Research Article

Improved Delay-Dependent Robust Stability Criteria for a Class of Uncertain Neutral Type Lur'e Systems with Discrete and Distributed Delays

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This paper is concerned with the problem of delay-dependent robust stability analysis for a class of uncertain neutral type Lur'e systems with mixed time-varying delays. The system has not only time-varying uncertainties and sector-bounded nonlinearity, but also discrete and distributed delays, which has never been discussed in the previous literature. Firstly, by employing one effective mathematical technique, some less conservative delay-dependent stability results are established without employing the bounding technique and the mode transformation approach. Secondly, by constructing an appropriate new type of Lyapunov-Krasovskii functional with triple terms, improved delay-dependent stability criteria in terms of linear matrix inequalities (LMIs) derived in this paper are much brief and valid. Furthermore, both nonlinearities located in finite sector and infinite one have been also fully taken into account. Finally, three numerical examples are presented to illustrate lesser conservatism and the advantage of the proposed main results.

1. Introduction

Time delay arises naturally in connection with system process and information flow for different part of dynamic systems. Dynamical systems with time delays, also called dynamic systems with hereditary systems, after effect or dead time, equations with deviating argument, have been investigated extensively for the past few decades due to their wide applications in many research fields, covering nuclear reactor, manual control, engineering systems, neural networks, and other scientific areas [1–42]. The main reason is that time-delay phenomenon frequently happens in various engineering systems and is also a key source of instability and oscillation. Hence, the delayed systems have been widely considered by a great quantity of research results in the recent years. To mention a few, stability analysis is carried out in [1, 3, 6, 7, 11, 12, 15, 17, 25, 32–34, 36]; H_∞ controllers and H_∞ estimation are investigated in [31, 38, 41]; Lur'e

dynamical systems are addressed in [4–6, 26–29]; neutral systems with discrete and distributed delays are researched in [32–34]; passivity analysis and reliable fuzzy control are considered, respectively, in [39, 42]. For the new results on time-delay systems, see the works [3, 21, 29, 34, 38–42] and references cited therein. Therefore, it is of both theoretical and practical importance to study the problem of stability analysis on delayed systems. So far in the literatures, delayed systems can be classified into two types: retarded type and neutral type [1–12]. The retarded type system only depends on the state delay, while the neutral type system not only defines the derivative term of the current state but also explains the derivative term of the past state. And what is more, the delayed stability criteria are also classified into two categories according to their dependence on the size of the delays, namely, delay-independent stability criteria and delay-dependent stability criteria [13–18]. It is well known that delay-dependent stability criteria are less conservative

than delay-independent ones when the sizes of time-delays are small.

As everyone knows, many nonlinear physical systems can be expressed as a feedback connection of a linear dynamical system and nonlinearity. One of the important classes of non-linear systems is the Lur'e dynamical system whose nonlinear element satisfies certain sector constraints. Since the notion of absolute stability was introduced by Lur'e [19], stability analysis for the Lur'e systems has been extensively studied for several decades [20–29]. Recently, various approaches have been proposed by many researchers to obtain robust stability criteria for time-delay Lur'e control systems. Therefore, many methods have been proposed in these results to reduce the conservatism of the stability criteria, such as model transformation method, the bounding technique, free-weighting matrix method, the method of constructing Lyapunov-Krasovskii functionals, delay decomposition technique, and weighting-matrix decomposition method. By using suitable Lyapunov-Krasovskii functionals and free-weighting matrices, a new delay-dependent robust stability criterion of the class of uncertain mixed neutral and Lur'e systems has been proposed in [21]. Some conditions have been derived by employing Lyapunov-Krasovskii functionals and generalized convex combination in [26]. Sufficient conditions for the robustly asymptotic stability have been achieved by eliminating nonlinearity and removing free-weighting matrices in [29]. In addition, delayed systems often have a spatial extent because of the presence of an amount of parallel pathways of varying axon sizes and lengths. Then, there may exist either a distribution of conduction velocities along these pathways or a distribution of propagation delays over a period of time in some cases, which may cause another type of time delays, namely, distributed time delays in delayed systems. Hence, the stability problem for neutral systems with nonlinearity perturbation and time delay was thus of interest to a great number of researchers. The results in [30] are neutral-delay-independent by a method based on the equivalent equation of zero and the Leibniz-Newton formula in the derivative of the Lyapunov-Krasovskii functionals. In [32], Li and Zhu have obtained the discrete-, neutral-, and distributed-delay-dependent stability conditions for uncertain neutral system with discrete and distributed delays by constructing an augmented Lyapunov-Krasovskii functional and using the free-weighting matrices combined with the bounding technique. In [34], by constructing a proper Lyapunov-Krasovskii functional, Chen et al. have derived some new delay-dependent stability criteria for uncertain neutral system with mixed constant delays by making full use of the free-weighting matrices method and without bounding technique bringing much more conservatism used to deal with these triple-integral terms in [33]; however, the research of uncertain neutral system with mixed constant delays has a lot of limitations because of the delay-dependent robust stability criteria for uncertain neutral systems often connected with time-varying delay. Since time delay and parameter uncertainty exist in Lur'e systems in practice, the robust stability analysis for uncertain Lur'e systems with time delays still requires further consideration. And theoretically, except the traditional free-weighting matrices approach, how to get rid of the rigorous

constraint that the time derivatives of time-varying delays must be less than one still needs much more study.

Motivated by the above discussion, in the paper, it is the first attempt to investigate one effective mathematical technique [29] to extend for a class of uncertain neutral type Lur'e systems with mixed time-varying delays which has not only time-varying uncertainties and sector-bounded nonlinearity, but also discrete and distributed delays. The effective mathematical technique is that dividing $[h_L, h_U]$ into $[h_L, h_L + h_{LU}(1)/s) \cup \cdots \cup [h_L + h_{LU}(i - 1)/s, h_L + h_{LU}(i)/s) \cup \cdots \cup [h_L + h_{LU}(s - 1)/s, h_U]$ is defined as a delay segmentation technique and supposing $h_L + h_{LU}(i - 1)/s \leq h(t) \leq h_L + h_{LU}i/s$ $(i = 1, 2, \ldots, s,$ $h_{LU} = h_U - h_L)$. Based on the mathematical technique, the derivative of $\int_{t-h_U}^{t} (s - (t - h_U))\dot{x}^T(s)Q_2\dot{x}(s)ds$ was estimated by $h_U\dot{x}^T(t)Q_2\dot{x}(t) - (1/h_L)\int_{t-h_L}^{t} \dot{x}^T(s)dsQ_1\int_{t-h_L}^{t} \dot{x}(s)ds - (s/h_{LU}(s - i + 1))\int_{t-h_U}^{t-h(t)} \dot{x}^T(s)dsQ_2\int_{t-h_U}^{t-h(t)} \dot{x}(s)ds - (s/h_{LU}i)\int_{t-h(t)}^{t-h_L} \dot{x}^T(s)dsQ_2\int_{t-h(t)}^{t-h_L} \dot{x}(s)ds$, which plays an important role in the improvement of less conservative results. Secondly, by constructing a new type of Lyapunov-Krasovskii functional with triple terms, improved delay-dependent stability criteria in terms of linear matrix inequalities (LMIs) derived in this paper are brief and effective, which can be easily solved by using MATLAB LIM control toolbox [37]. Furthermore, both nonlinearities located in a finite sector and infinite one have been also fully taken into account. Finally, three numerical examples are given to illustrate the less conservatism of the proposed method.

Notation. Notations used in this paper are fairly standard: R^n denotes the n-dimensional Euclidean space; $R^{n \times m}$ is the set of all $n \times m$ dimensional matrices; I denotes the identity matrix of appropriate dimensions; T stands for matrix transposition; the notation $X > 0$ (resp., $X \geq 0$), for $X \in R^{n \times n}$, means that the matrix is real symmetric positive definite (resp., positive semidefinite); diag$\{r_1, r_2, \ldots, r_n\}$ denotes block diagonal matrix with diagonal elements r_i, $i = 1, 2, \ldots, m$; the symbol $*$ represents the elements below the main diagonal of a symmetric matrix.

2. Preliminaries

Consider the following class of uncertain neutral type Lur'e systems with mixed time-varying delays:

$$
\begin{aligned}
\dot{x}(t) =\ & (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t - h(t)) \\
& + (C + \Delta C(t))\dot{x}(t - \tau(t)) \\
& + (D + \Delta D(t))\int_{t-r}^{t} x(s)ds + (B + \Delta B(t))f(\sigma(t)),
\end{aligned}
$$

$$
\sigma(t) = H^T x(t) = [h_1, h_2, \ldots, h_m]^T x(t), \quad t \geq 0,
$$

$$
x(t) = \phi(t), \quad t \in [-\max\{h_U, \tau_U, r\}, 0],
$$

$$\tag{1}$$

where $x(t) \in R^n$ is the state vector, $\sigma(t) \in R^m$ is the output vector, $A \in R^{n \times n}$, $A_d \in R^{n \times n}$, $D \in R^{n \times n}$, $C \in R^{n \times n}$, $B \in R^{n \times m}$,

and $H \in R^{n\times m}$ are known matrices, and $f(\sigma(t)) \in R^m$ is the nonlinear function in the feedback path, which is denoted as f for simplicity in the sequel. Its form is formulated as

$$
\begin{aligned}
f(\sigma(t)) &= \left[f_1(\sigma_1(t)) \ \ f_2(\sigma_2(t)) \ \cdots \ f_m(\sigma_m(t)) \right]^T \\
\sigma(t) &= \left[\sigma_1(t) \ \ \sigma_2(t) \ \cdots \ \sigma_m(t) \right]^T \\
&= H^T x(t) = \left[h_1 \ \ h_2 \ \cdots \ h_m \right]^T x(t),
\end{aligned}
\tag{2}
$$

wherein each term $f_i(\sigma_i(t))$, $i = 1, 2, \ldots, m$, satisfies one of the following sector conditions:

$$
\begin{aligned}
f_i(\sigma_i(t)) &\in K_{[0,k_i]} \\
&= \left\{ f_i(\sigma_i(t)) \mid f_i(0) = 0, \ 0 < \frac{f_i(\sigma_i(t))}{\sigma_i(t)} \le k_i, \ \sigma_i(t) \ne 0 \right\}
\end{aligned}
\tag{3}
$$

or

$$
\begin{aligned}
f_i(\sigma_i(t)) &\in K_{[0,\infty]} \\
&= \left\{ f_i(\sigma_i(t)) \mid f_i(0) = 0, \ \frac{f_i(\sigma_i(t))}{\sigma_i(t)} > 0, \ \sigma_i(t) \ne 0 \right\},
\end{aligned}
\tag{4}
$$

where $\Delta A(t)$, $\Delta A_d(t)$, $\Delta C(t)$, $\Delta D(t)$, and $\Delta B(t)$ are time-varying uncertain matrices of appropriate dimensions, which are assumed to be of the following form:

$$
\begin{aligned}
&\left[\Delta A(t) \ \ \Delta A_d(t) \ \ \Delta C(t) \ \ \Delta D(t) \ \ \Delta B(t) \right] \\
&\quad = NF(t) \left[E_1 \ \ E_2 \ \ E_3 \ \ E_4 \ \ E_5 \right],
\end{aligned}
\tag{5}
$$

where N, E_1, E_2, E_3, E_4, and E_5 are known real constant matrices of appropriate dimensions and $F(t)$ is a time-varying uncertain matrix satisfying

$$
F(t)^T F(t) \le I, \quad \forall t \ge 0.
\tag{6}
$$

By using (5), the uncertain system (1) can be rewritten as follows:

$$
\begin{aligned}
\dot{x}(t) &= (A + \Delta A(t)) x(t) + (A_d + \Delta A_d(t)) x(t - h(t)) \\
&\quad + (C + \Delta C(t)) \dot{x}(t - \tau(t)) \\
&\quad + (D + \Delta D) \int_{t-r}^t x(s)\,ds + (B + \Delta B(t)) f(\sigma(t)) \\
&= Ax(t) + A_d x(t - h(t)) + C\dot{x}(t - \tau) \\
&\quad + D \int_{t-r}^t x(s)\,ds + Bf(\sigma(t)) + NP(t),
\end{aligned}
\tag{7}
$$

where

$$
\xi^T(t) = \left[x^T(t) \ \ x^T(t - h_L) \ \ x^T(t - h(t)) \ \ x^T(t - h_U) \ \ \dot{x}^T(t - \tau(t)) \ \ \int_{t-r}^t x^T(s)\,ds \ \ f^T(\sigma(t)) \ \ P^T(t) \right],
$$

$$
\xi = \left[E_1 \ \ 0 \ \ E_2 \ \ 0 \ \ E_3 \ \ E_4 \ \ E_5 \ \ 0 \right], \qquad P(t) = F(t)\xi\xi(t).
\tag{8}
$$

The delays $h(t)$ and $\tau(t)$ are time-delaying continuous functions that satisfy

$$
\begin{aligned}
0 \le h_L \le h(t) \le h_U, \quad \dot{h}(t) \le h_D, \\
0 \le \tau(t) \le \tau_U, \quad \dot{\tau}(t) \le \tau_D < 1,
\end{aligned}
\tag{9}
$$

where h_L, h_U, h_D, τ_U, and τ_D are known scalars.

The following fact and lemmas are introduced, which will be used in the proof of the main results.

Fact 1 (Schur complement). For a given symmetric matrix $S = S^T = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}$, where $S_{11} \in R^{n\times n}$, the following conditions are equivalent:

(1) $S < 0$;

(2) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(3) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 1 (see [35]). *For any constant matrix $0 < R = R^T \in R^{n\times n}$, a scalar $r > 0$, a vector function $x : [0, r] \to R^n$ such that the integrations concerned are well defined, and then*

$$
-r \int_0^r x^T(s) Rx(s)\,ds \le -\left(\int_0^r x(s)ds \right)^T R \left(\int_0^r x(s)\,ds \right).
\tag{10}
$$

Lemma 2 (see [36]). *For any constant matrix $R \in R^{n\times n}$, $R = R^T > 0$, a scalar function $h := h(t) > 0$, and a vector-valued function $\dot{x} : [-h, 0] \to R^n$ such that the following integrations are well defined:*

$$
-h \int_{t-h}^t \dot{x}(s) R\dot{x}(s)\,ds \le \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix},
$$

$$
-\frac{h^2}{2} \int_{-h}^0 \int_{t+\theta}^t \dot{x}^T(s) R\dot{x}(s)\,ds\,d\theta
$$

$$
\le \begin{bmatrix} hx(t) \\ \int_{t-h}^t x(s)ds \end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} hx(t) \\ \int_{t-h}^t x(s)\,ds \end{bmatrix}.
\tag{11}
$$

3. Main Results

3.1. Stability Analysis in the Finite Sector $K_{[0,k_i]}$. In this section, we will give sufficient conditions under which the system (1) is robustly asymptotically stable.

Theorem 3. *For a given positive integer s and some scalars h_L, h_U, h_D, τ_U, τ_D, and ϵ with $h_U > h_L > 0$, $\tau_D < 1$, and $\epsilon > 0$, the system (1) with a nonlinear function $f(\sigma(t))$ satisfying the finite sector condition (3) and time-varying delays $h(t)$ and $\tau(t)$ satisfying (9) is robustly asymptotically stable if there exist $P_i^T = P_i > 0$ ($i = 1, 2, \ldots, 5$), $Q_i^T = Q_i > 0$, $M_i^T = M_i > 0$ ($i = 1, 2$), diagonal matrices $R = \text{diag}\{r_1, r_2, \ldots, r_m\} \geq 0$, and $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\} \geq 0$ with dimensions such that the following symmetric linear matrix inequality holds:*

$$\begin{bmatrix} \Phi + \Pi_i & \Gamma^T P_5 & h_L \Gamma^T Q_1 & \Gamma^T Q_2 & \frac{r^2}{2}\Gamma^T M_2 \\ * & -P_5 & 0 & 0 & 0 \\ * & * & -Q_1 & 0 & 0 \\ * & * & * & -\frac{1}{h_U}Q_2 & 0 \\ * & * & * & * & -M_2 \end{bmatrix} < 0, \tag{12}$$

$$(i = 1, 2, \ldots, s),$$

where

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} & \Phi_{17} & \Phi_{18} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} & \Phi_{27} & \Phi_{28} \\ * & * & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36} & \Phi_{37} & \Phi_{38} \\ * & * & * & \Phi_{44} & \Phi_{45} & \Phi_{46} & \Phi_{47} & \Phi_{48} \\ * & * & * & * & \Phi_{55} & \Phi_{56} & \Phi_{57} & \Phi_{58} \\ * & * & * & * & * & \Phi_{66} & \Phi_{67} & \Phi_{68} \\ * & * & * & * & * & * & \Phi_{77} & \Phi_{78} \\ * & * & * & * & * & * & * & \Phi_{88} \end{bmatrix},$$

$$\Pi_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & \phi_{i1} & \phi_{i2} & 0 & 0 & 0 & 0 & 0 \\ * & * & \phi_{i3} & \phi_{i4} & 0 & 0 & 0 & 0 \\ * & * & * & \phi_{i5} & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & 0 \end{bmatrix},$$

$$\Phi_{11} = P_1 A + A^T P_1 + P_2 + P_3 + P_4 - Q_1 - \frac{1}{h_L}Q_2$$
$$+ rM_1 - r^2 M_2 + \epsilon E_1^T E_1,$$

$$\Phi_{12} = Q_1 + \frac{1}{h_L}Q_2, \qquad \Phi_{13} = P_1 A_d + \epsilon E_1^T E_2,$$

$$\Phi_{15} = P_1 C + \epsilon E_1^T E_3, \qquad \Phi_{16} = P_1 D + rM_2 + \epsilon E_1^T E_4,$$

$$\Phi_{17} = P_1 B + A^T H\Lambda + HKR + \epsilon E_1^T E_5,$$

$$\Phi_{18} = P_1 N, \qquad \Phi_{22} = -P_2 - Q_1 - \frac{1}{h_L}Q_2,$$

$$\Phi_{33} = -(1 - h_D) P_3 + \epsilon E_2^T E_2, \qquad \Phi_{35} = \epsilon E_2^T E_3,$$

$$\Phi_{36} = \epsilon E_2^T E_4, \qquad \Phi_{37} = A_d^T H\Lambda + \epsilon E_2^T E_5,$$

$$\Phi_{44} = -P_4, \qquad \Phi_{55} = -(1 - \tau_D) P_5 + \epsilon E_3^T E_3,$$

$$\Phi_{56} = \epsilon E_3^T E_4, \qquad \Phi_{57} = C^T H\Lambda + \epsilon E_3^T E_3,$$

$$\Phi_{66} = -\frac{1}{r}M_1 - M_2 + \epsilon E_4^T E_4, \qquad \Phi_{67} = D^T H\Lambda + \epsilon E_4^T E_5,$$

$$\Phi_{77} = \Lambda H^T B + B^T H\Lambda - 2R + \epsilon E_5^T E_5, \qquad \Phi_{78} = \Lambda H^T N,$$

$$\Phi_{88} = -\epsilon I, \qquad \phi_{i1} = -\frac{s}{h_{LU}i}Q_2, \qquad \phi_{i2} = \frac{s}{h_{LU}i}Q_2,$$

$$\phi_{i3} = -\left(\frac{s}{h_{LU}i} + \frac{s}{h_{LU}(s-i+1)}\right)Q_2,$$

$$\phi_{i4} = \frac{s}{h_{LU}(s-i+1)}Q_2, \qquad \phi_{i5} = -\frac{s}{h_{LU}(s-i+1)}Q_2,$$

$$\Gamma = \begin{bmatrix} A & 0 & A_d & 0 & C & D & B & N \end{bmatrix},$$

$$h_{LU} = h_U - h_L, \text{ other terms are } 0. \tag{13}$$

Proof. Consider a novel augmented Lyapunov-Krasovskii functional for the system (1) as follows:

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) + V_4(x_t) + V_5(x_t)$$
$$+ V_6(x_t) + V_7(x_t) + V_8(x_t) + V_9(x_t) + V_{10}(x_t), \tag{14}$$

where

$$V_1(x_t) = x^T(t) P_1 x(t),$$

$$V_2(x_t) = \int_{t-h_L}^t x^T(s) P_2 x(s) \, ds,$$

$$V_3(x_t) = \int_{t-h(t)}^t x^T(s) P_3 x(s) \, ds,$$

$$V_4(x_t) = \int_{t-h_U}^t x^T(s) P_4 x(s) \, ds,$$

$$V_5(x_t) = \int_{t-\tau(t)}^t \dot{x}^T(s) P_5 \dot{x}(s) \, ds,$$

$$V_6(x_t) = \int_{t-h_L}^t h_L(s - (t - h_L)) \dot{x}^T(s) Q_1 \dot{x}(s) \, ds,$$

$$V_7(x_t) = \int_{t-h_U}^t (s - (t - h_U)) \dot{x}^T(s) Q_2 \dot{x}(s) \, ds,$$

$$V_8(x_t) = \int_{t-r}^t (s - (t - r)) x^T(s) M_1 x(s) \, ds,$$

$$V_9(x_t) = \frac{r^2}{2} \int_{-r}^0 \int_0^\theta \int_{t+\lambda}^t \dot{x}^T(s) M_2 x(s) \, ds \, d\lambda \, d\theta,$$

$$V_{10}(x_t) = 2\sum_{i=1}^N \lambda_i \int_0^{\sigma_i} f_i(s) \, ds. \tag{15}$$

The derivative of $V(x_t)$ with respect to time along the trajectory (1) is

$$\dot{V}(x_t) = \dot{V}_1(x_t) + \dot{V}_2(x_t) + \dot{V}_3(x_t) + \dot{V}_4(x_t) + \dot{V}_5(x_t)$$
$$+ \dot{V}_6(x_t) + \dot{V}_7(x_t) + \dot{V}_8(x_t) + \dot{V}_9(x_t) + \dot{V}_{10}(x_t). \tag{16}$$

By utilizing (7), the time derivative of $V_1(x_t)$ is obtained as

$$\dot{V}_1(x_t)$$
$$= 2x^T(t) P_1 \dot{x}(t)$$
$$= 2x^T(t) P_1 \left[Ax(t) + A_d x(t - h(t)) + C\dot{x}(t - \tau(t)) \right.$$
$$\left. + D \int_{t-r}^{t} x(s)\, ds + Bf(\sigma(t)) + NP(t) \right]$$
$$= x^T(t) \left(P_1 A + A^T P_1 \right) x(t) + 2x^T(t) P_1 A_d x(t - h(t))$$
$$+ 2x^T(t) P_1 C\dot{x}(t - \tau(t)) + 2x^T(t) P_1 D \int_{t-r}^{t} x(s)\, ds$$
$$+ 2x^T(t) P_1 Bf(\sigma(t)) + 2x^T(t) P_1 NP(t),$$

$$\dot{V}_2(x_t) = x^T(t) P_2 x(t) - x^T(t - h_L) P_2 x(t - h_L),$$

$$\dot{V}_3(x_t)$$
$$= x^T(t) P_3 x(t) - \left(1 - \dot{h}(t) \right) x^T(t - h(t)) P_3 x(t - h(t))$$
$$\le x^T(t) P_3 x(t) - \left(1 - h_D \right) x^T(t - h(t)) P_3 x(t - h(t)),$$

$$\dot{V}_4(x_t) = x^T(t) P_4 x(t) - x^T(t - h_U) P_4 x(t - h_U),$$

$$\dot{V}_5(x_t)$$
$$= \dot{x}^T(t) P_5 \dot{x}(t) - (1 - \dot{\tau}(t)) \dot{x}^T(t - \tau(t)) P_5 \dot{x}(t - \tau(t))$$
$$\le \dot{x}^T(t) P_5 \dot{x}(t) - (1 - \tau_D) \dot{x}^T(t - \tau(t)) P_5 \dot{x}(t - \tau(t)). \tag{17}$$

By utilizing Lemma 2, the time derivative of $V_6(x_t)$ is obtained as

$$\dot{V}_6(x_t)$$
$$= h_L^2 \dot{x}^T(t) Q_1 \dot{x}(t) - \int_{t-h_L}^{t} h_L \dot{x}^T(s) Q_1 \dot{x}(s)\, ds$$
$$\le h_L^2 \dot{x}^T(t) Q_1 \dot{x}(t) - \left(\int_{t-h_L}^{t} \dot{x}(s)\, ds \right)^T Q_1 \left(\int_{t-h_L}^{t} \dot{x}(s)\, ds \right)$$
$$= h_L^2 \dot{x}^T(t) Q_1 \dot{x}(t) - [x(t) - x(t - h_L)]^T$$
$$\times Q_1 [x(t) - x(t - h_L)]$$
$$= h_L^2 \dot{x}^T(t) Q_1 \dot{x}(t) - x^T(t) Q_1 x(t) + 2x^T(t) Q_1 x(t - h_L)$$
$$- x^T(t - h_L) Q_1 x(t - h_L). \tag{18}$$

By utilizing Lemma 2, for every $i = 1, 2, \ldots, s$, if $h_L + h_{LU}(i - 1)/s \le h(t) \le h_L + h_{LU} i/s$, $h_{LU} = h_U - h_L$, the time derivative of $V_7(x_t)$ is obtained as

$$\dot{V}_7(x_t)$$
$$= h_U \dot{x}^T(t) Q_2 \dot{x}(t) - \int_{t-h_U}^{t} \dot{x}^T(s) Q_2 \dot{x}(s)\, ds$$
$$= h_U \dot{x}^T(t) Q_2 \dot{x}(t) - \int_{t-h_L}^{t} \dot{x}^T(s) Q_1 \dot{x}(s)\, ds$$
$$- \int_{t-h(t)}^{t-h_L} \dot{x}^T(s) Q_2 \dot{x}(s)\, ds$$
$$- \int_{t-h_U}^{t-h(t)} \dot{x}^T(s) Q_2 \dot{x}(s)\, ds$$
$$\le h_U \dot{x}^T(t) Q_2 \dot{x}(t) - \frac{1}{h_L} [x(t) - x(t - h_L)]^T$$
$$\times Q_2 [x(t) - x(t - h_L)] - \frac{s}{h_{LU}(s - i + 1)}$$
$$\times [x(t - h(t)) - x(t - h_U)]^T$$
$$\times Q_2 [x(t - h(t)) - x(t - h_U)]$$
$$- \frac{s}{h_{LU} i} [x(t - h_L) - x(t - h(t))]^T$$
$$\times Q_2 [x(t - h_L) - x(t - h(t))]$$
$$= h_U \dot{x}^T(t) Q_2 \dot{x}(t) - \frac{1}{h_L} x^T(t) Q_2 x(t)$$
$$+ \frac{2}{h_L} x^T(t) Q_2 x(t - h_L)$$
$$- x^T(t - h_L) \left(\frac{1}{h_L} + \frac{s}{h_{LU} i} \right) Q_2 x(t - h_L)$$
$$+ \frac{2s}{h_{LU} i} x^T(t - h_L) Q_2 x(t - h(t)) - x^T(t - h(t))$$
$$\times \left(\frac{s}{h_{LU}(s - i + 1)} + \frac{s}{h_{LU} i} \right) Q_2 x(t - h(t))$$
$$+ \frac{2s}{h_{LU}(s - i + 1)} x^T(t - h(t)) Q_2 x(t - h_U)$$
$$- \frac{s}{h_{LU}(s - i + 1)} x^T(t - h_U) Q_2 x(t - h_U),$$

$$\dot{V}_8(x_t)$$
$$= rx^T(t) M_1 x(t) - \int_{t-r}^{t} x^T(s) M_1 x(s)\, ds$$
$$\le rx^T(t) M_1 x(t)$$
$$- \frac{1}{r} \left(\int_{t-r}^{t} x(s)\, ds \right)^T M_1 \left(\int_{t-r}^{t} x(s)\, ds \right),$$

$$\dot{V}_9(x_t) = \frac{r^4}{4}\dot{x}^T(t)M_2\dot{x}(t) - \frac{r^2}{2}\int_{-r}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)M_2\dot{x}(s)\,ds\,d\theta$$

$$\leq \frac{r^4}{4}\dot{x}^T(t)M_2\dot{x}(t) - r^2x^T(t)M_2x(t)$$

$$+ 2rx^T(t)M_2\int_{t-r}^{t}x(s)\,ds$$

$$- \left(\int_{t-r}^{t}x(s)\,ds\right)^T M_2\left(\int_{t-r}^{t}x(s)\,ds\right),$$

$$\dot{V}_{10}(x_t)$$

$$= 2\sum_{i=1}^{m}\lambda_i\dot{\sigma}_i(t)f_i(\sigma_i) = 2f^T(\sigma(t))\Lambda H^T\dot{x}(t)$$

$$= 2f^T(\sigma(t))\Lambda H^T$$

$$\times\Big[Ax(t) + A_dx(t-h(t)) + C\dot{x}(t-\tau)$$

$$+ D\int_{t-r}^{t}x(s)\,ds + Bf(\sigma(t)) + NP(t)\Big]$$

$$= 2x^T(t)A^TH\Lambda f + 2x^T(t-h(t))A_d^TH\Lambda f$$

$$+ 2\dot{x}(t-\tau(t))C^TH\Lambda f + \int_{t-r}^{t}x^T(s)\,ds D^TH\Lambda f$$

$$+ f^T\left(\Lambda H^TB + B^TH\Lambda\right)f + f^T\Lambda H^TNP(t).$$

$$(19)$$

From system (1) with nonlinearity located in the sectors $K_{[0,k_i]}$ $(i = 1, 2, \ldots, m)$, if there exists $R = \text{diag}\{r_1, r_2, \ldots, r_m\}$, it is easy to achieve

$$r_if_i(\sigma_i)\left[k_ih_i^Tx(t) - f_i(\sigma_i)\right] \geq 0, \quad i = 1, 2, \ldots, m, \quad (20)$$

which is equivalent to

$$2\left[x^T(t)HKRf(\sigma(t)) - f^T(\sigma(t))Rf(\sigma(t))\right] \geq 0; \quad (21)$$

also, for any positive scalar ϵ, the following inequality is satisfied:

$$\epsilon\xi^T(t)\xi^T\xi\xi(t) - \epsilon P^T(t)P(t) \geq 0. \quad (22)$$

The combination of (17)–(22) gives that

$$\dot{V}(x_t)$$

$$\leq \xi^T(t)\left[\Phi + \Pi_i + \Gamma^T\left(P_5 + h_L^2Q_1 + h_UQ_2 + \frac{r^4}{4}M_2\right)\Gamma\right]\xi(t),$$

$$i = 1, 2, \ldots, s. \quad (23)$$

By Fact 1, (23) is equivalent to (12), which implies $\dot{V}(x_t) < 0$, which guarantees the robustly asymptotic stability of the system (1). This completes the proof. □

When $h_L = 0$, by simplifying the Lyapunov-Krasovskii functional $V(x_t)$ used in the proof of Theorem 3 to

$$\overline{V}(x_t) = V_1(x_t) + V_3(x_t) + V_4(x_t) + V_5(x_t) + V_7(x_t)$$

$$+ V_8(x_t) + V_9(x_t) + V_{10}(x_t). \quad (24)$$

Theorem 4. *For a given positive integer s and some scalars h_U, h_D, τ_U, τ_D, and ϵ with $0 < h_U, h_D < 1$, $\tau_D < 1$, and $\epsilon > 0$, the system (1) with a nonlinear function $f(\sigma(t))$ satisfying the finite sector condition (3) and time-varying delays $h(t)$ and $\tau(t)$ satisfying (9) is robustly asymptotically stable if there exist $P_i^T = P_i > 0$ $(i = 1, 3, 4, 5)$, $Q_2^T = Q_2 > 0$, $M_i^T = M_i > 0$ $(i = 1, 2)$, $R = \text{diag}\{r_1, r_2, \ldots, r_m\} \geq 0$, and diagonal matrices $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\} \geq 0$ with dimensions such that the following symmetric linear matrix inequality holds:*

$$\begin{bmatrix} \Psi + \Lambda_i & \Sigma^TP_5 & \Sigma^TQ_2 & \dfrac{r^2}{2}\Sigma^TM_2 \\ * & -P_5 & 0 & 0 \\ * & * & -\dfrac{1}{h_U}Q_2 & 0 \\ * & * & * & -M_2 \end{bmatrix} < 0, \quad (25)$$

$$(i = 1, 2, \ldots, s),$$

where

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & \Psi_{16} & \Psi_{17} \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} & \Psi_{26} & \Psi_{27} \\ * & * & \Psi_{33} & \Psi_{34} & \Psi_{35} & \Psi_{36} & \Psi_{37} \\ * & * & * & \Psi_{44} & \Psi_{45} & \Psi_{46} & \Psi_{47} \\ * & * & * & * & \Psi_{55} & \Psi_{56} & \Psi_{57} \\ * & * & * & * & * & \Psi_{66} & \Psi_{67} \\ * & * & * & * & * & * & \Psi_{77} \end{bmatrix},$$

$$\Lambda_i = \begin{bmatrix} \varphi_{i1} & \varphi_{i2} & 0 & 0 & 0 & 0 & 0 \\ * & \varphi_{i3} & \varphi_{i4} & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix},$$

$$\Psi_{11} = P_1A + A^TP_1 + P_3 + P_4 + rM_1 - r^2M_2 + \epsilon E_1^TE_1,$$

$$\Psi_{12} = P_1A_d + \epsilon E_1^TE_2, \qquad \Psi_{14} = P_1C + \epsilon E_1^TE_3,$$

$$\Psi_{15} = P_1D + rM_2 + \epsilon E_1^TE_4,$$

$$\Psi_{16} = P_1B + A^TH\Lambda + HKR + \epsilon E_1^TE_5,$$

$$\Psi_{17} = P_1N, \qquad \Psi_{22} = -(1-h_D)P_3 + \epsilon E_2^TE_2,$$

$$\Psi_{24} = \epsilon E_2^TE_3, \qquad \Psi_{25} = \epsilon E_2^TE_{24},$$

$$\Psi_{26} = A_dH^T\Lambda + \epsilon E_2^TE_5, \qquad \Psi_{33} = -P_4,$$

$$\Psi_{44} = -(1-\tau_D)P_5 + \epsilon E_3^TE_3,$$

$$\Psi_{45} = \epsilon E_3^TE_4, \qquad \Psi_{46} = C^TH\Lambda + \epsilon E_3^TE_5,$$

$$\Psi_{55} = -\frac{1}{r}M_1 - M_2 + \epsilon E_4^TE_4, \qquad \Psi_{56} = D^TH\Lambda + \epsilon E_4^TE_5,$$

$$\Psi_{66} = \Lambda H^T B + B^T H \Lambda - 2R + \epsilon E_5^T E_5, \qquad \Psi_{67} = \Lambda H^T N,$$

$$\Psi_{77} = -\epsilon I, \qquad \Psi_{i1} = -\frac{s}{h_U i} Q_2, \qquad \Psi_{i2} = \frac{s}{h_U i} Q_2,$$

$$\Psi_{i3} = -\left(\frac{s}{h_U i} + \frac{s}{h_U (s-i+1)} \right) Q_2,$$

$$\Psi_{i4} = \frac{s}{h_U (s-i+1)} Q_2,$$

$$\Sigma = \begin{bmatrix} A & A_d & 0 & C & D & B & N \end{bmatrix},$$

$$h_{LU} = h_U - h_L, \quad \textit{other terms are } 0.$$

$$(26)$$

When the nonlinear function $f(\sigma(t))$ vanishes, the system (1) reduces to

$$\dot{x}(t) = (A + \Delta A(t)) x(t) + (A_d + \Delta A_d(t)) x(t - h(t))$$

$$+ (C + \Delta C(t)) \dot{x}(t - \tau(t)) + (D + \Delta D) \int_{t-r}^{t} x(s) \, ds,$$

$$x(t) = \phi(t), \quad t \in [- \max\{h_U, r\}, 0].$$

$$(27)$$

Similarly, based on Theorem 3, we can obtain the robustly asymptotical stability for system (27) as follows.

Theorem 5. *For a given positive integer s and some scalars h_L, h_U, h_D, τ_U, τ_D, and ϵ with $h_U > h_L > 0$, $h_D < 1$, $\tau_D < 1$, and $\epsilon > 0$, the system (27) with time-varying delays $h(t)$ and $\tau(t)$ satisfying (9) is robustly asymptotically stable if there exist $P_i^T = P_i > 0$ ($i = 1, 2, \ldots, 5$), $Q_i^T = Q_i > 0$, and $M_i^T = M_i > 0$ ($i = 1, 2$) with dimensions such that the following symmetric linear matrix inequality holds:*

$$\begin{bmatrix} \Xi + \Delta_i & \Upsilon^T P_5 & h_L \Upsilon^T Q_1 & \Upsilon^T Q_2 & \frac{r^2}{2} \Upsilon^T M_2 \\ * & -P_5 & 0 & 0 & 0 \\ * & * & -Q_1 & 0 & 0 \\ * & * & * & -\frac{1}{h_U} Q_2 & 0 \\ * & * & * & * & -M_2 \end{bmatrix} < 0,$$

$$(i = 1, 2, \ldots, s),$$

$$(28)$$

where

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} & \Xi_{27} \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} & \Xi_{37} \\ * & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} & \Xi_{47} \\ * & * & * & * & \Xi_{55} & \Xi_{56} & \Xi_{57} \\ * & * & * & * & * & \Xi_{66} & \Xi_{67} \\ * & * & * & * & * & * & \Xi_{77} \end{bmatrix},$$

$$\Delta_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & \psi_{i1} & \psi_{i2} & 0 & 0 & 0 & 0 \\ * & * & \psi_{i3} & \psi_{i4} & 0 & 0 & 0 \\ * & * & * & \phi_{i5} & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix},$$

$$\Xi_{11} = P_1 A + A^T P_1 + P_2 + P_3 + P_4 - Q_1 - \frac{1}{h_L} Q_2$$

$$+ r M_1 - r^2 M_2 + \epsilon E_1^T E_1,$$

$$\Xi_{12} = Q_1 + \frac{1}{h_L} Q_2, \qquad \Xi_{13} = P_1 A_d + \epsilon E_1^T E_2,$$

$$\Xi_{15} = P_1 C + \epsilon E_1^T E_3, \qquad \Xi_{16} = P_1 D + r M_2 + \epsilon E_1^T E_4,$$

$$\Xi_{17} = P_1 N, \qquad \Xi_{22} = -P_2 - Q_1 - \frac{1}{h_L} Q_2,$$

$$\Xi_{33} = -(1 - h_D) P_3 + \epsilon E_2^T E_2, \qquad \Xi_{35} = \epsilon E_2^T E_3,$$

$$\Xi_{36} = \epsilon E_2^T E_4, \qquad \Xi_{44} = -P_4,$$

$$\Xi_{55} = -(1 - \tau_D) P_5 + \epsilon E_3^T E_3,$$

$$\Xi_{56} = \epsilon E_3^T E_4, \qquad \Xi_{66} = -\frac{1}{r} M_1 - M_2 + \epsilon E_4^T E_4,$$

$$\Xi_{77} = -\epsilon I, \qquad \psi_{i1} = -\frac{s}{h_{LU} i} Q_2, \qquad \psi_{i2} = \frac{s}{h_{LU} i} Q_2,$$

$$\psi_{i3} = -\left(\frac{s}{h_{LU} i} + \frac{s}{h_{LU} (s-i+1)} \right) Q_2,$$

$$\psi_{i4} = \frac{s}{h_{LU} (s-i+1)} Q_2,$$

$$\psi_{i5} = -\frac{s}{h_{LU} (s-i+1)} Q_2, \qquad h_{LU} = h_U - h_L,$$

$$\Upsilon = \begin{bmatrix} A & 0 & A_d & 0 & C & D & N \end{bmatrix}, \quad \textit{other terms are } 0.$$

$$(29)$$

3.2. Stability Analysis in the Infinite Sector $K_{[0,\infty]}$

Theorem 6. *For a given positive integer s and scalars h_L, h_U, h_D, τ_U, τ_D, and ϵ with $h_U > h_L > 0$, $h_D < 1$, $\tau_D < 1$, and $\epsilon > 0$, the system (1) with a nonlinear function $f(\sigma(t))$ satisfying the finite sector condition (4) and time-varying delays $h(t)$ and $\tau(t)$ satisfying (9) is robustly asymptotically stable if there exist $P_i^T = P_i > 0$ ($i = 1, 2, \ldots, 5$), $Q_i^T = Q_i > 0$, $M_i^T = M_i > 0$ ($i = 1, 2$), a scalar $\alpha > 0$, and diagonal matrix $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\} \geq 0$ with dimensions such that the following symmetric linear matrix inequality holds:*

$$\begin{bmatrix} \overline{\Phi} + \Pi_i & \Gamma^T P_5 & h_L \Gamma^T Q_1 & \Gamma^T Q_2 & \frac{r^2}{2} \Gamma^T M_2 \\ * & -P_5 & 0 & 0 & 0 \\ * & * & -Q_1 & 0 & 0 \\ * & * & * & -\frac{1}{h_U} Q_2 & 0 \\ * & * & * & * & -M_2 \end{bmatrix},$$

$$(i = 1, 2, \ldots, s),$$

$$(30)$$

where

$$\overline{\Phi} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} & \overline{\Phi}_{17} & \Phi_{18} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} & \Phi_{27} & \Phi_{28} \\ * & * & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36} & \Phi_{37} & \Phi_{38} \\ * & * & * & \Phi_{44} & \Phi_{45} & \Phi_{46} & \Phi_{47} & \Phi_{48} \\ * & * & * & * & \Phi_{55} & \Phi_{56} & \Phi_{57} & \Phi_{58} \\ * & * & * & * & * & \Phi_{66} & \Phi_{67} & \Phi_{68} \\ * & * & * & * & * & * & \overline{\Phi}_{77} & \Phi_{78} \\ * & * & * & * & * & * & * & \Phi_{88} \end{bmatrix},$$

$$\overline{\Phi}_{17} = P_1 B + A^T H \Lambda + \alpha H, \qquad \overline{\Phi}_{77} = \Lambda H^T B + B^T H \Lambda. \tag{31}$$

The remaining terms are defined as in Theorem 3.

Proof. For system (1) with nonlinearity located in the sector $K_{[0,\infty]}$, the condition (4) is equivalent to

$$2\alpha x^T(t) H f(\sigma(t)) > 0. \tag{32}$$

Now, replacing (21) by (32), then combining (17)–(22) and (32) yields

$$\dot{V}(x_t) \le \xi^T(t) \Psi_i \xi(t), \quad i = 1, 2, \dots, s. \tag{33}$$

Then, similarly to the proof of Theorem 3, the result follows immediately. This completes the proof. □

When $h_L = 0$, we can get a delay-dependent asymptotical stability criterion for the system (1) using the same method of Theorem 4. This result is shown in Theorem 7.

Theorem 7. *For a given positive integer s and scalars h_U, h_D, τ_U, τ_D, and ϵ with $0 < h_U$, $h_D < 1$, $\tau_D < 1$, and $\epsilon > 0$, the system (1) with a nonlinear function $f(\sigma(t))$ satisfying the finite sector condition (4) and time-varying delays $h(t)$ and $\tau(t)$ satisfying (9) is robustly asymptotically stable if there exist $P_i^T = P_i > 0$ $(i = 1, 3, 4, 5)$, $Q_2^T = Q_2 > 0$, $M_i^T = M_i > 0$ $(i = 1, 2)$, a scalar $\alpha > 0$, and diagonal matrices $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_m\} \ge 0$ with dimensions such that the following symmetric linear matrix inequality holds:*

$$\begin{bmatrix} \widetilde{\Psi} + \Pi_i & \Sigma^T P_5 & \Sigma^T Q_2 & \dfrac{r^2}{2}\Sigma^T M_2 \\ * & -P_5 & 0 & 0 \\ * & * & -\dfrac{1}{h_U}Q_2 & 0 \\ * & * & * & -M_2 \end{bmatrix} < 0, \tag{34}$$

$$(i = 1, 2, \dots, s),$$

where

$$\widetilde{\Psi} = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & \widetilde{\Psi}_{16} & \Psi_{17} \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} & \Psi_{26} & \Psi_{27} \\ * & * & \Psi_{33} & \Psi_{34} & \Psi_{35} & \Psi_{36} & \Psi_{37} \\ * & * & * & \Psi_{44} & \Psi_{45} & \Psi_{46} & \Psi_{47} \\ * & * & * & * & \Psi_{55} & \Psi_{56} & \Psi_{57} \\ * & * & * & * & * & \widetilde{\Psi}_{66} & \Psi_{67} \\ * & * & * & * & * & * & \Psi_{77} \end{bmatrix},$$

$$\widetilde{\Psi}_{16} = P_1 B + A^T H \Lambda + \alpha H, \qquad \widetilde{\Psi}_{66} = \Lambda H^T B + B^T H \Lambda. \tag{35}$$

Remark 8. Recently, Wang et al. [29] present the effective mathematical technique to improve significantly the stability criterion. Motivated by this technique, it is the first attempt of the integral partitioning method to extend to a class of uncertain neutral type Lur'e systems with discrete and distributed delays, which has never been mentioned in the previous literatures.

Remark 9. In the augmented Lyapunov-Krasovskii functional $V(x_t)$, this term $(r^2/2) \int_{-r}^{0} \int_{0}^{\theta} \int_{t+\lambda}^{t} \dot{x}^T(s) M_2 \dot{x}(s) ds\, d\lambda\, d\theta$ plays a key role in reducing the conservatism of our results.

Remark 10. Compared with those in previous methods, this method proposed in this paper has two differences: (i) the item $\int_{t-h_U}^{t} \dot{x}^T(s) Q_2 \dot{x}(s) ds$ is added into the Lyapunov functional, which will reduce the conservativeness of results; (ii) the Jensen inequality technique is used, which results in LMIs criteria with less variables than free-weighting matrix approach.

4. Numerical Examples

In this section, numerical simulation examples are given to show the effectiveness and correctness of the derived main results.

Example 11. Consider the following class of uncertain neutral type Lur'e systems with mixed time-varying delays:

$$\begin{aligned} \dot{x}(t) = {} & (A + \Delta A(t)) x(t) + (A_d + \Delta A_d(t)) x(t - h(t)) \\ & + (C + \Delta C(t)) \dot{x}(t - \tau(t)) \\ & + (D + \Delta D(t)) \int_{t-r}^{t} x(s)\, ds + (B + \Delta B(t)) f(\sigma(t)), \end{aligned} \tag{36}$$

where

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, \qquad A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} -0.12 & 0 \\ -0.12 & 0.12 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.3 \\ -0.2 \end{bmatrix}, \qquad H = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix},$$

$$N = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}, \qquad E_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad E_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$E_4 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad E_5 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \qquad \Delta A(t) = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix},$$

$$\Delta A_d(t) = \begin{bmatrix} \alpha_3 & 0 \\ 0 & \alpha_4 \end{bmatrix}, \qquad \Delta C(t) = \begin{bmatrix} \alpha_5 & 0 \\ 0 & \alpha_6 \end{bmatrix},$$

$$\Delta C(t) = \begin{bmatrix} \alpha_7 & 0 \\ 0 & \alpha_8 \end{bmatrix}, \qquad \Delta B(t) = \begin{bmatrix} \alpha_9 \\ 0 \end{bmatrix}. \tag{37}$$

TABLE 1: Allowable time delay h_U for $h_L = 0.1$, $h_D = 0.1$, and $r = 1$ and different values of τ_D in Example 11.

τ_D	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Theorem 6	3.3587	3.1749	3.1258	3.0987	2.9859	2.9525	2.9012	2.8753	2.5537
Theorem 3	3.5201	3.5016	3.4857	3.4512	3.4185	3.3125	3.1517	2.9513	2.8718

TABLE 2: Allowable time delay h_U for $h_L = 0$, $\tau_D = 0.1$, and $r = 1$ and different values of h_D in Example 11.

h_D	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Theorem 7	4.4132	4.2052	4.1231	4.0532	3.8962	3.7132	3.4891	3.2567	3.1032
Theorem 4	4.5658	4.3562	4.2578	4.1365	4.0596	3.9862	3.7253	3.5329	3.3568
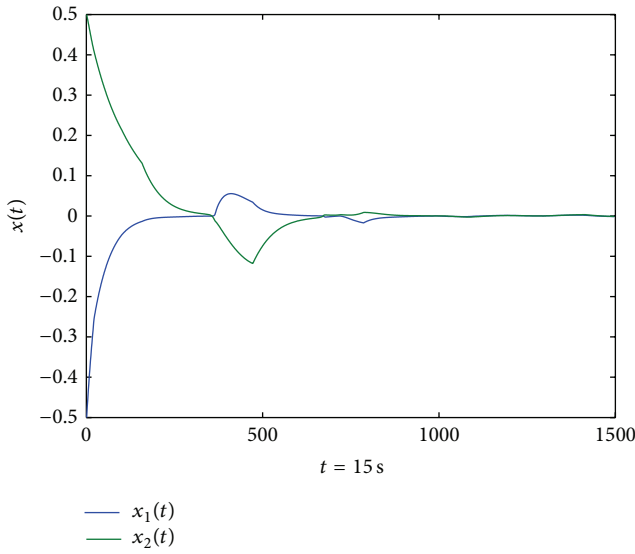
FIGURE 1: State trajectories of $x(t)$ in the plane ($h_U = 3.5201$, $h_L = 0.1$, $h_D = 0.1$, $\tau_D = 0.1$, and $r = 1$, in Example 11, where the initial value is $[-0.5 \quad 0.5]^T$).
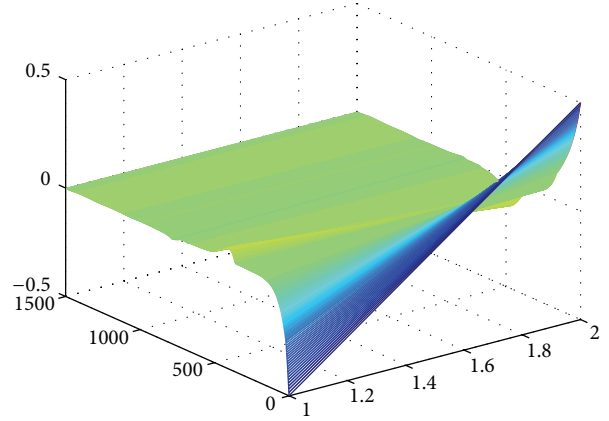

FIGURE 2: State trajectories of $x(t)$ on the space ($h_U = 3.5201$, $h_L = 0.1$, $h_D = 0.1$, $\tau_D = 0.1$, and $r = 1$, in Example 11, where the initial value is $[-0.5 \quad 0.5]^T$).

α_i $(i = 1, 2, \ldots, 9)$ denote the uncertainties which satisfy

$$-1.6 < \alpha_1 < 1.6, \qquad -0.05 < \alpha_2 < 0.05,$$
$$-1.8 < \alpha_3 < 1.8, \qquad -0.15 < \alpha_4 < 0.15,$$
$$-1.65 < \alpha_5 < 1.65, \qquad -0.25 < \alpha_6 < 0.25, \qquad (38)$$
$$-1.5 < \alpha_7 < 1.5, \qquad -0.2 < \alpha_8 < 0.2,$$
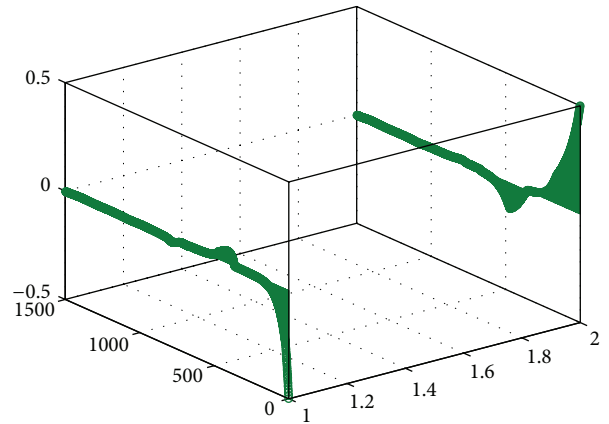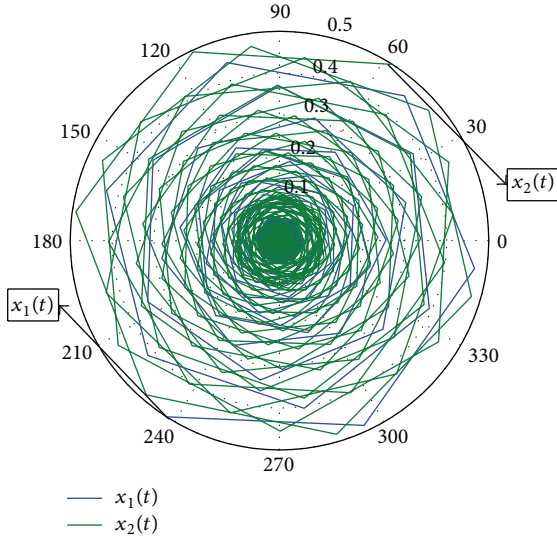$$-1.7 < \alpha_9 < 1.7.$$

FIGURE 3: State trajectories of $x(t)$ on the space ($h_U = 3.5201$, $h_L = 0.1$, $h_D = 0.1$, $\tau_D = 0.1$, and $r = 1$, in Example 11, where the initial value is $[-0.5 \quad 0.5]^T$).

Remark 12. For Example 11, our results of the maximum allowable delay bound (MADB) h_U for different values of h_D and τ_D are listed in Tables 1 and 2. It is clear to show that the maximum allowable delay bound (MADB) h_U is dependent on different values of h_D and τ_D. The values of h_D and τ_D increase, respectively, as the maximum allowable delay bound (MADB) h_U decreases. In order to verify the stability condition, the simulation results are given as $h_L = 0.1$, $h_D = 0.1$, $r = 1$, and $f(\sigma(t)) = [f_1(\sigma(s)) = 0.1(|s + 1| - |s - 1|) \quad f_2(\sigma(s)) = 0.1(|s + 1| - |s - 1|)]^T$ with initial

condition $[-0.5 \quad 0.5]^T$ in Figures 1–4. It is clear that the state trajectories approach to zero asymptotically.

Remark 13. Table 3 shows the maximum values of h_U which guarantee stability of this system by applying Theorems 3 and 6 in this paper, where $A_1 = K[0, 0.5]$, $A_2 = K[0, 10]$, $A_3 = K[0, 1000]$, and $A_4 = K[0, \infty)$.

TABLE 3: Allowable time delay h_U for $h_L = 0.1$, $h_d = 0.1$, and $r = 1$ and different values of τ_D in Example 11.

τ_D	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Theorem 3 ($f \in A_1$)	3.5127	3.4125	3.3652	3.2158	3.1569	3.0237	2.9853	2.9356	2.8765
Theorem 3 ($f \in A_2$)	3.4419	3.3589	3.2518	3.1321	3.0128	2.9512	2.8758	2.8133	2.7567
Theorem 3 ($f \in A_3$)	3.2158	3.1675	3.1291	3.0962	2.9866	2.8534	2.7962	2.7531	2.6158
Theorem 6 ($f \in A_4$)	3.2013	3.1326	3.0987	2.9765	2.9215	2.8312	2.7538	2.7312	2.5983

FIGURE 4: State trajectories of $x(t)$ in polar coordinates ($h_U = 3.5201$, $h_L = 0.1$, $h_D = 0.1$, $\tau_D = 0.1$, and $r = 1$, in Example 11, where the initial value is $[-0.5 \quad 0.5]^T$).

TABLE 4: Maximum allowable delay bound (MADB) r for $\tau = 0.1$ and different values of h in Example 14.

h	0.1	0.5	1	1.5	1.6	1.7
Li and Zhu [32]	6.64	5.55	1.62	—	—	—
Sun et al. [33]	6.67	6.12	2.75	1.31	0.93	0.42
Theorem 3.5 in Chen et al. [34]	6.60	4.32	0.91	—	—	—
Theorem 3.4 in Chen et al. [34]	6.67	6.02	3.19	1.50	1.25	1.02
Theorem 5 ($s = 18$)	7.32	7.13	5.89	4.56	4.08	3.16
Theorem 5 ($s = 28$)	8.16	7.96	7.68	6.58	5.32	4.65

TABLE 5: Maximum allowable delay bound (MADB) h for $\tau = 0.1$ and different values of r in Example 14.

r	1	2	3	4	5	6
Li and Zhu [32]	1.12	0.93	0.77	0.65	0.55	0.43
Sun et al. [33]	1.58	1.20	0.95	0.77	0.64	0.51
Theorem 3.5 in Chen et al. [34]	0.97	0.77	0.63	0.52	0.44	0.35
Theorem 3.4 in Chen et al. [34]	1.71	1.71	1.09	0.83	0.68	0.51
Theorem 5 ($s = 18$)	2.12	2.09	1.98	1.87	1.54	1.12
Theorem 5 ($s = 28$)	2.65	2.56	2.35	2.13	1.87	1.52

Example 14. Consider the following uncertain system:

$$\dot{x}(t) = (A + \Delta A(t)) x(t) + (A_d + \Delta A_d(t)) x(t - h)$$

$$+ (C + \Delta C(t)) \dot{x}(t - \tau) + (D + \Delta D) \int_{t-r}^{t} x(s) \, ds,$$

(39)

where

$$A = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}, \qquad A_d = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix},$$

$$C = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}, \qquad D = \begin{bmatrix} -0.12 & 0 \\ -0.12 & 0.12 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad E_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$E_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad E_4 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad (40)$$

$$N = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}, \qquad \Delta A(t) = \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix},$$

$$\Delta A_d(t) = \begin{bmatrix} \beta_3 & 0 \\ 0 & \beta_4 \end{bmatrix}, \qquad \Delta C(t) = \begin{bmatrix} \beta_5 & 0 \\ 0 & \beta_6 \end{bmatrix},$$

$$\Delta C(t) = \begin{bmatrix} \beta_7 & 0 \\ 0 & \beta_8 \end{bmatrix}.$$

β_i ($i = 1, 2, \ldots, 8$) denote the uncertainties which satisfy

$$-1.6 < \beta_1 < 1.6, \qquad -0.05 < \beta_2 < 0.05,$$

$$-1.5 < \beta_3 < 1.5, \qquad -0.06 < \beta_4 < 0.06,$$

$$-1.6 < \beta_5 < 1.6, \qquad -0.2 < \beta_6 < 0.2,$$

(41)

$$-1.4 < \beta_7 < 1.4, \qquad -0.1 < \beta_8 < 0.1.$$

Remark 15. By applying Theorem 5 and solving the LMI (28) using MATLAB LMI control toolbox, we obtain the maximum allowable upper bounds which are listed in Tables 4 and 5. From Tables 4 and 5, it can be seen that our results show significant improvements over the results obtained in [15, 33, 34]. Further, if the values of s become larger, then the upper bound of the delay will also become larger which is the advantage of the delay partitioning approach used in this paper. Figures 5–8 show clearly that the state vector $x(t)$ stabilizes to zero asymptotically.

Example 16. Consider the system (1) with the following parameters [7]:

$$A = \begin{bmatrix} -2 & 0.5 \\ 0 & -1 \end{bmatrix}, \qquad A_d = \begin{bmatrix} 1 & 0.4 \\ 0.4 & -1 \end{bmatrix},$$
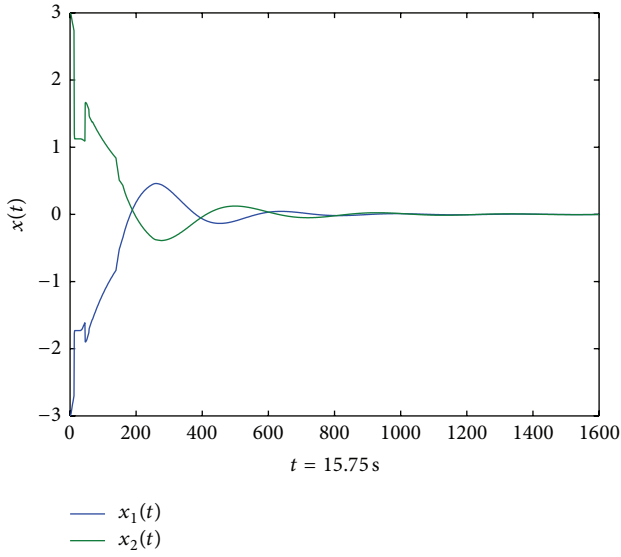
FIGURE 5: State trajectories of $x(t)$ in the plane ($r = 8.16$, $h = 0.1$, and $\tau = 0.1$ in Example 14, where the initial value is $[-3 \quad 3]^T$).
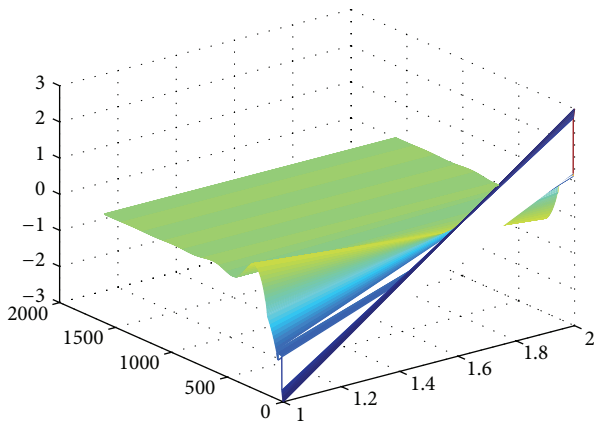

FIGURE 6: State trajectories of $x(t)$ on the space ($r = 8.16$, $h = 0.1$, and $\tau = 0.1$ in Example 14, where the initial value is $[-3 \quad 3]^T$).

$$C = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.5 \\ -0.75 \end{bmatrix}, \qquad H = \begin{bmatrix} 0.2 \\ 0.6 \end{bmatrix},$$

$$\Delta A(t) = \Delta A_d(t) = \Delta B(t) = \Delta C(t) = \Delta D(t) = D = 0. \tag{42}$$

Remark 17. When $h_L = 0$, the maximum upper bounds h_U of the delay $h(t)$ for different h_D and τ_D obtained from [7] and Theorem 3 of this paper are listed in Tables 6, 7, and 8. From the tables, we can see that the proposed result in this paper is less conservative than the one in [7]. Besides, Figures 9–12 show the simulation results for the state trajectories with initial conditions $[-3 \quad 3]^T$, $f(\sigma(t)) = [f_1(\sigma(s)) = 0.1(|s + 1| - |s - 1|) \quad f_2(\sigma(s)) = 0.1(|s + 1| - |s - 1|)]^T$, and time-delay $h_U = 2.6052$. One can see that the state trajectories approach to zero asymptotically.
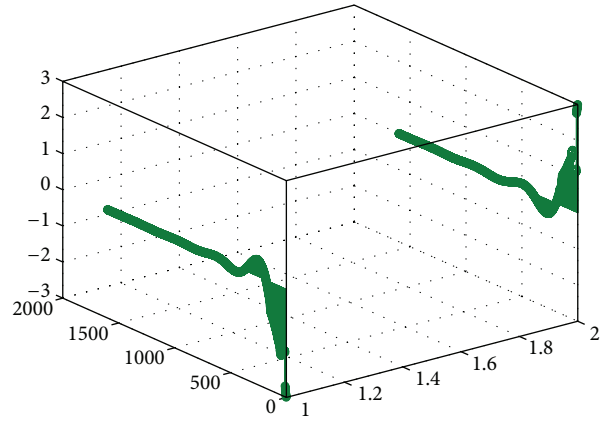

FIGURE 7: State trajectories of $x(t)$ on the space ($r = 8.16$, $h = 0.1$, and $\tau = 0.1$ in Example 14, where the initial value is $[-3 \quad 3]^T$).

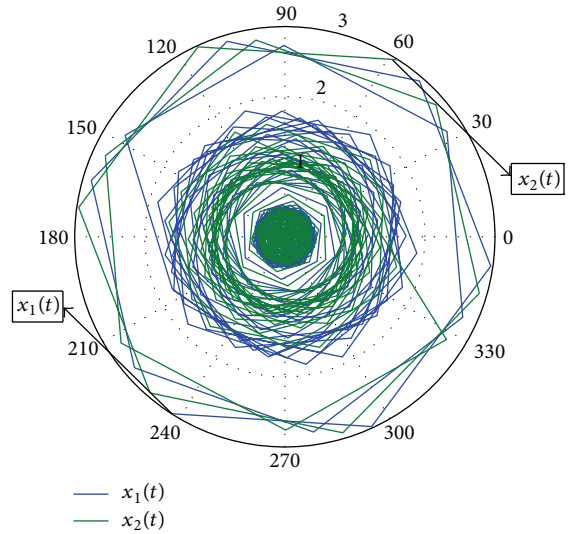FIGURE 8: State trajectories of $x(t)$ in polar coordinates ($r = 8.16$, $h = 0.1$, and $\tau = 0.1$ in Example 14, where the initial value is $[-3 \quad 3]^T$).

TABLE 6: Maximum allowable delay bound (MADB) h_U for different values of h_D in Example 16.

h_D ($h_L = 0, \tau_D = 0.1$)	0.2	0.4	0.6	0.8
Gao et al. [7]	2.5405	1.8002	1.3605	0.7314
Theorem 3 ($s = 18$)	2.5617	1.9216	1.5845	1.0123
Theorem 3 ($s = 28$)	2.6052	1.9538	1.6232	1.1659

Finally, in order to better understand the effectiveness of the paper, we give some explanation about Figures 1–12 presented for Examples 11, 14, and 16.

Figures 1, 5, and 9 are 2D line graph using linear axes (vectors create a single line; matrices create one line per column). Plotted variables are

(1) single variable: plot a vector or each column of a matrix as one line versus its index;

(2) N variable pairs: plot each pair of variables in the selected sequence. For example, the sequence var1,
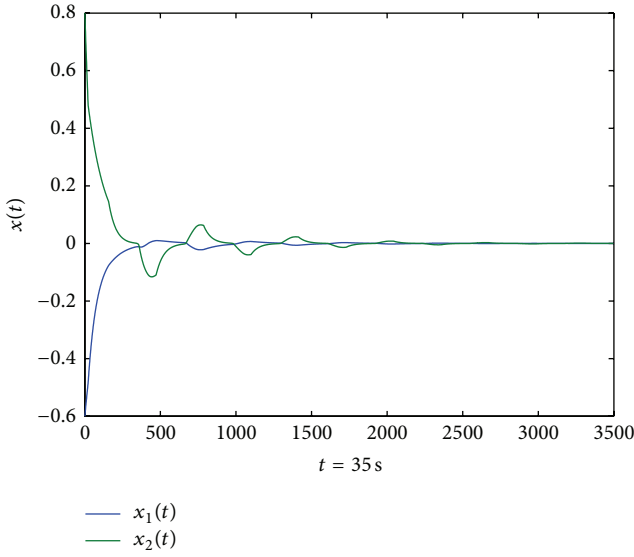
FIGURE 9: State trajectories of $x(t)$ in the plane ($h_U = 2.6052$, $h_L = 0$, $h_D = 0.2$, and $\tau = 0.1$ in Example 16, where the initial value is $[-0.6 \quad 0.8]^T$).
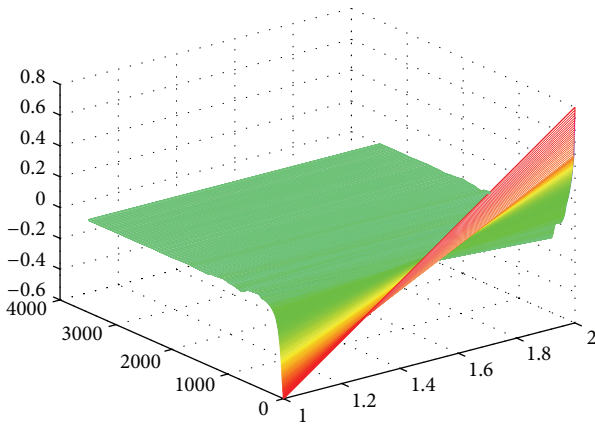

FIGURE 10: State trajectories of $x(t)$ on the space ($h_U = 2.6052$, $h_L = 0$, $h_D = 0.2$, and $\tau = 0.1$ in Example 16, where the initial value is $[-0.6 \quad 0.8]^T$).
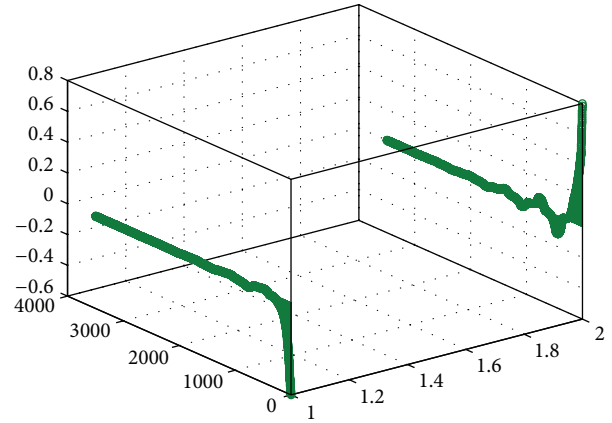

FIGURE 11: State trajectories of $x(t)$ on the space ($h_U = 2.6052$, $h_L = 0$, $h_D = 0.2$, and $\tau = 0.1$ in Example 16, where the initial value is $[-0.6 \quad 0.8]^T$).
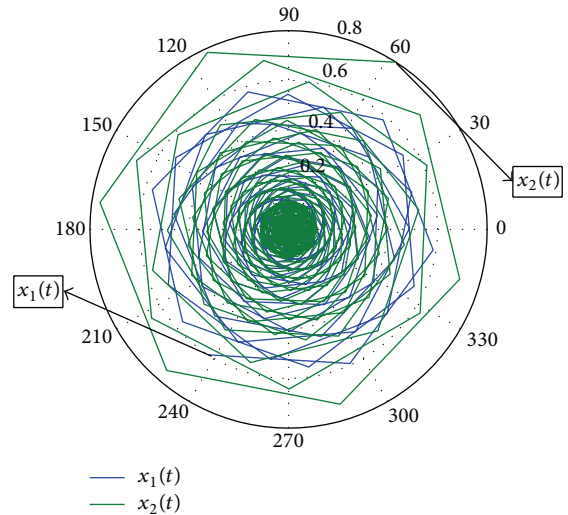

FIGURE 12: State trajectories of $x(t)$ in polar coordinates ($h_U = 2.6052$, $h_L = 0$, $h_D = 0.2$, and $\tau = 0.1$ in Example 16, where the initial value is $[-0.6 \quad 0.8]^T$).

var2, var3, and var4 is plotted as var2 versus var1, var4 versus var3, and so on. Both variables in associated pairs must contain the same number of elements.

Figures 2, 6, and 10 are 3D mesh plot displaying a matrix as a wireframe surface. Plotted variables are as follows:

(1) single variable (z): plot matrix values as heights above the x-y plane and map data values to colormap;

(2) three variables (X, Y, Z): X and Y are vectors or matrices defining the x and y components of a surface.

Figures 3, 7, and 11 are 3D stem graph (display lines extending from the x-y-plane, terminating in circular markers). Plotted variables are as follows:

(1) single variable (z): when z is a row vector, plot all elements at equally spaced x-values against the same

y-value. When z is a column vector, plot all elements at equally spaced y-values against the same x-value;

(2) three variables (x, y, z): plot z at values specified by x and y. x, y, and z must be of the same size.

Figures 4, 8, and 12 are line graph in polar coordinates. Plotted variables are as follows:

(1) two variables (theta, rho): polar coordinate plot of angle theta (radians) versus radius rho (data units).

Figures 1–12 show clearly that the state trajectories of the system (1), (27) are asymptotically stable from different angles, respectively.

5. Conclusions

This paper is focused on the problem of some improved delay-dependent robust stability criteria for a class of uncertain

TABLE 7: Maximum allowable delay bound (MADB) h_U for different values of h_D in Example 16.

h_D ($h_L = 0, \tau_D = 0.5$)	0.2	0.4	0.6	0.8
Gao et al. [7]	2.1066	1.5581	1.0701	0.6329
Theorem 3 ($s = 18$)	2.1258	1.6689	1.3210	0.9836
Theorem 3 ($s = 28$)	2.1532	1.6986	1.3687	1.0066

TABLE 8: Maximum allowable delay bound (MADB) h_U for different values of h_D in Example 16.

h_D ($h_L = 0, \tau_D = 0.9$)	0.2	0.4	0.6	0.8
Gao et al. [7]	0.1086	0.1022	0.0989	0.0972
Theorem 3 ($s = 18$)	0.1219	0.1086	0.1082	0.1082
Theorem 3 ($s = 28$)	0.1221	0.1087	0.1083	0.1083

neutral type Lur'e systems with discrete and distributed delays. In the first place, by using one effective mathematical technique, some less conservative delay-dependent stability results are established without employing the bounding technique and the mode transformation approach. In the second place, by constructing an appropriate Lyapunov-Krasovskii functional with triple terms playing an important role in reducing the conservatism of our results, improved delay-dependent stability criteria in terms of linear matrix inequalities (LMIs) derived in this paper are much simple and valid. What is more, both nonlinearities located in a finite sector and infinite one have been also fully taken into account. Finally, three numerical examples are given to illustrate the less conservatism of the proposed main results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

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