Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2015, Article ID 494162, 12 pages
http://dx.doi.org/10.1155/2015/494162

Research Article

Condition-Based Maintenance Strategy for Production Systems Generating Environmental Damage

L. Tlili,[1] M. Radhoui,[2] and A. Chelbi[1]

[1] *Ecole Nationale Supérieure d'Ingénieurs de Tunis, Université de Tunis, CEREP, Tunis, Tunisia*
[2] *Ecole Nationale d'Ingénieurs de Carthage, Université de Carthage, CEREP, Tunis, Tunisia*

Correspondence should be addressed to A. Chelbi; anis.chelbi@planet.tn

Received 21 December 2014; Revised 10 March 2015; Accepted 12 March 2015

Academic Editor: Elena Benvenuti

We consider production systems which generate damage to environment as they get older and degrade. The system is submitted to inspections to assess the generated environmental damage. The inspections can be periodic or nonperiodic. In case an inspection reveals that the environmental degradation level has exceeded the critical level U, the system is considered in an advanced deterioration state and will have generated significant environmental damage. A corrective maintenance action is then performed to renew the system and clean the environment and a penalty has to be paid. In order to prevent such an undesirable situation, a lower threshold level L is considered to trigger a preventive maintenance action to bring back the system to a state as good as new at a lower cost and without paying the penalty. Two inspection policies are considered (periodic and nonperiodic). For each one of them, a mathematical model and a numerical procedure are developed to determine simultaneously the preventive maintenance (PM) threshold L^* and the inspection sequence which minimize the average long-run cost per time unit. Numerical calculations are performed to illustrate the proposed maintenance policies and highlight their main characteristics with respect to relevant input parameters.

1. Introduction

Because of the degradation of the environment in the world and the growing public pressure, governments imposed several constraining and penalizing measures to companies whose production processes potentially generate any form of environmental damage. These measures have been taken globally (Kyoto Protocol in 1997) and nationally. Moreover, the companies' greenhouse gas emissions (primarily CO_2) are severely limited and penalized financially. These constraints led decision-makers to establish and implement effective environmental sustainability policies considering environmental and cost criteria.

Numerous efforts have emerged to address the environment issues related to environmental impact assessment within the company. A set of qualitative tools such as FMEA [1], Fault Tree Analysis (FTA) [2], and HAZOP [3] are used as well as quantitative models [4–6]. It is clearly established that preventive maintenance plays a crucial role in meeting the environmental legislation requirements by limiting and preventing equipment degradation which may lead to environmental damage. Martorell et al. [7] proposed integrated multicriteria decision-making (IMCDM) in order to achieve better scores in reliability, availability, maintainability, safety, and cost (RAMS+C) at nuclear power plants. The proposed methodology integrates, among others, the risk to environment. The IMCDM is formulated as a multiobjective optimization where the decision variables are preventive maintenance interval, surveillance test interval, and maximum allowed outage time. Vassiliadis and Pistikopoulos [8] developed an optimization framework which takes into account the environmental risks and the operability characteristics at the early stage of process design. This framework permits the identification of optimal preventive maintenance schedules.

The excess of environmental damages generated by production systems is, in numerous situations, caused indirectly by the deterioration of those systems (e.g., degradation and/or corrosion yielding leakages, wear causing excess of energy consumption, etc.). For instance, in a nuclear power plant, the degradation of the mechanical shaft seal of the refrigeration compressor induces toxic refrigerant leakages [9]. Yan and Hua [10] showed that the degradation of machine tools causes an increase of energy consumption which can be converted to carbon emission by referring to the standard developed by the Climate Registry.

Therefore, when it is technically possible, condition-based monitoring, repair, and maintenance are the most appropriate activities to be adopted. It allows monitoring the degradation level and the resulting environmental damage in order to take the appropriate preventive actions and limit the risk of penalties and sometimes catastrophes. In the literature, two approaches are proposed to monitor the degradation: continuous monitoring [11, 12] and inspection [13]. In many situations in practice, the continuous monitoring of the environmental damage generated by production systems is costly and sometimes difficult to perform (e.g., gas emissions, wastes, etc.). The sequential inspection can be more effective. It is widely used. Numerous efforts have been conducted to deal with the determination of inspection sequences for a given critical threshold of system deterioration, optimizing a certain objective like maintenance total cost, or system's availability [14]. Chelbi and Ait-Kadi [15] studied a system subject to deterioration. They determined optimal inspection sequences that minimize the expected total cost per time unit over an infinite time span. Castanier et al. [16] introduced a condition-based maintenance policy for a repairable system subject to a continuous-state gradual deterioration monitored by sequential nonperiodic inspections. They propose availability and cost models where the inspection sequence is derived.

Golmakani and Fattahipour [17] proposed a cost-effective age-based inspection scheme (with nonconstant inspection intervals) for a single unit system. Their model is based on the control limit policy proposed by Makis and Jardine [18] to determine the optimal replacement threshold.

Recently, Chouikhi et al. [9] developed a maintenance optimization model taking into account environmental deterioration. They considered a single-unit system subject to random deterioration which impacts the quality of the environment. The inspection sequence is optimized in order to minimize the long-run average maintenance cost.

To make condition-based maintenance more effective, both the inspection sequence and the alarm threshold level of degradation need to be optimized. In this context, Grall et al. [19] studied a system subject to a random deterioration process. They developed a model based on a stationary process to determine both the preventive maintenance threshold and inspection dates that minimize the average long-run cost rate. Optimization of alarm threshold and sequential inspection scheme were also the aim of the work of Jiang [20]. They developed an algorithm to that purpose.

In the same context of condition-based policies optimizing alarm threshold levels and inspection schedules, we focus in this paper on production systems whose degradation generates directly (or indirectly) environmental damage. A condition-based maintenance strategy is proposed for such systems considering two threshold levels related to the amount of environmental damage: a critical one, U, that is known and which yields a significant penalty in case it is exceeded and a lower one, L, to be determined. In case inspection reveals the latter is exceeded, a preventive maintenance action is undertaken avoiding the penalty. We propose a new modeling approach based on the fact that the degradation process is modeled by the Wiener process (discussed in Section 2). Thus, the first hitting time and the remaining useful life of the system conditional to its environmental degradation level are considered. Two types of inspection policy, periodic and nonperiodic, can be used to reveal the level of degradation. We develop a mathematical model and a numerical procedure for each policy. The objective for both strategies is to determine the threshold level L^* and the inspection sequence which minimize the total expected cost per time unit over an infinite time horizon.

The remainder of this paper is organized as follows. The main notations used in the mathematical model are presented in Notations. Other notations will be introduced throughout the following sections. The next section is devoted to the modeling of the environmental degradation process. The detailed problem description and the definition of the assumptions are given in Section 3. The fourth section is dedicated to the modeling of the periodic inspection policy, and an optimization algorithm is developed to find the optimal inspection period and the PM threshold level. In Section 5, we extend the model presented in Section 4 considering nonperiodic inspections. We develop a numerical procedure allowing the generation of a nearly optimal solution and we discuss obtained numerical results. Concluding remarks together with some indications about extensions currently under consideration are provided in the last section.

2. Analysis of the Degradation Process

Stochastic processes are appropriate to model the degradation process involving independent increments. Many stochastic processes have been studied in the literature (for more details see [21, 22]), that is, Gamma process [23] and Wiener process [24].

The Wiener process (also called Brownian motion process) had firstly been used to model the irregular motion of the pollen particles floating in water. The mathematical theory of Brownian motion was discussed in detail in Wiener's dissertation in 1918 and in papers that followed. The theory was completed by Lévy, Ito, McKean, and others [25]. Moreover, the Wiener process has been adopted for many applications, for example, finance [26], wind energy [27], electrical devices [28], demography, linguistics, employment service, and chemical reactions [29]. In addition, it has been found useful to analyze degradation data [30, 31].

The Wiener process as a degradation model is based on the consideration that the degradation increment in an infinitesimal time interval might be viewed as an additive superposition of a large number of small external effects [32]. Moreover, it is flexible in incorporating random effects and explanatory variables that take in consideration the heterogeneities commonly observed in degradation problems. It is typically used for modeling degradation processes in a random environment and where the degradation increases linearly with time.

Suppose that the degradation process, $X(t)$, $t > 0$, obeys a Wiener process and is written as

$$X(t) = \lambda t + \sigma W(t), \tag{1}$$

where λ is the drift coefficient, $\sigma > 0$ is the diffusion coefficient, and $W(t)$ is the standard Brownian motion.

$X(t)$ has the following properties [33].

(a) $X(0) = 0$ almost surely.

(b) For any time sequence t_i, $i = 1, 2, \ldots, n$, with $0 < t_1 < t_2 < \cdots < t_n$, the random increments $X(t_1), X(t_2 - t_1), \ldots, X(t_n - t_{n-1})$, where $X(t_{j-1} - t_j) = X(t_{j-1}) - X(t_j)$, are independent, and any $X(t) - X(r)$ $(t > 0, r > 0)$ follows a normal distribution $N(0, \sigma^2|t-r|)$.

(c) The paths of $X(t)$ are continuous with probability one.

The Wiener process is used in modeling degradation processes and has shown some advantages regarding the mathematical properties. As an example, the inverse Gaussian (IG) distribution is used to formulate analytically the first hitting process of the Wiener process when the degradation path is monotonic and gradual [30].

The failure time is defined as the first hitting time (FHT) which is the time from present time to the instant T_U at which the degradation first hits a critical level U. Based on the concept of FHT, the failure time can be defined as

$$T_U = \inf\{t \geq 0 \mid X(t) \geq U\}. \tag{2}$$

Furthermore, the FHT or failure time of the process to a threshold, U, follows the inverse Gaussian distribution with a pdf [24]:

$$f_{T_U}(t) = \frac{U}{\sqrt{2\pi t^3 \sigma^2}} \exp\left(-\frac{(U-\lambda t)^2}{2\sigma^2 t}\right) \tag{3}$$

and the cdf is given by [24]

$$F_{T_U}(t) = \Phi\left(\frac{-U+\lambda t}{\sigma\sqrt{t}}\right) + \exp\left(\frac{2\lambda U}{\sigma^2}\right)\Phi\left(\frac{-U-\lambda t}{\sigma\sqrt{t}}\right), \tag{4}$$

where Φ denotes the standard normal cdf.

We define the remaining useful life (RUL) of a deteriorating system associated with the degradation process given that the degradation level $X(t)$ is L, as

$$T_s = T_U - T_L = \inf\{s \geq 0 \mid X(t+s) \geq U\} \quad \text{if } L < U; \tag{5}$$

otherwise $T_s = 0$.

Based on property (b) of the Wiener process, the degradation increments are independent; hence, if $L < U$, T_s can be written as follows:

$$\begin{aligned} T_s &= \inf\{s \geq 0 \mid X(t) + X(s) \geq U\} \\ &= \inf\{s \geq 0 \mid X(s) \geq U - L\}. \end{aligned} \tag{6}$$

Thus, the remaining useful life (RUL) distribution is derived utilizing the property that the sum of Gaussian variables is Gaussian again; then the pdf of the RUL of T_s from time zero is expressed as follows [34, 35]:

$$f_{T_U-T_L}(t) = \frac{U-L}{\sqrt{2\pi t^3\sigma^2}} \exp\left(-\frac{(U-L-\lambda t)^2}{2\sigma^2 t}\right). \tag{7}$$

And the cdf of the RUL is given by

$$\begin{aligned} F_{T_U-T_L}(t) &= \Phi\left(\frac{-(U-L)+\lambda t}{\sigma\sqrt{t}}\right) + \exp\left(\frac{2\lambda(U-L)}{\sigma^2}\right)\Phi \\ &\quad \cdot \left(\frac{-(U-L)-\lambda t}{\sigma\sqrt{t}}\right). \end{aligned} \tag{8}$$

This (RUL) distribution will be used in the cost model which will be developed below.

3. Problem Description

We consider a production system assimilated to a single unit that causes a random amount of damage to environment as it gets older and degrades. It is assumed that the environmental degradation process is modeled by Wiener process. The value of the environmental degradation level (damage) can be known (measured) only by inspection. Hence, the system is submitted to inspections to assess the generated environmental damage. In case an inspection reveals that the environmental degradation level has exceeded a critical level U (generally known and fixed by legislation), the system is considered in advanced deterioration state and will have generated significant environmental damage subject to a penalty incurred from the instant at which the critical level U has been exceeded. A CM action is then performed to renew the system and clean the environment and the resulting penalty has to be paid. In order to lower the chances to be in such an undesirable situation, a lower threshold level L has to be considered to trigger a PM to bring back the system to a state as good as new at a lower cost and without paying the penalty (Figure 1). In case inspection reveals that the environmental degradation level is lower than L, no maintenance action is done.

The following working assumptions are considered.

(i) The degradation of the system induces the degradation of the environment.

(ii) Inspections are perfect and their duration is negligible.

(iii) The durations of PM and CM actions are also negligible.

(iv) After each inspection, only one of the three following events is possible: do nothing, perform a PM action, or perform a CM action.

(v) Both PM and CM actions renew or bring back the system to a state as good as new.

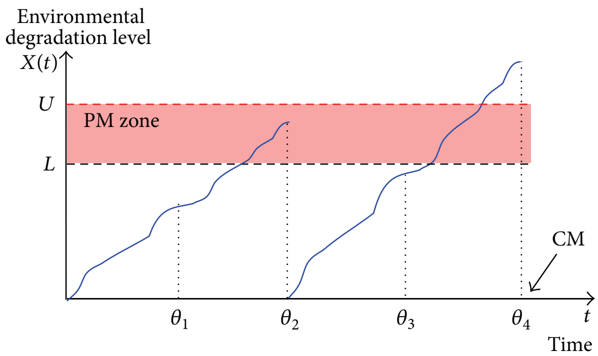

FIGURE 1: The condition-based maintenance strategy.

(vi) All costs related to maintenance, inspection, and environmental penalty are considered as average costs. They are known and constant.

(vii) The resources necessary for the achievement of maintenance actions are always available.

4. Periodic Inspection Optimization Model

In this section, we assume that periodic inspections are performed at times $i\tau$, $i = 1, 2, \ldots$. Our objective is to determine simultaneously the optimal PM threshold level L^* as well as the interinspection period τ^* that minimizes the average long-run cost rate.

By using classical renewal arguments, the total average cost per time unit can be expressed over a renewal cycle S defined as the period between consecutive maintenance actions. In fact, as previously stated, the system is considered to be as good as new after both maintenance actions (PM or CM).

Hence, the expression of the long-run average cost per unit of time is given by

$$EC^1(L,\tau) = \frac{E\left[C_T^1(L,\tau)\right]}{E\left[T_{\text{cycle}}^1(L,\tau)\right]}, \tag{9}$$

where $E[C_T^1]$ represents the expected total cost incurred within a cycle and $E[T_{\text{cycle}}^1]$ is the expected cycle length.

The following analysis will lead to the expression of the average long-run cost per time unit.

4.1. Expected Total Cost within a Cycle. The expected total cost during a cycle $E[C_T^1]$ can be expressed as follows:

$$E\left[C_T^1(L,\tau)\right] = C_c P_c^1 + C_p P_p^1 + C_i E\left[N^1\right] + C_{\text{env}} E\left[T_\zeta^1\right]. \tag{10}$$

The three first terms are the PM and CM and inspection average costs during a cycle, respectively. The last term represents the average penalty cost related to excess environmental damage during a cycle.

The analytical expressions of these different components of the total expected cost are developed below.

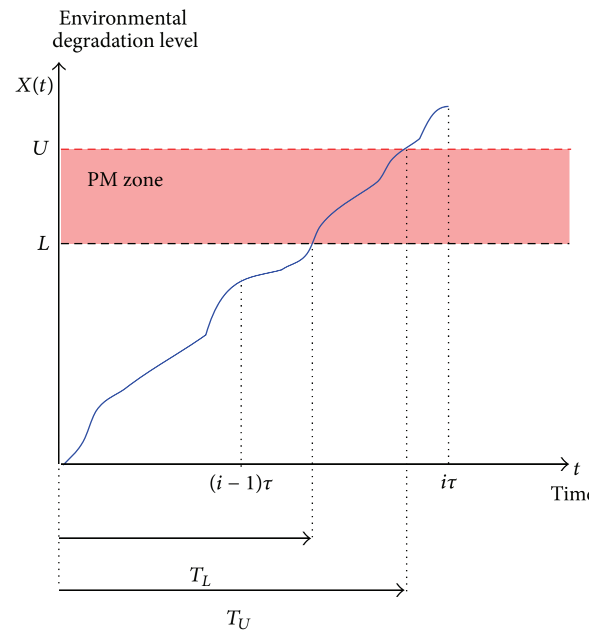

FIGURE 2: Example of cycle that ends by CM action.

(a) The Probability P_c^1 That the Cycle Ends with a Corrective Maintenance Action. Recall that a CM action is undertaken in case following an inspection, the measured amount of environment degradation is found higher than the critical known level U. Let us consider any inspection interval $](i-1)\tau, i\tau]$. The probability $\text{PR}_c^1(i)$ that this inspection interval ends with a CM action can be expressed as follows (Figure 2):

$$\begin{aligned}\text{PR}_c^1(i) &= P\left\{(i-1)\tau < T_L < T_U \le i\tau\right\} \\ &= \int_{(i-1)\tau}^{i\tau} F_{T_U - T_L}(i\tau - y) f_{T_L}(y)\,dy.\end{aligned} \tag{11}$$

Hence, P_c^1 is given by

$$\begin{aligned}P_c^1 &= \sum_{i=1}^{\infty} \text{PR}_c^1(i) \\ &= \sum_{i=1}^{\infty} \int_{(i-1)\tau}^{i\tau} F_{T_U - T_L}(i\tau - y) f_{T_L}(y)\,dy.\end{aligned} \tag{12}$$

(b) The Probability P_p^1 That the Cycle Ends with a Preventive Maintenance Action. The PM action is performed whenever the measured environmental degradation $X(t)$ is found between L and U ($L \le X(t) < U$) (Figure 3). Therefore, the probability $\text{PR}_p^1(i)$ that the inspection interval $](i-1)\tau, i\tau]$ ends with a PM action can be expressed as

$$\begin{aligned}\text{PR}_p^1(i) &= P\left\{(i-1)\tau < T_L \le i\tau < T_U\right\} \\ &= \int_{(i-1)\tau}^{i\tau} \left(1 - F_{T_U - T_L}(i\tau - y)\right) f_{T_L}(y)\,dy.\end{aligned} \tag{13}$$

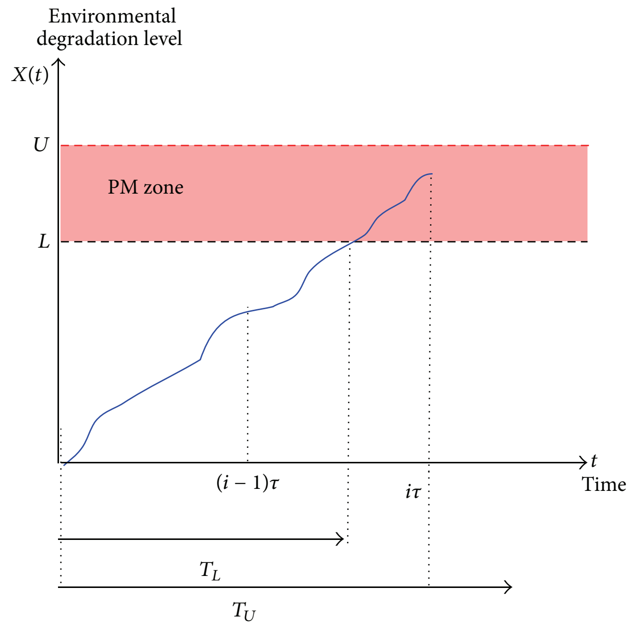

Figure 3: Example of cycle that ends by PM action.

Thus,

$$P_p^1 = \sum_{i=1}^{\infty} \mathrm{PR}_p^1(i) = \sum_{i=1}^{\infty} \int_{(i-1)\tau}^{i\tau} \left(1 - F_{T_U - T_L}(i\tau - y)\right) f_{T_L}(y)\,dy. \tag{14}$$

(c) The Expected Number of Inspections during a Cycle. The expected number of inspections during a cycle, $E[N^1]$, is given by

$$E\left[N^1\right] = \sum_{i=1}^{\infty} iP\{N = i\}, \tag{15}$$

where $P\{N = i\}$ is the probability of having a total of i inspections within a cycle.

Performing i inspections within a cycle is equivalent to the fact that at the last inspection which is carried out at $i\tau$ (ith inspection), the observed environmental degradation level has reached either the critical level U or the PM threshold level L. Consequently, $P\{N = i\}$ is given by

$$P\{N = i\} = \mathrm{PR}_c^1(i) + \mathrm{PR}_p^1(i). \tag{16}$$

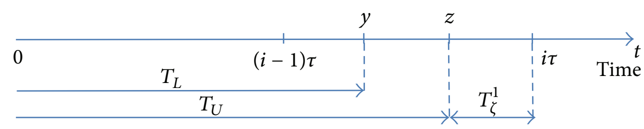

Figure 4: The duration of generation of excess amount of damage.

As a result, using (11) and (13), we have

$$\begin{aligned} E\left[N^1\right] &= \sum_{i=1}^{\infty} i \int_{(i-1)\tau}^{i\tau} F_{T_U - T_L}(i\tau - y) f_{T_L}(y)\,dy \\ &\quad + \sum_{i=1}^{\infty} i \int_{(i-1)\tau}^{i\tau} \left(1 - F_{T_U - T_L}(i\tau - y)\right) f_{T_L}(y)\,dy, \\ E\left[N^1\right] &= \sum_{i=1}^{\infty} i \int_{(i-1)\tau}^{i\tau} f_{T_L}(y)\,dy. \end{aligned} \tag{17}$$

Then,

$$E\left[N^1\right] = \sum_{i=1}^{\infty} i \left(F_{T_L}(i\tau) - F_{T_L}((i-1)\tau)\right). \tag{18}$$

(d) The Average Time $E[T_\zeta^1]$ of Generation of Excess Amount of Environmental Damage during a Cycle. Let us consider the inspection instant $i\tau$, $i = 1, 2, \ldots$. If $i\tau \geq T_U$, the system will have generated an excess amount of damage to environment during a period $T_\zeta^1(i) = i\tau - T_U$. Let $E[T_\zeta^1]$ denote the average time between the instant when the amount of environment degradation exceeds the critical level U and the moment of the inspection that reveals it (Figure 4).

$E[T_\zeta^1]$ is expressed as follows:

$$\begin{aligned} E\left[T_\zeta^1\right] &= \sum_{i=1}^{\infty} P\left\{(i-1)\tau < T_L < T_U \leq i\tau \mid T_\zeta = i\tau - T_U\right\} \\ &= \sum_{i=1}^{\infty} \int_{(i-1)\tau}^{i\tau} \int_{(i-1)\tau}^{z} (i\tau - z) F_{T_U - T_L}(z - y) f_{T_L}(y)\,dy\,dz. \end{aligned} \tag{19}$$

4.2. The Expected Renewal Cycle Length. The cycle is considered to be the interval between consecutive maintenance activities either PM or CM. Therefore, the expected cycle length is given as follows:

$$E\left[T_{\text{cycle}}^1(L, \tau)\right] = \sum_{i=1}^{\infty} i\tau \mathrm{PR}_c^1(i) + \sum_{i=1}^{\infty} i\tau \mathrm{PR}_p^1(i). \tag{20}$$

Using (11) and (13), we obtain the following expression:

$$E\left[T_{\text{cycle}}^1(L, \tau)\right] = \sum_{i=1}^{\infty} i\tau \int_{(i-1)\tau}^{i\tau} f_{T_L}(y)\,dy, \tag{21}$$

$$E\left[T_{\text{cycle}}^1(L, \tau)\right] = \sum_{i=1}^{\infty} i\tau \left(F_{T_L}(i\tau) - F_{T_L}((i-1)\tau)\right). \tag{22}$$

TABLE 1: Input data.

C_c (\$)	C_p (\$)	C_i (\$)	C_{env} (\$/week)	U (Tons)
900	500	100	10000	10

Hence, the average long-run cost rate function $EC^1(L,\tau)$ can be obtained by combining (12), (14), (18), (19), and (22).

It is expressed below as a function of the decision variables which are the inspection period τ and the PM threshold value L:

$$
\begin{aligned}
EC^1&(L,\tau)\\
&= \Bigg[C_p\sum_{i=1}^{\infty}\left(F_{T_L}(i\tau)-F_{T_L}((i-1)\tau)\right)\\
&\quad+\left(C_c-C_p\right)\sum_{i=1}^{\infty}\int_{(i-1)\tau}^{i\tau}F_{T_U-T_L}(i\tau-y)\,f_{T_L}(y)\,dy\\
&\quad+C_i\sum_{i=1}^{\infty}i\left(F_{T_L}(i\tau)-F_{T_L}((i-1)\tau)\right)+C_{env}\\
&\quad\cdot\sum_{i=1}^{\infty}\int_{(i-1)\tau}^{i\tau}\int_{(i-1)\tau}^{z}(i\tau-z)\,F_{T_U-T_L}(z-y)\,f_{T_L}(y)\,dy\,dz\Bigg]\\
&\quad\cdot\left(\sum_{i=1}^{\infty}i\tau\left(F_{T_L}(i\tau)-F_{T_L}((i-1)\tau)\right)\right)^{-1}.
\end{aligned}
\tag{23}
$$

The following simple numerical iterative procedure has been used to obtain the optimal condition-based maintenance policy. This procedure looks for the optimum values of the PM threshold level L^* and the interinspection interval τ^* for $L\in(0,U]$ and $\tau\in[\tau_{min},\tau_{max}]$.

4.3. Numerical Example. Consider a production system subject to continuous CO_2 gas emissions (it could be any other source of environmental damage). We suppose that the amount of environmental damage generated $X(t)$ follows a Wiener process with drift coefficient λ and diffusion coefficient σ.

The following input parameters of the problem have been arbitrarily chosen. The parameters of the process's pdf are $\lambda = 1.3$ and $\sigma = 0.35$. The costs of maintenance actions, inspection cost, environmental penalty cost, and the critical level of damage to environment are presented in Table 1.

Hence, the pdf of the FHT for the critical level L (3) is computed by

$$f_{T_L}(t)=\frac{L}{\sqrt{0.09\pi t^3}}\exp\left(-\frac{(L-t)^2}{0.09t}\right). \tag{24}$$

The cdf (4) is given by

$$F_{T_L}(t)=\Phi\left(\frac{-L+t}{0.15\sqrt{t}}\right)+\exp\left(\frac{2L}{0.045}\right)\Phi\left(\frac{-L-t}{0.15\sqrt{t}}\right). \tag{25}$$

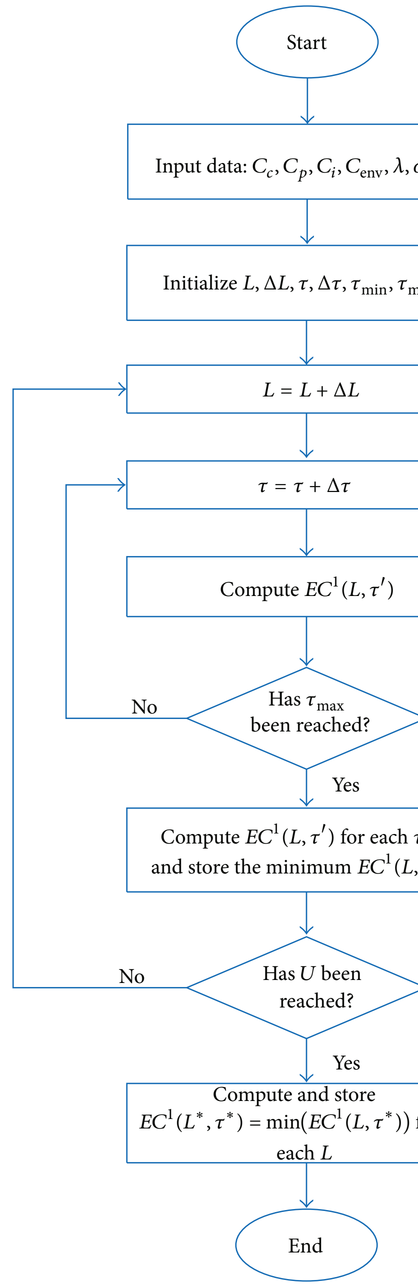

FIGURE 5: Numerical procedure for the periodic strategy.

The cdf of the remaining useful life (RUL) $T_U - T_L$ (8) is expressed as follows:

$$
\begin{aligned}
F_{T_U-T_L}(t)&=\Phi\left(\frac{10-L-t}{0.15\sqrt{t}}\right)+\exp\left(\frac{20-2L}{0.045}\right)\Phi\\
&\quad\cdot\left(\frac{-(10-L)-t}{0.15\sqrt{t}}\right).
\end{aligned}
\tag{26}
$$

Given the above input parameters, we applied the numerical procedure of Figure 5 with $\Delta L = 1$ ton of CO_2 gas emissions, $\Delta\tau = 1$ week, $\tau_{min} = 1$ week, and $\tau_{max} = 12$ weeks. The obtained optimal solution is presented in Table 2.

Table 2: The obtained optimal solution for periodic inspection policy.

L^* (Ton)	τ^* (week)	P_c^1 (%)	CM cost ($)	P_p^1 (%)	PM cost ($)	$E[N^1]$	Inspection cost ($)	$E[T_\xi^1]$	Environmental penalty cost ($)	$E[T_{\text{cycle}}^1]$	EC^1 ($/week)
2	**7**	17.76	160	82.24	412	1.08	108	0.02	200	7.1	123.94

Table 3: Effect of C_{env}.

C_{env}	τ^*	L^*	P_c^1 (%)	CM cost ($)	P_p^1 (%)	PM cost ($)	$E[N^1]$	Inspection cost ($)	$E[T_\xi^1]$	Environmental penalty cost ($)	$E[T_{\text{cycle}}^1]$ (week)	EC^1 ($/week)
0	**7**	**9**	61.43	553	38.57	193	1.44	144	7.1	0	10.06	88.47
10000	**7**	**2**	17.76	160	82.24	412	1.08	108	0.02	200	7.1	123.94
100000	**5**	**2**	0.43	3.87	99.57	498	1.00	100	$7\,10^{-3}$	700	5	260.37

Table 4: Effect of C_i.

C_i	τ^*	L^*	P_c^1 (%)	CM cost ($)	P_p^1 (%)	PM cost ($)	$E[N^1]$	Inspection cost ($)	$E[T_\xi^1]$	Environmental penalty cost ($)	$E[T_{\text{cycle}}^1]$ (week)	EC^1 ($/week)
0	**1**	**8**	0.21	2	99.79	499	6.65	0	$8\,10^{-6}$	$8\,10^{-2}$	6.65	75.35
100	**7**	**2**	17.76	160	82.24	412	1.08	108	0.02	200	7.1	123.94
1000	**9**	**2**	95.25	857	4.75	24	1.00	1000	1.13	11300	9	1464.56

From the results presented in Table 2, the maintenance crew has to inspect the system every 7 weeks. If an inspection reveals that the environmental damage level is between 2 and 10 tons of CO_2, the maintenance crew has to undergo immediately a preventive maintenance action. The total average cost per time unit of this strategy is 123.94 $/week. The optimal strategy preconizes to perform more PM actions (P_p^1 = 82.24%) than CM actions (P_c^1 = 17.76%).

In what follows, while keeping the original combination of input parameters, the unitary cost of environmental penalty C_{env}, the inspection cost C_i, and the CM actions costs are varied, respectively, in order to investigate their influence on the optimal solution. The obtained results are presented in Tables 3, 4, and 5.

From Table 3, we can notice that as the environmental penalty cost increases, the inspection period remains rather constant whereas the PM threshold level is reduced enabling a PM action to be taken earlier to reduce the probability to generate excess CO_2 emissions and be penalized. These results confirm the relevance of adopting a condition-based maintenance policy.

From Table 4, we can clearly see that for a relatively low inspection cost, interinspection interval is rather low, the PM threshold level is relatively high and the cycle is most likely to finish with a PM action.

In case of costly inspections, interinspection interval is high (less frequent inspections), the PM threshold level is reduced, and the cycle is most likely to finish with a corrective action and an important penalty.

Finally, Table 5 shows the variation of the cost of a corrective maintenance action. One can see that as this cost increases, the proposed policy suggests undergoing rather frequent inspections and encourages PM actions (higher probability P_p^1). This is similar to the effect of costly environmental penalty.

5. Nonperiodic Inspection Optimization Model

In this section, it is assumed that the system is submitted to nonperiodic inspections to assess the generated environmental damage. Let $\Theta = (\theta_1, \theta_2, \ldots, \theta_N, \ldots)$ denote the inspection instants sequence. The purpose is to determine simultaneously the optimal PM threshold value L^* as well as the inspection instants sequence Θ^* that minimizes the average long-run cost rate.

The expression of the expected total cost per time unit is developed as follows.

(i) The probability that the cycle ends with a corrective maintenance action is

$$P_c^2 = \sum_{i=1}^{\infty} \int_{\theta_{i-1}}^{\theta_i} F_{T_U - T_L}(\theta_i - y) f_{T_L}(y)\, dy. \tag{27}$$

(ii) The probability that the cycle ends with a preventive maintenance action is

$$P_p^2 = \sum_{i=1}^{\infty} \int_{\theta_{i-1}}^{\theta_i} \left(1 - F_{T_U - T_L}(\theta_i - y)\right) f_{T_L}(y)\, dy. \tag{28}$$

(iii) The expected number of inspections during a cycle is

$$E\left[N^2\right] = \sum_{i=1}^{\infty} i \left(F_{T_L}(\theta_i) - F_{T_L}(\theta_{i-1})\right). \tag{29}$$

Table 5: Effect of C_c.

C_c	τ^*	L^*	P_c^1 (%)	CM cost ($)	P_p^1 (%)	PM cost ($)	$E[N^1]$	Inspection cost ($)	$E[T_\xi^1]$	Environmental penalty cost ($)	$E[T_{\text{cycle}}^1]$ (week)	EC^1 ($/week)
600	***8***	***4***	67.47	405	32.53	163	1.00	100	0.0262	262	8	116.25
900	***7***	***2***	17.76	160	82.24	412	1.08	108	0.02	200	7.1	123.94
1100	***5***	***2***	0.43	3.87	99.57	498	1.00	100	$7\,10^{-3}$	70	5	134.37

(iv) The average time of generation of excess amount of environmental damage during a cycle is

$$E\left[T_\zeta^2\right] = \sum_{i=1}^{\infty} \int_{\theta_{i-1}}^{\theta_i} \int_{\theta_{i-1}}^{z} (\theta_i - z) F_{T_U - T_L}(z-y) f_{T_L}(y)\, dy\, dz. \quad (30)$$

(v) The expected renewal cycle length is

$$E\left[T_{\text{cycle}}^2(L,\Theta)\right] = \sum_{i=1}^{\infty} \theta_i \left(F_{T_L}(\theta_i) - F_{T_L}(\theta_{i-1})\right). \quad (31)$$

By collecting (27), (28), (29), (30), and (31), the expected long-run rate cost is written as follows:

$$\begin{aligned} EC^2(L,\Theta) &= \left[C_p \sum_{i=1}^{\infty} \left(F_{T_L}(\theta_i) - F_{T_L}(\theta_{i-1})\right) \right. \\ &\quad + \left(C_c - C_p\right) \sum_{i=1}^{\infty} \int_{\theta_{i-1}}^{\theta_i} F_{T_U - T_L}(\theta_i - y) f_{T_L}(y)\, ds \\ &\quad + C_i \sum_{i=1}^{\infty} i \left(F_{T_L}(\theta_i) - F_{T_L}(\theta_{i-1})\right) + C_{\text{env}} \\ &\quad \left. \cdot \sum_{i=1}^{\infty} \int_{\theta_{i-1}}^{\theta_i} \int_{\theta_{i-1}}^{z} (\theta_i - z) F_{T_U - T_L}(z-y) f_{T_L}(y)\, dy\, dz \right] \\ &\quad \cdot \left(\sum_{i=1}^{\infty} \theta_i \left(F_{T_L}(\theta_i) - F_{T_L}(\theta_{i-1})\right) \right)^{-1}. \end{aligned} \quad (32)$$

Due to the complexity of the analytical model, a numerical procedure is developed in the next section in order to find a nearly optimal solution (L^*, Θ^*) for any instance of the problem.

5.1. Numerical Procedure. The developed procedure is based on the Nelder-Mead algorithm. It is presented in Figure 6 followed by the description of the use of Nelder-Mead algorithm at one of the steps. Basically, the proposed numerical procedure consists in searching for each value of L the best inspection sequence Θ using the Nelder-Mead algorithm [36]. It is one of the best known algorithms for multidimensional unconstrained optimization without derivatives and it is especially appreciated for its robustness, its simplicity, its low use of memory (few variables), and its short computing time [37].

It consists in an iterative procedure comparing the values of the objective function at the $(n+1)$ vertices and moving gradually to a quasioptimal point. Its movement is achieved by using four operations, known as *reflection*, *expansion*, *contraction*, and *shrink*. The steps of Nelder-Mead algorithm applied to our model are described as follows.

Step 1. Sort the simplex vertices according to the function value at that point:

$$EC^2(\Theta_1) \le EC^2(\Theta_2) \le \cdots \le EC^2(\Theta_{n+1}). \quad (33)$$

We refer to Θ_1 as the best vertex and to Θ_{n+1} as the *worst* vertex.

Step 2. Compute the centroid $\overline{\Theta}$ using all points except Θ_{n+1}, the worst point. Consider $\overline{\Theta} = \sum_{i=1}^{n} \Theta_i$.

Step 3. Compute the reflection point

$$\Theta_r = \overline{\Theta} + \alpha\left(\overline{\Theta} - \Theta_{n+1}\right) \quad \text{(reflection)}. \quad (34)$$

Step 3.1. If $EC^2(\Theta_1) \le EC^2(\Theta_r) \le EC^2(\Theta_n)$, replace Θ_{n+1} with Θ_r; go to Step 1.

Step 3.2. If $EC^2(\Theta_r) \le EC^2(\Theta_1)$, then compute the expansion point

$$\Theta_e = \overline{\Theta} + \beta\left(\Theta_r - \overline{\Theta}\right) \quad \text{(expansion)} \quad (35)$$

and evaluate; if $EC^2(\Theta_e) < EC^2(\Theta_r)$, then replace Θ_{n+1} with Θ_e; else replace Θ_{n+1} with Θ_r; go to Step 1.

Step 3.3. If $EC^2(\Theta_n) \le EC^2(\Theta_r) < EC^2(\Theta_{n+1})$ compute the outside contraction point

$$\Theta_{\text{oc}} = \Theta_{n+1} + \gamma\left(\overline{\Theta} - \Theta_{n+1}\right) \quad \text{(outside contraction)}. \quad (36)$$

If $EC^2(\Theta_n) \le EC^2(\Theta_r)$, then use Θ_{oc} and reject Θ_{n+1}; go to Step 1. Else go to Step 4.

Step 3.4. If $EC^2(\Theta_r) \ge EC^2(\Theta_{n+1})$ compute the inside contraction point

$$\Theta_{\text{ic}} = \Theta_{n+1} - \gamma\left(\overline{\Theta} - \Theta_{n+1}\right) \quad \text{(inside contraction)}. \quad (37)$$

If $EC^2(\Theta_{\text{ic}}) < EC^2(\Theta_{n+1})$, then use Θ_{ic} and reject Θ_{n+1}; go to Step 1. Else go to Step 4.

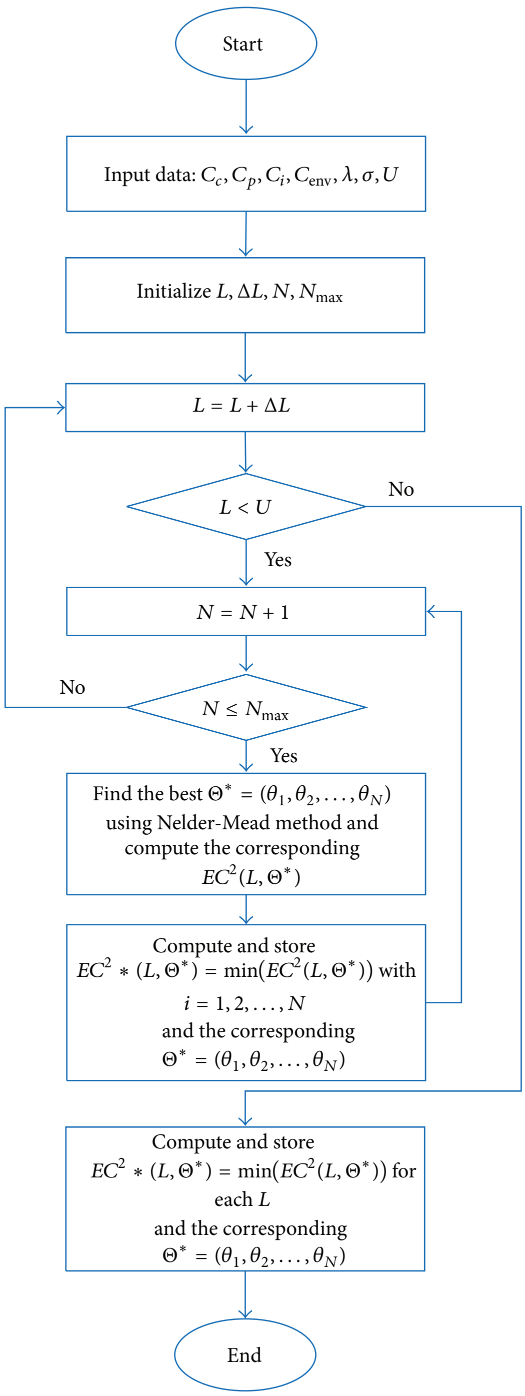

FIGURE 6: Numerical procedure for the nonperiodic strategy.

Step 4. Compute new n vertices keeping only the best one, Θ_1. Consider

$$\Theta_i = \Theta_1 + \eta\left(\Theta_i - \Theta_1\right), \quad \forall 2 \leq i \leq n+1 \,(\text{shrink}); \quad (38)$$

go to Step 1.

The convergence of the algorithm is achieved when the standard deviation of the objective function at the $(n+1)$ vertices is smaller than a specific value ε; that is,

$$\sqrt{\frac{1}{n+1}\sum_{i=1}^{n+1}\left[EC^2(\Theta_i) - \overline{EC^2}\right]^2} < \varepsilon. \quad (39)$$

The standard values of Nelder-Mead parameters are chosen to be $\alpha = 1$, $\beta = 2$, $\gamma = 0.5$, and $\eta = 0.5$ [37, 38].

In the numerical procedure described in Figure 6, the following notations were adopted:

(i) ΔL: increment of environmental degradation;

(ii) N: number of inspections;

(iii) N_{max}: maximum number of inspections.

5.2. Numerical Example (Nonperiodic Inspections). We consider the same example of Section 4.3 with the same input data. The pdf and cdf of T_L given by (24) and (25), respectively, and the cumulative function of the RUL $T_U - T_L$ given by (26) are considered.

Given the above input parameters, we applied the procedure of Figure 6 with $\Delta L = 1$, $N = 1$, and $N_{max} = 10$. The obtained nearly optimal solution is $L^* = 2$ tons of CO_2 gas emissions and $\Theta^* = 6.6, 7.1, 7.4$ (see Table 6).

Note that the first inspection instant is 6.6 weeks. We give only the first three values in this particular example because the cycle average duration is 6.7 weeks and the average number of inspections per cycle is 1.09. Therefore, having more than three inspections per cycle is nearly impossible.

PM action should be performed whenever an inspection reveals that the environmental degradation level has exceeded 2 tons of CO_2 gas emissions. By adopting this strategy, it would cost in average a total of 100.46 \$/week.

In comparison with the periodic inspection policy, one can notice that, in the case of nonperiodic inspections, the probability that the cycle ends with a CM action and the average period of excess CO_2 gas emissions are lower. This indicates that with the nonperiodic inspection policy PM actions are more likely to be performed in order to avoid the exceeding of the critical level and therefore the reduction of the emission of excess damage to environment.

Moreover, the nonperiodic inspection policy has the lowest average cost rate due to the reduction of the expected CM cost as well as the environmental penalty cost. Hence, even with a nearly optimal solution, the nonperiodic inspection policy is more economical than the periodic inspection policy. This can be explained by the fact that, in the case of the nonperiodic strategy, sequential inspections are scheduled in accordance with the evolution of environmental damage generation.

6. Conclusion

In this paper, we have considered a condition-based maintenance policy for production systems which degrade as they get older and generate environmental damage. We have

Table 6: The obtained nearly optimal solution for the nonperiodic inspection policy.

L^* (Ton)	Θ^* (week)	P_c^2 (%)	CM cost ($)	P_p^2 (%)	PM cost ($)	$E[N^2]$	Inspection cost ($)	$E[T_\xi^2]$	Environmental penalty cost ($)	$E[T_{\text{cycle}}^2]$ (week)	EC^2 ($/week)
2	***6.6, 7.1, 7.4***	6.27	56.41	93.73	468.67	1.09	109	0.0039	39	6.7	100.46

proposed a new modeling approach based on the fact that the degradation process is modeled by the Wiener process. Thus, the first hitting time and the remaining useful life of the system conditional to its environmental degradation level are considered. Moreover, two types of inspection policies, periodic and nonperiodic, can be used to reveal the level of environmental damage and act consequently. According to the observed amount of environmental degradation at each inspection, one decides to undertake or not maintenance actions (PM or CM) on the system.

CM action is undertaken following inspections that reveal the exceeding of a known critical level of environmental degradation. In such situation, an environmental penalty is incurred due to the excess amount of environmental degradation generated. To prevent such event, a lower threshold level has to be considered to trigger a PM action to renew the system at a lower cost and without paying the penalty. This lower threshold level and the inspection schedule were considered as the decision variables.

For the two proposed inspection policies, the total expected cost per time unit has been mathematically modeled and the optimal inspection schedules and PM threshold levels were derived using two numerical procedures.

The developed condition-based preventive maintenance models permit highlighting the role of preventive maintenance in reducing environmental damage and its consequences that could be caused by the degradation of production systems. The proposed models can be relatively easily used by decision-makers in the perspective of implementing an effective green maintenance.

This work can be improved in several ways. First, it would be of interest to consider situations in which maintenance actions durations are not negligible and where resources are not always immediately available to perform maintenance actions. Moreover, in many real situations, production systems may not generate only a single damage to environment, but several kinds of damages at the same time at different rates with different impacts. It would be interesting to investigate these issues taking the present model as a start.

Notations

U: Critical level of environmental degradation

L: PM threshold level of environmental degradation

C_p: Preventive maintenance (PM) action cost

C_c: Corrective maintenance (CM) action cost

C_{env}: Penalty cost per time unit related to excess environmental damage incurred once the critical level U is exceeded

C_i: Inspection cost

T_U: The time at which the environmental damage of the system exceeds the critical level U for the first time

T_L: The time at which the environmental damage of the system exceeds the PM threshold level L for the first time

$T_{\text{cycle}}^1(T_{\text{cycle}}^2)$: The system renewal cycle length within the periodic (nonperiodic) inspection policy; a cycle is the time between consecutive maintenance actions (either preventive or corrective)

$f_{T_U}(t)$, $F_{T_U}(t)$: Probability density function (pdf) and cumulative distribution function (cdf) associated with T_U

$f_{T_L}(t)$, $F_{T_L}(t)$: pdf and cdf associated with T_L

$N^1(N^2)$: Discrete random variable associated with the total number of inspections during a cycle within the periodic (nonperiodic) inspection policy

θ_i: The th inspection instant

$\Theta = (\theta_1, \theta_2, \ldots, \theta_N, \ldots)$: Inspection instants sequence

$P_p^1(P_p^2)$: Probability that the cycle ends with a PM action within the periodic (nonperiodic) inspection policy

$P_c^1(P_c^2)$: Probability that the cycle ends with a CM action within the periodic (nonperiodic) inspection policy

$E[N^1](E[N^2])$: The expected number of inspections during a cycle in the case of the periodic (nonperiodic) inspection policy

T_ζ^i: Period during which excess of damage is generated between the instant when the amount of environment degradation exceeds the critical level U and the moment of the inspection that reveals it ($i = 1$ for the periodic inspection policy, $i = 2$ for the nonperiodic inspection policy)

$EC^1(L,\tau)$: The long-run average cost per unit of time corresponding to the periodic inspection policy

$EC^2(L,\Theta)$: The long-run average cost per unit of time corresponding to the nonperiodic inspection policy.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] R. K. Sharma, D. Kumar, and P. Kumar, "Predicting uncertain behavior of industrial system using FM—a practical case," *Applied Soft Computing Journal*, vol. 8, no. 1, pp. 96–109, 2008.

[2] M. Čepin and B. Mavko, "Fault tree developed by an object-based method improves requirements specification for safety-related systems," *Reliability Engineering and System Safety*, vol. 63, no. 2, pp. 111–125, 1999.

[3] T. C. McKelvey, "How to improve the effectiveness of hazard & operability analysis," *IEEE Transactions on Reliability*, vol. 37, no. 2, pp. 167–170, 1988.

[4] P. R. S. da Silva and F. G. Amaral, "An integrated methodology for environmental impacts and costs evaluation in industrial processes," *Journal of Cleaner Production*, vol. 17, no. 15, pp. 1339–1350, 2009.

[5] S. K. Stefanis, A. G. Livingston, and E. N. Pistikopoulos, "Minimizing the environmental impact of process plants: a process systems methodology," *Computers & Chemical Engineering*, vol. 19, supplement 1, pp. 39–44, 1995.

[6] B. Gjorgiev, D. Kančev, and M. Čepin, "A new model for optimal generation scheduling of power system considering generation units availability," *International Journal of Electrical Power and Energy Systems*, vol. 47, no. 1, pp. 129–139, 2013.

[7] S. Martorell, J. F. Villanueva, S. Carlos et al., "RAMS+C informed decision-making with application to multi-objective optimization of technical specifications and maintenance using genetic algorithms," *Reliability Engineering and System Safety*, vol. 87, no. 1, pp. 65–75, 2005.

[8] C. G. Vassiliadis and E. N. Pistikopoulos, "Maintenance-based strategies for environmental risk minimization in the process industries," *Journal of Hazardous Materials*, vol. 71, no. 1–3, pp. 481–501, 2000.

[9] H. Chouikhi, A. Khatab, and N. Rezg, "A condition-based maintenance policy for a production system under excessive environmental degradation," *Journal of Intelligent Manufacturing*, vol. 25, no. 4, pp. 727–737, 2014.

[10] J. Yan and D. Hua, "Energy consumption modeling for machine tools after preventive maintenance," in *Proceedings of the IEEE International Conference on Industrial Engineering and Engineering Management (IEEM '10)*, pp. 2201–2205, IEEE, Macao, China, December 2010.

[11] V. A. Kopnov, "Optimal degradation processes control by two-level policies," *Reliability Engineering and System Safety*, vol. 66, no. 1, pp. 1–11, 1999.

[12] X. Zhou, L. Xi, and J. Lee, "Reliability-centered predictive maintenance scheduling for a continuously monitored system subject to degradation," *Reliability Engineering & System Safety*, vol. 92, no. 4, pp. 530–534, 2007.

[13] B. Castanier, A. Grall, and C. Bérenguer, "A condition-based maintenance policy with non-periodic inspections for a two-unit series system," *Reliability Engineering and System Safety*, vol. 87, no. 1, pp. 109–120, 2005.

[14] A. Chelbi and D. Ait-Kadi, "Inspection strategies for randomly failing systems," in *Handbook of Maintenance Management and Engineering*, chapter 13, pp. 303–335, Springer, London, UK, 2009.

[15] A. Chelbi and D. Ait-Kadi, "An optimal inspection strategy for randomly failing equipment," *Reliability Engineering and System Safety*, vol. 63, no. 2, pp. 127–131, 1999.

[16] B. Castanier, C. Bérenguer, and A. Grall, "A sequential condition-based repair/replacement policy with non-periodic inspections for a system subject to continuous wear," *Applied Stochastic Models in Business and Industry*, vol. 19, no. 4, pp. 327–347, 2003.

[17] H. R. Golmakani and F. Fattahipour, "Age-based inspection scheme for condition-based maintenance," *Journal of Quality in Maintenance Engineering*, vol. 17, no. 1, pp. 93–110, 2011.

[18] V. Makis and A. K. S. Jardine, "Optimal replacement in the proportional hazards model," *Information Systems and Operational Research*, vol. 30, no. 1, 1992.

[19] A. Grall, C. Bérenguer, and L. Dieulle, "A condition-based maintenance policy for stochastically deteriorating systems," *Reliability Engineering & System Safety*, vol. 76, no. 2, pp. 167–180, 2002.

[20] R. Jiang, "Optimization of alarm threshold and sequential inspection scheme," *Reliability Engineering and System Safety*, vol. 95, no. 3, pp. 208–215, 2010.

[21] M. Grigoriu, *Stochastic Calculus: Applications in Science and Engineering*, Birkhauser, Boston, Mass, USA, 2002.

[22] F. C. Klebaner, *Introduction To Stochastic Calculus With Applications*, Imperial College Press, 3rd edition, 2012.

[23] M. Abdel-Hameed, "A gamma wear process," *IEEE Transactions on Reliability*, vol. 24, no. 2, pp. 152–153, 1975.

[24] R. Chhikara and L. Folks, *The Inverse Gaussian Distribution: Theory, Methodology, and Applications*, Marcel Dekker, New York, NY, USA, 1989.

[25] S. Sato and J. Inoue, "Inverse Gaussian distribution and its application," *Electronics and Communications in Japan, Part III: Fundamental Electronic Science*, vol. 77, no. 1, pp. 32–42, 1994.

[26] R. C. Merton, "On the role of the Wiener process in finance theory and practice: the case of replicating portfolios," in *The Legacy of Norbert Wiener: A Centennial Symposium*, vol. 60, American Mathematical Society, 1997.

[27] W. E. Bardsley, "Note on the use of the inverse Gaussian distribution for wind energy applications," *Journal of Applied Meteorology*, vol. 19, no. 9, pp. 1126–1130, 1980.

[28] G. A. Whitmore and F. Schenkelberg, "Modelling accelerated degradation data using wiener diffusion with a time scale transformation," *Lifetime Data Analysis*, vol. 3, no. 1, pp. 27–45, 1997.

[29] T. Z. Fahidy, "Applying the inverse Gaussian distribution to the assessment of chemical reactor performance," *International Journal of Chemistry*, vol. 4, no. 2, 2012.

[30] X. Wang and D. Xu, "An inverse gaussian process model for degradation data," *Technometrics*, vol. 52, no. 2, pp. 188–197, 2010.

[31] G. A. Whitmore, "Estimating degradation by a wiener diffusion process subject to measurement error," *Lifetime Data Analysis*, vol. 1, no. 3, pp. 307–319, 1995.

[32] Z.-S. Ye and N. Chen, "The inverse Gaussian process as a degradation model," *Technometrics*, vol. 56, no. 3, pp. 302–311, 2014.

[33] Z. Brzezniak and T. Zastawniak, *Basic Stochastic Process: A Course Through Exercise*, Springer, London, UK, 2002.

[34] D. R. Cox and H. D. Miller, *The Theory of Stochastic Processes*, Methuen and Company, London, UK, 1965.

[35] C. Guo, W. Wang, B. Guo, and X. Si, "A maintenance optimization model for mission-oriented systems based on Wiener degradation," *Reliability Engineering & System Safety*, vol. 111, pp. 183–194, 2013.

[36] J. A. Nelder and R. Mead, "A simplex method for function minimization," *The Computer Journal*, vol. 7, no. 4, pp. 308–313, 1965.

[37] O. Roux, D. Duvivier, G. Quesnel, and E. Ramat, "Optimization of preventive maintenance through a combined maintenance-production simulation model," in *Proceedings of the International Conference on Industrial Engineering and Systems Management*, Montréal, Canada, May 2009.

[38] L. Han and M. Neumann, "Effect of dimensionality on the Nelder-Mead simplex method," *Optimization Methods and Software*, vol. 21, no. 1, pp. 1–16, 2006.