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Research Article

Global Exponential Stability of Almost Periodic Solutions for SICNNs with Continuously Distributed Leakage Delays

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Shunting inhibitory cellular neural networks (SICNNs) are considered with the introduction of continuously distributed delays in the leakage (or forgetting) terms. By using the Lyapunov functional method and differential inequality techniques, some sufficient conditions for the existence and exponential stability of almost periodic solutions are established. Our results complement with some recent ones.

1. Introduction

It is well known that a neural network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths; it is desired to model them by introducing continuously distributed delays over a certain duration of time [1–4]. In particular, shunting inhibitory cellular neural networks (SICNNs) with continuously distributed delays can be described by

$$\begin{aligned} x'_{ij}(t) = & -a_{ij}(t)\,x_{ij}(t) \\ & - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)\left(x_{kl}(t-\tau(t))\right)x_{ij}(t) \\ & - \sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(t) \\ & \cdot \int_0^\infty K_{ij}(u)\,g\left(x_{kl}(t-u)\right)du\,x_{ij}(t) \\ & + L_{ij}(t), \quad i=1,2,\ldots,m,\ j=1,2,\ldots,n, \end{aligned} \tag{1}$$

where C_{ij} denotes the cell at the (i,j) position of the lattice. The r-neighborhood $N_r(i,j)$ of is given as

$$\begin{aligned} N_r(i,j) = \{ & C_{kl} : \max\left(|k-i|,|l-j|\right) \le r, \\ & 1\le k\le m, 1\le l\le n\}, \end{aligned} \tag{2}$$

where $N_q(i,j)$ is similarly specified, x_{ij} is the activity of the cell C_{ij}, $L_{ij}(t)$ is the external input to C_{ij}, the constant $a_{ij} > 0$ represents the passive decay rate of the cell activity, $C_{ij}^{kl}(t)$ and $B_{ij}^{kl}(t)$ are the connection or coupling strengths of postsynaptic activity of the cell transmitted to the cell C_{ij}, the activity functions $f(\cdot)$ and $g(\cdot)$ are continuous functions representing the output or firing rate of the cell C_{kl}, and $\tau(t) \ge 0$ corresponds to the transmission delay.

Since SICNNs (1) have been introduced as a new cellular neural networks (CNNs) in Bouzerdout and Pinter in [5–7], it has been extensively applied in psychophysics, speech, perception, robotics, adaptive pattern recognition, vision, and image processing. Hence, there have been extensive results on the problem of the existence and stability of the equilibrium point, periodic and almost periodic solutions of SICNNs with continuously distributed delays in the literature. We refer the reader to [8–12] and references cited therein.

As pointed out in Gopalsamy [13], the first term in each of the right side of (1) corresponds to a stabilizing negative feedback of the system which acts instantaneously without time delay; these terms are variously known as "forgettin" or leakage terms (see, e.g., Kosko [14] and Haykin [15]). It is known from the literature on population dynamics and neural networks dynamics (see Gopalsamy [16]) that time delays in the stabilizing negative feedback terms will have a tendency to destabilize a system. Therefore, the authors of [17–21] dealt with the existence and stability of equilibrium and periodic solutions for neuron networks model involving leakage delays. Since leakage delays can have a destabilizing influence on the dynamical behaviors of neural networks and the incorporation of time delays in the leakage terms are usually not easy to handle, it necessary to investigate leakage delay effects on the stability of neural networks. On the other hand, as pointed out in [22, 23], periodically varying environment and almost periodically varying environment are foundations for the theory of nature selection. Compared with periodic effects, almost periodic effects are more frequent. Hence, the effects of the almost periodic environment on the evolutionary theory have been the object of intensive analysis by numerous authors, and some of these results can be found in [8, 9, 11] and references cited therein. However, to the best of our knowledge, few authors have considered the existence and exponential stability of almost periodic solutions of SICNNs with continuously distributed delays in the leakage terms. Motivated by the above discussions, in this present paper, we will consider the following SICNNs with continuously distributed leakage delays:

$$\begin{aligned} x'_{ij}(t) = & -a_{ij}(t)\int_0^{\infty} h_{ij}(s)\,x_{ij}(t-s)\,ds \\ & - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)\,f\left(x_{kl}(t-\tau(t))\right)x_{ij}(t) \\ & - \sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(t)\int_0^{\infty} K_{ij}(u)\,g\left(x_{kl}(t-u)\right)du\,x_{ij}(t) \\ & + L_{ij}(t), \end{aligned} \tag{3}$$

where $i = 1,2,\ldots,m,\ j = 1,2,\ldots,n,\ a_{ij} : R \to (0\ +\infty), \tau : R \to [0\ +\infty)$, and $L_{ij}, C_{ij}^{kl}, B_{ij}^{kl} : R \to R$ are almost periodic functions, $\tau(t)$ denotes transmission delay, the leakage delay kernels $h_{ij} : [0,\infty) \to [0,\infty)$ are continuous and integrable, respectively, and the delay kernels $K_{ij} : [0,\infty) \to [0,\infty)$ are continuous and integrable.

The main purpose of this paper is to give the conditions for the existence and exponential stability of the almost periodic solutions for system (3). By applying the Lyapunov functional method and differential inequality techniques, we derive some new sufficient conditions ensuring the existence, uniqueness, and exponential stability of the almost periodic solution for system (3), which are new and complement previously known results. Moreover, an example is also provided to illustrate the effectiveness of our results.

Throughout this paper, for $ij \in J := \{11, 12, \ldots, 1n, 21, 22, \ldots, 2n, \ldots, m1, m2, \ldots, mn\}$, delay kernels $h_i(s)$ and $K_{ij}(u)$ are continuous functions, and there exist constants a_{ij}^+ and η_{ij}^+ such that

$$a_{ij}^+ = \sup_{t\in R} a_{ij}(t), \qquad \eta_{ij}^+ = \int_0^{\infty} s h_{ij}(s)\,ds. \tag{4}$$

From the theory of almost periodic functions in [22, 23], it follows that, for any $\epsilon > 0$, it is possible to find a real number $l = l(\epsilon) > 0$, for any interval with length $l(\epsilon)$, and there exists a number $\delta = \delta(\epsilon)$ in this interval such that

$$\begin{gathered} \left|a_{ij}(t+\delta) - a_{ij}(t)\right| < \epsilon, \\ \left|C_{ij}^{kl}(t+\delta) - C_{ij}^{kl}(t)\right| < \epsilon, \\ \left|B_{ij}^{kl}(t+\delta) - B_{ij}^{kl}(t)\right| < \epsilon, \\ \left|\tau(t+\delta) - \tau(t)\right| < \epsilon, \\ \left|L_{ij}(t+\delta) - L_{ij}(t)\right| < \epsilon, \end{gathered} \tag{5}$$

for all $t \in R, kl, ij \in J$.

We set

$$\begin{aligned} \left\{x_{ij}(t)\right\} = & \left(x_{11}(t), \ldots, x_{1n}(t), \ldots, x_{i1}(t), \ldots, x_{in}(t), \ldots,\right. \\ & \left. x_{m1}(t), \ldots, x_{mn}(t)\right) \in R^{m\times n}. \end{aligned} \tag{6}$$

For any $x(t) = \{x_{ij}(t)\} \in R^{m\times n}$, we define the norm $\|x(t)\| = \max_{(i,j)}\{|x_{ij}(t)|\}$. We also assume that the following conditions (T_1) and (T_2) hold.

(T_1) $f : R \to R$ and $g : R \to R$ are nonincreasing functions on $[0, +\infty)$, and there exist constants M_f, M_g, μ_f, and μ_g such that

$$\begin{aligned} & \left|f(u) - f(v)\right| \le \mu_f\,|u-v|, \\ & \left|f(u)\right| \le M_f, \quad \left|g(u) - g(v)\right| \le \mu_g\,|u-v|, \\ & \left|g(u)\right| \le M_g, \\ & \qquad \forall u, v \in R. \end{aligned} \tag{7}$$

(T_2) For $ij \in J,\ 1 - a_{ij}^+\eta_{ij}^+ > 0$,

$$\begin{aligned} \delta_{ij}(t) = & \left\{ \left[a_{ij}(t)\int_0^{\infty} h_{ij}(s)\,ds\left(1 - 2a_{ij}^+\eta_{ij}^+\right) \right.\right. \\ & \left. - \int_0^{\infty} h_{ij}(s)\left|a_{ij}(t) - a_{ij}(t-s)\right| ds \right] \\ & - \sum_{C_{kl}\in N_r(i,j)} \left|C_{ij}^{kl}(t)\right| M_f \end{aligned}$$

$$
\begin{aligned}
&- \sum_{C_{kl}\in N_q(i,j)} \left|B_{ij}^{kl}(t)\right| \int_0^{\infty} \left|K_{ij}(u)\right| du M_g \Bigg\} \\
&\quad \times \frac{1}{1-a_{ij}^{+}\eta_{ij}^{+}} \\
&> 0,
\end{aligned} \tag{8}
$$

and there exist positive constants η and λ such that

$$
\begin{aligned}
&\lambda < a_{ij}(t)\int_0^{\infty} h_{ij}(s)\,ds, \\
&\int_0^{\infty} h_{ij}(s) e^{\lambda s} ds < +\infty, \\
&\int_0^{\infty} \left|K_{ij}(s)\right| e^{\lambda s} ds < +\infty, \\
&-\Bigg[\left(a_{ij}(t)\int_0^{\infty} h_{ij}(s)\,ds - \lambda\right)\left(1-2a_{ij}^{+}\eta_{ij}^{+}\right) \\
&\quad - \int_0^{\infty} h_{ij}(s)\left|a_{ij}(t)e^{\lambda s} - a_{ij}(t-s)\right| ds\Bigg] \frac{1}{1-a_{ij}^{+}\eta_{ij}^{+}} \\
&\quad + \sum_{C_{kl}\in N_r(i,j)} \left|C_{ij}^{kl}(t)\right| \mu_f e^{\lambda\tau(t)} \frac{1}{1-a_{kl}^{+}\eta_{kl}^{+}} \frac{1}{1-a_{ij}^{+}\eta_{ij}^{+}} \left(\frac{L_{ij}}{\delta_{ij}}\right)^{+} \\
&\quad + \sum_{C_{kl}\in N_r(i,j)} \left|C_{ij}^{kl}(t)\right| M_f \frac{1}{1-a_{ij}^{+}\eta_{ij}^{+}} \\
&\quad + \sum_{C_{kl}\in N_q(i,j)} \left|B_{ij}^{kl}(t)\right| \int_0^{\infty} e^{\lambda u} \left|K_{ij}(u)\right| du \\
&\qquad \times \mu_g \frac{1}{1-a_{kl}^{+}\eta_{kl}^{+}} \\
&\qquad \times \frac{1}{1-a_{ij}^{+}\eta_{ij}^{+}} \left(\frac{L_{ij}}{\delta_{ij}}\right)^{+} \\
&\quad + \sum_{C_{kl}\in N_q(i,j)} \left|B_{ij}^{kl}(t)\right| \int_0^{\infty} \left|K_{ij}(u)\right| du M_g \frac{1}{1-a_{ij}^{+}\eta_{ij}^{+}} \\
&\quad < -\eta,
\end{aligned} \tag{9}
$$

where $t \ge 0$, and $(L_{ij}/\delta_{ij})^{+} = \sup_{t\in R}|L_{ij}(t)/\delta_{ij}(t)|$.

The initial conditions associated with system (3) are of the form

$$
x_{ij}(s) = \varphi_{ij}(s), \quad s \in (-\infty, 0], \ ij \in J, \tag{10}
$$

where $\varphi_{ij}(\cdot)$ denotes real-valued bounded continuous function defined on $(-\infty, 0]$.

Definition 1 (see [22, 23]). Let $u(t) : R \to R^{m\times n}$ be continuous in t. $u(t)$ is said to be almost periodic on R if, for any $\varepsilon > 0$, the set $T(u,\varepsilon) = \{\delta : \|u(t+\delta) - u(t)\| < \varepsilon, \forall t \in R\}$ is relatively dense; that is, for any $\varepsilon > 0$, it is possible to find a real number $l = l(\varepsilon) > 0$, for any interval with length $l(\varepsilon)$, and there exists a number $\delta = \delta(\varepsilon)$ in this interval such that $\|u(t+\delta) - u(t)\| < \varepsilon$, for all $t \in R$.

The remaining part of this paper is organized as follows. In Section 2, we will derive some new sufficient conditions for checking the existence of bounded solutions. In Section 3, we present some new sufficient conditions for the existence, uniqueness and exponential stability of the positive almost periodic solution of (3). In Section 4, we will give some examples and remarks to illustrate our results obtained in previous sections.

2. Preliminary Results

The following lemmas will be useful to prove our main results in Section 3.

Lemma 2. *Let (T_1) and (T_2) hold. Suppose that $x(t) = \{x_{ij}(t)\}$ is a solution of system (3) with initial conditions*

$$
\begin{aligned}
&x_{ij}(s) = \varphi_{ij}(s), \\
&\left|\varphi_{ij}(t) - \int_0^{\infty} h_{ij}(s)\int_{t-s}^{t} a_{ij}(\theta)\varphi_{ij}(\theta)\,d\theta\,ds\right| < \left(\frac{L_{ij}}{\delta_{ij}}\right)^{+},
\end{aligned} \tag{11}
$$

where $s, t \in (-\infty, 0], ij \in J$. Then

$$
\begin{aligned}
&\left|x_{ij}(t) - \int_0^{\infty} h_{ij}(s)\int_{t-s}^{t} a_{ij}(\theta)x_{ij}(\theta)\,d\theta\,ds\right| \\
&\quad \le \left(\frac{L_{ij}}{\delta_{ij}}\right)^{+}, \quad \forall t \ge 0, \ ij \in J,
\end{aligned} \tag{12}
$$

$$
\left|x_{ij}(t)\right| \le \frac{1}{1-a_{ij}^{+}\eta_{ij}^{+}}\left(\frac{L_{ij}}{\delta_{ij}}\right)^{+}, \quad \forall t \ge 0, \ ij \in J. \tag{13}
$$

Proof. Assume, by way of contradiction, that (12) does not hold. Then, there exist $ij \in J$, $\gamma > (L_{ij}/\delta_{ij})^{+}$, and $t_* > 0$ such that

$$
\left|X_{ij}(t_*)\right| = \gamma, \quad \left|X_{ij}(t)\right| < \gamma, \quad \forall t \in (-\infty, t_*), \tag{14}
$$

where

$$
X_{ij}(t) = x_{ij}(t) - \int_0^{\infty} h_{ij}(s)\int_{t-s}^{t} a_{ij}(\theta)x_{ij}(\theta)\,d\theta\,ds. \tag{15}
$$

It follows that

$$
\begin{aligned}
&\left|x_{ij}(t)\right| \\
&\quad \le \left|x_{ij}(t) - \int_0^{\infty} h_{ij}(s)\int_{t-s}^{t} a_{ij}(\theta)x_{ij}(\theta)\,d\theta\,ds\right| \\
&\qquad + \left|\int_0^{\infty} h_{ij}(s)\int_{t-s}^{t} a_{ij}(\theta)x_{ij}(\theta)\,d\theta\,ds\right| \\
&\quad \le \gamma + a_{ij}^{+}\eta_{ij}^{+} \sup_{s\in(-\infty,t_*]} \left|x_{ij}(s)\right|, \quad \forall t \in (-\infty, t_*].
\end{aligned} \tag{16}
$$

Consequently, in view of (16) and the fact $a_{ij}^{+}\eta_{ij}^{+} < 1\ (ij \in J)$, we have

$$\begin{aligned} \left|x_{ij}(t)\right| &\le \sup_{s\in(-\infty,t_*]} \left|x_{ij}(s)\right| \\ &\le \frac{1}{1-a_{ij}^{+}\eta_{ij}^{+}}\gamma, \quad \forall t \in (-\infty, t_*]. \end{aligned} \tag{17}$$

From system (3), we derive

$$\begin{aligned} &\frac{d}{dt}\left(x_{ij}(t) - \int_0^{\infty} h_{ij}(s) \int_{t-s}^{t} a_{ij}(\theta) x_{ij}(\theta) d\theta\, ds\right) \\ &\quad = x_{ij}'(t) - a_{ij}(t) \int_0^{\infty} h_{ij}(s) ds x_{ij}(t) \\ &\quad\quad + \int_0^{\infty} h_{ij}(s) a_{ij}(t-s) x_{ij}(t-s) ds \\ &\quad = -a_{ij}(t) \int_0^{\infty} h_{ij}(s) ds \\ &\quad\quad \times \left(x_{ij}(t) - \int_0^{\infty} h_{ij}(s) \int_{t-s}^{t} a_{ij}(\theta) x_{ij}(\theta) d\theta\, ds\right) \\ &\quad\quad - a_{ij}(t) \int_0^{\infty} h_{ij}(s) ds \int_0^{\infty} h_{ij}(s) \\ &\quad\quad \times \int_{t-s}^{t} a_{ij}(\theta) x_{ij}(\theta) d\theta\, ds \\ &\quad\quad - \int_0^{\infty} h_{ij}(s) \left[a_{ij}(t) - a_{ij}(t-s)\right] x_{ij}(t-s) ds \\ &\quad\quad - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t) f\left(x_{kl}(t-\tau(t))\right) x_{ij}(t) \\ &\quad\quad - \sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(t) \int_0^{\infty} K_{ij}(u) g\left(x_{kl}(t-u)\right) du x_{ij}(t) \\ &\quad\quad + L_{ij}(t), \quad ij \in J. \end{aligned} \tag{18}$$

Calculating the upper left derivative of $|X_{ij}(t)|$, together with (14), (17), (18), (T_1), and (T_2), we obtain

$$\begin{aligned} 0 &\le D^{-}\left|X_{ij}(t_*)\right| \\ &\le -a_{ij}(t_*) \int_0^{\infty} h_{ij}(s) ds \left|X_{ij}(t_*)\right| \\ &\quad + \left| -a_{ij}(t_*) \int_0^{\infty} h_{ij}(s) ds \int_0^{\infty} h_{ij}(s) \right. \\ &\quad \times \int_{t_*-s}^{t_*} a_{ij}(\theta) x_{ij}(\theta) d\theta\, ds \\ &\quad - \int_0^{\infty} h_{ij}(s) \left[a_{ij}(t_*) - a_{ij}(t_*-s)\right] x_{ij}(t_*-s) ds \\ &\quad - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t_*) f\left(x_{kl}(t_*-\tau(t_*))\right) x_{ij}(t_*) \\ &\quad - \sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(t_*) \\ &\quad\quad \times \int_0^{\infty} K_{ij}(u) g\left(x_{kl}(t_*-u)\right) du x_{ij}(t_*) \\ &\quad \left. + L_{ij}(t_*) \right| \\ &\le -a_{ij}(t_*) \int_0^{\infty} h_{ij}(s) ds\gamma + a_{ij}(t_*) \\ &\quad \times \int_0^{\infty} h_{ij}(s) ds a_{ij}^{+}\eta_{ij}^{+} \frac{1}{1-a_{ij}^{+}\eta_{ij}^{+}}\gamma \\ &\quad + \int_0^{\infty} h_{ij}(s) \left|a_{ij}(t_*) - a_{ij}(t_*-s)\right| ds \frac{\gamma}{1-a_{ij}^{+}\eta_{ij}^{+}} \\ &\quad + \sum_{C_{kl}\in N_r(i,j)} \left|C_{ij}^{kl}(t_*)\right| M_f \frac{1}{1-a_{ij}^{+}\eta_{ij}^{+}}\gamma \\ &\quad + \sum_{C_{kl}\in N_q(i,j)} \left|B_{ij}^{kl}(t_*)\right| \int_0^{\infty} \left|K_{ij}(u)\right| du M_g \frac{1}{1-a_{ij}^{+}\eta_{ij}^{+}}\gamma \\ &\quad + \left|L_{ij}(t_*)\right| \\ &= \left\{ -\left[a_{ij}(t_*) \int_0^{\infty} h_{ij}(s) ds \left(1 - 2a_{ij}^{+}\eta_{ij}^{+}\right) \right.\right. \\ &\quad\quad \left. - \int_0^{\infty} h_{ij}(s) \left|a_{ij}(t_*) - a_{ij}(t_*-s)\right| ds \right] \\ &\quad\quad + \sum_{C_{kl}\in N_r(i,j)} \left|C_{ij}^{kl}(t_*)\right| M_f \\ &\quad\quad \left. + \sum_{C_{kl}\in N_q(i,j)} \left|B_{ij}^{kl}(t_*)\right| \int_0^{\infty} \left|K_{ij}(u)\right| du M_g \right\} \frac{1}{1-a_{ij}^{+}\eta_{ij}^{+}}\gamma \\ &\quad + \left|L_{ij}(t_*)\right| \\ &\le -\delta_{ij}(t_*) \left[\gamma - \left(\frac{L_{ij}}{\delta_{ij}}\right)^{+}\right] \\ &< 0. \end{aligned} \tag{19}$$

It is a contradiction and shows that (12) holds. Then, using a similar argument as in the proof of (16) and (17), we can show that (13) holds. The proof of Lemma 2 is now completed. □

Remark 3. In view of the boundedness of this solution, from the theory of functional differential equations with infinite delay in [21], it follows that the solution of system (3) with initial conditions (11) can be defined on $[0, \infty)$.

Lemma 4. *Suppose that (T_1) and (T_2) hold. Moreover, assume that $x(t) = \{x_{ij}(t)\}$ is a solution of system (3) with initial function $\varphi_{ij}(\cdot)$ satisfying (11), and $\varphi'_{ij}(\cdot)$ is bounded continuous on $(-\infty, 0]$. Then, for any $\epsilon > 0$, there exists $l = l(\epsilon) > 0$, such that every interval $[\alpha, \alpha + l]$ contains at least one number δ for which there exists $N > 0$ which satisfies*

$$\|x(t+\delta) - x(t)\| \leq \epsilon, \quad \forall t > N. \tag{20}$$

Proof. For $ij \in J$, set

$$\begin{aligned}
\epsilon_{ij}(\delta, t) = & -\left[a_{ij}(t+\delta) - a_{ij}(t)\right] \\
& \times \int_0^\infty h_{ij}(s)\, x_{ij}(t+\delta-s)\, ds \\
& - \sum_{C_{kl}\in N_r(i,j)} \left[C_{ij}^{kl}(t+\delta) - C_{ij}^{kl}(t)\right] \\
& \times \left(f\left(x_{kl}(t-\tau(t+\delta)+\delta)\right) x_{ij}(t+\delta)\right) \\
& - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)\left[f\left(x_{kl}(t-\tau(t+\delta)+\delta)\right)\right. \\
& \left. - f\left(x_{kl}(t-\tau(t)+\delta)\right)\right] \\
& \times x_{ij}(t+\delta) \\
& - \sum_{C_{kl}\in N_q(i,j)} \left[B_{ij}^{kl}(t+\delta) - B_{ij}^{kl}(t)\right] \\
& \times \int_0^\infty K_{ij}(u)\, g\left(x_{kl}(t+\delta-u)\right) du\, x_{ij}(t+\delta) \\
& + \left[L_{ij}(t+\delta) - L_{ij}(t)\right].
\end{aligned} \tag{21}$$

By Lemma 2, the solution $x(t) = \{x_{ij}(t)\}$ is bounded and

$$\left|x_{ij}(t)\right| \leq \frac{1}{1 - a_{ij}^{+}\eta_{ij}^{+}}\left(\frac{L_{ij}}{\delta_{ij}}\right)^{+}, \quad \forall t \in [0, +\infty), \; ij \in J. \tag{22}$$

Thus, the right side of (3) is also bounded, which implies that $x(t)$ is uniformly continuous on R. From (5), for any $\epsilon > 0$, there exists $l = l(\epsilon) > 0$, such that every interval $[\alpha, \alpha + l]$, $\alpha \in R$, contains a δ for which

$$\left|\epsilon_{ij}(\delta, t)\right| \leq \frac{1}{2}\eta \min_{\widetilde{ij}\in J}\left\{1 - a_{\widetilde{ij}}^{+}\eta_{\widetilde{ij}}^{+}\right\}\epsilon, \quad \text{where } ij \in J, t \in R. \tag{23}$$

Let $N_0 \geq 0$ be sufficiently large such that $t + \delta \geq 0$, for $t \geq N_0$, and denote $u_{ij}(t) = x_{ij}(t+\delta) - x_{ij}(t)$. We obtain

$$\begin{aligned}
\frac{du_{ij}(t)}{dt} = & -a_{ij}(t)\int_0^\infty h_{ij}(s)\, u_{ij}(t-s)\, ds \\
& - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)\left(f\left(x_{kl}(t-\tau(t)+\delta)\right)\right. \\
& \left. - f\left(x_{kl}(t-\tau(t))\right)\right) \\
& \times x_{ij}(t+\delta) \\
& - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)\, f\left(x_{kl}(t-\tau(t))\right) \\
& \cdot \left(x_{ij}(t+\delta) - x_{ij}(t)\right) \\
& - \sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(t)\int_0^\infty K_{ij}(u) \\
& \times \left(g\left(x_{kl}(t+\delta-u)\right)\right. \\
& \left. - g\left(x_{kl}(t-u)\right)\right) du\, x_{ij}(t+\delta) \\
& - \sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(t)\int_0^\infty K_{ij}(u)\, g\left(x_{kl}(t-u)\right) du \\
& \cdot \left(x_{ij}(t+\delta) - x_{ij}(t)\right) + \epsilon_{ij}(\delta, t), \\
& \forall t \geq N_0, \; ij \in J,
\end{aligned} \tag{24}$$

which yields

$$\begin{aligned}
& \frac{d}{dt}\left(e^{\lambda t}u_{ij}(t) - \int_0^\infty h_{ij}(s)\int_{t-s}^{t} a_{ij}(\theta)\, e^{\lambda\theta}u_{ij}(\theta)\, d\theta\, ds\right) \\
& = \lambda e^{\lambda t}u_{ij}(t) + e^{\lambda t}u'_{ij}(t) \\
& \quad - a_{ij}(t)\int_0^\infty h_{ij}(s)\, ds\, e^{\lambda t}u_{ij}(t) \\
& \quad + \int_0^\infty h_{ij}(s)\, a_{ij}(t-s)\, e^{\lambda(t-s)}u_{ij}(t-s)\, ds \\
& = -\left(a_{ij}(t)\int_0^\infty h_{ij}(s)\, ds - \lambda\right) \\
& \quad \times \left(e^{\lambda t}u_{ij}(t) - \int_0^\infty h_{ij}(s)\int_{t-s}^{t} a_{ij}(\theta)\, e^{\lambda\theta}u_{ij}(\theta)\, d\theta\, ds\right) \\
& \quad - \left(a_{ij}(t)\int_0^\infty h_{ij}(s)\, ds - \lambda\right)\int_0^\infty h_{ij}(s) \\
& \quad \times \int_{t-s}^{t} a_{ij}(\theta)\, e^{\lambda\theta}u_{ij}(\theta)\, d\theta\, ds \\
& \quad - \int_0^\infty h_{ij}(s)\left[a_{ij}(t)\, e^{\lambda s} - a_{ij}(t-s)\right] \\
& \qquad \times e^{\lambda(t-s)}u_{ij}(t-s)\, ds
\end{aligned}$$

$$+e^{\lambda t}\Bigg\{-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)\times\left(f\left(x_{kl}\left(t-\tau(t)+\delta\right)\right)-f\left(x_{kl}\left(t-\tau(t)\right)\right)\right)x_{ij}(t+\delta)$$
$$-\sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)\, f\left(x_{kl}\left(t-\tau(t)\right)\right)\cdot\left(x_{ij}(t+\delta)-x_{ij}(t)\right)$$
$$-\sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(t)\int_0^\infty K_{ij}(u)\times\left(g\left(x_{kl}(t+\delta-u)\right)-g\left(x_{kl}(t-u)\right)\right)du\,x_{ij}(t+\delta)$$
$$-\sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(t)\int_0^\infty K_{ij}(u)\,g\left(x_{kl}(t-u)\right)du\cdot\left(x_{ij}(t+\delta)-x_{ij}(t)\right)+\epsilon_{ij}(\delta,t)\Bigg\},\quad \forall t\ge N_0,\ ij\in J. \tag{25}$$

Set

$$U(t)=\left\{U_{ij}(t)\right\}, \tag{26}$$

where

$$U_{ij}(t)=e^{\lambda t}u_{ij}(t)-\int_0^\infty h_{ij}(s)\int_{t-s}^t a_{ij}(\theta)\,e^{\lambda\theta}u_{ij}(\theta)\,d\theta\,ds,\quad ij\in J. \tag{27}$$

Let $(ij)_t$ be such an index that

$$\left|U_{(ij)_t}(t)\right|=\|U(t)\|. \tag{28}$$

Calculating the upper left derivative of $|U_{(ij)_s}(s)|$ along (25), we have

$$D^-\left(\left|U_{(ij)_s}(s)\right|\right)\Big|_{s=t}$$
$$\le-\left(a_{(ij)_t}(t)\int_0^\infty h_{(ij)_t}(s)\,ds-\lambda\right)\left|U_{(ij)_t}(t)\right|$$
$$+\Bigg|-\left(a_{(ij)_t}(t)\int_0^\infty h_{(ij)_t}(s)\,ds-\lambda\right)\int_0^\infty h_{(ij)_t}(s)\times\int_{t-s}^t a_{(ij)_t}(\theta)\,e^{\lambda\theta}u_{(ij)_t}(\theta)\,d\theta\,ds$$
$$-\int_0^\infty h_{(ij)_t}(s)\left[a_{(ij)_t}(t)\,e^{\lambda s}-a_{(ij)_t}(t-s)\right]\times e^{\lambda(t-s)}u_{(ij)_t}(t-s)\,ds$$
$$+e^{\lambda t}\Bigg\{-\sum_{C_{kl}\in N_r(i,j)_t} C_{(ij)_t}^{kl}(t)\times\left(f\left(x_{kl}\left(t-\tau(t)+\delta\right)\right)-f\left(x_{kl}\left(t-\tau(t)\right)\right)\right)\times x_{(ij)_t}(t+\delta)$$
$$-\sum_{C_{kl}\in N_r(i,j)_t} C_{(ij)_t}^{kl}(t)\, f\left(x_{kl}\left(t-\tau(t)\right)\right)\cdot\left(x_{(ij)_t}(t+\delta)-x_{(ij)_t}(t)\right)$$
$$-\sum_{C_{kl}\in N_q(i,j)_t} B_{(ij)_t}^{kl}(t)\times\int_0^\infty K_{(ij)_t}(u)\times\left(g\left(x_{kl}(t+\delta-u)\right)-g\left(x_{kl}(t-u)\right)\right)du\,x_{(ij)_t}(t+\delta)$$
$$-\sum_{C_{kl}\in N_q(i,j)_t} B_{(ij)_t}^{kl}(t)\times\int_0^\infty K_{(ij)_t}(u)\,g\left(x_{kl}(t-u)\right)du\cdot\left(x_{(ij)_t}(t+\delta)-x_{(ij)_t}(t)\right)+\epsilon_{(ij)_t}(\delta,t)\Bigg\}\Bigg|. \tag{29}$$

Let

$$M(t)=\sup_{s\le t}\{\|U(s)\|\}. \tag{30}$$

It is obvious that $\|U(t)\|\le M(t)$, and $M(t)$ is nondecreasing. In particular,

$$e^{\lambda\rho}\left|u_{ij}(\rho)\right|\le\left|e^{\lambda\rho}u_{ij}(\rho)-\int_0^\infty h_{ij}(s)\int_{\rho-s}^\rho a_{ij}(\theta)\,e^{\lambda\theta}u_{ij}(\theta)\,d\theta\,ds\right|+\left|\int_0^\infty h_{ij}(s)\int_{\rho-s}^\rho a_{ij}(\theta)\,e^{\lambda\theta}u_{ij}(\theta)\,d\theta\,ds\right|$$
$$\le M(t)+a_{ij}^+\eta_{ij}^+\sup_{\theta\in(-\infty,t]} e^{\lambda\theta}\left|u_{ij}(\theta)\right|,\quad \forall t\ge\rho,\ ij\in J. \tag{31}$$

Consequently, in view of (31) and the fact $a_{ij}^+\eta_{ij}^+ < 1\ (ij \in J)$, we have

$$e^{\lambda s}\left|u_{ij}(s)\right| \le \sup_{\theta\in(-\infty,t]} e^{\lambda\theta}\left|u_{ij}(\theta)\right| \le \frac{M(t)}{1-a_{ij}^+\eta_{ij}^+}, \quad \text{where } s \in (-\infty,t],\ ij \in J. \tag{32}$$

Now, we consider two cases.

Case (i). If

$$M(t) > \|U(t)\|, \quad \forall t \ge N_0, \tag{33}$$

then, we claim that

$$M(t) \equiv M(N_0) \text{ is a constant}, \quad \forall t \ge N_0. \tag{34}$$

Assume, by way of contradiction, that (34) does not hold. Then, there exists $t_1 > N_0$, such that $M(t_1) > M(N_0)$, since

$$\|U(t)\| \le M(N_0), \quad \forall t \le N_0. \tag{35}$$

There must exist $\beta \in (N_0, t_1)$ such that

$$\|U(\beta)\| = M(t_1) \ge M(\beta), \tag{36}$$

which contradicts (33). This contradiction implies that (34) holds. It follows from (32) that there exists $t_2 > N_0$ such that

$$\begin{aligned}\|u(t)\| &= \max_{ij\in J}\left|u_{ij}(t)\right| \\ &\le \max_{ij\in J}\frac{e^{-\lambda t}M(t)}{1-a_{ij}^+\eta_{ij}^+} \\ &= \max_{ij\in J}\frac{e^{-\lambda t}M(N_0)}{1-a_{ij}^+\eta_{ij}^+} \\ &< \epsilon, \quad \forall t \ge t_2.\end{aligned} \tag{37}$$

Case (ii). If there is such a point $t_0 \ge N_0$ that $M(t_0) = \|U(t_0)\|$, then, in view of (8), (22), (23), (29), (32), (T_1), and (T_2), we get

$$\begin{aligned}
0 \le\ & D^-\left(\left|U_{(ij)_s}(s)\right|\right)\Big|_{s=t_0} \\
\le\ & -\left(a_{(ij)_{t_0}}(t_0)\int_0^\infty h_{(ij)_{t_0}}(s)\,ds-\lambda\right)\left|U_{(ij)_{t_0}}(t_0)\right| \\
& + \left(a_{(ij)_{t_0}}(t_0)\int_0^\infty h_{(ij)_{t_0}}(s)\,ds-\lambda\right)\int_0^\infty h_{(ij)_{t_0}}(s) \\
& \times \int_{t_0-s}^{t_0} a_{(ij)_{t_0}}^+ e^{\lambda\theta}\left|u_{(ij)_{t_0}}(\theta)\right| d\theta\, ds \\
& + \int_0^\infty h_{(ij)_{t_0}}(s)\left|a_{(ij)_{t_0}}(t_0)e^{\lambda s}-a_{(ij)_{t_0}}(t_0-s)\right| \\
& \times e^{\lambda(t_0-s)}\left|u_{(ij)_{t_0}}(t_0-s)\right| ds \\
& + \sum_{C_{kl}\in N_r(i,j)_{t_0}}\left|C_{(ij)_{t_0}}^{kl}(t_0)\right| e^{\lambda t_0} \\
& \quad \times \left|f\left(x_{kl}\left(t_0-\tau(t_0)+\delta\right)\right)-f\left(x_{kl}\left(t_0-\tau(t_0)\right)\right)\right| \\
& \quad \times \left|x_{(ij)_{t_0}}(t_0+\delta)\right| \\
& + \sum_{C_{kl}\in N_r(i,j)_{t_0}}\left|C_{(ij)_{t_0}}^{kl}(t_0)\right|\left|f\left(x_{kl}\left(t_0-\tau(t_0)\right)\right)\right| \\
& \quad \cdot e^{\lambda t_0}\left|x_{(ij)_{t_0}}(t_0+\delta)-x_{(ij)_{t_0}}(t_0)\right| \\
& + \sum_{C_{kl}\in N_q(i,j)_{t_0}}\left|B_{(ij)_{t_0}}^{kl}(t_0)\right| \\
& \times \int_0^\infty\left|K_{(ij)_{t_0}}(u)\right| \\
& \quad \times e^{\lambda t_0}\left|g\left(x_{kl}(t_0+\delta-u)\right)-g\left(x_{kl}(t_0-u)\right)\right| du \\
& \times \left|x_{(ij)_{t_0}}(t_0+\delta)\right| \\
& + \sum_{C_{kl}\in N_q(i,j)_{t_0}}\left|B_{(ij)_{t_0}}^{kl}(t_0)\right| \\
& \quad \times \int_0^\infty\left|K_{(ij)_{t_0}}(u)\right|\left|g\left(x_{kl}(t_0-u)\right)\right| du \\
& \quad \cdot e^{\lambda t_0}\left|x_{(ij)_{t_0}}(t_0+\delta)-x_{(ij)_{t_0}}(t_0)\right| \\
& + \left|\epsilon_{(ij)_{t_0}}(\delta,t_0)\right| e^{\lambda t_0} \\
\le\ & -\left(a_{(ij)_{t_0}}(t_0)\int_0^\infty h_{(ij)_{t_0}}(s)\,ds-\lambda\right)M(t_0) \\
& + \left(a_{(ij)_{t_0}}(t_0)\int_0^\infty h_{(ij)_{t_0}}(s)\,ds-\lambda\right)a_{(ij)_{t_0}}^+\eta_{(ij)_{t_0}}^+ \\
& \times \frac{M(t_0)}{1-a_{(ij)_{t_0}}^+\eta_{(ij)_{t_0}}^+} \\
& + \int_0^\infty h_{(ij)_{t_0}}(s)\left|a_{(ij)_{t_0}}(t_0)e^{\lambda s}-a_{(ij)_{t_0}}(t_0-s)\right| ds \\
& \quad \times \frac{M(t_0)}{1-a_{(ij)_{t_0}}^+\eta_{(ij)_{t_0}}^+} \\
& + \sum_{C_{kl}\in N_r(i,j)_{t_0}}\left|C_{(ij)_{t_0}}^{kl}(t_0)\right| \\
& \quad \times \mu_f e^{\lambda\tau(t_0)}e^{\lambda(t_0-\tau(t_0))}\left|u_{kl}\left(t_0-\tau(t_0)\right)\right| \\
& \quad \times \frac{1}{1-a_{(ij)_{t_0}}^+\eta_{(ij)_{t_0}}^+}\left(\frac{L_{(ij)_{t_0}}}{\delta_{(ij)_{t_0}}}\right)^+ \\
& + \sum_{C_{kl}\in N_r(i,j)_{t_0}}\left|C_{(ij)_{t_0}}^{kl}(t_0)\right| M_f e^{\lambda t_0}\left|u_{(ij)_{t_0}}(t_0)\right|
\end{aligned}$$

$$
\begin{aligned}
&+ \sum_{C_{kl}\in N_q(i,j)_{t_0}} \left|B^{kl}_{(ij)_{t_0}}(t_0)\right| \\
&\times \int_0^\infty e^{\lambda u} \left|K_{(ij)_{t_0}}(u)\right| \mu_g e^{\lambda(t_0-u)} \left|u_{kl}(t_0-u)\right| du \\
&\times \frac{1}{1-a^+_{(ij)_{t_0}}\eta^+_{(ij)_{t_0}}} \left(\frac{L_{(ij)_{t_0}}}{\delta_{(ij)_{t_0}}}\right)^+ \\
&+ \sum_{C_{kl}\in N_q(i,j)_{t_0}} \left|B^{kl}_{(ij)_{t_0}}(t_0)\right| \\
&\times \int_0^\infty \left|K_{(ij)_{t_0}}(u)\right| du M_g e^{\lambda t_0} \left|u_{(ij)_{t_0}}(t_0)\right| \\
&+ \left|\epsilon_{(ij)_{t_0}}(\delta, t_0)\right| e^{\lambda t_0} \\
\le & \Bigg\{ -\Bigg[\left(a_{(ij)_{t_0}}(t_0) \int_0^\infty h_{(ij)_{t_0}}(s)\, ds - \lambda \right) \\
&\quad \times \left(1 - 2a^+_{(ij)_{t_0}}\eta^+_{(ij)_{t_0}}\right) \\
&\quad - \int_0^\infty h_{(ij)_{t_0}}(s) \left| a_{(ij)_{t_0}}(t_0) e^{\lambda s} - a_{(ij)_{t_0}}(t_0-s) \right| ds \Bigg] \\
&\quad \times \frac{1}{1-a^+_{(ij)_{t_0}}\eta^+_{(ij)_{t_0}}} \\
&\quad + \sum_{C_{kl}\in N_r(i,j)_{t_0}} \left|C^{kl}_{(ij)_{t_0}}(t_0)\right| \mu_f e^{\lambda\tau(t_0)} \\
&\quad \times \frac{1}{1-a^+_{kl}\eta^+_{kl}} \frac{1}{1-a^+_{(ij)_{t_0}}\eta^+_{(ij)_{t_0}}} \left(\frac{L_{(ij)_{t_0}}}{\delta_{(ij)_{t_0}}}\right)^+ \\
&\quad + \sum_{C_{kl}\in N_r(i,j)_{t_0}} \left|C^{kl}_{(ij)_{t_0}}(t_0)\right| M_f \frac{1}{1-a^+_{(ij)_{t_0}}\eta^+_{(ij)_{t_0}}} \\
&\quad + \sum_{C_{kl}\in N_q(i,j)_{t_0}} \left|B^{kl}_{(ij)_{t_0}}(t_0)\right| \int_0^\infty e^{\lambda u} \left|K_{(ij)_{t_0}}(u)\right| du \\
&\quad \times \mu_g \frac{1}{1-a^+_{kl}\eta^+_{kl}} \frac{1}{1-a^+_{(ij)_{t_0}}\eta^+_{(ij)_{t_0}}} \left(\frac{L_{(ij)_{t_0}}}{\delta_{(ij)_{t_0}}}\right)^+ \\
&\quad + \sum_{C_{kl}\in N_q(i,j)_{t_0}} \left|B^{kl}_{(ij)_{t_0}}(t_0)\right| \\
&\quad \times \int_0^\infty \left|K_{(ij)_{t_0}}(u)\right| du M_g \frac{1}{1-a^+_{(ij)_{t_0}}\eta^+_{(ij)_{t_0}}} \Bigg\} M(t_0) \\
&+ \eta \min_{ij\in J}\left\{1-a^+_{ij}\eta^+_{ij}\right\} \epsilon e^{\lambda t_0} \\
< & -\eta M(t_0) + \eta \min_{ij\in J}\left\{1-a^+_{ij}\eta^+_{ij}\right\} \epsilon e^{\lambda t_0},
\end{aligned} \tag{38}
$$

which yields that

$$
\begin{gathered}
\|U(t_0)\| = M(t_0) < \min_{ij\in J}\left\{1-a^+_{ij}\eta^+_{ij}\right\} \epsilon e^{\lambda t_0}, \\
\|u(t_0)\| \le \max_{ij\in J} \frac{e^{-\lambda t_0} M(t_0)}{1-a^+_{ij}\eta^+_{ij}} < \epsilon.
\end{gathered} \tag{39}
$$

For any $t > t_0$, by the same approach used in the proof of (39), we have

$$
\begin{aligned}
&\|U(t)\| = M(t) < \min_{ij\in J}\left\{1-a^+_{ij}\eta^+_{ij}\right\} \epsilon e^{\lambda t}, \\
&\|u(t)\| < \epsilon, \\
&\qquad\qquad \text{if } M(t) = \|U(t)\|.
\end{aligned} \tag{40}
$$

On the other hand, if $M(t) > \|U(t)\|$ and $t > t_0$, we can choose $t_0 \le t_3 < t$ such that

$$
M(t_3) = \|U(t_3)\|, \qquad M(s) > \|U(s)\|, \quad \forall s \in (t_3, t], \tag{41}
$$

which, together with (40), yields that

$$
\begin{gathered}
M(t_3) = \|U(t_3)\| < \min_{ij\in J}\left\{1-a^+_{ij}\eta^+_{ij}\right\} \epsilon e^{\lambda t_3}, \\
\|u(t_3)\| < \epsilon.
\end{gathered} \tag{42}
$$

Using a similar argument as in the proof of Case (i), we can show that

$$
M(s) \equiv M(t_3) \ \text{ is a constant}, \quad \forall s \in (t_3, t], \tag{43}
$$

which implies that

$$
\begin{aligned}
\|u(t)\| &\le \max_{ij\in J} \frac{e^{-\lambda t} M(t)}{1-a^+_{ij}\eta^+_{ij}} \\
&= \max_{ij\in J} \frac{e^{-\lambda t} M(t_3)}{1-a^+_{ij}\eta^+_{ij}} \\
&< \max_{ij\in J} \frac{e^{-\lambda t} \min_{ij\in J}\left\{1-a^+_{ij}\eta^+_{ij}\right\} \epsilon e^{\lambda t_3}}{1-a^+_{ij}\eta^+_{ij}} \\
&< \epsilon.
\end{aligned} \tag{44}
$$

In summary, there must exist $N > \max\{t_0, N_0, t_2\}$ such that $\|u(t)\| \le \epsilon$ holds, for all $t > N$. The proof of Lemma 4 is now complete. □

3. Main Results

In this section, we establish some results for the existence, uniqueness, and exponential stability of the almost periodic solution of (3).

Theorem 5. *Suppose that (T_1) and (T_2) are satisfied. Then system (3) has exactly one almost periodic solution $Z^*(t)$. Moreover, $Z^*(t)$ is globally exponentially stable.*

Proof. Let $v(t) = \{v_{ij}(t)\}$ be a solution of system (3) with initial function $\varphi_{ij}^{v}(\cdot)$ satisfying (11), and $(\varphi_{ij}^{v}(\cdot))'$ is bounded continuous on $(-\infty, 0]$.

Set

$$\begin{aligned}\epsilon_{ij,k}(t) = & -\left[a_{ij}(t+t_k) - a_{ij}(t)\right] \\ & \times \int_0^{\infty} h_{ij}(s)\, v_{ij}(t+t_k-s)\, ds \\ & - \sum_{C_{kl}\in N_r(i,j)} \left[C_{ij}^{kl}(t+t_k) - C_{ij}^{kl}(t)\right] \\ & \times \left(f\left(x_{kl}\left(t-\tau(t+t_k)+t_k\right)\right)\right) x_{ij}(t+t_k) \\ & - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t) \\ & \qquad \times \left[f\left(x_{kl}\left(t-\tau(t+t_k)+t_k\right)\right)\right. \\ & \qquad\qquad \left. - f\left(x_{kl}\left(t-\tau(t)+t_k\right)\right)\right] \\ & \qquad \times x_{ij}(t+t_k) \\ & - \sum_{C_{kl}\in N_q(i,j)} \left[B_{ij}^{kl}(t+t_k) - B_{ij}^{kl}(t)\right] \\ & \times \int_0^{\infty} K_{ij}(u)\, g\left(x_{kl}(t+t_k-u)\right) du\, x_{ij}(t+t_k) \\ & + \left[L_{ij}(t+t_k) - L_{ij}(t)\right], \quad ij \in J,\end{aligned} \tag{45}$$

where $\{t_k\}$ is any sequence of real numbers. By Lemma 2, the solution $v(t)$ is bounded and

$$\left|v_{ij}(t)\right| \le \frac{1}{1-a_{ij}^{+}\eta_{ij}^{+}} \left(\frac{L_{ij}}{\delta_{ij}}\right)^{+}, \quad \forall t \in R,\ ij \in J, \tag{46}$$

which implies that the right side of (3) is also bounded, and $v'(t)$ is a bounded function on R. Thus, $v(t)$ is uniformly continuous on R. Then, from the almost periodicity of $a_{ij}, \tau, C_{ij}^{kl}$, and B_{ij}^{kl}, we can select a sequence $\{t_k\} \to +\infty$ such that

$$\begin{aligned}\left|a_{ij}(t+t_k) - a_{ij}(t)\right| &\le \frac{1}{k}, \\ \left|C_{ij}^{kl}(t+t_k) - C_{ij}^{kl}(t)\right| &\le \frac{1}{k}, \\ \left|B_{ij}^{kl}(t+t_k) - B_{ij}^{kl}(t)\right| &\le \frac{1}{k}, \\ \left|\tau(t+t_k) - \tau(t)\right| &\le \frac{1}{k}, \\ \left|\epsilon_{ij,k}(t)\right| &\le \frac{1}{k},\end{aligned} \tag{47}$$

for all $ij, kl \in J$, $t \in R$.

Since $\{v(t+t_k)\}_{k=1}^{+\infty}$ is uniformly bounded and equiuniformly continuous, by the Arzala-Ascoli Lemma and diagonal selection principle, we can choose a subsequence $\{t_{k_j}\}$ of $\{t_k\}$, such that $v(t+t_{k_j})$ (for convenience, we still denote by $v(t+t_k)$) uniformly converges to a continuous function $Z^*(t) = \{x_{ij}^*(t)\}$ on any compact set of R, and

$$\left|x_{ij}^*(t)\right| \le \frac{1}{1-a_{ij}^{+}\eta_{ij}^{+}} \left(\frac{L_{ij}}{\delta_{ij}}\right)^{+}, \quad \forall t \in R,\ ij \in J. \tag{48}$$

Now, we prove that $Z^*(t)$ is a solution of (3). In fact, for any $t > 0$ and $\Delta t \in R$, from (47), we have

$$\begin{aligned} & x_{ij}^*(t+\Delta t) - x_{ij}^*(t) \\ & = \lim_{k\to+\infty} \left[v_{ij}(t+\Delta t+t_k) - v_{ij}(t+t_k)\right] \\ & = \lim_{k\to+\infty} \int_t^{t+\Delta t} \Bigg\{ - a_{ij}(\mu+t_k) \\ & \qquad \times \int_0^{\infty} h_{ij}(s)\, v_{ij}(\mu+t_k-s)\, ds \\ & \qquad - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(\mu+t_k) \\ & \qquad \times f\left(v_{kl}\left(\mu+t_k-\tau(\mu+t_k)\right)\right) \\ & \qquad \times v_{ij}(\mu+t_k) \\ & \qquad - \sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(\mu+t_k) \\ & \qquad\qquad \times \int_0^{\infty} K_{ij}(u)\, g \\ & \qquad\qquad\quad \times \left(v_{kl}(\mu+t_k-u)\right) du \\ & \qquad\qquad\quad \times v_{ij}(\mu+t_k) \\ & \qquad + L_{ij}(\mu+t_k) \Bigg\} d\mu \\ & = \lim_{k\to+\infty} \int_t^{t+\Delta t} \Bigg\{ -a_{ij}(\mu) \int_0^{\infty} h_{ij}(s)\, v_{ij}(\mu+t_k-s)\, ds \\ & \qquad - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(\mu)\, f \\ & \qquad\qquad \times \left(v_{kl}\left(\mu-\tau(\mu)+t_k\right)\right) \\ & \qquad\qquad \times v_{ij}(\mu+t_k) \\ & \qquad - \sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(\mu) \\ & \qquad\qquad \times \int_0^{\infty} K_{ij}(u)\, g\left(v_{kl}(\mu+t_k-u)\right) \\ & \qquad\qquad\quad \times du\, v_{ij}(\mu+t_k) \\ & \qquad + L_{ij}(\mu) + \epsilon_{ij,k}(\mu) \Bigg\} d\mu \end{aligned}$$

$$
\begin{aligned}
&= \int_t^{t+\Delta t} \Big\{ -a_{ij}(\mu) \int_0^\infty h_{ij}(s) x_{ij}^*(\mu - s)\, ds \\
&\qquad - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(\mu) f\left(x_{kl}^*(\mu - \tau(\mu))\right) x_{ij}^*(\mu) \\
&\qquad - \sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(\mu) \\
&\qquad \times \int_0^\infty K_{ij}(u) g\left(x_{kl}^*(\mu - u)\right) du x_{ij}^*(\mu) \\
&\qquad + L_{ij}(\mu) \Big\} d\mu \\
&\quad + \lim_{k\to+\infty} \int_t^{t+\Delta t} \epsilon_{ij,k}(\mu)\, d\mu \\
&= \int_t^{t+\Delta t} \Big\{ -a_{ij}(\mu) \int_0^\infty h_{ij}(s) x_{ij}^*(\mu - s)\, ds \\
&\qquad - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(\mu) f\left(x_{kl}^*(\mu - \tau(\mu))\right) x_{ij}^*(\mu) \\
&\qquad - \sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(\mu) \\
&\qquad\quad \times \int_0^\infty K_{ij}(u) g\left(x_{kl}^*(\mu - u)\right) du x_{ij}^*(\mu) \\
&\qquad + L_{ij}(\mu) \Big\} d\mu,
\end{aligned} \tag{49}
$$

which implies that

$$
\begin{aligned}
\frac{d}{dt}\left\{x_{ij}^*(t)\right\} = &-a_{ij}(t) \int_0^\infty h_{ij}(s) x_{ij}^*(t - s)\, ds \\
&- \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t) f\left(x_{kl}^*(t - \tau(t))\right) x_{ij}^*(t) \\
&- \sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(t) \\
&\qquad \times \int_0^\infty K_{ij}(u) g\left(x_{kl}^*(t - u)\right) du x_{ij}^*(t) \\
&+ L_{ij}(t), \quad ij \in J.
\end{aligned} \tag{50}
$$

Therefore, $Z^*(t)$ is a solution of (3).

Secondly, we prove that $Z^*(t)$ is an almost periodic solution of (3). From Lemma 4, for any $\varepsilon > 0$, there exists $l = l(\varepsilon) > 0$, such that every interval $[\alpha, \alpha + l]$ contains at least one number δ for which there exists $N > 0$ which satisfies

$$
\left|v_{ij}(t+\delta) - v_{ij}(t)\right| \le \varepsilon, \quad \forall t > N,\ ij \in J. \tag{51}
$$

Then, for any fixed $s \in R$, we can find a sufficiently large positive integer $N_1 > N$ such that, for any $k > N_1$,

$$
\begin{gathered}
s + t_k > N, \\
\left|v_{ij}(s + t_k + \delta) - v_{ij}(s + t_k)\right| \le \varepsilon, \quad ij \in J.
\end{gathered} \tag{52}
$$

Let $k \to +\infty$; we obtain

$$
\left|x_{ij}^*(s+\delta) - x_{ij}^*(s)\right| \le \varepsilon, \quad ij \in J, \tag{53}
$$

which implies that $Z^*(t)$ is an almost periodic solution of (3).

Finally, we prove that $Z^*(t)$ is globally exponentially stable.

Let $Z^*(t) = \{x_{ij}^*(t)\}$ be the positive almost periodic solution of system (3) with initial value $\varphi^* = \{\varphi_{ij}^*(t)\}$ and $Z(t) = \{x_{ij}(t)\}$ an arbitrary solution of system (3) with initial value $\varphi = \{\varphi_{ij}(t)\}$, and set $y(t) = \{y_{ij}(t)\} = \{x_{ij}(t) - x_{ij}^*(t)\} = Z(t) - Z^*(t)$. Then

$$
\begin{aligned}
y_{ij}'(t) = &-a_{ij}(t) \int_0^\infty h_{ij}(s) y_{ij}(t-s)\, ds \\
&- \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t) \\
&\qquad \times \left[f\left(x_{kl}(t - \tau(t))\right) x_{ij}(t) \right. \\
&\qquad\quad \left. - f\left(x_{kl}^*(t - \tau(t))\right) x_{ij}^*(t) \right] \\
&- \sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(t) \\
&\qquad \times \left[\int_0^\infty K_{ij}(u) g\left(x_{kl}(t-u)\right) du x_{ij}(t) \right. \\
&\qquad\quad \left. - \int_0^\infty K_{ij}(u) g\left(x_{kl}^*(t-u)\right) du x_{ij}^*(t) \right],
\end{aligned} \tag{54}
$$

which yields

$$
\begin{aligned}
&\frac{d}{dt}\left(e^{\lambda t} y_{ij}(t) - \int_0^\infty h_{ij}(s) \int_{t-s}^t a_{ij}(\theta) e^{\lambda\theta} y_{ij}(\theta)\, d\theta\, ds \right) \\
&\quad = \lambda e^{\lambda t} y_{ij}(t) + e^{\lambda t} y_{ij}'(t) \\
&\qquad - a_{ij}(t) \int_0^\infty h_{ij}(s)\, ds e^{\lambda t} y_{ij}(t) \\
&\qquad + \int_0^\infty h_{ij}(s) a_{ij}(t-s) e^{\lambda(t-s)} y_{ij}(t-s)\, ds \\
&\quad = -\left(a_{ij}(t) \int_0^\infty h_{ij}(s)\, ds - \lambda \right) \\
&\qquad \times \left(e^{\lambda t} y_{ij}(t) - \int_0^\infty h_{ij}(s) \int_{t-s}^t a_{ij}(\theta) e^{\lambda\theta} y_{ij}(\theta)\, d\theta\, ds \right)
\end{aligned}
$$

$$
\begin{aligned}
& -\left(a_{ij}(t)\int_0^\infty h_{ij}(s)\,ds-\lambda\right)\int_0^\infty h_{ij}(s)\\
&\times\int_{t-s}^{t} a_{ij}(\theta)e^{\lambda\theta}y_{ij}(\theta)\,d\theta\,ds\\
&-\int_0^\infty h_{ij}(s)\left[a_{ij}(t)e^{\lambda s}-a_{ij}(t-s)\right]\\
&\times e^{\lambda(t-s)}y_{ij}(t-s)\,ds\\
&+e^{\lambda t}\Bigg\{-\sum_{C_{kl}\in N_r(i,j)}C_{ij}^{kl}(t)\\
&\qquad\times\left[f\left(x_{kl}(t-\tau(t))\right)-f\left(x_{kl}^{*}(t-\tau(t))\right)\right]x_{ij}(t)\\
&\qquad-\sum_{C_{kl}\in N_r(i,j)}C_{ij}^{kl}(t)\,f\left(x_{kl}^{*}(t-\tau(t))\right)\\
&\qquad\times\left[x_{ij}(t)-x_{ij}^{*}(t)\right]\\
&\qquad-\sum_{C_{kl}\in N_q(i,j)}B_{ij}^{kl}(t)\int_0^\infty K_{ij}(u)\\
&\qquad\times\left[g\left(x_{kl}(t-u)\right)-g\left(x_{kl}^{*}(t-u)\right)\right]du\,x_{ij}(t)\\
&\qquad-\sum_{C_{kl}\in N_q(i,j)}B_{ij}^{kl}(t)\int_0^\infty K_{ij}(u)\,g\\
&\qquad\times\left(x_{kl}^{*}(t-u)\right)du\left[x_{ij}(t)-x_{ij}^{*}(t)\right]\Bigg\},
\end{aligned}
\tag{55}
$$

where $ij\in J$.

Let

$$
\begin{aligned}
Y_{ij}(t)=\Bigg|&e^{\lambda t}y_{ij}(t)-\int_0^\infty h_{ij}(s)\\
&\int_{t-s}^{t}a_{ij}(\theta)e^{\lambda\theta}y_{ij}(\theta)\,d\theta\,ds\Bigg|,\quad ij\in J.
\end{aligned}
\tag{56}
$$

We define a positive constant M as follows:

$$
M=\max_{ij\in J}\left\{\sup_{s\in(-\infty,0]}Y_{ij}(s)\right\}.
\tag{57}
$$

Let K be a positive number such that

$$
Y_{ij}(t)\le M<M+1=K,\quad \forall t\in(-\infty,0],\ ij\in J.
\tag{58}
$$

We claim that

$$
Y_{ij}(t)<K,\quad \forall t>0,\ i=1,2,\dots,n.
\tag{59}
$$

Otherwise, there must exist $ij\in J$ and $\varsigma>0$ such that

$$
Y_{ij}(\varsigma)=K,\quad Y_{\tilde{i}\tilde{j}}(t)<K,\quad \forall t\in(-\infty,\varsigma),\ \tilde{i}\tilde{j}\in J.
\tag{60}
$$

It follows that

$$
\begin{aligned}
e^{\lambda t}\left|y_{\tilde{i}\tilde{j}}(t)\right|\le\ &\Bigg|e^{\lambda t}y_{\tilde{i}\tilde{j}}(t)-\int_0^\infty h_{\tilde{i}\tilde{j}}(s)\\
&\times\int_{t-s}^{t}a_{\tilde{i}\tilde{j}}(\theta)e^{\lambda\theta}y_{\tilde{i}\tilde{j}}(\theta)\,d\theta\,ds\Bigg|\\
&+\left|\int_0^\infty h_{\tilde{i}\tilde{j}}(s)\int_{t-s}^{t}a_{\tilde{i}\tilde{j}}(\theta)e^{\lambda\theta}y_{\tilde{i}\tilde{j}}(\theta)\,d\theta\,ds\right|\\
\le\ & K+a_{\tilde{i}\tilde{j}}^{+}\eta_{\tilde{i}\tilde{j}}^{+}\sup_{s\in(-\infty,\varsigma]}e^{\lambda s}\left|y_{\tilde{i}\tilde{j}}(s)\right|,\\
&\forall t\in(-\infty,\varsigma],\ \tilde{i}\tilde{j}\in J.
\end{aligned}
\tag{61}
$$

Consequently, in view of (61) and the fact $a_{\tilde{i}\tilde{j}}^{+}\eta_{\tilde{i}\tilde{j}}^{+}<1(\tilde{i}\tilde{j}\in J)$, we have

$$
\begin{aligned}
e^{\lambda t}\left|y_{\tilde{i}\tilde{j}}(t)\right|&\le\sup_{s\in(-\infty,\varsigma]}e^{\lambda s}\left|y_{\tilde{i}\tilde{j}}(s)\right|\\
&\le\frac{K}{1-a_{\tilde{i}\tilde{j}}^{+}\eta_{\tilde{i}\tilde{j}}^{+}},\quad \forall t\in(-\infty,\varsigma],\ \tilde{i}\tilde{j}\in J.
\end{aligned}
\tag{62}
$$

Calculating the upper left derivative of $Y_{ij}(t)$, together with (13), (55), (60), (62), (T_1), and (T_2), we obtain

$$
\begin{aligned}
0&\le D^{-}Y_{ij}(\varsigma)\\
&\le-\left(a_{ij}(\varsigma)\int_0^\infty h_{ij}(s)\,ds-\lambda\right)Y_{ij}(\varsigma)\\
&\quad+\Bigg|-\left(a_{ij}(\varsigma)\int_0^\infty h_{ij}(s)\,ds-\lambda\right)\\
&\qquad\times\int_0^\infty h_{ij}(s)\int_{\varsigma-s}^{\varsigma}a_{ij}(\theta)e^{\lambda\theta}y_{ij}(\theta)\,d\theta\,ds\\
&\qquad-\int_0^\infty h_{ij}(s)\left[a_{ij}(\varsigma)e^{\lambda s}-a_{ij}(\varsigma-s)\right]\\
&\qquad\times e^{\lambda(\varsigma-s)}y_{ij}(\varsigma-s)\,ds\\
&\qquad+e^{\lambda\varsigma}\Bigg\{-\sum_{C_{kl}\in N_r(i,j)}C_{ij}^{kl}(\varsigma)\\
&\qquad\qquad\times\left[f\left(x_{kl}(\varsigma-\tau(\varsigma))\right)-f\left(x_{kl}^{*}(\varsigma-\tau(\varsigma))\right)\right]\\
&\qquad\qquad\times x_{ij}(\varsigma)\\
&\qquad\qquad-\sum_{C_{kl}\in N_r(i,j)}C_{ij}^{kl}(\varsigma)\,f\left(x_{kl}^{*}(\varsigma-\tau(\varsigma))\right)
\end{aligned}
$$

$$
\begin{aligned}
&\qquad\qquad \times \left[x_{ij}(\varsigma) - x_{ij}^{*}(\varsigma)\right] \\
&\quad - \sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(\varsigma) \\
&\quad \times \int_0^{\infty} K_{ij}(u) \\
&\qquad \times \left[g\left(x_{kl}(\varsigma - u)\right) - g\left(x_{kl}^{*}(\varsigma - u)\right)\right] \\
&\qquad \times du x_{ij}(\varsigma) \\
&\quad - \sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(\varsigma) \\
&\quad \times \int_0^{\infty} K_{ij}(u)\, g\left(x_{kl}^{*}(\varsigma - u)\right) du \\
&\quad \times \left[x_{ij}(\varsigma) - x_{ij}^{*}(\varsigma)\right] \Bigg\} \Bigg| \\
&\le -\left(a_{ij}(\varsigma)\int_0^{\infty} h_{ij}(s)\, ds - \lambda\right) Y_{ij}(\varsigma) \\
&\quad + \left(a_{ij}(\varsigma)\int_0^{\infty} h_{ij}(s)\, ds - \lambda\right) \frac{K}{1 - a_{ij}^{+}\eta_{ij}^{+}} a_{ij}^{+}\eta_{ij}^{+} \\
&\quad + \int_0^{\infty} h_{ij}(s) \left|a_{ij}(\varsigma) e^{\lambda s} - a_{ij}(\varsigma - s)\right| \\
&\qquad \times e^{\lambda(\varsigma - s)} \left|y_{ij}(\varsigma - s)\right| ds \\
&\quad + \sum_{C_{kl}\in N_r(i,j)} \left|C_{ij}^{kl}(\varsigma)\right| \mu_f e^{\lambda\tau(\varsigma)} e^{\lambda(\varsigma - \tau(\varsigma))} \\
&\quad \times \left|y_{kl}(\varsigma - \tau(\varsigma))\right| \left|x_{ij}(\varsigma)\right| \\
&\quad + \sum_{C_{kl}\in N_r(i,j)} \left|C_{ij}^{kl}(\varsigma)\right| M_f e^{\lambda\varsigma} \left|y_{ij}(\varsigma)\right| \\
&\quad + \sum_{C_{kl}\in N_q(i,j)} \left|B_{ij}^{kl}(\varsigma)\right| \int_0^{\infty} e^{\lambda u} \\
&\quad \times \left|K_{ij}(u)\right| \mu_g e^{\lambda(\varsigma - u)} \left|y_{kl}(\varsigma - u)\right| du \left|x_{ij}(\varsigma)\right| \\
&\quad + \sum_{C_{kl}\in N_q(i,j)} \left|B_{ij}^{kl}(\varsigma)\right| \\
&\quad \times \int_0^{\infty} \left|K_{ij}(u)\right| du \\
&\qquad \times M_g e^{\lambda\varsigma} \left|y_{ij}(\varsigma)\right| \\
&\le \Bigg\{ - \Bigg[\left(a_{ij}(\varsigma)\int_0^{\infty} h_{ij}(s)\, ds - \lambda\right)\left(1 - 2a_{ij}^{+}\eta_{ij}^{+}\right) \\
&\qquad - \int_0^{\infty} h_{ij}(s) \left|a_{ij}(\varsigma) e^{\lambda s} - a_{ij}(\varsigma - s)\right| ds \Bigg] \\
&\qquad \times \frac{1}{1 - a_{ij}^{+}\eta_{ij}^{+}} \\
&\qquad + \sum_{C_{kl}\in N_r(i,j)} \left|C_{ij}^{kl}(\varsigma)\right| \mu_f e^{\lambda\tau(\varsigma)} \\
&\qquad \times \frac{1}{1 - a_{kl}^{+}\eta_{kl}^{+}} \frac{1}{1 - a_{ij}^{+}\eta_{ij}^{+}} \left(\frac{L_{ij}}{\delta_{ij}}\right)^{+} \\
&\qquad + \sum_{C_{kl}\in N_r(i,j)} \left|C_{ij}^{kl}(\varsigma)\right| M_f \frac{1}{1 - a_{ij}^{+}\eta_{ij}^{+}} \\
&\qquad + \sum_{C_{kl}\in N_q(i,j)} \left|B_{ij}^{kl}(\varsigma)\right| \int_0^{\infty} e^{\lambda u} \left|K_{ij}(u)\right| du \\
&\qquad \times \mu_g \frac{1}{1 - a_{kl}^{+}\eta_{kl}^{+}} \frac{1}{1 - a_{ij}^{+}\eta_{ij}^{+}} \left(\frac{L_{ij}}{\delta_{ij}}\right)^{+} \\
&\qquad + \sum_{C_{kl}\in N_q(i,j)} \left|B_{ij}^{kl}(\varsigma)\right| \int_0^{\infty} \left|K_{ij}(u)\right| du \\
&\qquad \times M_g \frac{1}{1 - a_{ij}^{+}\eta_{ij}^{+}} \Bigg\} K \\
&< -\eta K \\
&< 0,
\end{aligned}
\tag{63}
$$

which is a contradiction and implies that (59) holds.

Consequently, using a similar argument as in (61)-(62), we know that

$$
\left|y_{ij}(t)\right| e^{\lambda t} \le \frac{K}{1 - a_{ij}^{+}\eta_{ij}^{+}}, \quad \forall t \in R,\ ij \in J. \tag{64}
$$

This completes the proof. □

4. An Example

In this section, we give an example to demonstrate the results obtained in the previous sections.

Example 6. Consider the following SICNNs with continuously distributed delays in the leakage terms:

$$
\begin{aligned}
\frac{dx_{ij}}{dt} = &- a_{ij}(t) \int_0^{\infty} h_{ij}(s)\, x_{ij}(t - s)\, ds \\
&- \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)\, f\left(x_{kl}(t - \tau(t))\right) x_{ij}(t) \\
&- \sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(t) \int_0^{\infty} K_{ij}(u)\, g\left(x_{kl}(t - u)\right) du x_{ij}(t) \\
&+ L_{ij}(t), \quad i, j = 1, 2, 3,
\end{aligned}
\tag{65}
$$

$$\begin{bmatrix} a_{11}(t) & a_{12}(t) & a_{13}(t) \\ a_{21}(t) & a_{22}(t) & a_{23}(t) \\ a_{31}(t) & a_{32}(t) & a_{33}(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 \\ 3 & 1 & 3 \\ 3 & 1 & 3 \end{bmatrix},$$

$$\begin{bmatrix} B_{11}(t) & B_{12}(t) & B_{13}(t) \\ B_{21}(t) & B_{22}(t) & B_{23}(t) \\ B_{31}(t) & B_{32}(t) & B_{33}(t) \end{bmatrix} = \begin{bmatrix} C_{11}(t) & C_{12}(t) & C_{13}(t) \\ C_{21}(t) & C_{22}(t) & C_{23}(t) \\ C_{31}(t) & C_{32}(t) & C_{33}(t) \end{bmatrix} = \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 0.2 & 0 & 0.2 \\ 0.1 & 0.2 & 0.1 \end{bmatrix},$$

$$\begin{bmatrix} L_{11}(t) & L_{12}(t) & L_{13}(t) \\ L_{21}(t) & L_{22}(t) & L_{23}(t) \\ L_{31}(t) & L_{32}(t) & L_{33}(t) \end{bmatrix} = 0.05 \times \begin{bmatrix} 0.7+0.24\sin^2\sqrt{2}t & 0.41+0.5\cos^2 t & 0.74+0.2\sin^2 t \\ 0.61+0.2\cos^2 t & 0.67+0.2\sin^2 t & 0.75+0.2\sin^2 t \\ 0.59+0.4\cos^4 t & 0.5+0.41\sin^2 t & 0.76+0.2\cos^2 t \end{bmatrix}. \tag{66}$$

Set $\lambda = 0.001$, $\eta = 0.05$, $r = q = 1$, $h_{ij}(s) = (1/5)e^{-s^2}$, $K_{ij}(u) = |\sin u|e^{-u}$, $i = 1, 2$, and 3. $j = 1, 2$, and 3, and $f(x) = g(x) = (1/4000)(|x-1| - |x+1|)$, $\tau(t) = (1/100)\sin^2 t$; clearly, $M_f = M_g = 0.0005$, $\mu_f = \mu_g = 0.0005$, $\sum_{C_{kl}\in N_1(1,1)} C_{11}^{kl} = \sum_{C_{kl}\in N_1(1,1)} B_{11}^{kl} = 0.5$, $\sum_{C_{kl}\in N_1(1,2)} C_{12}^{kl} = \sum_{C_{kl}\in N_1(1,2)} B_{12}^{kl} = 0.8$, $\sum_{C_{kl}\in N_1(1,3)} C_{13}^{kl} = \sum_{C_{kl}\in N_1(1,3)} B_{13}^{kl} = 0.5$, $\sum_{C_{kl}\in N_1(2,1)} C_{21}^{kl} = \sum_{C_{kl}\in N_1(2,1)} B_{21}^{kl} = 0.8$, $\sum_{C_{kl}\in N_1(2,2)} C_{22}^{kl} = \sum_{C_{kl}\in N_1(2,2)} B_{22}^{kl} = 1.2$, $\sum_{C_{kl}\in N_1(2,3)} C_{23}^{kl} = \sum_{C_{kl}\in N_1(2,3)} B_{23}^{kl} = 0.8$, $\sum_{C_{kl}\in N_1(3,1)} C_{31}^{kl} = \sum_{C_{kl}\in N_1(3,1)} B_{31}^{kl} = 0.5$, $\sum_{C_{kl}\in N_1(3,2)} C_{32}^{kl} = \sum_{C_{kl}\in N_1(3,2)} B_{32}^{kl} = 0.8$, $\sum_{C_{kl}\in N_1(3,3)} C_{33}^{kl} = \sum_{C_{kl}\in N_1(3,3)} B_{33}^{kl} = 0.5$, $1 \le a_{ij}^+ \le 3$, $\eta_{ij}^+ = 0.1$, $a_{ij}^+\eta_{ij}^+ \le 0.3 < 1$, and $ij \in J = \{11, 12, 13, 21, 22, 23, 31, 32, 33\}$.

Consider,

$$\begin{aligned}
\min_{ij\in J} \delta_{ij}(t) = \min_{ij\in J} \Bigg\{ & \left[a_{ij}(t) \int_0^\infty h_{ij}(s)\,ds \left(1 - 2a_{ij}^+\eta_{ij}^+\right) - \int_0^\infty h_{ij}(s) \left| a_{ij}(t) - a_{ij}(t-s) \right| ds \right] \\
& - \sum_{C_{kl}\in N_r(i,j)} \left| C_{ij}^{kl}(t) \right| M_f - \sum_{C_{kl}\in N_q(i,j)} \left| B_{ij}^{kl}(t) \right| \int_0^\infty \left| K_{ij}(u) \right| du M_g \Bigg\} \\
& \times \frac{1}{1 - a_{ij}^+\eta_{ij}^+} \\
> 0.0715 > 0, \quad & \forall t \ge 0.
\end{aligned}$$

$$\begin{aligned}
\max_{ij\in J} \Bigg\{ & -\left[\left(a_{ij}(t) \int_0^\infty h_{ij}(s)\,ds - \lambda \right) \left(1 - 2a_{ij}^+\eta_{ij}^+\right) - \int_0^\infty h_{ij}(s) \left| a_{ij}(t) e^{\lambda s} - a_{ij}(t-s) \right| ds \right] \\
& \times \frac{1}{1 - a_{ij}^+\eta_{ij}^+} \\
& + \sum_{C_{kl}\in N_r(i,j)} \left| C_{ij}^{kl}(t) \right| \mu_f e^{\lambda\tau(t)} \times \frac{1}{1 - a_{kl}^+\eta_{kl}^+} \frac{1}{1 - a_{ij}^+\eta_{ij}^+} \left(\frac{L_{ij}}{\delta_{ij}} \right)^+ \\
& + \sum_{C_{kl}\in N_r(i,j)} \left| C_{ij}^{kl}(t) \right| M_f \frac{1}{1 - a_{ij}^+\eta_{ij}^+} \\
& + \sum_{C_{kl}\in N_q(i,j)} \left| B_{ij}^{kl}(t) \right| \int_0^\infty e^{\lambda u} \left| K_{ij}(u) \right| du \times \mu_g \frac{1}{1 - a_{kl}^+\eta_{kl}^+} \frac{1}{1 - a_{ij}^+\eta_{ij}^+} \left(\frac{L_{ij}}{\delta_{ij}} \right)^+ \\
& + \sum_{C_{kl}\in N_q(i,j)} \left| B_{ij}^{kl}(t) \right| \int_0^\infty \left| K_{ij}(u) \right| du \times M_g \frac{1}{1 - a_{ij}^+\eta_{ij}^+} \Bigg\} \\
< -0.0556 < -0.05 = -\eta, \quad & \forall t \ge 0.
\end{aligned} \tag{67}$$

It follows that system (65) satisfies all the conditions in Theorem 5. Hence, system (65) has exactly one almost periodic solution. Moreover, the almost periodic solution is globally exponentially stable.

Remark 7. Since [1–11] only dealt with SICNNs without leakage delays, [12–21] give no opinions about the problem of almost periodic solutions for SICNNs with leakage delays. One can observe that all the results in these literature and the references therein can not be applicable to prove the existence and exponential stability of almost periodic solutions for SICNNs (65).

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