Research Article

Global Exponential Stability of Learning-Based Fuzzy Networks on Time Scales

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We investigate a class of fuzzy neural networks with Hebbian-type unsupervised learning on time scales. By using Lyapunov functional method, some new sufficient conditions are derived to ensure learning dynamics and exponential stability of fuzzy networks on time scales. Our results are general and can include continuous-time learning-based fuzzy networks and corresponding discrete-time analogues. Moreover, our results reveal some new learning behavior of fuzzy synapses on time scales which are seldom discussed in the literature.

1. Introduction

It is well known that many applications of neural networks exist in diverse areas such as optimization, signal and image processing, pattern recognition, and control system. These applications are based on stability of equilibrium points of the network models. Hence, stability criteria of equilibrium points of networks have been greatly investigated in the literature [1–4]. Meanwhile, more recently, there have been several publications on the theme of neural networks where fuzzy logic is used. Yang and Yang [2, 5] and Yang et al. [6] have proposed a fuzzy cellular neural network to include and analyze the ambiguity or vagueness inherent in the inputs and outputs of neural networks. Further analysis of this type of networks can be found in the works of Yuan et al. [4], Liu and Tang [7], Huang and Zhang [8], Huang [9, 10], Chen and Liao [11], and the references therein.

It is welknown that the theory of time scales has a tremendous potential for applications in some mathematical models of real processes and phenomena studied in physics, population dynamics, biotechnology, economics, and so on. Meanwhile, it is unsuitable to study the stability for continuous and discrete system, respectively. Therefore, it is meaningful to study that on time scales which can unify the continuous and discrete situations. Many authors incorporate time scales into stability analysis of neural network models; we can refer to [12–15].

Stimulated by [16], we consider a class of networks of somatically crisp neurons with fuzzy learnable synapses on time scales $\mathbb{T}$ described by

$$
\begin{aligned}
u_i^\Delta(t) = {} & -a_i u_i(t) + \sum_{j=1}^{n} a_{ij} f_j\left(u_j(t)\right) + B_i \sum_{j=1}^{n} m_{ij}(t)\, p_j \\
& + b_i \bigvee_{j=1}^{n} b_{ij} f_j\left(u_j\left(t - \tau_j\right)\right) \\
& + c_i \bigwedge_{j=1}^{n} c_{ij} f_j\left(u_j\left(t - \tau_j\right)\right) + J_i, \\
m_{ij}^\Delta(t) = {} & -\alpha_i m_{ij}(t) \\
& + \left[\beta_i f_i\left(u_i(t)\right) + \gamma_i \bigwedge_{j=1}^{n} \gamma_{ij} f_j\left(u_j\left(t - \sigma_j\right)\right) \right. \\
& \left. + \delta_i \bigvee_{j=1}^{n} \delta_{ij} f_j\left(u_j\left(t - \sigma_j\right)\right) \right] p_j,
\end{aligned}
\tag{1}
$$

where $i \in \mathbb{N} = \{1, 2, \ldots, n\}$, $u_i(t)$ denotes the state of neuron i at time t, a_i denotes the passive negative stabilizing feedback of neuron i, a_{ij}, b_{ij}, c_{ij}, γ_{ij}, and δ_{ij} denote the synaptic weights of the various fuzzy and nonfuzzy synapses of neuron i, B_i, b_i, c_i, β_i, γ_i, δ_i are disposable constants, and m_{ij} denotes a learnable synaptic weight of neuron when it is presented with a constant input signal vector $p = (p_1, p_2, \ldots, p_n)$; the external bias to the network is denoted by the constant vector $J = (J_1, J_2, \ldots, J_n)$. The operators $\bigvee$ and $\bigwedge$ denote, respectively, the "max" and "min" operators used in fuzzy logic. The learning equation is based on the Hebbian-type [16, 17] unsupervised algorithm modified by the introduction of a forgetting term as proposed by Amari [18]. By using auxiliary variables $v_i(t) = \sum_{j=1}^{n} m_{ij}(t) p_i$, one gets

$$
\begin{aligned}
u_i^\Delta (t) = & -a_i u_i (t) + \sum_{j=1}^{n} a_{ij} f_j \left(u_j (t) \right) + B_i v_i (t) \\
& + b_i \bigvee_{j=1}^{n} b_{ij} f_j \left(u_j \left(t - \tau_j \right) \right) \\
& + c_i \bigwedge_{j=1}^{n} c_{ij} f_j \left(u_j \left(t - \tau_j \right) \right) + J_i, \\
v_i^\Delta (t) = & -\alpha_i v_i (t) \\
& + \left[\beta_i f_i \left(u_i (t) \right) + \gamma_i \bigwedge_{j=1}^{n} \gamma_{ij} f_j \left(u_j \left(t - \sigma_j \right) \right) \right. \\
& \left. + \delta_i \bigvee_{j=1}^{n} \delta_{ij} f_j \left(u_j \left(t - \sigma_j \right) \right) \right] c
\end{aligned}
\tag{2}
$$

$$
i = 1, 2, \ldots, n, \ t > 0,
$$

where $c = \sum_{j=1}^{n} p_j^2$, $i \in \mathbb{N} = \{1, 2, \ldots, n\}$, $t \in \mathbb{T}^+$. Equation (2) is quite general and it includes several well known neural networks [16] and its difference analogue is

$$
\begin{aligned}
\Delta u_i (m) = & -a_i u_i (m) + \sum_{j=1}^{n} a_{ij} f_j \left(u_j (m) \right) + B_i v_i (m) \\
& + b_i \bigvee_{j=1}^{n} b_{ij} f_j \left(u_j \left(m - \tau_j \right) \right) \\
& + c_i \bigwedge_{j=1}^{n} c_{ij} f_j \left(u_j \left(m - \tau_j \right) \right) + J_i, \\
\Delta v_i (m) = & -\alpha_i v_i (m) \\
& + \left[\beta_i f_i \left(u_i (m) \right) + \gamma_i \bigwedge_{j=1}^{n} \gamma_{ij} f_j \left(u_j \left(m - \sigma_j \right) \right) \right. \\
& \left. + \delta_i \bigvee_{j=1}^{n} \delta_{ij} f_j \left(u_j \left(m - \sigma_j \right) \right) \right] c,
\end{aligned}
\tag{3}
$$

where $\Delta u_i(m) = u_i(m + 1) - u_i(m)$ and $\Delta v_i(m) = v_i(m + 1) - v_i(m)$ are the forward difference operators and $m, \tau_j, \sigma_j \in \mathbb{Z}$, $i \in \mathbb{N}$.

For convenience, we let $b_{ij}^\ell := b_i b_{ij}$, $c_{ij}^\ell := c_i c_{ij}$, $\gamma_{ij}^\ell := \gamma_i \gamma_{ij}$, $\delta_{ij}^\ell := \delta_i \delta_{ij}$, and $c = 1$. Then, (2) reduces to

$$
\begin{aligned}
u_i^\Delta (t) = & -a_i u_i (t) + \sum_{j=1}^{n} a_{ij} f_j \left(u_j (t) \right) + \bigvee_{j=1}^{n} b_{ij}^\ell f_j \left(u_j \left(t - \tau_j \right) \right) \\
& + \bigwedge_{j=1}^{n} c_{ij}^\ell f_j \left(u_j \left(t - \tau_j \right) \right) + B_i v_i (t) + J_i, \\
v_i^\Delta (t) = & -\alpha_i v_i (t) + \beta_i f_i \left(u_i (t) \right) + \bigwedge_{j=1}^{n} \gamma_{ij}^\ell f_j \left(u_j \left(t - \sigma_j \right) \right) \\
& + \bigvee_{j=1}^{n} \delta_{ij}^\ell f_j \left(u_j \left(t - \sigma_j \right) \right).
\end{aligned}
\tag{4}
$$

Correspondingly, synaptic dynamic equation is as follows:

$$
\begin{aligned}
m_{ij}^\Delta (t) = & -\alpha_i m_{ij} (t) \\
& + \left[\beta_i f_i \left(u_i (t) \right) + \bigwedge_{j=1}^{n} \gamma_{ij}^\ell f_j \left(u_j \left(t - \sigma_j \right) \right) \right. \\
& \left. + \bigvee_{j=1}^{n} \delta_{ij}^\ell f_j \left(u_j \left(t - \sigma_j \right) \right) \right] p_j,
\end{aligned}
\tag{5}
$$

where $\sum_{j=1}^{n} p_j^2 = 1$. In this paper, we will study learning-based fuzzy networks (4) on time scales. Without the learning component and $\mathbb{T} = \mathbb{R}$, (4) will include fuzzy networks discussed by several authors recently (see [2, 4, 7, 9–11]). In the absence of fuzzy synapses, our model reduces to the most commonly studied Hopfield-type neural network. Moreover, by using the calculus theory on time scale to unify and generalize discrete-time and continuous-time learning-based fuzzy networks, we can establish new sufficient conditions to ensure existence and global exponential stability of equilibrium of (4).

The paper is organized as follows. In Section 2, we present some basic definitions concerning the calculus on time scales. In Section 3, we develop Lyapunov functions technique on time scale to give some sufficient conditions of global exponential stability for (4). In Section 4, an example is given to illustrate the effectiveness of our main results. Conclusions remarks are given in Section 5.

2. Preliminaries on Time Scales

The basic calculus theory on time scales was initiated by Hilger [19, 20], and Agarwal et al. summarize and organize much of relative results in monograph [21–23]. In this section, we will introduce some basic definitions and lemmas.

Definition 1. A time scale $\mathbb{T}$ is arbitrary nonempty closed subset of the real set $\mathbb{R}$ with the topology and ordering inherited from $\mathbb{R}$.

Definition 2. On any time scale $\mathbb{T}$, one defines the forward and backward jump operators by $\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}$ and $\rho(t) := \sup\{s \in \mathbb{T} : s < t\}$; one puts $\inf \emptyset := \sup \mathbb{T}$ and $\sup \emptyset := \inf \mathbb{T}$, where $\emptyset$ denotes the empty set. A point t is said to be left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, left-scattered if $\rho(t) < t$, and right-scattered if $\sigma(t) > t$. The graininess function μ for a time scale $\mathbb{T}$ is defined by $\mu(t) := \sigma(t) - t$. If $\mathbb{T}$ has a left-scattered maximum m, then one defined $\mathbb{T}^k$ to be $\mathbb{T} - m$. Otherwise, $\mathbb{T}^k = \mathbb{T}$.

Definition 3. For a function $f : \mathbb{T} \to \mathbb{R}$ (the range $\mathbb{R}$ of f may be actually replaced by Banach space), the (delta) derivative is defined by

$$f^\Delta = \frac{f(\sigma(t)) - f(t)}{\sigma(t) - t}, \tag{6}$$

if f is continuous at t and t is right-scattered. If t is not right-scattered then the derivative is defined by

$$f^\Delta = \lim_{s \to t} \frac{f(t) - f(s)}{t - s} \tag{7}$$

provided this limit exists.

Definition 4. A function $F : \mathbb{T}^k \to \mathbb{R}$ is called a delta-antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^\Delta = f$ holds for all $t \in \mathbb{T}^k$. In this case, one defines the integral of f by

$$\int_a^t f(s)\,\Delta s = F(t) - F(a) \quad \forall t \in \mathbb{T}, \tag{8}$$

and one has the following formula:

$$\int_t^{\sigma(t)} f(s)\,\Delta s = \mu(t)f(t) \quad \forall t \in \mathbb{T}^k. \tag{9}$$

Definition 5. A function $f : \mathbb{T} \to \mathbb{R}$ is called right-dense continuous provided it is continuous at right-dense points of $\mathbb{T}$ and the left sided limit exists (finite) at left-dense point of $\mathbb{T}$. The set of all right-dense continuous functions on $\mathbb{T}$ is defined by $C_{rd} = C_{rd}(\mathbb{T}) = C_{rd}(\mathbb{T}, \mathbb{R})$.

Definition 6. One says that a function $p : \mathbb{T} \to \mathbb{R}$ is regressive provided $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}$. The set of all regressive functions on a time scale $\mathbb{T}$ forms an Abelian group under the addition $\oplus$ defined by $p \oplus q := p + q + \mu pq$. The additive inverse in this group is denoted by $\ominus p := -p/(1 + \mu p)$. One then defines subtraction $\ominus$ on the set of regressive functions by $p \ominus q := p \oplus (\ominus q)$. It can be shown that $p \oplus (\ominus q) = -(p - q)/(1 + \mu q)$. The set of all regressive and right-dense continuous functions will be denoted by $\mathcal{R}$.

Definition 7. One defines the set $\mathcal{R}^+$ of all positively regressive elements of $\mathcal{R}$ by $\mathcal{R}^+ = \{f \in \mathcal{R} : 1 + \mu(t)f(t) > 0 \text{ for all } t \in \mathbb{T}\}$.

Next, we give the definition of the exponential function and list some of its properties.

Definition 8. If $p(t) \in \mathcal{R}$, one defines the generalized exponential function as

$$
\begin{aligned}
e_p(t,s) &= \exp\left(\int_s^t \xi_{\mu(\tau)}(p(\tau))\,\Delta\tau\right) \\
&= \exp\left(\int_s^t \frac{\mathrm{Log}(1 + \mu(\tau)p(\tau))}{\mu(\tau)}\Delta\tau\right) \quad \text{for } t, \tau \in \mathbb{T},
\end{aligned} \tag{10}
$$

where $\xi_h(x) = \mathrm{Log}(1 + xh)/h$, $h > 0$, and $\xi_0(x) = x$.

Remark 9. The exponential function $e_p(\cdot, t_0)$ is the unique solution of the IVP $x^\Delta = p(t)x$, $x(t_0) = 1$ for $p(t) \in \mathcal{R}$. As $\mathbb{T} = \mathbb{R}$, $e_p(t, t_0) = \exp(\int_{t_0}^t p(s)ds)$.

Lemma 10. *If $p, q \in \mathcal{R}$, then*

(i) $e_0(t,s) \equiv 1$ and $e_p(t,t) = 1$;

(ii) $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t,s)$;

(iii) $e_p(t,s)e_p(s,r) = e_p(t,r)$;

(iv) $e_p(t,s) = 1/e_p(s,t) = e_{\ominus p}(s,t)$;

(v) $e_p(t,s) > 0$, for $p \in \mathcal{R}^+$;

(vi) $e_p(t,s)e_q(t,s) = e_{p \oplus q}(t,s)$;

(vii) $e_p(t,s)/e_q(t,s) = e_{p \ominus q}(t,s)$.

Lemma 11 (see [3]). *If $p \in \mathcal{R}$ and $a, b, c \in \mathbb{T}$, then*

$$\int_a^b p(t)e_p(c, \sigma(t))\,\Delta t = e_p(c,a) - e_p(c,b). \tag{11}$$

If $-p \in \mathcal{R}^+$, then

$$e_{-p}(t,s) \leq \exp\left\{-\int_s^t p(u)\,du\right\}, \quad t \geq s. \tag{12}$$

Lemma 12 (see [22]). *Let $y, f \in C_{rd}$ and $p \in \mathcal{R}^+$. Then, $y^\Delta(t) \leq p(t)y(t) + f(t)$, $t \in \mathbb{T}$, implies*

$$y(t) \leq y(t_0)e_p(t, t_0) + \int_{t_0}^t e_p(t, \sigma(\tau))f(\tau)\,\Delta\tau, \quad t \in \mathbb{T}. \tag{13}$$

Lemma 13 (see [2, 5]). *Suppose $x = (x_1, x_2, \ldots, x_n)$, $y = (y_1, y_2, \ldots, y_n)$ are any two vectors in $\mathbb{R}^n$:*

$$\left|\bigwedge_{j=1}^n \alpha_{ij} f_j(x_j) - \bigwedge_{j=1}^n \alpha_{ij} f_j(y_j)\right| \leq \sum_{j=1}^n |\alpha_{ij}| |f_j(x_j) - f_j(y_j)|,$$

$$\left|\bigvee_{j=1}^n \beta_{ij} f_j(x_j) - \bigvee_{j=1}^n \beta_{ij} f_j(y_j)\right| \leq \sum_{j=1}^n |\beta_{ij}| |f_j(x_j) - f_j(y_j)|,$$

$$\forall i \in \mathbb{N}. \tag{14}$$

Throughout this paper, we make the following basic assumptions:

$(\mathscr{A}_1)$ The functions g_i, $f_j (i, j \in \mathbb{N})$ are Lipschitz continuous on $\mathbb{R}$ with the Lipschitz constants $\widetilde{L_i}$ and L_j, respectively; that is, $|f_j(y) - f_j(x)| \leq L_j|y - x|$, $|g_i(y) - g_i(x)| \leq \widetilde{L_i}|y - x|$.

3. Main Results

In this section, we study the global exponential stability of the unique equilibrium for (4) on time scale by using Lyapunov method.

Theorem 14. *Suppose that* (4) *satisfies* $(\mathscr{A}_1)$; *if there exist positive constants* λ_i, ξ_i, *and* p *such that*

$(\mathscr{A}_2)$

$$\lambda_i p + (1 + \mu(t)p)$$

$$\times \left[-2\lambda_i a_i + \lambda_i \left(\sum_{j=1}^n \left(|a_{ij}| + \alpha_{ij} \right) L_j + |B_i| \right) \right.$$

$$+ \xi_i |\beta_i| L_i + \sum_{j=1}^n \lambda_j |a_{ji}| L_i$$

$$+ \mu(t) \left[\lambda_i a_i^2 + \lambda_i a_i \left(\sum_{j=1}^n \left(|a_{ij}| + \alpha_{ij} \right) L_j + |B_i| \right) \right.$$

$$+ \xi_i |\beta_i| L_i \left(|\beta_i| L_i + \alpha_i + \sum_{j=1}^n \beta_{ij} L_j \right)$$

$$+ \sum_{j=1}^n \lambda_j |a_{ji}| L_i$$

$$\left. \left. \times \left(n |a_{ji}| L_i + a_j + \alpha_{ji} L_i + \left| B_j \right| \right) \right) \right] \right]$$

$$+ \sum_{j=1}^n \lambda_j \left(1 + \mu(t + \tau_i) p \right) e_p(t + \tau_i, t)$$

$$\times \alpha_{ji} L_i \left[1 + \mu(t + \tau_i) \left(n \alpha_{ji} L_i + a_j + |a_{ji}| L_i + |B_j| \right) \right]$$

$$+ \sum_{j=1}^n \xi_j \left(1 + \mu(t + \sigma_i) p \right) e_p(t + \sigma_i, t) \beta_{ji} L_i$$

$$\times \left[1 + \mu(t + \sigma_i) \left(n \beta_{ji} L_i + \alpha_j + |\beta_j| L_j \right) \right] < 0,$$

$$\xi_i p + (1 + \mu(t)p)$$

$$\times \left[\lambda_i |B_i| - 2\xi_i \alpha_i + \xi_i |\beta_i| L_i + \xi_i \sum_{j=1}^n \beta_{ij} L_j \right.$$

$$+ \mu(t) \left[\lambda_i |B_i| \left(|B_i| + a_i + \sum_{j=1}^n \left(|a_{ij}| + \alpha_{ij} \right) L_j \right) \right.$$

$$\left. \left. + \alpha_i \xi_i \left(\alpha_i + |\beta_i| L_i + \sum_{j=1}^n \beta_{ij} L_j \right) \right] \right] < 0,$$

$$(15)$$

where $|b_{ij}^\ell| + |c_{ij}^\ell| = \alpha_{ij}$ *and* $|\gamma_{ij}^\ell| + |\delta_{ij}^\ell| = \beta_{ij}$, $i, j \in \mathbb{N}$, *then there exists a unique equilibrium* $(u^*, v^*) = (u_1^*, \ldots, u_n^*, v_1^*, \ldots, v_n^*)$ *of* (4) *which is globally exponentially stable; that is, every solution* $(u, v) = (u_1, \ldots, u_n, v_1, \ldots, v_n)$ *of* (4) *satisfies*

$$\sum_{i=1}^n \left(u_i(t) - u_i^* \right)^2 + \sum_{i=1}^n \left(v_i(t) - v_i^* \right)^2 \leq M e_{\ominus p}(t, 0) \Phi_\theta \qquad (16)$$

$$\longrightarrow 0 \quad as \ t \longrightarrow +\infty,$$

where $M > 0$, $\theta = \max_{j \in \mathbb{N}}(\tau_j, \sigma_j)$, *and*

$$\Phi_\theta := \left[\sum_{i=1}^n \sup_{i - \theta \leq s \leq 0} \left(u_i(s) - u_i^* \right)^2 + \sum_{i=1}^n \sup_{i - \theta \leq l \leq 0} \left(v_i(l) - v_i^* \right)^2 \right].$$

$$(17)$$

Proof. Similar to the proof of [16], we can prove (4) possesses a unique equilibrium $(u^*, v^*) = (u_1^*, \ldots, u_n^*, v_1^*, \ldots, v_n^*)$. Let $w_i(t) = u_i(t) - u_i^*$ and let $\rho_i(t) = v_i(t) - v_i^*$; then, we can rewrite (4) into

$$w_i^\Delta(t) \leq - a_i w_i(t) + \sum_{j=1}^n |a_{ij}| L_j |w_j(t)|$$

$$+ \sum_{j=1}^n |b_{ij}^\ell| L_j |w_j(t - \tau_j)|$$

$$+ \sum_{j=1}^n |c_{ij}^\ell| L_j |w_j(t - \tau_j)| + |B_i| |\rho_i(t)|$$

$$= - a_i w_i(t) + \sum_{j=1}^n |a_{ij}| L_j |w_j(t)|$$

$$+ \sum_{j=1}^n \left(|b_{ij}^\ell| + |c_{ij}^\ell| \right) L_j |w_j(t - \tau_j)| + |B_i| |\rho_i(t)|$$

$$= - a_i w_i(t) + \sum_{j=1}^n |a_{ij}| L_j |w_j(t)|$$

$$+ \sum_{j=1}^n \alpha_{ij} L_j |w_j(t - \tau_j)| + |B_i| |\rho_i(t)|,$$

$$\rho_i^\Delta(t) \le -\alpha_i \rho_i(t) + |\beta_i| L_i |w_i(t)| + \sum_{j=1}^n |\gamma_{ij}^\ell| L_j |w_j(t-\sigma_j)|$$

$$+ \sum_{j=1}^n |\delta_{ij}^\ell| L_j |w_j(t-\sigma_j)|$$

$$= |\beta_i| L_i |w_i(t)| + \sum_{j=1}^n \left(|\gamma_{ij}^\ell| + |\delta_{ij}^\ell|\right) L_j |w_j(t-\sigma_j)|$$

$$- \alpha_i \rho_i(t)$$

$$= |\beta_i| L_i |w_i(t)| + \sum_{j=1}^n \beta_{ij} L_j |w_j(t-\sigma_j)| - \alpha_i \rho_i(t). \tag{18}$$

Now, we construct the Lyapunov function $F(t) = F_1(t) + F_2(t) + F_3(t) + F_4(t)$, where

$$F_1(t) = \sum_{i=1}^n \lambda_i w_i^2(t) e_p(t,0),$$

$$F_2(t) = \sum_{i=1}^n \xi_i \rho_i^2(t) e_p(t,0)$$

$$F_3(t) = \sum_{i=1}^n \sum_{j=1}^n \lambda_i \int_{t-\tau_j}^t \Big\{ \alpha_{ij} L_j$$

$$\times \left[1 + \mu(s+\tau_j)\right.$$

$$\times \left(n\alpha_{ij} L_j + a_i + |a_{ij}| L_j + |B_i|\right)\Big]\Big\}$$

$$\times \left(1 + \mu(s+\tau_j) p\right) e_p(s+\tau_j, 0) w_j^2(s) \Delta s$$

$$F_4(t) = \sum_{i=1}^n \sum_{j=1}^n \xi_i \int_{t-\sigma_j}^t \Big\{ \beta_{ij} L_j \left[1 + \mu(s+\sigma_j)\right.$$

$$\times \left(n\beta_{ij} L_j + \alpha_i + |\beta_i| L_i\right)\Big]\Big\}$$

$$\times \left(1 + \mu(s+\sigma_j) p\right) e_p(s+\sigma_j, 0) w_j^2(s) \Delta s$$

$$F_1^\Delta(t) = \sum_{i=1}^n \lambda_i \left[\left(w_i^2(t)\right)^\Delta e_p(\sigma(t), 0) + w_i^2(t) e_p^\Delta(t, 0)\right]$$

$$= \sum_{i=1}^n \lambda_i \Big\{ \left[w_i(t) + w_i(\sigma(t))\right] w_i^\Delta(t) e_p(\sigma(t), 0)$$

$$+ w_i^2(t) p e_p(t, 0) \Big\}$$

$$= \sum_{i=1}^n \lambda_i \Big\{ \left[2w_i(t) w_i^\Delta(t) + \mu(t) \left(w_i^\Delta(t)\right)^2\right]$$

$$\times e_p(\sigma(t), 0) + w_i^2(t) p e_p(t, 0) \Big\}$$

$$\le \sum_{i=1}^n \lambda_i \Big\{ 2w_i(t) \left[-a_i w_i(t) + \sum_{j=1}^n |a_{ij}| L_j |w_j(t)|\right.$$

$$\left.+ \sum_{j=1}^n \alpha_{ij} L_j |w_j(t-\tau_j)| + |B_i| |\rho_i(t)|\right]$$

$$\times e_p(\sigma(t), 0)$$

$$+ \mu(t) \left[-a_i w_i(t) + \sum_{j=1}^n |a_{ij}| L_j |w_j(t)|\right.$$

$$+ \sum_{j=1}^n \alpha_{ij} L_j \times |w_j(t-\tau_j)|$$

$$\left.+ |B_i| |\rho_i(t)|\right]^2 e_p(\sigma(t), 0)$$

$$+ w_i^2(t) p e_p(t, 0) \Big\}$$

$$\le \sum_{i=1}^n \lambda_i e_p(\sigma(t), 0)$$

$$\times \Big\{ \left[-2a_i w_i^2(t) + \sum_{j=1}^n |a_{ij}| L_j \left(w_i^2(t) + w_j^2(t)\right)\right.$$

$$+ \sum_{j=1}^n \alpha_{ij} L_j \times \left(w_i^2(t) + w_j^2(t-\tau_j)\right)$$

$$\left.+ |B_i| \left(w_i^2(t) + \rho_i^2(t)\right)\right]$$

$$+ \mu(t) \left[a_i^2 w_i^2(t) + n \sum_{j=1}^n a_{ij}^2 L_j^2 w_j^2(t)\right.$$

$$+ n \sum_{j=1}^n \alpha_{ij}^2 L_j^2 w_j^2(t-\tau_j) + B_i^2 \rho_i^2(t)$$

$$+ a_i \sum_{j=1}^n |a_{ij}| L_j \left(w_i^2(t) + w_j^2(t)\right)$$

$$+ a_i \sum_{j=1}^n \alpha_{ij} L_j \left(w_i^2(t) + w_j^2(t-\tau_j)\right)$$

$$+ a_i |B_i| \left(w_i^2(t) + \rho_i^2(t)\right)$$

$$+ \sum_{j=1}^{n} |a_{ij}| \alpha_{ij} L_j^2$$

$$\times \left(w_j^2(t) + w_j^2(t - \tau_j) \right)$$

$$+ \sum_{j=1}^{n} |a_{ij}| L_j |B_i| \left(w_j^2(t) + \rho_i^2(t) \right)$$

$$+ |B_i|$$

$$\times \sum_{j=1}^{n} \alpha_{ij} L_j \left(w_j^2(t - \tau_j) + \rho_i^2(t) \right) \Bigg] \Bigg\}$$

$$+ \sum_{i=1}^{n} \lambda_i p\, e_p(t,0)\, w_i^2(t)$$

$$= \sum_{i=1}^{n} \lambda_i e_p(t,0)$$

$$\times \Bigg\{ p + (1 + \mu(t)\, p)$$

$$\times \Bigg[-2a_i + \mu(t)\, a_i^2$$

$$+ \left(\sum_{j=1}^{n} (|a_{ij}| + \alpha_{ij}) L_j + |B_i| \right)$$

$$\times (1 + \mu(t)\, a_i) \Bigg] \Bigg\} w_i^2(t)$$

$$+ \sum_{i=1}^{n} \lambda_i e_p(t,0)(1 + \mu(t)\, p)$$

$$\times \Bigg\{ \sum_{j=1}^{n} |a_{ij}| L_j$$

$$\times \Big[1 + \mu(t)$$

$$\times \left(n |a_{ij}| L_j + a_i + \alpha_{ij} L_j + |B_i| \right) \Big]$$

$$\times w_j^2(t)$$

$$+ \sum_{j=1}^{n} \alpha_{ij} L_j$$

$$\times \Big[1 + \mu(t)$$

$$\times \left(n \alpha_{ij} L_j + a_i + |a_{ij}| L_j + |B_i| \right) \Big]$$

$$\times w_j^2(t - \tau_j)$$

$$+ |B_i| \Bigg[1 + \mu(t)$$

$$\times \Bigg(|B_i| + a_i$$

$$+ \sum_{j=1}^{n} (|a_{ij}| + \alpha_{ij}) L_j \Bigg) \Bigg]$$

$$\times \rho_i^2(t) \Bigg\}$$

$$F_2^\Delta(t) = \sum_{i=1}^{n} \xi_i \left[\left(\rho_i^2(t) \right)^\Delta e_p(\sigma(t),0) + \rho_i^2(t)\, e_p^\Delta(t,0) \right]$$

$$= \sum_{i=1}^{n} \xi_i \Bigg\{ \left[2\rho_i(t)\, \rho_i^\Delta(t) + \mu(t) \left(\rho_i^\Delta(t) \right)^2 \right]$$

$$\times e_p(\sigma(t),0) + \rho_i^2(t)\, p\, e_p(t,0) \Bigg\}$$

$$\leq \sum_{i=1}^{n} \xi_i \Bigg\{ 2\rho_i(t) \Bigg[-\alpha_i \rho_i(t) + |\beta_i| L_i |w_i(t)|$$

$$+ \sum_{j=1}^{n} \beta_{ij} L_j |w_j(t - \sigma_j)| \Bigg]$$

$$\times e_p(\sigma(t),0)$$

$$+ \mu(t) \Bigg[-\alpha_i \rho_i(t) + |\beta_i| L_i |w_i(t)|$$

$$+ \sum_{j=1}^{n} \beta_{ij} L_j |w_j(t - \sigma_j)| \Bigg]^2$$

$$\times e_p(\sigma(t),0) + \rho_i^2(t)\, p\, e_p(t,0) \Bigg\}$$

$$\leq \sum_{i=1}^{n} \xi_i e_p(\sigma(t),0)$$

$$\times \Bigg\{ \Bigg[-2\alpha_i \rho_i^2(t) + |\beta_i| L_i \left(\rho_i^2(t) + w_i^2(t) \right)$$

$$+ \sum_{j=1}^{n} \beta_{ij} L_j \left(\rho_i^2(t) + w_j^2(t - \sigma_j) \right) \Bigg]$$

$$+ \mu(t)\left[\alpha_i^2 \rho_i^2(t) + \beta_i^2 L_i^2 w_i^2(t)\right.$$

$$+ n\sum_{j=1}^n \beta_{ij}^2 L_j^2 w_j^2(t - \sigma_j)$$

$$+ \alpha_i |\beta_i| L_i \left(\rho_i^2(t) + w_i^2(t)\right)$$

$$+ \alpha_i L_j \sum_{j=1}^n \beta_{ij}\left(\rho_i^2(t) + w_j^2(t-\sigma_j)\right)$$

$$+ |\beta_i| L_i$$

$$\left.\times \sum_{j=1}^n \beta_{ij} L_j \left(w_i^2(t) + w_j^2(t-\sigma_j)\right)\right]\Bigg\}$$

$$+ \sum_{i=1}^n \xi_i p\, e_p(t,0)\rho_i^2(t)$$

$$= \sum_{i=1}^n \xi_i e_p(t,0)$$

$$\times \Bigg\{ p + (1+\mu(t)p)$$

$$\times \left[-2\alpha_i + \mu(t)\alpha_i^2 + \left(|\beta_i| L_i + \sum_{j=1}^n \beta_{ij} L_j\right)\right.$$

$$\left.\times (1+\mu(t)\alpha_i)\right]\Bigg\}\rho_i^2(t)$$

$$+ \sum_{i=1}^n \xi_i e_p(t,0)(1+\mu(t)p)$$

$$\times \Bigg\{ |\beta_i| L_i$$

$$\times \left[1 + \mu(t)\left(|\beta_i| L_i + \alpha_i + \sum_{j=1}^n L_j \beta_{ij}\right)\right]$$

$$\times w_i^2(t)$$

$$+ \sum_{j=1}^n \beta_{ij} L_j$$

$$\times \left[1 + \mu(t)(n\beta_{ij}L_j + \alpha_i + |\beta_i| L_i)\right]$$

$$\times w_j^2(t-\sigma_j)\Bigg\}$$

$$F_3^\Delta(t) = \sum_{i=1}^n \sum_{j=1}^n \lambda_i e_p\left(t+\tau_j, 0\right)\left(1+\mu\left(t+\tau_j\right)p\right)$$

$$\times \left\{\alpha_{ij} L_j\right.$$

$$\times \left[1+\mu\left(t+\tau_j\right)\right.$$

$$\left.\times \left(n\alpha_{ij}L_j + a_i + |a_{ij}| L_j + |B_i|\right)\right]\}$$

$$\times w_j^2(t)$$

$$- \sum_{i=1}^n \sum_{j=1}^n \lambda_i (1+\mu(t)p)\,e_p(t,0)$$

$$\times \left\{\alpha_{ij} L_j\right.$$

$$\times \left[1+\mu(t)\right.$$

$$\left.\times \left(n\alpha_{ij}L_j + a_i + |a_{ij}| L_j + |B_i|\right)\right]\}$$

$$\times w_j^2\left(t-\tau_j\right)$$

$$F_4^\Delta(t) = \sum_{i=1}^n \sum_{j=1}^n \xi_i e_p\left(t+\sigma_j,0\right)\left(1+\mu\left(t+\sigma_j\right)p\right)$$

$$\times \left\{\beta_{ij} L_j \left[1 + \mu\left(t+\sigma_j\right)\right.\right.$$

$$\left.\left.\times \left(n\beta_{ij}L_j + \alpha_i + |\beta_i| L_i\right)\right]\right\} w_j^2(t)$$

$$- \sum_{i=1}^n \sum_{j=1}^n \xi_i e_p(t,0)(1+\mu(t)p)$$

$$\times \left\{\beta_{ij} L_j \left[1 + \mu(t)\right.\right.$$

$$\left.\left.\times \left(n\beta_{ij}L_j + \alpha_i + |\beta_i| L_i\right)\right]\right\}$$

$$\times w_j^2\left(t-\sigma_j\right)$$

$$F^\Delta(t) = F_1^\Delta(t) + F_2^\Delta(t) + F_3^\Delta(t) + F_4^\Delta(t)$$

$$\leq \sum_{i=1}^n \lambda_i e_p(t,0)$$

$$\times \Bigg\{ p + (1+\mu(t)p)$$

$$\times \left[-2a_i + \mu(t)a_i^2\right.$$

$$+ \left(\sum_{j=1}^n \left(|a_{ij}| + \alpha_{ij}\right) L_j + |B_i|\right)$$

$$\left.\times (1+\mu(t)a_i)\right]\Bigg\} w_i^2(t)$$

$$+ \sum_{i=1}^{n} \lambda_i e_p(t,0)(1+\mu(t)p)$$

$$\times \left\{ \sum_{j=1}^{n} |a_{ij}| L_j \right.$$

$$\times \left[1 + \mu(t) \right.$$

$$\times \left(n|a_{ij}|L_j + a_i + \alpha_{ij}L_j + |B_i| \right) \left] \right.$$

$$\times w_j^2(t)$$

$$+ |B_i| \left[1 + \mu(t) \right.$$

$$\times \left(|B_i| + a_i \right.$$

$$\left. + \sum_{j=1}^{n} \left(|a_{ij}| + \alpha_{ij} \right) L_j \right) \right]$$

$$\left. \times \rho_i^2(t) \right\}$$

$$+ \sum_{i=1}^{n} \xi_i e_p(t,0)$$

$$\times \left\{ p + (1+\mu(t)p) \right.$$

$$\times \left[-2\alpha_i + \mu(t)\alpha_i^2 + \left(|\beta_i| L_i + \sum_{j=1}^{n} \beta_{ij} L_j \right) \right.$$

$$\left. \times (1+\mu(t)\alpha_i) \right] \left. \right\} \rho_i^2(t)$$

$$+ \sum_{i=1}^{n} \xi_i e_p(t,0)(1+\mu(t)p)$$

$$\times \left\{ |\beta_i| L_i \left[1 + \mu(t) \right. \right.$$

$$\times \left(|\beta_i| L_i + \alpha_i + \sum_{j=1}^{n} \beta_{ij} L_j \right) \left] \right.$$

$$\times w_i^2(t)$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_i e_p(t+\tau_j,0)(1+\mu(t+\tau_j)p)$$

$$\times \left\{ \alpha_{ij} L_j \left[1 + \mu(t+\tau_j) \right. \right.$$

$$\left. \times \left(n\alpha_{ij} L_j + a_i + |a_{ij}| L_j + |B_i| \right) \right] \right\}$$

$$\times w_j^2(t)$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{n} \xi_i e_p(t+\sigma_j,0)(1+\mu(t+\sigma_j)p)$$

$$\times \left\{ \beta_{ij} L_j \right.$$

$$\left. \times \left[1 + \mu(t+\sigma_j)\left(n\beta_{ij} L_j + \alpha_i + |\beta_i| L_i \right) \right] \right\}$$

$$\times w_j^2(t)$$

$$= \sum_{i=1}^{n} e_p(t,0)$$

$$\times \left\{ \lambda_i \left[p + (1+\mu(t)p) \right. \right.$$

$$\times \left(-2a_i + \mu(t)a_i^2 \right.$$

$$+ \left(\sum_{j=1}^{n} \left(|a_{ij}| + \alpha_{ij} \right) L_j + |B_i| \right)$$

$$\left. \times (1+\mu(t)a_i) \right) \left] \right.$$

$$+ \xi_i (1+\mu(t)p)|\beta_i| L_i$$

$$\left. \times \left[1 + \mu(t)\left(|\beta_i| L_i + \alpha_i + \sum_{j=1}^{n} \beta_{ij} L_j \right) \right] \right\}$$

$$\times w_i^2(t)$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{n} e_p(t,0)$$

$$\times \left\{ \lambda_i (1+\mu(t)p)|a_{ij}| L_j \right.$$

$$\times \left[1 + \mu(t) \right.$$

$$\times \left(n|a_{ij}| L_j + a_i + \alpha_{ij} L_j + |B_i| \right) \right]$$

$$+ \lambda_i (1+\mu(t+\tau_j)p) e_p(t+\tau_j,t)\alpha_{ij} L_j$$

$$\times \left[1 + \mu\left(t + \tau_j\right)\right.$$

$$\left. \times \left(n\alpha_{ij}L_j + a_i + \left|a_{ij}\right|L_j + |B_i|\right)\right]$$

$$+ \xi_i\left(1 + \mu\left(t + \sigma_j\right)p\right)$$

$$\times e_p\left(t + \sigma_j, t\right)\beta_{ij}L_j$$

$$\times \left[1 + \mu\left(t + \sigma_j\right)\right.$$

$$\left.\left. \times \left(n\beta_{ij}L_j + \alpha_i + \left|\beta_i\right|L_i\right)\right]\right\}$$

$$\times w_j^2(t)$$

$$+ \sum_{i=1}^{n} e_p(t, 0)$$

$$\times \left\{\lambda_i\left(1 + \mu(t)p\right)|B_i|\right.$$

$$\times \left[1 + \mu(t)\right.$$

$$\left. \times \left(|B_i| + a_i + \sum_{j=1}^{n}\left(\left|a_{ij}\right| + \alpha_{ij}\right)L_j\right)\right]$$

$$+ \xi_i\left[p + \left(1 + \mu(t)p\right)\right.$$

$$\times \left(-2\alpha_i + \mu(t)\alpha_i^2\right.$$

$$+ \left(\left|\beta_i\right|L_i + \sum_{j=1}^{n}\beta_{ij}L_j\right)$$

$$\left.\left.\left. \times \left(1 + \mu(t)\alpha_i\right)\right)\right]\right\}\rho_i^2(t)$$

$$= \sum_{i=1}^{n} e_p(t, 0)$$

$$\times \left\{\lambda_i\left[p + \left(1 + \mu(t)p\right)\right.\right.$$

$$\times \left(-2a_i + \mu(t)a_i^2\right.$$

$$+ \left(\sum_{j=1}^{n}\left(\left|a_{ij}\right| + \alpha_{ij}\right)L_j + |B_i|\right)$$

$$\left.\left. \times \left(1 + \mu(t)a_i\right)\right)\right]$$

$$+ \xi_i\left(1 + \mu(t)p\right)\left|\beta_i\right|L_i$$

$$\times \left[1 + \mu(t)\left(\left|\beta_i\right|L_i + \alpha_i + \sum_{j=1}^{n}\beta_{ij}L_j\right)\right]$$

$$+ \sum_{j=1}^{n}\lambda_j\left(1 + \mu(t)p\right)\left|a_{ji}\right|L_i$$

$$\times \left[1 + \mu(t)\right.$$

$$\left. \times \left(n\left|a_{ji}\right|L_i + a_j + \alpha_{ji}L_i + \left|B_j\right|\right)\right]$$

$$+ \sum_{j=1}^{n}\lambda_j \times \left(1 + \mu\left(t + \tau_i\right)p\right)$$

$$\times e_p\left(t + \tau_i, t\right)\alpha_{ji}L_i$$

$$\times \left[1 + \mu\left(t + \tau_i\right)\right.$$

$$\left. \times \left(n\alpha_{ji}L_i + a_j + \left|a_{ji}\right|L_i + \left|B_j\right|\right)\right]$$

$$+ \sum_{j=1}^{n}\xi_j\left(1 + \mu\left(t + \sigma_i\right)p\right)e_p\left(t + \sigma_i, t\right)\beta_{ji}L_i$$

$$\times \left[1 + \mu\left(t + \sigma_i\right)\right.$$

$$\left.\left. \times \left(n\beta_{ji}L_i + \alpha_j + \left|\beta_j\right|L_j\right)\right]\right\}w_i^2(t)$$

$$+ \sum_{i=1}^{n} e_p(t, 0)$$

$$\times \left\{\lambda_i\left(1 + \mu(t)p\right)|B_i|\right.$$

$$\times \left[1 + \mu(t)\right.$$

$$\left. \times \left(|B_i| + a_i + \sum_{j=1}^{n}\left|a_{ij}\right|L_j + \sum_{j=1}^{n}\alpha_{ij}L_j\right)\right]$$

$$+ \xi_i\left[p + \left(1 + \mu(t)p\right)\right.$$

$$\times \left(-2\alpha_i + \mu(t)\alpha_i^2\right.$$

$$+ \left(\left|\beta_i\right|L_i + \sum_{j=1}^{n}\beta_{ij}L_j\right)$$

$$\left.\left.\left. \times \left(1 + \mu(t)\alpha_i\right)\right)\right]\right\}\rho_i^2(t)$$

$$= \sum_{i=1}^{n} e_p(t,0)$$

$$\times \left\{ \lambda_i p + (1 + \mu(t) p) \right.$$

$$\times \left[-2\lambda_i a_i \right.$$

$$+ \lambda_i \left(\sum_{j=1}^{n} \left(|a_{ij}| + \alpha_{ij} \right) L_j + |B_i| \right)$$

$$+ \xi_i |\beta_i| L_i + \sum_{j=1}^{n} \lambda_j |a_{ji}| L_i$$

$$+ \mu(t) \left[\lambda_i a_i^2 \right.$$

$$+ \lambda_i a_i$$

$$\times \left(\sum_{j=1}^{n} \left(|a_{ij}| + \alpha_{ij} \right) L_j + |B_i| \right)$$

$$+ \xi_i |\beta_i| L_i$$

$$\times \left(|\beta_i| L_i + \alpha_i + \sum_{j=1}^{n} \beta_{ij} L_j \right)$$

$$+ \sum_{j=1}^{n} \lambda_j |a_{ji}| L_i$$

$$\times \left(n |a_{ji}| L_i + a_j + \alpha_{ji} L_i \right.$$

$$\left. \left. \left. + |B_j| \right) \right] \right]$$

$$+ \sum_{j=1}^{n} \lambda_j (1 + \mu(t + \tau_i) p)$$

$$\times e_p(t + \tau_i, t) \alpha_{ji} L_i$$

$$\times \left[1 + \mu(t + \tau_i) \right.$$

$$\left. \times \left(n\alpha_{ji} L_i + a_j + |a_{ji}| L_i + |B_j| \right) \right]$$

$$+ \sum_{j=1}^{n} \xi_j (1 + \mu(t + \sigma_i) p)$$

$$\times e_p(t + \sigma_i, t) \beta_{ji} L_i$$

$$\times \left[1 + \mu(t + \sigma_i) \right.$$

$$\left. \times z \left(n\beta_{ji} L_i + \alpha_j + |\beta_j| L_j \right) \right] \right\} w_i^2(t)$$

$$+ \sum_{i=1}^{n} e_p(t,0)$$

$$\times \left\{ \xi_i p + (1 + \mu(t) p) \right.$$

$$\times \left[\lambda_i |B_i| - 2\xi_i \alpha_i + \xi_i |\beta_i| L_i \right.$$

$$+ \xi_i \sum_{j=1}^{n} \beta_{ij} L_j + \mu(t)$$

$$\times \left[\lambda_i |B_i| \left(|B_i| + a_i \right. \right.$$

$$+ \sum_{j=1}^{n} \left(|a_{ij}| + \alpha_{ij} \right) L_j \right)$$

$$+ \alpha_i \xi_i \left(\alpha_i + |\beta_i| L_i \right.$$

$$\left. \left. \left. \left. + \sum_{j=1}^{n} \beta_{ij} L_j \right) \right] \right] \right\} \rho_i^2(t).$$

(19)

By using ($\mathscr{A}_2$), we can conclude that $F^\Delta(t) \leq 0$ which implies that $F(t) \leq F(0)$ for $t \in \mathbb{T}_0^+$.

Consider

$$F(0) = F_1(0) + F_2(0) + F_3(0) + F_4(0)$$

$$\leq \sum_{i=1}^{n} \left\{ \lambda_i \right.$$

$$+ \sum_{j=1}^{n} \lambda_j \alpha_{ji} L_i$$

$$\times \left[1 + \mu \left(n\alpha_{ji} L_i + a_j + |a_{ji}| L_i + |B_j| \right) \right]$$

$$\times (1 + p\mu^*) \int_{-\tau_i}^{0} e_p(s + \tau_i, 0) \Delta s$$

$$+ \sum_{j=1}^{n} \xi_j \beta_{ji} L_i$$

$$\times \left[1 + \mu \left(n\beta_{ji} L_i + \alpha_j + |\beta_j| L_j \right) \right]$$

$$\left. \times (1 + p\mu^*) \int_{-\sigma_i}^{0} e_p(s + \sigma_i, 0) \Delta s \right\}$$

$$\times \sup_{-\theta \leq s \leq 0} w_i^2(s) + \sum_{i=1}^{n} \xi_i \sup_{-\theta \leq l \leq 0} \rho_i^2(l)$$

$$\leq \max_{1 \leq i \leq n} \Bigg\{ \lambda_i$$

$$+ \sum_{j=1}^{n} \lambda_j \alpha_{ji} L_i$$

$$\times \left[1 + \mu \left(n \alpha_{ji} L_i + a_j + \left| a_{ji} \right| L_i \left| B_j \right| \right) \right]$$

$$\times \left(1 + p\mu \right) \tau e_p(\tau, 0)$$

$$+ \sum_{j=1}^{n} \xi_j \beta_{ji} L_i \left[1 + \mu \left(n \beta_{ji} L_i + \alpha_j \left| \beta_j \right| L_j \right) \right]$$

$$\times \left(1 + p\mu \right) \sigma e_p(\sigma, 0) \Bigg\} \sum_{i=1}^{n} \sup_{-\theta \leq s \leq 0} w_i^2(s)$$

$$+ \max_{1 \leq i \leq n} \xi_i \sum_{i=1}^{n} \sup_{-\theta \leq l \leq 0} \rho_i^2(l) \leq M_1 \Phi_\theta, \tag{20}$$

where $\left| \mu(t) \right| \leq \mu^*$, $\tau = \max_{1 \leq j \leq n}(\tau_j)$, $\sigma = \max_{1 \leq j \leq n}(\sigma_j)$, $\theta = \max_{1 \leq j \leq n}(\tau_j, \sigma_j)$, and

$$M_1 = \max_{1 \leq i \leq n} \Bigg\{ \max_{1 \leq i \leq n} \Bigg\{ \lambda_i$$

$$+ \sum_{j=1}^{n} \lambda_j \alpha_{ji} L_i$$

$$\times \left[1 + \mu \left(n \alpha_{ji} L_i + a_j + \left| a_{ji} \right| L_i \left| B_j \right| \right) \right]$$

$$\times \left(1 + p\mu \right) \tau e_p(\tau, 0)$$

$$+ \sum_{j=1}^{n} \xi_j \beta_{ji} L_i$$

$$\times \left[1 + \mu \left(n \beta_{ji} L_i + \alpha_j \left| \beta_j \right| L_j \right) \right]$$

$$\times \left(1 + p\mu \right) \sigma e_p(\sigma, 0) \Bigg\}, \max_{1 \leq i \leq n} \xi_i \Bigg\}. \tag{21}$$

Observe that

$$F(t) \geq \min_{1 \leq i \leq n} \lambda_i \sum_{i=1}^{n} w_i^2(t) e_p(t, 0) + \min_{1 \leq i \leq n} \xi_i \sum_{i=1}^{n} \rho_i^2(t) e_p(t, 0)$$

$$\geq M_2 \left(\sum_{i=1}^{n} w_i^2(t) + \sum_{i=1}^{n} \rho_i^2(t) \right) e_p(t, 0), \tag{22}$$

where $M_2 = \min_{1 \leq j \leq n} \{ \min_{1 \leq i \leq n} \lambda_i, \min_{1 \leq i \leq n} \xi_i \}$. Due to Lemma 11, we get

$$\sum_{i=1}^{n} w_i^2(t) + \sum_{i=1}^{n} \rho_i^2(t) \leq M \cdot e_{\ominus p}(t, 0) \Phi_\theta \longrightarrow 0 \tag{23}$$

$$\text{as } t \longrightarrow +\infty,$$

where $M = M_1/M_2 > 0$ is a constant. This completes the proof. $\square$

Remark 15. It is well known that few works have been done to report learning dynamics of fuzzy networks in the literature. If the time scale $\mathbb{T} = \mathbb{R}$, then $\mu(t) = 0$ and (4) reduces to common fuzzy networks with learning behavior reported by [16]. Removing learning variables, (4) is a generation form of [2, 4, 7]. From Theorem 14, we can immediately obtain the following corollary which contains relative results in [16].

Corollary 16. *Suppose that* (4) *satisfies* $(\mathscr{A}_1)$*,* $\mathbb{T} = \mathbb{R}$*, and if there exist constants* $\lambda_i > 0$*,* $\xi_i > 0$ *such that*

$$\lambda_i \left[-2a_i + \sum_{j=1}^{n} \left(\left| a_{ij} \right| + \alpha_{ij} \right) L_j + \left| B_i \right| \right] + \xi_i \left| \beta_i \right| L_i$$

$$+ \sum_{j=1}^{n} \lambda_j \left| a_{ji} \right| L_i + \sum_{j=1}^{n} \lambda_j \alpha_{ji} L_i + \sum_{j=1}^{n} \xi_j \beta_{ji} L_i < 0, \tag{24}$$

$$\lambda_i \left| B_i \right| + \xi_i \left[-2\alpha_i + \left| \beta_i \right| L_i + \sum_{j=1}^{n} \beta_{ij} L_j \right] < 0,$$

where $i \in \mathbb{N}$*, then there exists a unique equilibrium* (u^*, v^*) *of* (4) *which is globally exponentially stable.*

If the time scale $\mathbb{T} = Z$*, then* $\mu(t) = 1$ *and* (4) *reduces to*

$$u_i(n+1) = (1 - a_i) u_i(n) + \sum_{j=1}^{n} a_{ij} f_j(u_j(n))$$

$$+ \bigvee_{j=1}^{n} b_{ij}^\ell f_j(u_j(n - \tau_j))$$

$$+ \bigwedge_{j=1}^{n} c_{ij}^\ell f_j(u_j(n - \tau_j)) + B_i v_i(n) + J_i,$$

$$v_i(n+1) = (1 - \alpha_i) v_i(n) + \beta_i f_i(u_i(t))$$

$$+ \bigwedge_{j=1}^{n} \gamma_{ij}^\ell f_j(u_j(n - \sigma_j))$$

$$+ \bigvee_{j=1}^{n} \delta_{ij}^\ell f_j(u_j(n - \sigma_j)), \tag{25}$$

which includes discrete-time analogues of competitive networks [24] *as its special case when there is no fuzzy terms. From Theorem 14, we have the following corollary.*

Corollary 17. *Suppose that* (25) *satisfies* $(\mathscr{A}_1)$ *and if there exist constants* $\lambda_i > 0$, $\xi_i > 0$ *such that*

$$\lambda_i \left[-2a_i + a_i^2 + \left(\sum_{j=1}^n \left(|a_{ij}| + \alpha_{ij} \right) L_j + |B_i| \right) (1 + a_i) \right]$$

$$+ \xi_i |\beta_i| L_i \left[1 + |\beta_i| L_i + \alpha_i + \sum_{j=1}^n \beta_{ij} L_j \right]$$

$$+ \sum_{j=1}^n \lambda_j |a_{ji}| L_i \left[1 + n |a_{ji}| L_i + a_j + \alpha_{ji} L_i + |B_j| \right]$$

$$+ \sum_{j=1}^n \lambda_j \alpha_{ji} L_i$$

$$\times \left[1 + n\alpha_{ji} L_i + a_j + |a_{ji}| L_i + |B_j| \right] \qquad (26)$$

$$+ \sum_{j=1}^n \xi_j \beta_{ji} L_i \left[1 + n\beta_{ji} L_i + \alpha_j + |\beta_j| L_j \right] < 0,$$

$$\lambda_i |B_i| \left[1 + |B_i| + a_i + \sum_{j=1}^n |a_{ij}| L_j + \sum_{j=1}^n \alpha_{ij} L_j \right]$$

$$+ \xi_i \left[-2\alpha_i + \alpha_i^2 + \left(|\beta_i| L_i + \sum_{j=1}^n \beta_{ij} L_j \right) (1 + \alpha_i) \right]$$

$$< 0,$$

where $i \in \mathbb{N}$, *then there exists a unique equilibrium* (u^*, v^*) *of* (25) *which is globally exponentially stable.*

Remark 18. The result of Theorem 14 unifies the previous literature on fuzzy networks of discrete-time and continuous-time and reveals the discrepancies of results of continuous-time ($\mu(t) = 0$) and discrete-time ($\mu(t) = 1$) fuzzy networks [1–4, 16].

Finally, we should briefly investigate learning convergence of fuzzy networks. Consider learning dynamics governed by (5):

$$m_{ij}^{\Delta}(t) = - \alpha_i m_{ij}(t)$$

$$+ \left[\beta_i f_i (u_i(t)) + \bigwedge_{k=1}^n \gamma_{ik}^{\ell} f_k (u_k(t - \sigma_k)) \right. \qquad (27)$$

$$\left. + \bigvee_{k=1}^n \delta_{ik}^{\ell} f_k (u_k(t - \sigma_k)) \right] p_j,$$

where $i, j \in \mathbb{N}$ and $t \in \mathbb{T}_0^+$. It follows from Theorem 14 that (4) has a unique equilibrium $(u^*, v^*) = (u_1^*, \ldots, u_n^*, v_1^*, \ldots, v_n^*)$. Hence, (5) has a unique equilibrium satisfying

$$m_{ij}^* = \frac{p_j}{\alpha_i} \left[\beta_i f_i (u_i^*) + \bigwedge_{k=1}^n \gamma_{ik}^{\ell} f_k (u_k^*) + \bigvee_{k=1}^n \delta_{ik}^{\ell} f_k (u_k^*) \right],$$

$$(28)$$

$$i, j \in \mathbb{N}.$$

By (23), there exists a $\widetilde{M} > 0$ such that

$$|u_i(t) - u_i^*| \le \widetilde{M} \sqrt{e_{\ominus p}(t, 0)}, \quad i \in \mathbb{N}. \qquad (29)$$

It follows from (5), (28), and (29) that we get

$$\left(m_{ij}(t) - m_{ij}^* \right)^{\Delta} \le - \alpha_i \left(m_{ij}(t) - m_{ij}^* \right)$$

$$+ |p_j| |\beta_i| L_i |u_i(t) - u_i^*|$$

$$+ |p_j| \sum_{k=1}^n \beta_{ik} L_k |u_k(t - \sigma_k) - u_k^*|$$

$$\le - \alpha_i \left(m_{ij}(t) - m_{ij}^* \right) \qquad (30)$$

$$+ \widetilde{M} \left[|p_j| |\beta_i| L_i \sqrt{e_{\ominus p}(t, t - \sigma_k)} \right.$$

$$\left. + \sum_{k=1}^n |p_j| \beta_{ik} L_k \right]$$

$$\times \sqrt{e_{\ominus p}(t - \sigma_k, 0)}.$$

Let $\mu^M := \sup_{s \in \mathbb{T}} \mu(s)$. Since $F(\mu) = -\log(1 + p\mu)/\mu$ is an increasing function defined on $\mathbb{R}^+$, we know $\xi_{\mu(\tau)}(\ominus p) \le \max\{\ominus p, -\log(1 + p\mu^M)/\mu^M\} :\triangleq \widetilde{N}$ and

$$\sqrt{e_{\ominus p}(t, t - \sigma_k)} \le \sqrt{\exp\left(\widetilde{N} \sigma_k \right)},$$

$$\sqrt{e_{\ominus p}(t, 0)} \le \sqrt{\exp\left(\widetilde{N} t \right)}. \qquad (31)$$

Hence, for any given $\epsilon > 0$, there exists a $t_0 \in \mathbb{T}$ such that $\sqrt{e_{\ominus p}(t, 0)} \le \epsilon$ as $t \ge t_0$. From (30) and Lemma 13, one gets

$$m_{ij}(t) - m_{ij}^*$$

$$\le \left(m_{ij}(t_0) - m_{ij}^* \right) e_{-\alpha_i}(t, t_0)$$

$$+ \widetilde{M} \epsilon \left[|p_j| |\beta_i| L_i \sqrt{\exp\left(\widetilde{N} \sigma_k \right)} + \sum_{k=1}^n |p_j| \beta_{ik} L_k \right] \qquad (32)$$

$$\times \int_{t_0}^t e_{-\alpha_i}(t, \sigma(s)) \Delta s.$$

It follows from Lemma 11 that we have

$$
\begin{aligned}
& m_{ij}(t) - m_{ij}^* \\
& \quad \leq \left(m_{ij}(t_0) - m_{ij}^* \right) e_{-\alpha_i}(t, t_0) \\
& \qquad - \frac{\widetilde{M}\epsilon}{\alpha_i} \left[|p_j| |\beta_i| L_i \sqrt{\exp\left(\widetilde{N}\sigma_k\right)} + \sum_{k=1}^n |p_j| \beta_{ik} L_k \right] \\
& \qquad \times \left(e_{-\alpha_i}(t, t_0) - 1 \right) \\
& \quad \leq \left(m_{ij}(t_0) - m_{ij}^* \right) e^{-\alpha_i(t-t_0)} \\
& \qquad - \frac{\widetilde{M}\epsilon}{\alpha_i} \left[|p_j| |\beta_i| L_i \sqrt{\exp\left(\widetilde{N}\sigma_k\right)} + \sum_{k=1}^n |p_j| \beta_{ik} L_k \right] \\
& \qquad \times \left(e^{-\alpha_i(t-t_0)} - 1 \right)
\end{aligned}
\tag{33}
$$

which leads to $m_{ij}(t) - m_{ij}^* \to 0$ as $t \to +\infty$. That is, time-varying and learnable synaptic weights converge exponentially to stationary weights m_{ij}^* encoding the signal vector $p = (p_1, p_2, \ldots, p_n)$ in the sense of (28).

4. An Example

In this section, an example is shown to verify the effectiveness of the result obtained in the previous section. Consider the following fuzzy networks with delays on time scale $\mathbb{T}$:

$$
\begin{aligned}
u_i^\Delta(t) = & -a_i u_i(t) + \sum_{j=1}^2 a_{ij} f_j\left(u_j(t)\right) + \bigvee_{j=1}^2 b_{ij}^\ell f_j\left(u_j\left(t - \tau_j\right)\right) \\
& + \bigwedge_{j=1}^2 c_{ij}^\ell f_j\left(u_j\left(t - \tau_j\right)\right) + B_i v_i(t) + J_i,
\end{aligned}
$$

$$
\begin{aligned}
v_i^\Delta(t) = & -\alpha_i v_i(t) + \beta_i f_i\left(u_i(t)\right) + \bigwedge_{j=1}^2 \gamma_{ij}^\ell f_j\left(u_j\left(t - \sigma_j\right)\right) \\
& + \bigvee_{j=1}^2 \delta_{ij}^\ell f_j\left(u_j\left(t - \sigma_j\right)\right)
\end{aligned}
\tag{34}
$$

for $i = 1, 2$ and $t \in \mathbb{T}_0^+$, where $(a_1, a_2)^T = (0.3, 0.3)^T$, $(\alpha_1, \alpha_2)^T = (0.2, 0.2)^T$, $(B_1, B_2)^T = (0.1, 0.1)^T$, $(\beta_1, \beta_2)^T = (0.01, 0.02)^T$, $f_i(x) = (1/2)(|x+1| + |x-1|)$, and

$$
\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 0.01 & 0.01 \\ 0.01 & 0.01 \end{pmatrix},
$$

$$
\begin{pmatrix} b_{11}^\ell & b_{12}^\ell \\ b_{21}^\ell & b_{22}^\ell \end{pmatrix} = \begin{pmatrix} 0.01 & 0.02 \\ 0.02 & 0.01 \end{pmatrix},
$$

$$
\begin{pmatrix} c_{11}^\ell & c_{12}^\ell \\ c_{21}^\ell & c_{22}^\ell \end{pmatrix} = \begin{pmatrix} 0.02 & 0.01 \\ 0.01 & 0.02 \end{pmatrix},
$$

$$
\begin{pmatrix} \gamma_{11}^\ell & \gamma_{12}^\ell \\ \gamma_{21}^\ell & \gamma_{22}^\ell \end{pmatrix} = \begin{pmatrix} 0.02 & 0.02 \\ 0.02 & 0.02 \end{pmatrix},
$$

$$
\begin{pmatrix} \delta_{11}^\ell & \delta_{12}^\ell \\ \delta_{21}^\ell & \delta_{22}^\ell \end{pmatrix} = \begin{pmatrix} 0.01 & 0.01 \\ 0.02 & 0.02 \end{pmatrix},
$$

$$
\tau_i = \sigma_i = \frac{1}{2}, \qquad L_i = J_i = c = 1.
\tag{35}
$$

Choosing $\lambda_1 \xi_i = 1 \ (i = 1, 2)$, we can easily verify that the assumptions of Corollaries 16 and 17 are all satisfied, respectively. When $\mathbb{T} = \mathbb{R}$, that is, $\mu(t) = 0$,

$$
\begin{aligned}
& \lambda_1 \left[-2a_1 + \sum_{j=1}^2 |a_{1j}| L_j + \sum_{j=1}^2 \left(|b_{1j}^\ell| + |c_{1j}^\ell| \right) L_j + |B_1| \right] \\
& \quad + \xi_1 |\beta_1| L_1 + \sum_{j=1}^2 \lambda_j |a_{j1}| L_1 + \sum_{j=1}^2 \lambda_j \left(|b_{j1}^\ell| + |c_{j1}^\ell| \right) L_1 \\
& \quad + \sum_{j=1}^2 \xi_j \left(|\gamma_{j1}^\ell| + |\delta_{j1}^\ell| \right) L_1 = -0.26 < 0,
\end{aligned}
$$

$$
\begin{aligned}
& \lambda_2 \left[-2a_2 + \sum_{j=1}^2 |a_{2j}| L_j + \sum_{j=1}^2 \left(|b_{2j}^\ell| + |c_{2j}^\ell| \right) L_j + |B_2| \right] \\
& \quad + \xi_2 |\beta_2| L_2 + \sum_{j=1}^2 \lambda_j |a_{j2}| L_2 + \sum_{j=1}^2 \lambda_j \left(|b_{j2}^\ell| + |c_{j2}^\ell| \right) L_2 \\
& \quad + \sum_{j=1}^2 \xi_j \left(|\gamma_{j2}^\ell| + |\delta_{j2}^\ell| \right) L_2 = -0.25 < 0,
\end{aligned}
$$

$$
\begin{aligned}
& \lambda_1 |B_1| + \xi_1 \left[-2\alpha_1 + |\beta_1| L_1 + \sum_{j=1}^2 \left(|\gamma_{1j}^\ell| + |\delta_{1j}^\ell| \right) L_j \right] \\
& \quad = -0.23 < 0,
\end{aligned}
$$

$$
\begin{aligned}
& \lambda_2 |B_2| + \xi_2 \left[-2\alpha_2 + |\beta_2| L_2 + \sum_{j=1}^2 \left(|\gamma_{2j}^\ell| + |\delta_{2j}^\ell| \right) L_j \right] \\
& \quad = -0.2 < 0.
\end{aligned}
\tag{36}
$$

When $\mathbb{T} = \mathbb{Z}$, that is, $\mu(t) = 1$,

$$
\begin{aligned}
\lambda_1 \bigg[& -2a_1 + a_1^2 \\
& + \left(\sum_{j=1}^2 |a_{1j}| L_j + \sum_{j=1}^2 \left(|b_{1j}^\ell| + |c_{1j}^\ell| \right) L_j + |B_1| \right) \\
& \times (1 + a_1) \bigg] + \xi_1 |\beta_1| L_1
\end{aligned}
$$

$$
\times \left[1 + |\beta_1| L_1 + \alpha_1 + \sum_{j=1}^{2} \left(|\gamma_{1j}^{\ell}| + |\delta_{1j}^{\ell}| \right) L_j \right]
$$

$$
+ \sum_{j=1}^{2} \lambda_j |a_{j1}| L_1
$$

$$
\times \left[1 + 2 |a_{j1}| L_1 + a_j + \left(|b_{j1}^{\ell}| + |c_{j1}^{\ell}| \right) L_1 + |B_j| \right]
$$

$$
+ \sum_{j=1}^{2} \lambda_j \left(|b_{j1}^{\ell}| + |c_{j1}^{\ell}| \right) L_1
$$

$$
\times \left[1 + 2 \left(|b_{j1}^{\ell}| + |c_{j1}^{\ell}| \right) L_1 + a_j + |a_{j1}| L_1 + |B_j| \right]
$$

$$
+ \sum_{j=1}^{2} \xi_j \left(|\gamma_{j1}^{\ell}| + |\delta_{j1}^{\ell}| \right)
$$

$$
\times L_1 \left[1 + 2 \left(|\gamma_{j1}^{\ell}| + |\delta_{j1}^{\ell}| \right) L_1 + \alpha_j + |\beta_j| L_j \right]
$$

$$
= -0.056 < 0,
$$

$$
\lambda_2 \left[-2a_2 + a_2^2 \right.
$$

$$
+ \left(\sum_{j=1}^{2} |a_{2j}| L_j + \sum_{j=1}^{2} \left(|b_{2j}^{\ell}| + |c_{2j}^{\ell}| \right) L_j + |B_2| \right)
$$

$$
\left. \times (1 + a_2) \right]
$$

$$
+ \xi_2 |\beta_2| L_2 \left[1 + |\beta_2| L_2 + \alpha_2 + \sum_{j=1}^{2} \left(|\gamma_{2j}^{\ell}| + |\delta_{2j}^{\ell}| \right) L_j \right]
$$

$$
+ \sum_{j=1}^{2} \lambda_j |a_{j2}| L_2
$$

$$
\times \left[1 + 2 |a_{j2}| L_2 + a_j + \left(|b_{j2}^{\ell}| + |c_{j2}^{\ell}| \right) L_2 + |B_j| \right]
$$

$$
+ \sum_{j=1}^{2} \lambda_j \left(|b_{j2}^{\ell}| + |c_{j2}^{\ell}| \right) L_2
$$

$$
\times \left[1 + 2 \left(|b_{j2}^{\ell}| + |c_{j2}^{\ell}| \right) L_2 + a_j + |a_{j2}| L_2 + |B_j| \right]
$$

$$
+ \sum_{j=1}^{2} \xi_j \left(|\gamma_{j2}^{\ell}| + |\delta_{j2}^{\ell}| \right)
$$

$$
\times L_2 \left[1 + 2 \left(|\gamma_{j2}^{\ell}| + |\delta_{j2}^{\ell}| \right) L_2 + \alpha_j + |\beta_j| L_j \right]
$$

$$
= -0.0427 < 0,
$$

$$
\lambda_1 |B_1| \left[1 + |B_1| + a_1 + \sum_{j=1}^{2} |a_{1j}| L_j + \sum_{j=1}^{2} \left(|b_{1j}^{\ell}| + |c_{1j}^{\ell}| \right) L_j \right]
$$

$$
+ \xi_1 \left[-2\alpha_1 + \alpha_1^2 \right.
$$

$$
+ \left(|\beta_1| L_1 + \sum_{j=1}^{2} \left(|\gamma_{1j}^{\ell}| + |\delta_{1j}^{\ell}| \right) L_j \right) (1 + \alpha_1) \right]
$$

$$
= -0.128 < 0,
$$

$$
\lambda_2 |B_2| \left[1 + |B_2| + a_2 + \sum_{j=1}^{2} |a_{2j}| L_j + \sum_{j=1}^{2} \left(|b_{2j}^{\ell}| + |c_{2j}^{\ell}| \right) L_j \right]
$$

$$
+ \xi_2 \left[-2\alpha_2 + \alpha_2^2 \right.
$$

$$
+ \left(|\beta_2| L_2 + \sum_{j=1}^{2} \left(|\gamma_{2j}^{\ell}| + |\delta_{2j}^{\ell}| \right) L_j \right) (1 + \alpha_2) \right]
$$

$$
= -0.092 < 0.
$$

$$
\tag{37}
$$

It follows from Corollaries 16 and 17 that (34) has a unique equilibrium point which is globally exponentially stable.

5. Conclusion

By using the time scale calculus theory and the Lyapunov functional method, we derive some sufficient conditions to ensure the global exponential stability of learning-based fuzzy networks on time scales. The conditions possess highly important significance and can be easily checked in practice by simple algebraic method; the exponential convergence of the learning dynamics is also considered.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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