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MUON COOLING - EMITTANCE EXCHANGE

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Muon Cooling - Emittance Exchange [1]

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Abstract. Muon Cooling is the key factor in building of a Muon collider, (to a less degree) Muon storage ring, and a Neutrino Factory. Muon colliders potential to provide a probe for fundamental particle physics is very interesting, but may take a considerable time to realize, as much more work and study is needed. Utilizing high intensity Muon sources - Neutrino Factories , and other intermediate steps are very important and will greatly expand our abilities and confidence in the credibility of high energy muon colliders. To obtain the needed collider luminosity, the phase-space volume must be greatly reduced within the muon life time. The Ionization cooling is the preferred method used to compress the phase space and reduce the emittance to obtain high luminosity muon beams. We note that, the ionization losses results not only in damping, but also heating. The use of alternating solenoid lattices has been proposed, where the emittance are large. We present an overview of the cooling and discuss formalism, solenoid magnets and some beam dynamics.

INTRODUCTION

Alternating solenoid lattices has been proposed as desirable for use in the earlier cooling stages of Muon Colliders, where the emittances are large. Since the minimum $\beta_\perp$'s must decrease in order to obtain smaller transverse emittances as the muon beam travels down the cooling channel. This can be done by increasing the focusing fields and/or decreasing the muon momenta, where the current carrying lithium lenses may be used (to get a stronger radial focusing and to minimize the final emittance) for the last few cooling stages. The use of 'bent solenoids' may provide the required dispersion for the momentum measurement. Where the off-momentum muons are displaced vertically by an amount:

$$\Delta y \approx \frac{P}{eB_s}\frac{\Delta P}{P}\theta_{\text{bend}}, \tag{1}$$

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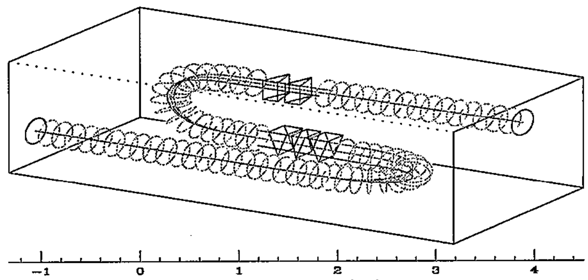

FIGURE 1. Example of bent solenoids and Wedges - for emittance exchange. May be used for muon collider longitudinal cooling

where B_s is the field of the bent solenoid and θ_{bend} is the bend angle. That eq. (1) describes the deflection of the 'guiding ray' (or axis) of the helical muon trajectory and not the trajectory itself. Also, the muon's momentum cannot be determined simply by measuring the height of its trajectory at the entrance and exit of a bent solenoid. Rather, the height of the guiding ray must be reconstructed at both places, which requires precise measurements of the helical trajectories. The momentum resolution of a bent solenoid spectrometer is given by:

$$\frac{\sigma_{D_P}}{P} \approx \frac{1}{\theta_{\text{bend}}} \frac{P}{eB_s} \frac{\sigma_{D_x}}{L^{5/2}} \sqrt{\frac{720}{n}}, \tag{2}$$

L is the length of the trajectory observed (before and after the bend) with n samples/m, each with transverse spatial resolution σ_{D_x}. Where the momentum resolution improves with increasing magnetic field. But, to reduce the cost of the solenoid channel one should try to use a relatively low magnetic field. The momentum resolution is improved by increasing θ_{bend} and by increasing L.

In Fig.1, the bending of the solenoid produce the dispersion required for the longitudinal to transverse emittance exchange. Where after one bend and one set of wedges the beam cross-section is asymmetric then the symmetry is restored by going through the second bend and wedge system (which is rotated by 90 degrees w.r.t. the first). [1].

FORMULATION AND MAPS FOR SOLENOIDS

The canonical equations in 2n-Dimensional phase space (e.g. 6 Dim., in our calculation) can be expressed as $\frac{d\psi_i}{dt} = [\psi_i, H]$, $i = 1, 2, \ldots 2n$, and in terms of the Lie transformations as

$$\frac{d\psi_i}{dt} = -:H:\psi_i, \qquad i = 1, 2, \ldots 2n \tag{3}$$

Where the Lie operator $(:H:)$ is generated by the Hamiltonian, (H), and Lie transformation, $M = e^{-t:H:}$, could generate the solution to Eq. (3) as $\psi_i = M\psi_i(0)$, where ψ_i is the value of $\psi_i(t)$ at $t > 0$ and $\psi_i(0)$ is the initial trajectory. The interest is to find solutions to equations of motion which differ slightly from the reference orbit Thus, one can choose the canonical variables, from the values for the reference trajectory (for small deviations) and Taylor expand the Hamiltonian (H) about the design trajectory $H = H_2 + H_3 + \ldots$. Where H_n is a homogeneous polynomial of degree n in the canonical variables. After transformations to the normalized dimensionless variables, one can obtain the effective Hamiltonian H^{New}, expressed as

$$H^{\text{New}} = F_2 + F_3 + F_4 \ldots . \tag{4}$$

Thus the particle trajectory $\vec{\psi} = (X, P_X, Y, P_Y, \tau, P_\tau)$ through a beamline element of length L can be described by $\psi_i^f = - : H^{\text{New}} : \psi_i$, $i = 1, 2, \ldots 2n$. The exact symplectic map that generates the particle trajectory through that element is $M = e^{-L:H^{\text{New}}:}$, where M describes the particle behavior through the element of length L. Using the factorization and expanding H^{New} as in Eq. (4), results in

$$M = e^{-L:H^{\text{New}}:} = e^{:f_2:} e^{:f_3:} e^{:f_4:} \ldots , \tag{5}$$

(e.g., for a map through 3rd order we need to include terms of f_2, f_3, and f_4).

To illustrate the above formalism, consider the evolution of the motion of particles in an external electromagnetic field described by the Hamiltonian

$$H = [m^2c^4 + c^2((p_x - qA_x)^2 + (p_y - qA_y)^2 + (p_z - qA_z)^2)]^{1/2} + e\phi(x, y, z; t)$$

,
where m and q are the rest mass and charge of the particle, A and ϕ are the vector and scalar potentials such that $\vec{B} = \nabla \times \vec{A}, \vec{E} = -\nabla\phi - \nabla\vec{A}/\partial t$.

Making a canonical transformation from H to H_1 and changing the independent variable from time t to z (for convenience) for a particle in magnetic field (e.g. of solenoid) results in
$p_z = [(p_z - qA_x)^2 + (p_y - qA_y)^2 + p_t^2/c^2 - m^2c^2]^{1/2}$.

Where $H = -p_t$, $H_1 = -p_z$ and $t = (z/v_{0z})$ the time as a function of z. We next make a canonical transformation from H_1 to H^{New}, with a dimensionless deviation variables (for convenience), $X = x/l$, $Y = y/l$, $\tau = c/l(t - z/v_{0z})$, $P_x = p_x/p_0$, $P_y = p_y/p_0$, $P_\tau = (p_t - p_0 t)/p_0 c$, where l is a length scale (taken as 1 m in our analysis), with $\mathbf{P} = \vec{P}_x + \vec{P}_y$ and $\mathbf{Q} = \vec{X} + \vec{Y}$ defined as two dimensional vectors [6], p_0 and $p_0 c$ are momentum and energy scales. Where p_0 is the design momentum, v_{0z} is the velocity on the design orbit and $p_0 t$ is a value of p_t on the design orbit ($p_{0t} = \sqrt{m^2c^4 + p_0^2c^2}$) (reminding that design orbit for the solenoid is along the z-axis). Thus, expanding the new Hamiltonian Eq. (4) leads to:

$$F_2 = \frac{P_\tau^2}{(2\beta^2\gamma^2)} - \frac{1}{2}B_0(\vec{Q} \times \vec{P}) \cdot \hat{+}\frac{1}{8}B_0^2 Q^2 + \frac{P^2}{2} \tag{6}$$

$$F_3 = \frac{P_\tau^3}{(2\beta^3\gamma^2)} - \frac{P_\tau}{2\beta}B_0(\vec{Q} \times \vec{P}) \cdot \hat{z}$$

$$+\frac{P_\tau}{8\beta}(B_0^2Q^2+4P^2) \tag{7}$$

$$\begin{aligned}F_4 = &\frac{P_\tau^4(5-\beta^2)}{8\beta^4\gamma^2}+\frac{P_\tau^2Q^2B_0^2(3-\beta^2)}{16\beta^2}\\ &-\frac{P_\tau^2}{2}(\vec{Q}\times\vec{P})\cdot\hat{z}\frac{B_0(3-\beta^2)}{2\beta^2}\\ &+\frac{P_\tau^2}{2}\frac{P^2(3-\beta^2)}{2\beta^2}+\frac{Q^4}{16}(B_0^4-4B_0B_2)/8\\ &+\frac{Q^2}{4}\frac{P^2 3B_0^2}{4}+\frac{Q^2}{4}(\vec{Q}\times\vec{P})\cdot\hat{z}(B_2-B_0^3)/4\\ &-\frac{1}{8}(\vec{P}\cdot\vec{Q})^2B_0-\frac{P^2}{4}(\vec{Q}\times\vec{P})\cdot\hat{z}B_0+\frac{P^4}{8}\end{aligned} \tag{8}$$

Following the Hamiltonian flow generated by:
$H^{\text{New}} = F_2 + F_3 \ldots$
from some initial ψ_0 to a final ψ_f coordinates we can calculate the transfer map M (Eq. (5)) for the solenoid. Where F_2, F_3, and F_4 would lead to the 1st, 2nd, and 3rd order maps. The effects of which can be seen from Eqs. (6–8). For example, the 2nd order effects due to solenoid transfer maps are purely chromatic aberrations Eq. (7). In addition, we note the third order geometric aberrations Eq. (8). As shown by Eqs. (6–8), the coupling between X, Y planes produced by a solenoid is rotation about the z-axis which is a consequence of rotational invariance of the Hamiltonian H^{New}, due to axial symmetry of the solenoid field. For beam simulations, M can be calculated to any order using numerical integration techniques such as Runge-Kutta method depending on the computer memory and space available [6].

In obtaining Eq. 8, the correlations were neglected (as e.g. in the Status Report see [1]), e.g. $\langle xP_x\rangle = 0$), and the relation $\langle x^2\rangle = \epsilon\beta_\perp = \frac{\epsilon_n\beta_\perp}{\gamma\beta}$ was used, which can not be assumed if the correlations are properly taken into account. Thus, if $\langle xP_x\rangle \neq 0$ then transverse cooling to be expressed as

$$\frac{d\epsilon_n}{ds} = \frac{1}{\beta^2}\frac{dE_\mu}{ds}\frac{\epsilon_n}{E_\mu} + \frac{1}{\beta^3}\frac{\langle x^2\rangle(0.014GeV)^2}{2\epsilon_n\, E_\mu m_\mu\, L_R} + \ldots, \tag{9}$$

As in Fig. 1, by introducing a transverse variation in the wedge (absorber) density or thickness, where there is dispersion (i.e. the transverse position is energy dependent), the energy spread, and the longitudinal emittance can be reduced. As we noted earlier, from theoretical point of view, a situation with ionization cooling completely corresponds to a situation with radiation cooling whose theory is well developed. For some standard "hierarchy" of methods for analyzing such systems see e.g. Ref. [5].

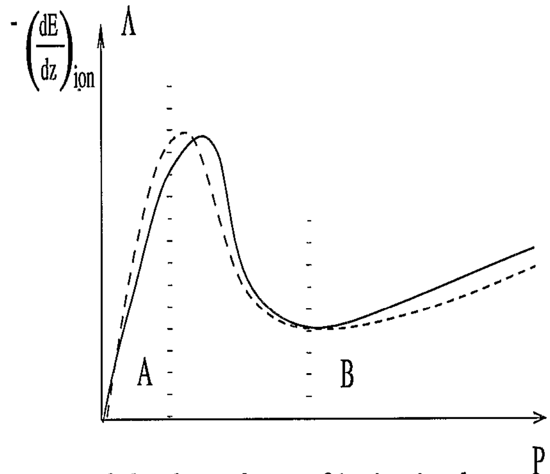

FIGURE 2. Schematic of the dependence of ionization losses on momenta.

IONIZATION COOLING

In ionization method, muons passing through a material medium lose momentum and energy through ionization interactions in transverse and longitudinal directions. The normalized emittance is reduced due to transvers energy losses. The curve in Fig. (2) shows the dependence of ionization losses on momenta. Damping rates (decrements) of individual particles in the absence of wedges (natural damping rate) are defined by the following formula:

$$\begin{aligned} \lambda_\perp &= -\frac{dE}{dz}_{ion}\,\frac{1}{2\beta^2\gamma mc^2} \\ \lambda_{||} &= -\frac{1}{z}\,\frac{d}{dp}\left[\left(\frac{dE}{dz}\right)_{\rm ion}\,\frac{1}{v}\right] \end{aligned} \tag{10}$$

Where $\lambda_\perp$ and $\lambda_{||}$ are natural transverse and longitudinal damping respectively. Here $\left(\frac{dE}{dz}\right)_{\rm ion}$ is the ionization losses of energy, m is the muon mass, β, γ are relativistic parameters, p, v are momentum and longitudinal velocity of muons being cooled. It was established, that the sum of all increments is invariant of the cooling system: $\Lambda = 2\lambda_\perp + \lambda_{||}$. This curve is also plotted in Fig. (2) (as the dotted line). In Fig. (2) we see that there are two natural regions for cooling: region A ("frictional cooling") and region B ("ionization cooling" for intermediate and high energies). Frictional Cooling is convenient only for cold (low energy) muons (e.g. Kinetic energy 10 to 150 KeV), and therefore it is difficult to use for high energy muon source, (in addition to big noises due to coulomb scattering etc.). Classical Ionization Cooling is usable for kinetic energy range of 30 to 100 MeV. Which due to absence of "natural" longitudinal cooling it is necessary to use "wedges" for which R & D is needed. A proposal for such studies is being considered [1].

COOLING FOR A NEUTRINO FACTORY

In a Neutrino Factory, a proton driver of moderate energy (< 50 GeV) and high average power,(e.g., 1-4 MW), similar to that required for a muon collider, but

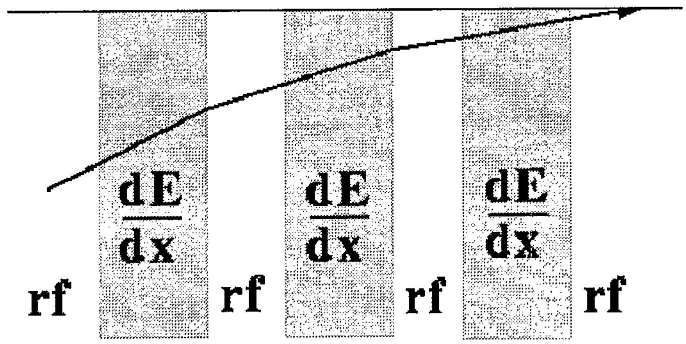

FIGURE 3. Schematic of Ionization Cooling concept (Ionization takes away momentum, and the RF acceleration puts momentum back along the z-axis, resulting in a Transverse Cooling).

with a less stringent requirements on the charge per bunch and power is needed. This is followed by a target and a pion-muons capture system. A longitudinal phase rotation is performed to reduce the muon energy spread at the expense of spreading it out over a longer time interval. The phase rotation system may be designed to correlate the muon polarization with time, allowing control of the relative intensity of muon and anti-electron neutrinos. Some cooling may be needed, to reduce phase space, about a factor of 50 in six dimensions. This is much smaller than the factor of 10^6 needed for a muon collider. Production is followed by fast muon acceleration to 50 GeV (for example), in a system of linac and two recirculating linear accelerators (RLA's), which may be identical to that for a first stage of muon collider such as a Higgs Factory.

Figure 3 shows a schematic of Ionization Cooling concept. Ionization cooling that has been proposed involves passing the beam through an absorber in which the muons lose transverse- and longitudinal-momentum by ionization loss (dE/dx). The longitudinal momentum is then restored by coherent re-acceleration, leaving a net loss of transverse momentum (transverse cooling). The process is repeated many times to achieve a large cooling factor. The beam energy spread can also be reduced using ionization cooling by introducing a transverse variation in the absorber density or thickness (e.g. a wedge) at a location where there is dispersion (the transverse position is energy dependent). Theoretical studies have shown that, assuming realistic parameters for the cooling hardware, ionization cooling can be expected to reduce the phase-space volume occupied by the initial muon beam by a factor of $10^5 - 10^6$. Ionization cooling is a new technique that has not yet been demonstrated. Special hardware needs to be developed to perform transverse and longitudinal cooling. It is recognized that understanding the feasibility of constructing an ionization cooling channel that can cool the initial muon beams by factors of $10^5 - 10^6$ is on the critical path to the overall feasibility of the muon collider concept. In Fig. 4, a schematic of the emittance exchange is shown.

MUON COOLING "MERIT FACTOR"

Luminosity of collider L is defined by the following expression:

$$L \sim \frac{N^2 f}{g_x g_y} = \frac{N^2 f}{\epsilon_\perp^f \cdot \beta_\perp^f} \tag{11}$$

Where N = a number of muons per bunch, f = mean repetition frequency of collisions, $\epsilon_\perp^f$ = emittance at collision point and $\beta_\perp^f$ = β-function at collision point. Usually $\beta_\perp^f$ is limited by condition: $\beta_\perp^f \geq \sigma_z^f$ where σ_z^f is a longitudinal bunch size. Let us assume, that: 1) $\frac{\Delta p_f}{p}$ is known (monochromatic experiments); 2) we can redistribute emittances inside a given six-dimensional phase volume. Then, taking into account losses in the cooling system, we can rewrite Eq. (11) in the following form:

$$L \sim \frac{N_0^2 \exp\left(-\frac{2}{cT_0}\int_0^z \frac{dz}{\gamma(z)}\right) D^2}{\sqrt{V_6^N \cdot \epsilon_{||}^f}} \cdot \left(\frac{\Delta p}{p}\right)_{||}^f \tag{12}$$

Here "N_0" is a number of particles at an entrance of the cooling system, "exp" describes muon decay, "D" describes muon losses in cooling section, and "V_6^N" is an invariant six-dimensional phase volume of muon beam.

Thus we can introduce "merit factor" which describes a quality of muon cooling system. We obtain

$$R = \frac{D^2 \exp\left[-\frac{2}{cT_0}\int_0^z \frac{d_z}{\gamma(z)}\right]}{\sqrt{V_6^N}} \tag{13}$$

Note that, the dependence on V_6^N may be stronger. With account of all the circumstances, we can write

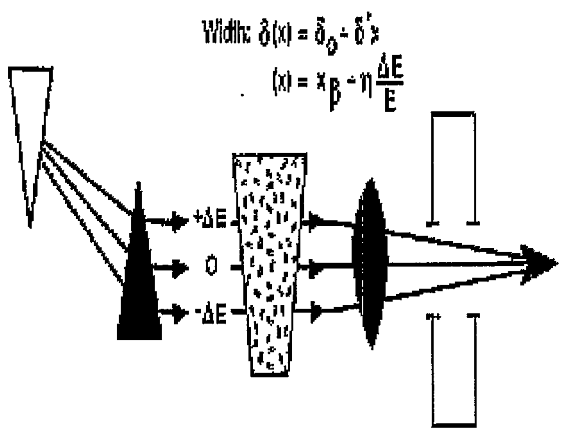

FIGURE 4. Schematic of the emittance exchange concept.

$$R \sim (V_6^N)^{\alpha} \tag{14}$$

with α in interval (0.5; 2/3). For more info. see references.

MUCOOL

The MUCOOL collaboration has been formed to pursue the development of a muon ionization cooling channel for a high luminosity muon collider. During the presentation we discussed the MUCOOL cooling studies and the Feasibility I studies for Fermilab side specific muon storage based neutrino facility, which we will not include here due to space limitation. We refer the reader to the MUCOOL - the muon ionization cooling experimental R&D page [1]. The examples presented include some of the scenarios being explored by our Neutrino Factory and Muon Collider Collaboration [12].

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