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Approximate $*$-derivations on fuzzy Banach $*$-algebras

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Abstract

In this paper, we establish functional equations of $*$-derivations and prove the stability of $*$-derivations on fuzzy Banach $*$-algebras. We also prove the superstability of $*$-derivations on fuzzy Banach $*$-algebras.

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1 Introduction

Let $\mathcal{A}$ be a Banach $*$-algebra. A linear mapping $\delta : D(\delta) \to \mathcal{A}$ is said to be a *derivation* on $\mathcal{A}$ if $\delta(ab) = \delta(a)b + a\delta(b)$ for all $a, b \in \mathcal{A}$, where $D(\delta)$ is a domain of δ and $D(\delta)$ is dense in $\mathcal{A}$. If δ satisfies the additional condition $\delta(a^*) = \delta(a)^*$ for all $a \in \mathcal{A}$, then δ is called a $*$-derivation on $\mathcal{A}$. It is well known that if $\mathcal{A}$ is a C^*-algebra and $D(\delta)$ is A, then the $*$-derivation δ is bounded. For several reasons, the theory of bounded derivations of C^*-algebras is very important in the theory of quantum mechanics and operator algebras [3, 4].

A functional equation is called *stable* if any function satisfying a functional equation "approximately" is near to a true solution of the functional equation. We say that a functional equation is *superstable* if every approximate solution is an exact solution of it.

In 1940, Ulam [24] proposed the following question concerning stability of group homomorphisms: *Under what condition is there an additive mapping near an approximately additive mapping?* Hyers [8] answered positively the problem of Ulam for the case where G_1 and G_2 are Banach spaces. A generalized version of the theorem of Hyers for an approximately linear mapping was given by ThM Rassias [20]. Since then, the stability problems of various functional equations have been extensively investigated by a number of authors (for instances, [1, 2, 9, 10, 19, 20]). In particular, those of the important functional equations are the following functional equations:

$$f(x+y) = f(x) + f(y), \tag{1.1}$$

$$2f\left(\frac{x+y}{2}\right) = f(x) + f(y), \tag{1.2}$$

which are called the Cauchy equation and the Jensen equation, respectively. Every solution of the functional equations (1.1) and (1.2) is said to be an *additive mapping*.

Since Katsaras [14] introduced the idea of fuzzy norm on a linear space, several definitions for a fuzzy norm on a linear space have been introduced and discussed from different points of view [5–7]. We use the definition of fuzzy normed spaces given in [5, 17] to investigate the stability of derivation in the fuzzy Banach $*$-algebra setting. The stability of functional equations in fuzzy normed spaces was begun by [17], after then lots of results of fuzzy stability were investigated [11, 13, 16, 18].

Definition 1.1 [5, 17, 21] Let X be a real vector space. A function $N : X \times \mathbb{R} \to [0,1]$ is called a *fuzzy norm* on X if for all $x, y \in X$ and all $s, t \in \mathbb{R}$,

(N_1) $N(x,t) = 0$ for $t \leq 0$;
(N_2) $x = 0$ if and only if $N(x,t) = 1$ for all $t > 0$;
(N_3) $N(cx,t) = N(x, \frac{t}{|c|})$ if $c \neq 0$;
(N_4) $N(x + y, s + t) \geq \min\{N(x,s), N(y,t)\}$;
(N_5) $N(x,\cdot)$ is a non-decreasing function of $\mathbb{R}$ and $\lim_{t\to\infty} N(x,t) = 1$;
(N_6) for $x \neq 0$, $N(x,\cdot)$ is continuous on $\mathbb{R}$.

The pair (X,N) is called a *fuzzy normed vector space*.

Furthermore, we can make (X,N) a fuzzy normed $*$-algebra if we add (N_7) and (N_8) as follows:

(N_7) $N(xy, st) \geq \min\{N(x,s), N(y,t)\}$;
(N_8) $N(x,t) = N(x^*, t)$.

The properties and examples of fuzzy normed vector spaces, fuzzy algebras, and fuzzy norms are given in [17, 18, 22, 23].

Definition 1.2 [5, 17, 21] Let (X,N) be a fuzzy normed vector space. A sequence $\{x_n\}$ in X is said to *be convergent* or *converge* if there exists an $x \in X$ such that $\lim_{n\to\infty} N(x_n - x, t) = 1$ for all $t > 0$. In this case, x is called the *limit* of the sequence $\{x_n\}$ and we denote it by N-$\lim_{n\to\infty} x_n = x$.

Definition 1.3 [5, 17, 21] Let (X,N) be a fuzzy normed vector space. A sequence $\{x_n\}$ in X is called *Cauchy* if for each $\varepsilon > 0$ and each $t > 0$ there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$ and all $p > 0$, we have $N(x_{n+p} - x_n, t) > 1 - \varepsilon$.

It is well known that every convergent sequence in a fuzzy normed vector space is Cauchy. If each Cauchy sequence is convergent, then the fuzzy norm is said to be *complete* and the fuzzy normed vector space is called a *fuzzy Banach space*.

We say that a mapping $f : X \to Y$ between fuzzy normed vector spaces X and Y is continuous at a point $x_0 \in X$ if for each sequence $\{x_n\}$ converging to x_0 in X, then the sequence $\{f(x_n)\}$ converges to $f(x_0)$. If $f : X \to Y$ is continuous at each $x \in X$, then $f : X \to Y$ is said to be *continuous* on X.

In this paper, using the functional equation of $*$-derivations

$$f(\lambda a + b + cd) = \lambda f(a) + f(b) + f(c)d + cf(d)$$

introduced in [12] we prove fuzzy version of the stability of $*$-derivations associated to the Cauchy functional equation and the Jensen functional equation. We also prove the superstability of $*$-derivations on fuzzy Banach $*$-algebras.

2 Stability of $*$-derivations on fuzzy Banach $*$-algebras

In this section, let $\mathcal{A}$ be a fuzzy Banach $*$-algebra.

Theorem 2.1 *Let $\varphi : \mathcal{A}^4 \to [0,\infty)$ and $\psi : \mathcal{A}^2 \to [0,\infty)$ be control functions such that*

$$\tilde{\varphi}(a,b,c,d) := \frac{1}{2}\sum_{n=0}^{\infty} 2^{-n}\varphi\left(2^n a, 2^n b, 2^n c, 2^n d\right) < \infty, \tag{2.1}$$

$$\lim_{n\to\infty} 2^{-n}\psi\left(2^n a, 2^n b\right) = 0. \tag{2.2}$$

Suppose that $f : \mathcal{A} \to \mathcal{A}$ is a mapping with $f(0) = 0$ satisfying the followings:

$$\lim_{t\to\infty} N\left(f(\lambda a + b + cd) - \lambda f(a) - f(b) - f(c)d - cf(d), t\varphi(a,b,c,d)\right) = 1 \tag{2.3}$$

uniformly on $\mathcal{A}^4$ and for all $\lambda \in \mathbb{T} := \{\lambda \in \mathbb{C} : |\lambda| = 1\}$

$$\lim_{t\to\infty} N\left(f(a)^* - f\left(a^*\right), t\psi\left(a, a^*\right)\right) = 1 \tag{2.4}$$

uniformly on $\mathcal{A}^2$. Then there exists a unique $*$-derivation δ on $\mathcal{A}$ satisfying

$$\lim_{t\to\infty} N\left(f(a) - \delta(a), t\tilde{\varphi}(a,a,0,0)\right) = 1 \tag{2.5}$$

for all $a \in \mathcal{A}$.

Proof Let $0 < \epsilon < 1$ be given. Setting $a = b$, $c = d = 0$ and $\lambda = 1$ in (2.3), we can find some $t_0 > 0$ such that

$$N\left(f(2a) - 2f(a), t\varphi(a,a,0,0)\right) \geq 1 - \epsilon$$

for all $a \in \mathcal{A}$ and $t \geq t_0$. One can use induction to show that

$$N\left(f\left(2^n a\right) - 2^n f(a), t\sum_{k=0}^{n-1} 2^{n-k-1}\varphi\left(2^k a, 2^k a, 0, 0\right)\right) \geq 1 - \epsilon. \tag{2.6}$$

Let $t = t_0$ and put $n = p$ then by replacing a with $2^n a$ in (2.6), we obtain

$$N\left(\frac{f(2^{n+p}a)}{2^{n+p}} - \frac{f(2^n a)}{2^n}, \frac{t_0}{2^{n+p}}\sum_{k=0}^{p-1} 2^{p-k-1}\varphi\left(2^{n+k}a, 2^{n+k}a, 0, 0\right)\right) \geq 1 - \epsilon \tag{2.7}$$

for all integers $n \geq 0$, $p \geq 0$. By the convergence of (2.1) there is $n_0 \in \mathbb{N}$ such that

$$\frac{t_0}{2}\sum_{k=n}^{n+p-1} 2^{-k}\varphi\left(2^k a, 2^k a, 0, 0\right) \leq \delta$$

for all $n \geq n_0$ and $p > 0$. Since the fuzzy norm $N(x,\cdot)$ is nondecreasing, we can have

$$N\left(\frac{f(2^{n+p}a)}{2^{n+p}} - \frac{f(2^n a)}{2^n}, \delta\right)$$

$$\geq N\left(\frac{f(2^{n+p}a)}{2^{n+p}} - \frac{f(2^n a)}{2^n}, \frac{t_0}{2^{n+p}}\sum_{k=0}^{p-1} 2^{p-k-1}\varphi\left(2^{n+k}a, 2^{n+k}a, 0, 0\right)\right) \geq 1 - \epsilon. \tag{2.8}$$

It follows from (2.8) and Definition 1.3 that the sequence $\{\frac{f(2^n a)}{2^n}\}$ is Cauchy. Due to the completeness of $\mathcal{A}$, this sequence is convergent. Define

$$\delta(a) := N - \lim_{n\to\infty}\frac{f(2^n a)}{2^n} \tag{2.9}$$

for all $a \in \mathcal{A}$. From the above equation, we have

$$\delta\left(\frac{1}{2^k}a\right) = N - \lim_{n\to\infty}\frac{1}{2^k}\frac{f(2^{n-k}a)}{2^{n-k}} = \frac{1}{2^k}\delta(a) \tag{2.10}$$

for each $k \in \mathbb{N}$. Moreover, letting $n = 0$ and passing the limit $p \to \infty$ in (2.8), we get

$$\lim_{t\to\infty} N\left(f(a) - \delta(a), t\tilde{\varphi}(a, a, 0, 0)\right) = 1 \tag{2.11}$$

for all $a \in \mathcal{A}$. Putting $c = d = 0$ and replacing a and b by $2^n a$ and $2^n b$, respectively, in (2.3), there exists $t_0 > 0$ such that

$$N\left(2^{-n}f\left(2^n(\lambda a + b)\right) - \lambda 2^{-n}f\left(2^n a\right) - 2^{-n}f\left(2^n b\right), t2^{-n}\varphi\left(2^n a, 2^n b, 0, 0\right)\right) \geq 1 - \epsilon$$

for all $t \geq t_0$. Let $a, b \in \mathcal{A}$. Temporarily fix $t > 0$. Since $\lim_{n\to\infty}\frac{1}{2^n}t\varphi(2^n a, 2^n b, 0, 0) = 0$, there exists $n_0 > 0$ such that

$$t\varphi\left(2^n a, 2^n a, 0, 0\right) \leq \frac{2^n t}{4},$$

for all $n \geq n_0$. Hence, we have

$$\begin{aligned}
&N\left(\delta(\lambda a + b) - \lambda\delta(a) - \delta(b), t\right) \\
&\quad\geq \min\left\{N\left(\delta(\lambda a + b) - 2^{-n}f\left(2^n(\lambda a + b)\right), \frac{t}{4}\right), N\left(\lambda\delta(a) - \lambda 2^{-n}f\left(2^n a\right), \frac{t}{4}\right),\right. \\
&\qquad\left. N\left(\delta(b) - 2^{-n}f\left(2^n b\right), \frac{4}{t}\right), N\left(f\left(2^n(\lambda a + b)\right) - \lambda f\left(2^n a\right) - f\left(2^n b\right), \frac{2^n 4}{t}\right)\right\}
\end{aligned}$$

for all $n \geq n_0$ and $t > 0$. The first three terms on the second and third lines of the above inequality tend to 1 as $n \to \infty$. Furthermore, the last term is greater than

$$N\left(f\left(2^n(\lambda a + b)\right) - \lambda f\left(2^n a\right) - f\left(2^n b\right), t_0\varphi\left(2^n a, 2^n b, 0, 0\right)\right),$$

which is greater than or equal to $1 - \epsilon$. Therefore,

$$N\left(\delta(\lambda a + b) - \lambda\delta(a) - \delta(b), t\right) \geq 1 - \epsilon$$

for all $t > 0$. It follows that $\delta(\lambda a + b) = \lambda\delta(a) + \delta(b)$ by (N_2) for all $a, b \in \mathcal{A}$ and all $\lambda \in \mathbb{T}$. Next, let $\lambda = \lambda_1 + i\lambda_2 \in \mathbb{C}$ where $\lambda_1, \lambda_2 \in \mathbb{R}$. Let $\gamma_1 = \lambda_1 - [\lambda_1]$ and $\gamma_2 = \lambda_2 - [\lambda_2]$, where $[\lambda]$ denotes the integer part of λ. Then $0 \leq \gamma_i < 1$ $(1 \leq i \leq 2)$. One can represent γ_i as $\gamma_i = \frac{\lambda_{i,1}+\lambda_{i,2}}{2}$ such that $\lambda_{i,j} \in \mathbb{T}$ $(1 \leq i, j \leq 2)$. From (2.10), we infer that

$$\begin{aligned}
\delta(\lambda x) &= \delta(\lambda_1 x) + \mathrm{i}\delta(\lambda_2 x) \\
&= \big([\lambda_1]\delta(x) + \delta(\gamma_1 x)\big) + \mathrm{i}\big([\lambda_2]\delta(x) + \delta(\gamma_2 x)\big) \\
&= \left([\lambda_1]\delta(x) + \frac{1}{2}\delta(\lambda_{1,1}x + \lambda_{1,2}x)\right) + \mathrm{i}\left([\lambda_2]\delta(x) + \frac{1}{2}\delta(\lambda_{2,1}x + \lambda_{2,2}x)\right) \\
&= \left([\lambda_1]\delta(x) + \frac{1}{2}\lambda_{1,1}\delta(x) + \frac{1}{2}\lambda_{1,2}\delta(x)\right) + \mathrm{i}\left([\lambda_2]\delta(x) + \frac{1}{2}\lambda_{2,1}\delta(x) + \frac{1}{2}\lambda_{2,2}\delta(x)\right) \\
&= \lambda_1\delta(x) + \mathrm{i}\lambda_2\delta(x) \\
&= \lambda\delta(x)
\end{aligned}$$

for all $x \in \mathcal{A}$. Hence, δ is $\mathbb{C}$-linear. Putting $a = b = 0$ and replacing c and d by $2^n c$ and $2^n d$, respectively, in (2.3), there exists $t_0 > 0$ such that

$$N\big(2^{-2n}f(2^{2n}cd) - 2^{-2n}f(2^n c)(2^n d) - 2^{-2n}(2^n c)f(2^n d), t2^{-2n}\varphi(0,0,2^n c, 2^n d)\big) \geq 1 - \epsilon$$

for all $t \geq t_0$. Fix $t(> 0)$ temporarily. By (2.1) there exists $n_0 > 0$ such that

$$t\varphi(0,0,2^n c, 2^n d) \leq \frac{2^{2n}t}{4}$$

for all $n \geq n_0$ and $t > 0$. We have

$$\begin{aligned}
&N\big(\delta(cd) - \delta(c)d - c\delta(d), t\big) \\
&\quad \geq \min\left\{N\left(\delta(cd) - 2^{-2n}f(2^{2n}cd), \frac{t}{4}\right), N\left(\delta(c)d - 2^{-2n}f(2^n c)(2^n d), \frac{t}{4}\right),\right. \\
&\qquad N\left(c\delta(d) - 2^{-2n}(2^n c)f(2^n d), \frac{t}{4}\right), \\
&\qquad \left. N\left(f(2^{2n}cd) - f(2^n c)(2^n d) - (2^n c)f(2^n d), \frac{2^{2n}4}{t}\right)\right\} \\
&\quad \geq \min\left\{N\left(\delta(cd) - 2^{-2n}f(2^{2n}cd), \frac{t}{4}\right), N\left(\delta(c)d - 2^{-2n}f(2^n c)(2^n d), \frac{t}{4}\right),\right. \\
&\qquad N\left(c\delta(d) - 2^{-2n}(2^n c)f(2^n d), \frac{t}{4}\right), \\
&\qquad \left. N\big(f(2^{2n}cd) - f(2^n c)(2^n d) - (2^n c)f(2^n d), t\varphi(0,0,2^n c, 2^n d)\big)\right\}
\end{aligned}$$

for all $n \geq n_0$ and $t > 0$. From the above computation

$$\delta(cd) = \delta(c)d + c\delta(d) \tag{2.12}$$

for all $c, d \in \mathcal{A}$. So it is a derivation on $\mathcal{A}$. Moreover, it follows from (2.7) with $n = 0$ and (2.9) that $\lim_{t\to\infty} N(\delta(a) - f(a), t\tilde{\varphi}(a,a,0,0)) = 1$ for all $a \in \mathcal{A}$. It is well known that the additive mapping δ satisfying (2.5) is unique (see [3] or [20]). Replacing a and a^* by $2^n a$ and $2^n a^*$, respectively, in (2.4) we can find $t_0 > 0$ such that

$$N\left(2^{-n}f\left(2^n a\right)^* - 2^{-n}f\left(2^n a^*\right), t2^{-n}\psi\left(2^n a, 2^n a^*\right)\right) \geq 1 - \epsilon$$

for all $a \in \mathcal{A}$ and all $t > t_0$. Since $\lim_{n\to\infty} 2^{-n}\psi(2^n a, 2^n a^*) = 0$, there exists some $n_0 > 0$ such that $t\psi(2^n a, 2^n a^*) < \frac{t2^n}{2}$ for all $n \geq n_0$. Hence,

$$\begin{aligned} &N\left(\delta(a)^* - \delta\left(a^*\right), t\right) \\ &\quad\geq \min\left\{N\left(\delta(a)^* - 2^{-n}f\left(2^n a\right)^*, \frac{t}{4}\right), N\left(\delta\left(a^*\right) - 2^{-n}f\left(2^n a^*\right), \frac{t}{4}\right),\right. \\ &\qquad \left. N\left(f\left(2^n a\right)^* - f\left(2^n a^*\right), \frac{2^n t}{2}\right)\right\}. \end{aligned}$$

The first two terms on the right-hand side of the above inequality tend to 1 as $n \to \infty$. Furthermore, the last term is greater than

$$N\left(f\left(2^n a\right)^* - f\left(2^n a^*\right), t\psi\left(2^n a, 2^n a^*\right)\right),$$

which is greater than or equal to $1 - \epsilon$. So, we have that $N(\delta(a)^* - \delta(a^*), t) > 1 - \epsilon$ for all $t > 0$. It follows from that $\delta(a^*) = \delta(a)^*$ for all $a \in \mathcal{A}$. So, δ is a $*$-derivation on $\mathcal{A}$. □

Theorem 2.2 *Suppose that $f : \mathcal{A} \to \mathcal{A}$ is a mapping with $f(0) = 0$ for which there exist functions $\varphi : \mathcal{A}^4 \to [0, \infty)$ and $\psi : \mathcal{A}^2 \to [0, \infty)$ such that*

$$\begin{aligned} &\tilde{\varphi}(a, b, c, d) := \frac{1}{2}\sum_{n=0}^{\infty} 2^n \varphi\left(2^{-n}a, 2^{-n}b, 2^{-n}c, 2^{-n}d\right) < \infty, \\ &\lim_{n\to\infty} 2^n \psi\left(2^{-n}a, 2^{-n}b\right) = 0, \\ &\lim_{t\to\infty} N\left(f(\lambda a + b + cd) - \lambda f(a) - f(b) - f(c)d - cf(d), t\varphi(a, b, c, d)\right) = 1, \\ &\lim_{t\to\infty} N\left(f(a)^* - f(a)^*, t\psi\left(a, a^*\right)\right) = 1 \end{aligned}$$

for all $\lambda \in \mathbb{T}$ and all $a, b, c, d \in \mathcal{A}$. Then there exists a unique $*$-derivation δ on $\mathcal{A}$ satisfying

$$\lim_{t\to\infty} N\left(f(a) - \delta(a), t\tilde{\varphi}(a, a, 0, 0)\right) = 1$$

for all $a \in \mathcal{A}$.

3 Stability of $*$-derivations associated to the Jensen equation

The stability of the Jensen equation has been studied first by Kominek and then by several other mathematicians: ([15]). In this section, we study the stability of $*$-derivation associated to the Jensen equation in a fuzzy Banach $*$-algebra $\mathcal{A}$.

Theorem 3.1 *Let $\mathcal{A}$ be a fuzzy Banach $*$-algebra. Suppose that $f : \mathcal{A} \to \mathcal{A}$ is a mapping with $f(0) = 0$ for which there exist functions $\varphi : \mathcal{A}^2 \to [0, \infty)$ and $\psi_i : \mathcal{A}^2 \to [0, \infty)$ $(1 \leq i \leq$*

2*) such that*

$$\widetilde{\varphi}(a,b) := \sum_{n=0}^{\infty} 3^{-n}\varphi\left(3^n a, 3^n b\right) < \infty, \tag{3.1}$$

$$\lim_{n\to\infty} 3^{-n}\psi_i\left(3^n a, 3^n b\right) = 0 \quad (1 \le i \le 2),$$

$$\lim_{t\to\infty} N\left(2f\left(\frac{\lambda a + \lambda b}{2}\right) - \lambda f(a) - \lambda f(b), t\varphi(a,b)\right) = 1, \tag{3.2}$$

$$\lim_{t\to\infty} N\left(f\left(a^*\right) - f(a)^*, t\psi_1\left(a,a^*\right)\right) = 1, \tag{3.3}$$

$$\lim_{t\to\infty} N\left(f(ab) - af(b) - f(a)b, t\psi_2(a,b)\right) = 1 \tag{3.4}$$

for all $a, b \in \mathcal{A}$ *and all* $\lambda \in \mathbb{T}$. *Then there exists a unique* $*$*-derivation* δ *on* $\mathcal{A}$ *satisfying*

$$\lim_{t\to\infty} N\left(f(a) - \delta(a), \frac{t}{3}\left(\tilde{\varphi}(a,-a) + \tilde{\varphi}(-a,3a)\right)\right) = 1 \tag{3.5}$$

for all $a \in \mathcal{A}$.

Proof Let $0 < \epsilon < 1$ be given. Letting $\lambda = -1$ and $b = -a$ in (3.2), we can find some $t_0 > 0$ such that

$$N\left(f(a) + f(-a), t\varphi(a,-a)\right) \ge 1 - \epsilon$$

for all $a \in \mathcal{A}$ and $t \ge t_0$. Letting $\lambda = 1$ and replacing a and b by $-a$ and $3a$, respectively, in (3.2), we get also $t_1 \ge t_0$ such that

$$N\left(2f(a) - f(-a) - f(3a), t\varphi(-a,3a)\right) \ge 1 - \epsilon$$

for all $a \in \mathcal{A}$ and $t \ge t_1$. Thus,

$$\begin{aligned} &N\left(f(a) - \frac{1}{3}f(3a), \frac{t}{3}\left(\varphi(a,-a) + \varphi(-a,3a)\right)\right) \\ &\quad \ge \min\left\{N\left(\frac{1}{3}\left(f(a) + f(-a)\right), \frac{t}{3}\varphi(a,-a)\right),\right. \\ &\qquad \left. N\left(\frac{1}{3}\left(2f(a) - f(-a) - f(3a)\right), \frac{t}{3}\varphi(-a,3a)\right)\right\} \ge 1 - \epsilon \end{aligned} \tag{3.6}$$

for all $a \in \mathcal{A}$. Replace a by $3^n a$ in (3.6)

$$N\left(\frac{f(3^n a)}{3^n} - \frac{f(3^{n+1}a)}{3^{n+1}}, \frac{t}{3^{n+1}}\left(\varphi\left(3^n a, -3^n a\right) + \varphi\left(-3^n a, 3^{n+1}a\right)\right)\right) \ge 1 - \epsilon.$$

Given $\delta > 0$, there exists an integer $n_0 > 0$ such that

$$\frac{t}{3}\sum_{j=m}^{n-1} 3^{-j}\left(\varphi\left(3^j a, -3^j a\right) + \varphi\left(-3^j a, 3^{j+1}a\right)\right) \le \delta$$

for all $n \ge m \ge n_0$.

So, we have

$$N\left(\frac{1}{3^n}f(3^n a)-\frac{1}{3^m}f(3^m a),\delta\right) \tag{3.7}$$

$$\geq N\left(\frac{1}{3^n}f(3^n a)-\frac{1}{3^m}f(3^m a),\frac{t}{3}\sum_{j=m}^{n-1}3^{-j}\left(\varphi\left(3^j a,-3^j a\right)+\varphi\left(-3^j a,3^{j+1}a\right)\right)\right) \tag{3.8}$$

$$\geq \min_{m\leq j\leq n-1}\left\{N\left(\frac{1}{3^j}f(3^j a)-\frac{1}{3^{j+1}}f(3^{j+1}a),\frac{t}{3}\left(\varphi\left(3^j a,-3^j a\right)+\varphi\left(-3^j a,3^{j+1}a\right)\right)\right)\right\}\geq 1-\epsilon$$

for all nonnegative integers n, m with $n \geq m \geq n_0$ and all $a \in \mathcal{A}$. It follows from Definition 1.3 that the sequence $\{\frac{1}{3^n}f(3^n a)\}$ is a Cauchy sequence for all $a \in \mathcal{A}$. Since $\mathcal{A}$ is complete, the sequence $\{\frac{1}{3^n}f(3^n a)\}$ is convergent. So, one can define the mapping $\delta : \mathcal{A} \to \mathcal{A}$ by

$$\delta(a) = N-\lim_{n\to\infty}\frac{1}{3^n}f(3^n a) \tag{3.9}$$

for all $a \in \mathcal{A}$. If we put $\lambda = 1$ and replace a, b with $3^n a$, $3^n b$, respectively, in (3.2), we can find some $t_0 > 0$ such that

$$N\left(2f\left(3^n\frac{a+b}{2}\right)-f(3^n a)-f(3^n b),3^{-n}t\varphi(3^n a,3^n b)\right)\geq 1-\epsilon$$

for all $t \geq t_0$. Fix $t > 0$ temporarily. Since $\lim_{n\to\infty}3^{-n}\varphi(3^n a,3^n b) = 0$, there is some $n_0 > 0$ such that $t\varphi(3^n a,3^n b) < \frac{3^n t}{4}$ for all $n \geq n_0$. Then we have

$$\begin{aligned}&N\left(2\delta\left(\frac{a+b}{2}\right)-\delta(a)-\delta(b),t\right)\\&\quad\geq \min\left\{N\left(2\delta\left(\frac{a+b}{2}\right)-\frac{1}{3^n}2f\left(3^n\frac{a+b}{2}\right),\frac{t}{4}\right),N\left(\delta(a)-\frac{f(3^n a)}{3^n},\frac{t}{4}\right),\right.\\&\qquad\left.N\left(\delta(b)-\frac{f(3^n b)}{3^n},\frac{t}{4}\right),N\left(2f\left(3^n\frac{a+b}{2}\right)-f(3^n a)-f(3^n b),\frac{3^n t}{4}\right)\right\}\end{aligned}$$

for all $a, b \in \mathcal{A}$ and $t > 0$. The first three terms on the second and third lines of the above inequality tend to 1 as $n \to \infty$. Furthermore, the last term is greater than

$$N\left(2f\left(3^n\frac{a+b}{2}\right)-f(3^n a)-f(3^n b),t\varphi(3^n a,3^n b)\right),$$

which is greater than or equal to $1-\epsilon$.

So, we have

$$N\left(2\delta\left(\frac{a+b}{2}\right)-\delta(a)-\delta(b),t\right)\geq 1-\epsilon$$

for all $t > 0$. By the definition of fuzzy norm, we have

$$2\delta\left(\frac{a+b}{2}\right)=\delta(a)+\delta(b) \tag{3.10}$$

for all $a, b \in \mathcal{A}$. Since $f(0) = 0$, we have $\delta(0) = 0$. Putting $b = 0$ in (3.10), we get $2\delta(\frac{a}{2}) = \delta(a)$ for each $a \in \mathcal{A}$ and, therefore, $\delta(a) + \delta(b) = 2\delta(\frac{a+b}{2}) = \delta(a + b)$ for all $a, b \in \mathcal{A}$. Moreover, letting $m = 0$ and passing the limit $n \to \infty$ in (3.8), we get

$$N\left(f(a) - \delta(a), \frac{t}{3}\big(\tilde{\varphi}(a, -a) + \tilde{\varphi}(-a, 3a)\big)\right) \geq 1 - \epsilon$$

for all $a \in \mathcal{A}$. So, we have Eq. (3.5). It is known that such an additive mapping δ is unique. Let $\lambda \in \mathbb{T}$. Replacing both a and b in (3.2) by $3^n a$ and dividing the both sides of the obtained inequality by 3^n, there exists some $t_0 > 0$ such that

$$N\big(3^{-n}f\big(\lambda 3^n a\big) - \lambda 3^{-n}f\big(3^n a\big), 3^{-n}t\varphi\big(3^n a, 3^n a\big)\big) \geq 1 - \epsilon$$

for all $a \in \mathcal{A}$ and all $t \geq t_0$. Fix $t > 0$ temporarily. Since $\lim_{n\to\infty} 3^{-n}\phi(3^n a, 3^n b) = 0$, there exists $n_0 > 0$ such that $3^{-n}\phi(3^n a, 3^n b) \leq \frac{t}{2}$ for all $n \geq n_0$.

If we consider the following inequality

$$\begin{aligned} &N\big(\delta(\lambda a) - \lambda\delta(a), t\big) \\ &\quad\geq \min\left\{N\left(\delta(\lambda a) - 3^{-n}f\big(\lambda 3^n a\big), \frac{t}{4}\right), N\left(\lambda\delta(a) - 3^{-n}f\big(\lambda 3^n a\big), \frac{t}{4}\right),\right. \\ &\qquad \left. N\left(3^{-n}f\big(\lambda 3^n a\big) - 3^{-n}f\big(\lambda 3^n a\big), \frac{t}{2}\right)\right\}, \end{aligned}$$

then the first two terms on the second line of the above inequality tend to 1 as $n \to \infty$ and the last term is greater than

$$N\big(3^{-n}f\big(\lambda 3^n a\big) - \lambda 3^{-n}f\big(3^n a\big), 3^{-n}t\varphi\big(3^n a, 3^n a\big)\big),$$

which is greater than or equal to $1 - \epsilon$. So, we can get $\delta(\lambda a) = \lambda\delta(a)$ for all $\lambda \in \mathbb{C}$ by the similar discussion in the proof Theorem 2.1. Replacing both a and a^* in (3.3) by $3^n a$ and $3^n a^*$, and then dividing the both sides of the obtained inequality by 3^n, we find some $t_0 > 0$ such that

$$N\big(3^{-n}f\big(3^n a\big)^* - 3^{-n}f\big(3^n a^*\big), t3^{-n}\psi_1\big(3^n a, 3^n a^*\big)\big) \geq 1 - \epsilon$$

for all $t \geq t_0$. Fix $t > 0$ temporarily. Since $\lim_{n\to\infty} 3^{-n}\psi_1(3^n a, 3^n a^*) = 0$, there exists $n_0 > 0$ such that $3^{-n}t\psi_1(3^n a, 3^n a^*) \leq \frac{t}{2}$ for all $n \geq n_0$. We consider the following inequality:

$$\begin{aligned} &N\big(\delta\big(a^*\big) - \delta(a)^*, t\big) \\ &\quad\geq \min\left\{N\left(\delta\big(a^*\big) - 3^{-n}f\big(3^n a^*\big), \frac{t}{4}\right), N\left(\delta(a)^* - 3^{-n}f\big(3^n a\big)^*, \frac{t}{4}\right),\right. \\ &\qquad \left. N\left(3^{-n}f\big(3^n a^*\big) - 3^{-n}f\big(3^n a\big)^*, \frac{t}{2}\right)\right\}. \end{aligned}$$

Then we get $\delta(a^*) = \delta(a)^*$ for all $a \in \mathcal{A}$. For the derivation property, replacing both a and b in (3.4) by $3^n a$ and $3^n b$, we can find some $t_0 > 0$ such that

$$N\left(\frac{f(3^{2n}ab)}{3^{2n}} - \frac{3^n af(3^n b)}{3^{2n}} - \frac{f(3^n a)(3^n b)}{3^{2n}}, 3^{-n}t\psi_2\big(3^n a, 3^n b\big)\right) \geq 1 - \epsilon$$

for all $t \geq t_0$. By (3.4), there exists $n_0 \in \mathbf{N}$ such that $3^{-n}t\psi_2(3^n a, 3^n b) \leq \frac{t}{4}$ for all $n \geq n_0$ and $t > 0$. We can get $\delta(ab) = \delta(a)b + a\delta(b)$ for all $a, b \in \mathcal{A}$ from the following computation:

$$\begin{aligned}
&N\bigl(\delta(ab) - a\delta(b) - \delta(a)b, t\bigr) \\
&\quad \geq \min\biggl\{N\biggl(\delta(ab) - \frac{f(3^{2n}ab)}{3^{2n}}, \frac{t}{4}\biggr), N\biggl(a\delta(b) - \frac{3^n af(3^n b)}{3^{2n}}, \frac{t}{4}\biggr), \\
&\qquad N\biggl(\delta(a)b - \frac{f(3^n a)(3^n b)}{3^{2n}}, \frac{t}{4}\biggr), N\biggl(\frac{f(3^{2n}ab)}{3^{2n}} - \frac{3^n af(3^n b)}{3^{2n}} - \frac{f(3^n a)(3^n b)}{3^{2n}}, \frac{t}{4}\biggr)\biggr\}.
\end{aligned}$$

Hence, δ is the $*$-derivation on $\mathcal{A}$ that we want. □

4 Superstability of $*$-derivations

In this section, we prove the superstability of $*$-derivations on a fuzzy Banach $*$-algebras. More precisely, we introduce the concept of (ψ, φ)-approximate $*$-derivation and show that any (ψ, φ)-approximate $*$-derivation is just a $*$-derivation.

Definition 4.1 Suppose that $\mathcal{A}$ is a $*$-normed algebra and $s \in \{-1, 1\}$. Let $\delta : \mathcal{A} \to \mathcal{A}$ be a mapping for which there exist a function $\varphi : \mathcal{A} \to \mathcal{A}$, and functions $\psi_i : \mathcal{A} \times \mathcal{A} \to \mathbb{R}$ $(1 \leq i \leq 3)$ satisfying

$$\lim_{n\to\infty} n^{-s}\psi_i\bigl(n^s a, b\bigr) = \lim_{n\to\infty} n^{-s}\psi_i\bigl(a, n^s b\bigr) = 0 \quad (a, b \in \mathcal{A}) \tag{4.1}$$

such that

$$\lim_{t\to\infty} N\bigl(\varphi(a)b - a\delta(b), t\psi_1(a, b)\bigr) = 1, \tag{4.2}$$

$$\lim_{t\to\infty} N\bigl(\varphi(a)cd - a\bigl(\delta(c)d - c\delta(d)\bigr), t\psi_2(a, cd)\bigr) = 1, \tag{4.3}$$

$$\lim_{t\to\infty} N\bigl(a\delta(b)^* - \varphi(a)b^*, t\psi_3(a, b)\bigr) = 1 \tag{4.4}$$

for all $a, b, c, d \in \mathcal{A}$. Then δ is called a (ψ, φ)-approximate $*$-derivation on $\mathcal{A}$.

Theorem 4.2 *Let $\mathcal{A}$ be a fuzzy Banach $*$-algebra with approximate unit. Then any (ψ, φ)-approximate $*$-derivation δ on $\mathcal{A}$ is a $*$-derivation.*

Proof We assume that (4.1) holds. An arbitrary $\epsilon > 0$ is given. Let $a, b \in \mathcal{A}$ and $\lambda \in \mathbb{C}$. For $n \in \mathbb{N}$ there exists $t_0 > 0$ by (4.2) such that

$$N\bigl(n^{-s}\bigl(n^s b\delta(\lambda a) - \varphi\bigl(n^s b\bigr)\lambda a\bigr), n^{-s}t\psi_1\bigl(n^s b, \lambda a\bigr)\bigr) \geq 1 - \epsilon,$$

$$N\bigl(n^{-s}\bigl(\varphi\bigl(n^s b\bigr)\lambda a - \lambda n^s b\delta(a)\bigr), n^{-s}t|\lambda|\psi_1\bigl(n^s b, a\bigr)\bigr) \geq 1 - \epsilon$$

for all $t \geq t_0$. Fix $t > 0$ temporarily. Since $\lim_{n\to\infty} n^{-s}\psi_1(n^s a, b) = \lim_{n\to\infty} n^{-s}\psi_1(a, n^s b) = 0$, there exists $n_0 > 0$ such that $tn^{-s}\psi_1(n^s b, \lambda a) \leq \frac{t}{2}$ and $n^{-s}t|\lambda|\psi_1(n^s b, a) \leq \frac{t}{2}$ for all $n \geq n_0$ and $t > 0$.

We have

$$\begin{aligned}
&N\big(b\big(\delta(\lambda a)-\lambda\delta(a)\big),t\big)\\
&\quad= N\big(n^{-s}\big(n^s b\delta(\lambda a)-\varphi\big(n^s b\big)\lambda a+\varphi\big(n^s b\big)\lambda a-\lambda n^s b\delta(a)\big),t\big)\\
&\quad\geq \min\left\{N\left(n^{-s}\big(n^s b\delta(\lambda a)-\varphi\big(n^s b\big)\lambda a\big),\frac{t}{2}\right),N\left(n^{-s}\big(\varphi\big(n^s b\big)\lambda a-\lambda n^s b\delta(a)\big),\frac{t}{2}\right)\right\}.
\end{aligned}$$

Since

$$N\left(n^{-s}\big(n^s b\delta(\lambda a)-\varphi\big(n^s b\big)\lambda a\big),\frac{t}{2}\right)\geq N\big(n^{-s}\big(n^s b\delta(\lambda a)-\varphi\big(n^s b\big)\lambda a\big),tn^{-s}\psi_1\big(n^s a,b\big)\big)$$

and

$$N\left(n^{-s}\big(\varphi\big(n^s b\big)\lambda a-\lambda n^s b\delta(a)\big),\frac{t}{2}\right)\geq N\big(n^{-s}\big(\varphi\big(n^s b\big)\lambda a-\lambda n^s b\delta(a)\big),tn^{-s}|\lambda|\psi_1\big(n^s b,a\big)\big),$$

it leads us to have a conclusion that $N(b(\delta(\lambda a)-\lambda\delta(a)),t)\geq 1-\epsilon$ for all $t>0$. Therefore, $b(\delta(\lambda a)-\lambda\delta(a))=0$ for all $b\in\mathcal{A}$ by (N_2). Let $\{e_i\}_{i\in I}$ be an approximate unit of $\mathcal{A}$. If we replace b with $\{e_i\}_{i\in I}$, then we have

$$e_i\big(\delta(\lambda a)-\lambda\delta(a)\big)=0$$

for all $i\in I$. So we conclude that $\delta(\lambda a)=\lambda\delta(a)$ for all $a\in A$ and $\lambda\in\mathbb{C}$. Next, we are going to prove the additivity of δ. By (4.2), there exists $t_0>0$ such that

$$\begin{aligned}
&N\big(n^{-s}\big(n^s c\delta(a+b)-\varphi\big(n^s c\big)(a+b)\big),n^{-s}t\psi_1\big(n^s c,a+b\big)\big)\geq 1-\epsilon,\\
&N\big(n^{-s}\big(n^s c\delta(a)-\varphi\big(n^s c\big)a\big),n^{-s}t\psi_1\big(n^s c,a\big)\big)\geq 1-\epsilon,
\end{aligned}$$

and

$$N\big(n^{-s}\big(n^s c\delta(b)-\varphi\big(n^s c\big)b\big),n^{-s}t\psi_1\big(n^s c,b\big)\big)\geq 1-\epsilon$$

for all $t\geq t_0$. Fix $t>0$ temporarily. By (4.1), we can find $n_0>0$ such that $n^{-s}t\psi_1(n^s c,a+b)\leq\frac{t}{3}$, $n^{-s}t\psi_1(n^s c,a)\leq\frac{t}{3}$, and $n^{-s}t\psi_1(n^s c,b)\leq\frac{t}{3}$ for all $n\geq n_0$.

For the additivity, we can have

$$\begin{aligned}
&N\big(c\big(\delta(a+b)-\delta(a)-\delta(b)\big),t\big)\\
&\quad= N(n^{-s}\big(n^s c\delta(a+b)-\varphi\big(n^s c\big)(a+b)\big)\\
&\qquad+n^{-s}\big(n^s c\delta(a)-\varphi\big(n^s c\big)a\big)+n^{-s}\big(n^s c\delta(b)-\varphi\big(n^s c\big)b\big),t)\\
&\quad\geq \min\left\{N\left(n^{-s}\big(n^s c\delta(a+b)-\varphi\big(n^s c\big)(a+b)\big),\frac{t}{3}\right),N\left(n^{-s}\big(n^s c\delta(a)-\varphi\big(n^s c\big)a\big),\frac{t}{3}\right),\right.\\
&\qquad\left.N\left(n^{-s}\big(n^s c\delta(b)-\varphi\big(n^s c\big)b\big),\frac{t}{3}\right)\right\}\\
&\quad\geq \min\big\{N\big(n^{-s}\big(n^s c\delta(a+b)-\varphi\big(n^s c\big)(a+b)\big),n^{-s}t\psi_1\big(n^s c,a+b\big)\big),
\end{aligned}$$

$$N\big(n^{-s}\big(n^s c\delta(a) - \varphi\big(n^s c\big)a\big), n^{-s}t\psi_1\big(n^s c, a\big)\big),$$
$$N\big(n^{-s}\big(n^s c\delta(b) - \varphi\big(n^s c\big)b\big), n^{-s}t\psi_1\big(n^s c, b\big)\big)\big\}.$$

Since all terms of the final inequality of the above inequality are larger than $1-\epsilon$, we can have $N(c(\delta(a+b)-\delta(a)-\delta(b)), t) > 1-\epsilon$ for all $t>0$. We can get $c(\delta(a+b)-\delta(a)-\delta(b)) = 0$ for all $a, b, c \in \mathcal{A}$ by (N_2). By using the approximate unit of $\mathcal{A}$, we have that $\delta(a+b) = \delta(a)+\delta(b)$ for all $a, b \in \mathcal{A}$. Next, we are going to show the derivation property of δ. From (4.2) and (4.1), there exists $t_0 > 0$ such that

$$N\big(n^{-s}\big(n^s z\delta(ab) - \varphi\big(n^s z\big)(ab)\big), n^{-s}t\psi_1\big(n^s z, ab\big)\big) \geq 1-\epsilon,$$
$$N\big(n^{-s}\big(\varphi\big(n^s z\big)ab - n^s z\big(\delta(a)b + a\delta(b)\big)\big), n^{-s}t\psi_2\big(n^s z, ab\big)\big) \geq 1-\epsilon$$

for all $t \geq t_0$. By (4.1), we can find $n_0 > 0$ such that $n^{-s}t\psi_1(n^s z, ab) \leq \frac{t}{2}$ and $n^{-s}t\psi_2(n^s z, ab) \leq \frac{t}{2}$ for all $n \geq n_0$. The following computation

$$\begin{aligned}
&N\big(z\big(\delta(ab) - \delta(a)b - a\delta(b)\big), t\big)\\
&\quad\geq \min\left\{N\left(n^{-s}\big(n^s z\delta(ab) - \varphi\big(n^s z\big)(ab)\big), \frac{t}{2}\right),\right.\\
&\qquad \left. N\left(n^{-s}\big(\varphi\big(n^s z\big)ab - n^s z\big(\delta(a)b + a\delta(b)\big)\big), \frac{t}{2}\right)\right\}\\
&\quad\geq \min\big\{N\big(n^{-s}\big(n^s z\delta(ab) - \varphi\big(n^s z\big)(ab)\big), n^{-s}t\psi_1\big(n^s z, ab\big)\big),\\
&\qquad N\big(n^{-s}\big(\varphi\big(n^s z\big)ab - n^s z\big(\delta(a)b + a\delta(b)\big)\big), n^{-s}t\psi_2\big(n^s z, ab\big)\big)\big\} \geq 1-\epsilon
\end{aligned}$$

yields that $\delta(ab) = \delta(a)b + a\delta(b)$ for all $a, b \in \mathcal{A}$. By (4.2) and (4.4) there exists $t_0 > 0$ such that

$$N\big(n^{-s}\big(n^s z\delta\big(a^*\big) - \varphi\big(n^s z\big)a^*\big), n^{-s}t\psi_1\big(n^s z, a^*\big)\big) \geq 1-\epsilon,$$
$$N\big(n^{-s}\big(\varphi\big(n^s z\big)a^* - n^s z\delta(a)^*\big), n^{-s}t\psi_3\big(n^s z, a\big)\big) \geq 1-\epsilon$$

for all $t \geq t_0$. For fixing $t > 0$ temporarily, there exists $n_0 > 0$ such that $n^{-s}t\psi_1(n^{-s}z, a^*) \leq \frac{t}{2}$ and $n^{-s}t\psi_3(n^s z, a) \leq \frac{t}{2}$ for $n \geq n_0$. From the following computation

$$\begin{aligned}
&N\big(z\big(\delta\big(a^*\big) - \delta(a)^*\big), t\big)\\
&\quad= N\big(n^{-s}\big(n^s z\delta\big(a^*\big) - \varphi\big(n^s z\big)a^*\big) + n^{-s}\big(\varphi\big(n^s z\big)a^* - n^s z\delta(a)^*\big), t\big)\\
&\quad\geq \min\left\{N\left(n^{-s}\big(n^s z\delta\big(a^*\big) - \varphi\big(n^s z\big)a^*\big), \frac{t}{2}\right), N\left(n^{-s}\big(\varphi\big(n^s z\big)a^* - n^s z\delta(a)^*\big), \frac{t}{2}\right)\right\}\\
&\quad\geq \min\big\{N\big(n^{-s}\big(n^s z\delta\big(a^*\big) - \varphi\big(n^s z\big)a^*\big), n^{-s}t\psi_1\big(n^{-s}z, a^*\big)\big),\\
&\qquad N\big(n^{-s}\big(\varphi\big(n^s z\big)a^* - n^s z\delta(a)^*\big), n^{-s}t\psi_3\big(n^s z, a\big)\big)\big\} > 1-\epsilon
\end{aligned}$$

we can have $N(z(\delta(a^*) - \delta(a)^*), t) > 1-\epsilon$ for all $t > 0$. By (N_2) and using approximate unit $\delta(a^*) = \delta(a)^*$ for all $a \in \mathcal{A}$. Thus, δ is a $*$-derivation on $\mathcal{A}$. □

Competing interests
Author declares that they have no competing interests.

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