 1, 2, and 3). We also obtain the condition of the permanence of system (2.2). If it is assumed that $x(0) = 2, y(0) = 2, a = 2, b = 1, d = 1, \beta = 0.6, k = 0.9, \mu_1 = 0.2, \mu_2 = 0.4, \mu = 3, l = 0.25, \tau = 1$, obviously, the permanent condition of system (2.2) is satisfied, then, system (2.1) is permanent (the numerical simulation can also be seen in Figures 4, 5, and 6). From results of the numerical simulation, we know that there exists an impulsive harvesting predator population threshold μ_2^{**} , which satisfies $0.4 < \mu_2^{**} < 0.6$. If $\mu_2 > \mu_2^{**}$, the predator-extinction periodic solution $(\widetilde{x(t)}, 0)$ of system (2.2) is globally asymptotically stable. If $\mu_2 < \mu_2^{**}$, system (2.2) is permanent.

From Theorems 4.1 and 4.3, we can easily guess that there must exist an impulsive harvesting predator population threshold μ_2^* . If $\mu_2 > \mu_2^*$, the predator-extinction periodic solution $(\widetilde{x(t)}, 0)$ of system (2.2) is globally asymptotically stable. If $\mu_2 < \mu_2^*$,

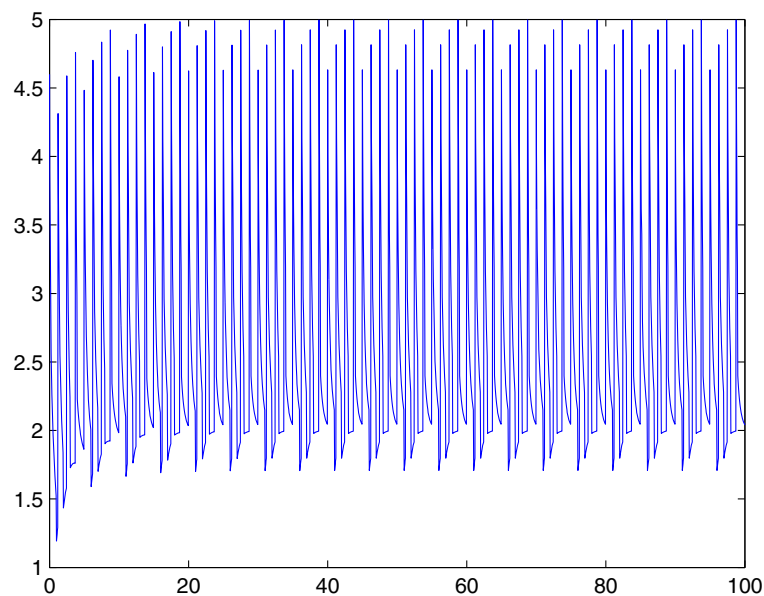


Figure 1 Time-series of $x(t)$ of globally asymptotic stability predator-extinction periodic solution of system (2.2) with $x(0) = 2, y(0) = 2, a = 2, b = 1, d = 1, \beta = 0.6, k = 0.9, \mu_1 = 0.2, \mu_2 = 0.6, \mu = 3, l = 0.25, \tau = 1$.

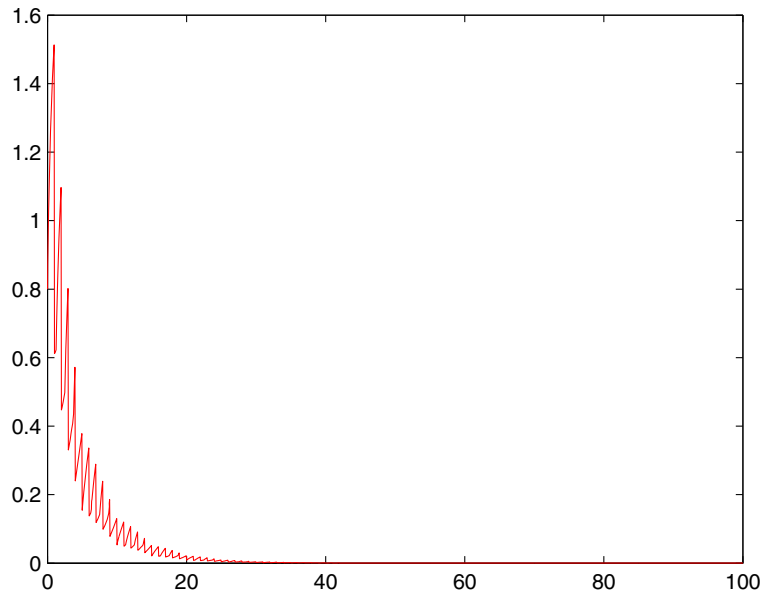


Figure 2 Time-series of $y(t)$ of globally asymptotic stability predator-extinction periodic solution of system (2.2) with $x(0) = 2$, $y(0) = 2$, $a = 2$, $b = 1$, $d = 1$, $\beta = 0.6$, $k = 0.9$, $\mu_1 = 0.2$, $\mu_2 = 0.6$, $\mu = 3$, $l = 0.25$, $\tau = 1$.

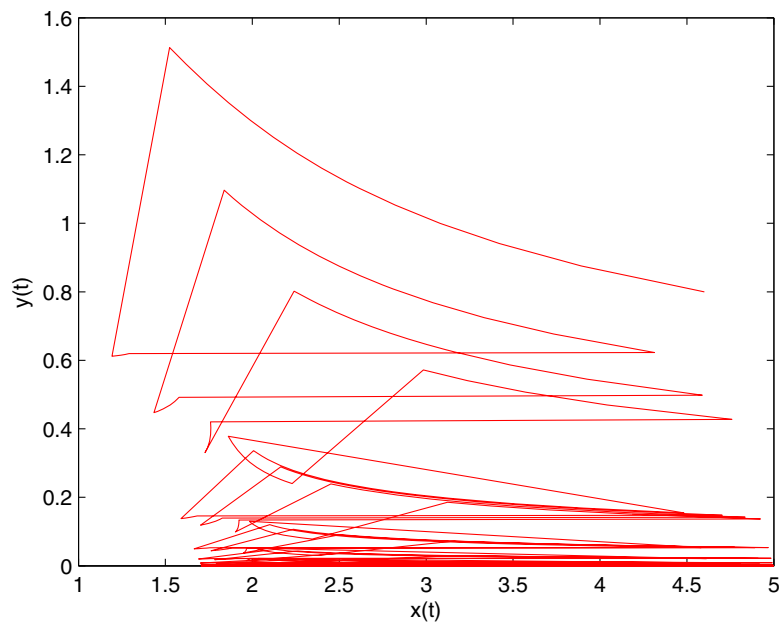


Figure 3 Phase diagram of globally asymptotic stability predator-extinction periodic solution of system (2.2) with $x(0) = 2$, $y(0) = 2$, $a = 2$, $b = 1$, $d = 1$, $\beta = 0.6$, $k = 0.9$, $\mu_1 = 0.2$, $\mu_2 = 0.6$, $\mu = 3$, $l = 0.25$, $\tau = 1$.

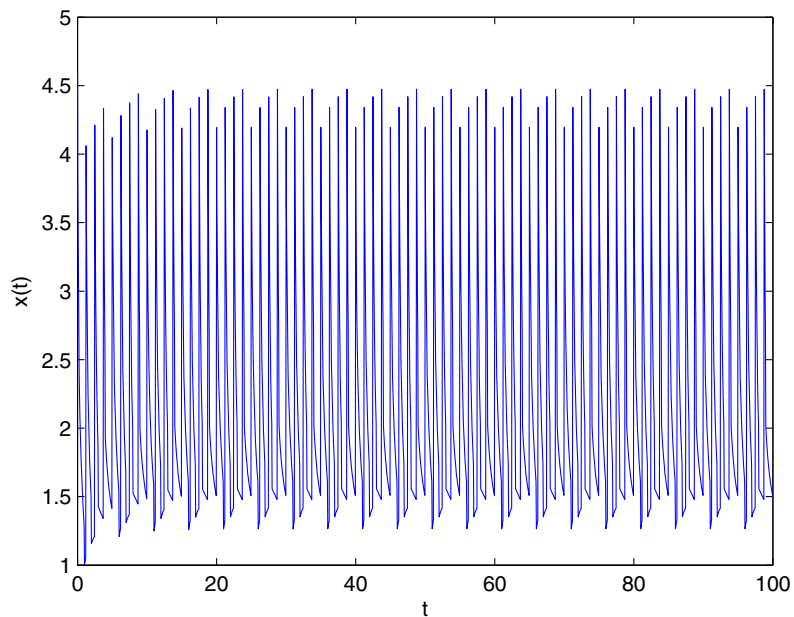


Figure 4 Time-series of $x(t)$ of permanence of system (2.2) with $x(0) = 2, y(0) = 2, a = 2, b = 1, d = 1, \beta = 0.6, k = 0.9, \mu_1 = 0.2, \mu_2 = 0.4, \mu = 3, l = 0.25, \tau = 1$.

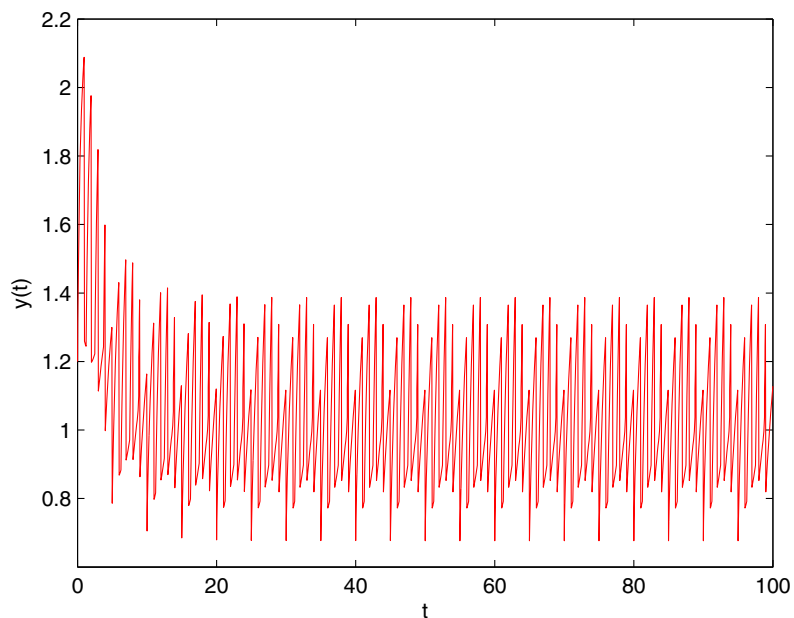
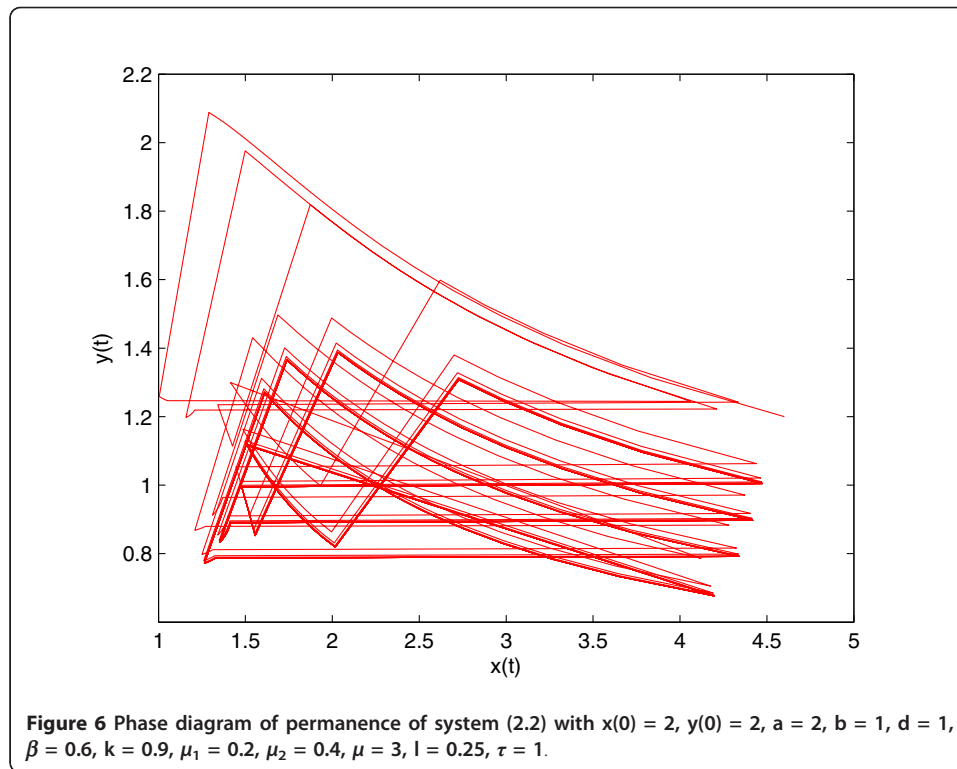


Figure 5 Time-series of $y(t)$ of permanence of system (2.2) with $x(0) = 2, y(0) = 2, a = 2, b = 1, d = 1, \beta = 0.6, k = 0.9, \mu_1 = 0.2, \mu_2 = 0.4, \mu = 3, l = 0.25, \tau = 1$.



system (2.2) is permanent. The same discussion can be applied to parameters μ_1 and τ . These results show that the impulsive effect plays an important role for the permanence of system (2.2). Our results provide reliable tactic basis for the practically biological resource management.

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Authors' contributions

JJ carried out the main part of this article, LC corrected the manuscript. SC brought forward some suggestion on this article. All authors have read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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