Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2015, Article ID 419643, 10 pages
http://dx.doi.org/10.1155/2015/419643

Research Article

Adaptive Control of MEMS Gyroscope Based on T-S Fuzzy Model

Yunmei Fang,[1] Shitao Wang,[2] Juntao Fei,[2] and Mingang Hua[2]

[1] *College of Mechanical and Electrical Engineering, Hohai University, Changzhou 213022, China*
[2] *College of IOT Engineering, Hohai University, Changzhou 213022, China*

Correspondence should be addressed to Juntao Fei; jtfei@yahoo.com

Received 23 November 2014; Revised 20 January 2015; Accepted 22 January 2015

Academic Editor: Manuel De la Sen

A multi-input multioutput (MIMO) Takagi-Sugeno (T-S) fuzzy model is built on the basis of a nonlinear model of MEMS gyroscope. A reference model is adjusted so that a local linear state feedback controller could be designed for each T-S fuzzy submodel based on a parallel distributed compensation (PDC) method. A parameter estimation scheme for updating the parameters of the T-S fuzzy models is designed and analyzed based on the Lyapunov theory. A new adaptive law can be selected to be the former adaptive law plus a nonnegative in variable to guarantee that the derivative of the Lyapunov function is smaller than zero. The controller output is implemented on the nonlinear model and T-S fuzzy model, respectively, for the purpose of comparison. Numerical simulations are investigated to verify the effectiveness of the proposed control scheme and the correctness of the T-S fuzzy model.

1. Introduction

Gyroscopes are commonly used sensors in navigation, automobile, and so forth. The performance of the MEMS gyroscope is often deteriorated by the effects of time varying parameters, quadrature errors, and external disturbances. Advanced control such as adaptive control and intelligent control are necessary to be adopted to control the MEMS gyroscope. In the last few years, various control approaches have been proposed to control the MEMS gyroscope. Increasing attention has been given to the tracking control of MEMS gyroscope. Leland [1] derived two adaptive controllers for a MEMS gyroscope that tune the drive axis natural frequency to a preselected frequency and drive the sense axis vibration to zero for force-to-rebalance operation. Sung et al. [2] proposed a phase-domain design approach to study the mode-matched control of gyroscope. Antonello et al. [3] derived extremum-seeking control to automatically match the vibration mode in MEMS vibrating gyroscopes. Park et al. [4] presented an adaptive controller for a MEMS gyroscope which drives both axes of vibration and controls the entire operation of the gyroscope. John and Vinay [5] developed a novel concept for an adaptively controlled triaxial angular velocity sensor device. Sliding mode control has been incorporated into adaptive controller to control the MEMS gyroscopes [6, 7]. Model uncertainties and disturbances are inevitable in actual engineering and require the controller to be either adaptive or robust to these model uncertainties. Adaptive fuzzy sliding mode controller can be utilized to compensate the model uncertainties and disturbances since it combines the merits of the sliding mode control, the fuzzy inference mechanism, and the adaptive algorithm. Wang [8] demonstrated that an arbitrary function of a certain set of functions can be approximated with arbitrary accuracy using fuzzy system on a compact domain using universal approximation theorem. Guo and Woo [9] derived adaptive fuzzy sliding mode controller for robot manipulator. Yoo and Ham [10] developed adaptive controller for robot manipulator using fuzzy compensator. Wai [11] proposed fuzzy sliding mode controller using adaptive tuning technique. Chang et al. [12] designed and implemented fuzzy sliding mode formation control for multirobot systems. Chien et al. [13] developed robust adaptive controller design for a class of uncertain nonlinear systems using online T-S fuzzy-neural modeling approach. Stabilizing controller design for uncertain nonlinear systems using fuzzy models was investigated in [14]. Park and Cho [15] designed T-S model based indirect adaptive fuzzy controller using online parameter estimation.

The system nonlinearities in MEMS gyroscope model have been described in [16–18]. Since there are system nonlinearities in MEMS gyroscope, it is necessary to use robust adaptive fuzzy controller to MEMS gyroscope and utilize T-S fuzzy model to represent the system nonlinearities.

Systematic stability analysis and controller design of the adaptive fuzzy controller using T-S fuzzy model for MEMS gyroscope have not been investigated before. The advantage of the proposed adaptive T-S fuzzy controller over other controllers is that it can represent system nonlinearities using T-S fuzzy model in the aspect of robust design. Thus, it can overcome the impacts on output tracking error that are caused by model uncertainties and external disturbances. The proposed T-S modeling method can provide a possibility for developing a systematic analysis and design method for complex nonlinear control systems, thus improving the tracking and compensation performance. Thus it is necessary to adopt the adaptive fuzzy control scheme to approximate the nonlinear system and compensate model uncertainties and external disturbances in the control of MEMS gyroscope using T-S fuzzy model.

In this paper, the Lyapunov-based robust adaptive fuzzy control strategy is applied to the tracking control of MEMS gyroscope using T-S fuzzy model. The paper integrates adaptive control and the nonlinear approximation of fuzzy control with T-S fuzzy model. The proposed adaptive fuzzy T-S controller can guarantee the asymptotic stability of the closed loop system and improve the robustness of control system in the presence of model uncertainties and external disturbances. The proposed controller has the following characteristics and contributions.

(1) Since the reference model is marginally stable, the reference model is required to be revised so that the local linear state feedback controller can be designed for each T-S fuzzy submodel based on PDC. The partial parameters of the reference control matrix are changed and then the reference input is changed accordingly to guarantee that the changed reference model is equivalent to previous one.

(2) To improve the performance of the control system, the corresponding improvement is made to the adaptive law. The new adaptive law can be chosen to be the previous adaptive law plus a robust term to guarantee that the derivative of the Lyapunov function is negative rather than nonpositive.

(3) The proposed estimator for the existing fuzzy state feedback controller can achieve a good robust performance against parameter uncertainties. The controller output is implemented in the T-S fuzzy model and nonlinear model, respectively, to verify the validity of the adaptive fuzzy control with parameter estimation scheme and prove the correctness of the T-S model and the feasibility of the proposed controller on the nonlinear model of gyroscope.

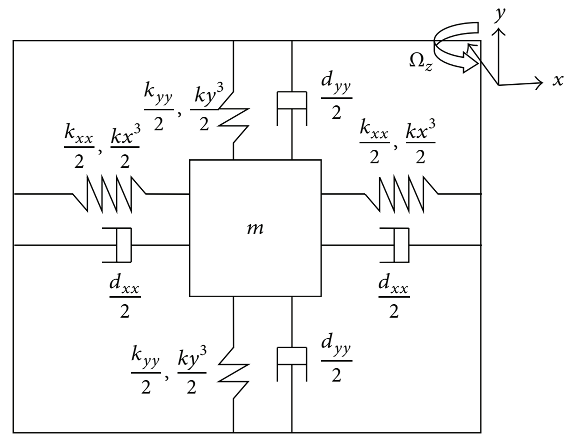

Figure 1: Z-axis vibratory MEMS gyroscope with nonlinear effective spring.

2. Dynamics of MEMS Gyroscope

Assume that the gyroscope is moving with a constant linear speed; the gyroscope is rotating at a constant angular velocity; the centrifugal forces are assumed negligible; the gyroscope undergoes rotations along z-axis as shown in Figure 1. Referring to [4, 16, 17], the dynamics equations of gyroscope system are as follows:

$$\begin{aligned} & m\ddot{x} + d_{xx}\dot{x} + \left(d_{xy} - 2m\Omega_z^*\right)\dot{y} \\ & \quad + \left(k_{xx} - m\Omega_z^{*2}\right)x + k_{xy}y + k_{x^3}x^3 = u_x, \\ & m\ddot{y} + d_{yy}\dot{y} + \left(d_{xy} + 2m\Omega_z^*\right)\dot{x} \\ & \quad + \left(k_{yy} - m\Omega_z^{*2}\right)y + k_{xy}x + k_{y^3}y^3 = u_y, \end{aligned} \tag{1}$$

where m is the mass of proof mass. Fabrication imperfections contribute mainly to the asymmetric spring term d_{xy}, and asymmetric damping terms k_{xy}, d_{xx}, and d_{yy} are damping terms; k_{xx}, k_{yy} are linear spring terms; k_{x^3}, k_{y^3} are nonlinear spring terms, Ω_z^* is angular velocity; u_x, u_y are the control forces.

Dividing the equation by the reference mass, and because of the nondimensional time $t^* = \omega_0 t$, dividing both sides of equation by reference frequency ω_0^2 and reference length q_0, and rewriting the dynamics in vector forms result in

$$\begin{aligned} & \frac{\ddot{q}_1^*}{q_0} + \frac{D^*}{m\omega_0}\frac{\dot{q}_1^*}{q_0} + 2\frac{S^*}{\omega_0}\frac{\dot{q}_1^*}{q_0} - \frac{\Omega_z^{*2}}{m\omega_0}\frac{q_1^*}{q_0} \\ & \quad + \frac{K_1^*}{m\omega_0^2}\frac{q_1^*}{q_0} + \frac{K_3^*}{m\omega_0^2}\frac{q_1^{*3}}{q_0} = \frac{u^*}{m\omega_0^2 q_0}, \end{aligned} \tag{2}$$

where

$$q_1^* = \begin{bmatrix} x \\ y \end{bmatrix}, \qquad u^* = \begin{bmatrix} u_x \\ u_y \end{bmatrix}, \qquad D^* = \begin{bmatrix} d_{xx} & d_{xy} \\ d_{xy} & d_{yy} \end{bmatrix},$$
$$S^* = \begin{bmatrix} 0 & -\Omega_z^* \\ \Omega_z^* & 0 \end{bmatrix}, \quad K_1^* = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{xy} & k_{yy} \end{bmatrix}, \quad K_3^* = \begin{bmatrix} k_{x^3} & 0 \\ 0 & k_{y^3} \end{bmatrix}. \tag{3}$$

Define new parameters as follows:

$$q_1 = \frac{q_1^*}{q_0}, \qquad u = \frac{u^*}{m\omega_0^2 q_0}, \qquad \Omega_z = \frac{\Omega_z^*}{\omega_0},$$
$$D = \frac{D^*}{m\omega_0}, \qquad S = \frac{S^*}{\omega_0}, \qquad K_1 = \frac{K_1^*}{m\omega_0^2}, \qquad K_3 = \frac{K_3^* q_0^2}{m\omega_0^2}. \tag{4}$$

For the convenience of notation, ignoring the superscript yields the final form of the nondimensional equation of motion for the z-axis gyroscope

$$\ddot{q}_1 = (2S - D)\,\dot{q}_1 + \left(\Omega_z^2 - K_1\right) q_1 - K_3 q_1^3 + u. \tag{5}$$

3. Adaptive Fuzzy Control Based on PDC

The MIMO T-S fuzzy model of the MEMS gyroscope, which is composed of i rules, is established based on the nonlinear model of MEMS gyroscope. Use the fuzzy implications and the fuzzy reasoning methods suggested by Takagi and Sugeno [19] to express a real plant model; the T-S fuzzy model uses fuzzy implication of the following form:

$$\begin{aligned} &\text{Rule } i\text{: IF } x \text{ is about } M_{i1} \text{ and } y \text{ is about } M_{i2} \text{ and } \dot{x} \\ &\qquad \text{is about } M_{i3} \text{ and } \dot{y} \text{ is about } M_{i4}, \\ &\qquad \text{THEN } \dot{q} = A_i q + B_i u, \quad i = 1, 2, \ldots, 9. \end{aligned} \tag{6}$$

The defuzzification fuzzy dynamic system model can be expressed using center-average defuzzifier, product inference, and singleton fuzzifier

$$\dot{q} = \frac{\sum_{i=1}^{9} \mu_i(\eta)\left[A_i q + B_i u\right]}{\sum_{i=1}^{9} \mu_i(\eta)}, \tag{7}$$

where

$$A_i = \begin{bmatrix} a_{11}^i & a_{12}^i & a_{13}^i & a_{14}^i \\ a_{21}^i & a_{22}^i & a_{23}^i & a_{24}^i \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$
$$B_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad \dot{q} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \dot{x} \\ \dot{y} \end{bmatrix}, \qquad q = \begin{bmatrix} \dot{x} \\ \dot{y} \\ x \\ y \end{bmatrix},$$
$$\mu_i(\eta) = M_{i1}(x)\, M_{i2}(y)\, M_{i3}(\dot{x})\, M_{i4}(\dot{y}). \tag{8}$$

$M_{i1}(x)$, $M_{i2}(y)$, $M_{i3}(\dot{x})$, and $M_{i4}(\dot{y})$ are membership function values of the fuzzy variable x, y, $\dot{x}$, and $\dot{y}$ with respect to fuzzy set M_{i1}, M_{i2}, M_{i3}, and M_{i4}, respectively.

Since the control target for MEMS gyroscope is to maintain the proof mass to oscillate in the x and y direction at given frequency and amplitude, $x = A_x \sin(\omega_x t)$, $y = A_y \sin(\omega_y t)$, generally the reference model can be defined as $\dot{q}_d = A_d q_d$, where

$$\dot{q}_d = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} A_x \omega_x^2 \sin(\omega_x t) \\ A_y \omega_y^2 \sin\left(\omega_y t\right) \\ A_x \omega_x \cos(\omega_x t) \\ A_y \omega_y \cos\left(\omega_y t\right) \end{bmatrix},$$
$$q_d = \begin{bmatrix} \dot{x} \\ \dot{y} \\ x \\ y \end{bmatrix} = \begin{bmatrix} A_x \omega_x \cos(\omega_x t) \\ A_y \omega_y \cos\left(\omega_y t\right) \\ A_x \sin(\omega_x t) \\ A_y \sin\left(\omega_y t\right) \end{bmatrix}, \tag{9}$$
$$A_d = \begin{bmatrix} 0 & 0 & -\omega_x^2 & 0 \\ 0 & 0 & 0 & -\omega_y^2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Since the reference model is nonasymptotically stable, the nonlinear T-S fuzzy model cannot be asymptotically stable. In order to design local linear state feedback controller for each T-S fuzzy submodel based on PDC, the reference model is adjusted to be $\dot{q}_m = A_m q_m + B_m r$, where

$$A_m = \begin{bmatrix} a_{11}^m & a_{12}^m & a_{13}^m & a_{14}^m \\ a_{21}^m & a_{22}^m & a_{23}^m & a_{24}^m \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -\omega_x^2 & 0 \\ 0 & -1 & 0 & -\omega_y^2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$
$$B_m = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad \dot{q}_m = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} A_x \omega_x^2 \sin(\omega_x t) \\ A_y \omega_y^2 \sin\left(\omega_y t\right) \\ A_x \omega_x \cos(\omega_x t) \\ A_y \omega_y \cos\left(\omega_y t\right) \end{bmatrix},$$

$$q_m = \begin{bmatrix} \dot{x} \\ \dot{y} \\ x \\ y \end{bmatrix} = \begin{bmatrix} A_x\omega_x \cos(\omega_x t) \\ A_y\omega_y \cos(\omega_y t) \\ A_x \sin(\omega_x t) \\ A_y \sin(\omega_y t) \end{bmatrix}, \quad r = \begin{bmatrix} A_x\omega_x \cos(\omega_x t) \\ A_y\omega_y \cos(\omega_y t) \end{bmatrix}. \tag{10}$$

Remark 1. Substituting A_m, B_m, q_m, r into $\dot{q}_m$ yields $\dot{q}_d$; we can get that the changed reference model $\dot{q}_m = A_m q_m + B_m r$ is equivalent to previous one $\dot{q}_d = A_d q_d$.

According to PDC [14], we design local state linear feedback controllers for each linear submodel. The fuzzy control rules are as the following forms:

$$\begin{aligned} &\text{Rule } i\text{: IF } x \text{ is about } M_{i1} \text{ and } y \text{ is about } M_{i2} \text{ and } \dot{x} \\ &\qquad \text{is about } M_{i3} \text{ and } \dot{y} \text{ is about } M_{i4}, \\ &\qquad \text{THEN } u(t) = -K_i(t)\,x(t) + r(t), \quad i = 1, 2, \ldots, 9. \end{aligned} \tag{11}$$

The controller output can be expressed using center-average defuzzifier, product inference, and singleton fuzzifier

$$u(t) = \frac{\sum_{i=1}^{9} \mu_i(\eta)\left[-K_i q(t) + r(t)\right]}{\sum_{i=1}^{9} \mu_i(\eta)}, \tag{12}$$

where

$$K_i = A_i - A_m = \begin{bmatrix} a_{11}^i - a_{11}^m & a_{12}^i - a_{12}^m & a_{13}^i - a_{13}^m & a_{14}^i - a_{14}^m \\ a_{21}^i - a_{21}^m & a_{22}^i - a_{22}^m & a_{23}^i - a_{23}^m & a_{24}^i - a_{24}^m \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad i = 1, 2, \ldots, 9. \tag{13}$$

Substituting the controller force (12) into the fuzzy system (7) yields the desired model

$$\dot{q}_m = A_m q_m + B_m r. \tag{14}$$

4. Parameter Estimation

The block diagram of the adaptive fuzzy control using T-S fuzzy model is shown as in Figure 2. Taking the parameter uncertainty into account, then the parameter estimation is applied to the adaptive fuzzy control. The adaptive law can guarantee the asymptotic convergence of the error between the plant model state and the estimation model state and make the closed loop system of fuzzy controller and estimation model be a desired linear model and, consequently, the plant model state can follow the state of the desired linear model. The controller output is implemented in the nonlinear model also to prove the correctness of the T-S model and the feasibility of the proposed control scheme on the nonlinear model of MEMS gyroscope.

With unknown parameters matrix A_i, the controller changes to

$$u(t) = \frac{\sum_{i=1}^{9} \mu_i(\eta)\left[-\widehat{K}_i q(t) + r(t)\right]}{\sum_{i=1}^{9} \mu_i(\eta)}, \tag{15}$$

where

$$\begin{aligned} \widehat{K}_i &= \widehat{A}_i - A_m = \begin{bmatrix} \widehat{a}_{11}^i - a_{11}^m & \widehat{a}_{12}^i - a_{12}^m & \widehat{a}_{13}^i - a_{13}^m & \widehat{a}_{14}^i - a_{14}^m \\ \widehat{a}_{21}^i - a_{21}^m & \widehat{a}_{22}^i - a_{22}^m & \widehat{a}_{23}^i - a_{23}^m & \widehat{a}_{24}^i - a_{24}^m \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \\ \widehat{A}_i &= \begin{bmatrix} \widehat{a}_{11}^i & \widehat{a}_{12}^i & \widehat{a}_{13}^i & \widehat{a}_{14}^i \\ \widehat{a}_{21}^i & \widehat{a}_{22}^i & \widehat{a}_{23}^i & \widehat{a}_{24}^i \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad i = 1, 2, \ldots, 9 \end{aligned} \tag{16}$$

is the estimate of A_i.

By considering the plant parameterization, (7) can be rewritten as

$$\dot{q}(t) = A_s q(t) + \frac{\sum_{i=1}^{9} \mu_i(\eta)\left[(A_i - A_s)\,q(t) + B_i u(t)\right]}{\sum_{i=1}^{9} \mu_i(\eta)}, \tag{17}$$

where A_s is an arbitrary stable matrix.

Define the estimation mode as the following formula:

$$\dot{\widehat{q}}(t) = A_s \widehat{q}(t) + \frac{\sum_{i=1}^{9} \mu_i(\eta)\left[\left(\widehat{A}_i - A_s\right) q(t) + B_i u(t)\right]}{\sum_{i=1}^{9} \mu_i(\eta)}. \tag{18}$$

The estimation model (18) can be the same one as by adopting the control input (15)

$$\dot{\widehat{q}}_m = A_m \widehat{q}_m + B_m r. \tag{19}$$

The estimation error $e(t) = q(t) - \widehat{q}_m(t)$ meets (19) and (20)

$$\begin{aligned} \dot{e}(t) &= A_s e(t) + \frac{\sum_{i=1}^{9} \mu_i(\eta)\left(A_i - \widehat{A}_i\right) q(t)}{\sum_{i=1}^{9} \mu_i(\eta)} \\ &= A_s e(t) - \frac{\sum_{i=1}^{9} \mu_i(\eta)\left[\widetilde{\alpha}_{1i} \;\; \widetilde{\alpha}_{2i} \;\; 0 \;\; 0\right]^T q(t)}{\sum_{i=1}^{9} \mu_i(\eta)}, \end{aligned} \tag{20}$$

$$\dot{e}^T(t) = e(t)\,A_s^T - \frac{\sum_{i=1}^{9} \mu_i(\eta)\, q^T(t)\left[\widetilde{\alpha}_{1i} \;\; \widetilde{\alpha}_{2i} \;\; 0 \;\; 0\right]}{\sum_{i=1}^{9} \mu_i(\eta)}, \tag{21}$$

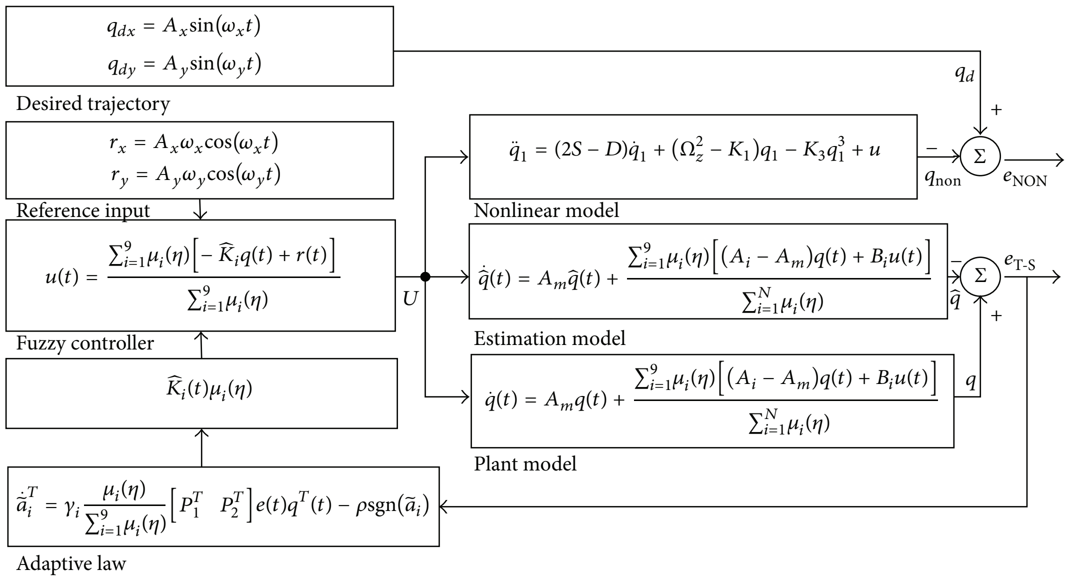

FIGURE 2: The block diagram of the adaptive fuzzy control using T-S fuzzy model.

where $e(t)$ actually is the tracking error between the plant model (7) and the estimation model (18), $\tilde{\alpha}_i(t) = [\tilde{\alpha}_{1i}(t)\ \tilde{\alpha}_{2i}(t)\ 0\ 0]$, $\tilde{\alpha}_{1i}(t) = \hat{\alpha}_{1i}(t) - \alpha_{1i}$, $\hat{\alpha}_{1i}(t) = \left[\hat{a}_{11}^i(t)\ \hat{a}_{12}^i(t)\ \hat{a}_{13}^i(t)\ \hat{a}_{14}^i(t)\right]$, $\alpha_{1i} = \left[a_{11}^i\ a_{12}^i\ a_{13}^i\ a_{14}^i\right]$, $\tilde{\alpha}_{2i}(t) = \hat{\alpha}_{2i}(t) - \alpha_{2i}$, $\hat{\alpha}_{2i} = \left[\hat{a}_{21}^i(t)\ \hat{a}_{22}^i(t)\ \hat{a}_{23}^i(t)\ \hat{a}_{24}^i(t)\right]$, $\alpha_{2i} = \left[a_{21}^i\ a_{22}^i\ a_{23}^i\ a_{24}^i\right]$.

Remark 2. The estimation error $e(t)$ can be thought of as the tracking error $e_{\text{T-S}}$ between the plant model (7) and the desired linear model (14) because $e(t) = q(t) - \hat{q}_m(t)$ and $e_{\text{T-S}}(t) = q(t) - q_m(t)$ have the same control matrix, while the tracking error e_{NON} between the reference model (14) and the nonlinear model (5) is used to prove the correctness of the T-S model and the feasibility of the proposed controller on the nonlinear model of gyroscope.

Define a Lyapunov function candidate as

$$V(t) = V(e, \tilde{a}_{1i}, \tilde{a}_{2i}) = e^T(t)Pe(t) + \sum_{i=1}^{9}\frac{\tilde{a}_{1i}^T(t)\tilde{a}_{1i}(t)}{\gamma_{1i}} + \sum_{i=1}^{9}\frac{\tilde{a}_{2i}^T(t)\tilde{a}_{2i}(t)}{\gamma_{2i}}, \tag{22}$$

where P meet $A_s^T P + PA_s = -I$.

The derivative of V with respect to time becomes

$$\dot{V}(t) = \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) + \sum_{i=1}^{9}2\frac{\dot{\tilde{a}}_{1i}^T(t)\tilde{a}_{1i}(t)}{\gamma_{1i}} + \sum_{i=1}^{9}2\frac{\dot{\tilde{a}}_{2i}^T(t)\tilde{a}_{2i}(t)}{\gamma_{2i}}. \tag{23}$$

Substituting (20), (21) into the derivative of V yields

$$\begin{aligned}\dot{V}(t) = &-e^T(t)\left(A_s^T P + PA_s\right)e(t) \\ &- 2\frac{\sum_{i=1}^{9}\mu_i(\eta)P_1^T e(t)q^T(t)\tilde{a}_{2i(t)}}{\sum_{i=1}^{9}\mu_i(\eta)} \\ &- 2\frac{\sum_{i=1}^{9}\mu_i(\eta)P_2^T e(t)q^T(t)\tilde{a}_{2i}(t)}{\sum_{i=1}^{9}\mu_i(\eta)} \\ &+ \sum_{i=1}^{9}2\frac{\dot{\tilde{a}}_{1i}^T(t)\tilde{a}_{1i}(t)}{\gamma_{1i}} + \sum_{i=1}^{9}2\frac{\dot{\tilde{a}}_{2i}^T(t)\tilde{a}_{2i}(t)}{\gamma_{2i}},\end{aligned} \tag{24}$$

where $P = \left[P_1\ \ P_2\ \ P_3\ \ P_4\right]$.

The adaptive law can be chosen as

$$\begin{aligned}\dot{\tilde{a}}_i^T = &\gamma_i\frac{\mu_i(\eta)}{\sum_{i=1}^{9}\mu_i(\eta)}\left[P_1^T\ \ P_2^T\right]e(t)q^T(t) \\ &- \rho\,\mathrm{sgn}\left(\tilde{a}_i(t)\right) \quad \rho > 0,\end{aligned} \tag{25}$$

where the sgn function can be defined as

$$\mathrm{sgn}\left(\tilde{a}_i\right) = \begin{cases} 1 & \tilde{a}_i > 0 \\ 0 & \tilde{a}_i = 0 \\ -1 & \tilde{a}_i < 0. \end{cases} \tag{26}$$

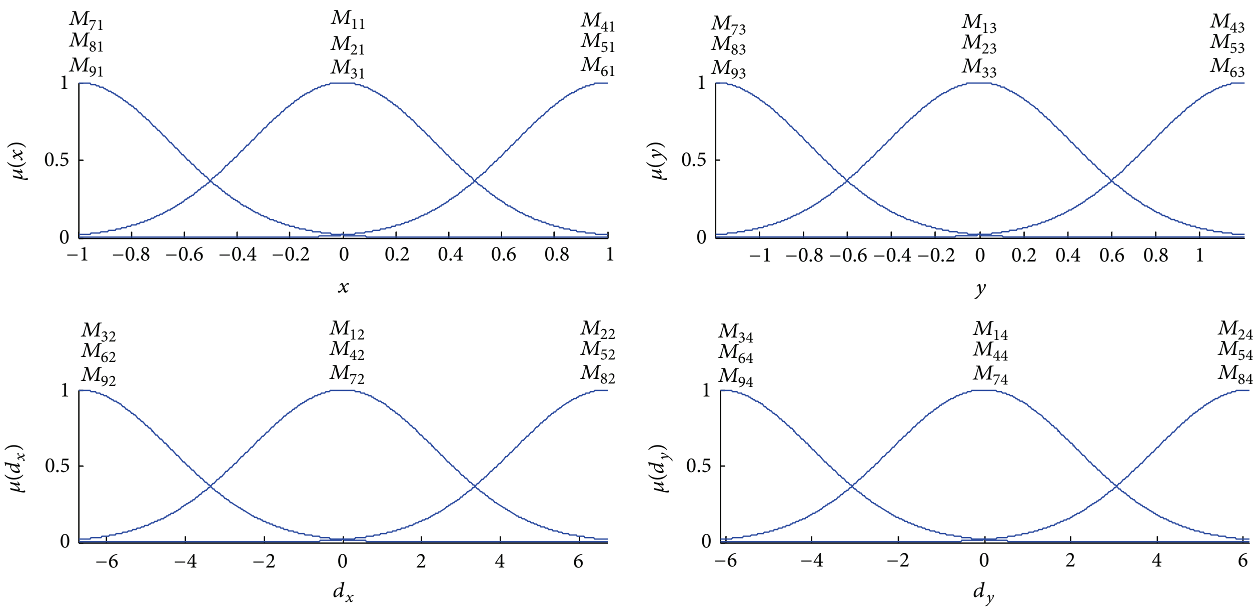

FIGURE 3: Membership functions.

Substituting (25) into (24) yields

$$\begin{aligned}\dot{V}(t) &= -e^T(t)e(t) - 2\rho\sum_{i=1}^{9}\frac{\operatorname{sgn}(\tilde{a}_{1i}(t))\tilde{a}_{1i}(t)}{\gamma_{1i}} \\ &\quad - 2\rho\sum_{i=1}^{9}\frac{\operatorname{sgn}(\tilde{a}_{2i}(t))\tilde{a}_{2i}(t)}{\gamma_{2i}} \\ &= -e^T(t)e(t) - 2\rho\sum_{i=1}^{9}\frac{|\tilde{a}_{1i}(t)|}{\gamma_{1i}} - 2\rho\sum_{i=1}^{9}\frac{|\tilde{a}_{2i}(t)|}{\gamma_{2i}} \\ &\le -e^T(t)e(t) \le -\|e(t)\|^2 \le 0.\end{aligned} \tag{27}$$

$V(t) \ge 0$ and $\dot{V}(t) \le 0$ imply the boundedness of $e(t)$ and $\tilde{\alpha}_i(t)$, the control is bounded for all time, (21) implies $\dot{e}(t)$ is also bounded, and inequality (27) implies $e(t) \in L^2$. Then, with $e(t) \in L^2 \cap L^\infty$ and $\dot{e}(t) \in L^\infty$, according to Barbalat's lemma [20], the tracking error $e(t)$ asymptotically converges to zero.

5. Simulation Analysis

In this section, we will evaluate the proposed adaptive fuzzy control using T-S model approach on the lumped MEMS gyroscope sensor model. Parameters of the MEMS gyroscope sensor are as follows:

$$\begin{gathered} m = 0.57e-8\,\text{kg}, \qquad \omega_0 = 1\,\text{kHz}, \qquad q_0 = 10e-6\,\text{m}, \\ d_{xx} = 0.429e-6\,\text{Ns/m}, \qquad d_{yy} = 0.0429e-6\,\text{Ns/m}, \\ d_{xy} = 0.0429e-6\,\text{Ns/m}, \qquad k_{xx} = 80.98\,\text{N/m}, \\ k_{yy} = 71.62\,\text{N/m}, \qquad k_{xy} = 5\,\text{N/m}, \\ k_{x^3} = 3.56e12\,\text{N/m}, \\ k_{y^3} = 3.56e12\,\text{N/m}, \qquad \Omega_z = 5.0\,\text{rad/s}. \end{gathered} \tag{28}$$

Since the general displacement range of the MEMS gyroscope sensor in each axis is submicrometer level, it is reasonable to choose $1\,\mu$m as the reference length q_0. Given that the usual natural frequency of each axle of a vibratory MEMS gyroscope sensor is in the kHz range, ω_0 is chosen as 1 kHz. The unknown angular velocity is assumed $\Omega_z = 5.0$ rad/s. The desired motion trajectories are $x_m = A_x\sin(\omega_x t)$, $y_m = A_y\sin(\omega_y t)$, where $A_x = 1$, $A_y = 1.2$, $\omega_x = 6.71$ kHz, and $\omega_y = 5.11$ kHz.

The plant parameters are adjusted online by adaptive law (24) where the adaptive gain $\gamma_i = 1$ and the adaptive parameter $\rho = 1$. The initial values of T-S model are $[0.2, 0.24, 0, 0]$, initial values of the estimation model are $[0, 0, 0, 0]$, and initial values of the nonlinear model are $[0.2, 0.24, 0, 0]$. The external disturbance is $d = \begin{bmatrix} d_x \\ d_y \end{bmatrix} = \begin{bmatrix} 10\sin(2\pi t) \\ 10\sin(2\pi t) \end{bmatrix}$. $A_m = \begin{bmatrix} -1 & 0 & -\omega_x^2 & 0 \\ 0 & -1 & 0 & -\omega_y^2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ and the eigenvalues of A_m are $\{-0.5 \pm 6.6916i, -0.5 \pm 5.0855i\}$. Although A_s can meet the requirement of A_s in $A_s^T P + PA_s = -I$, the values of first and second column in P which are $\begin{bmatrix} 0.51 & 0 \\ 0 & 0.52 \\ 0.01 & 0 \\ 0 & 0.02 \end{bmatrix}$ cannot adjust each parameter since partial parameters are zero which is used as adaptive gain in (24), so A_s is chosen as $A_s = \begin{bmatrix} -1 & -1 & -1 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$.

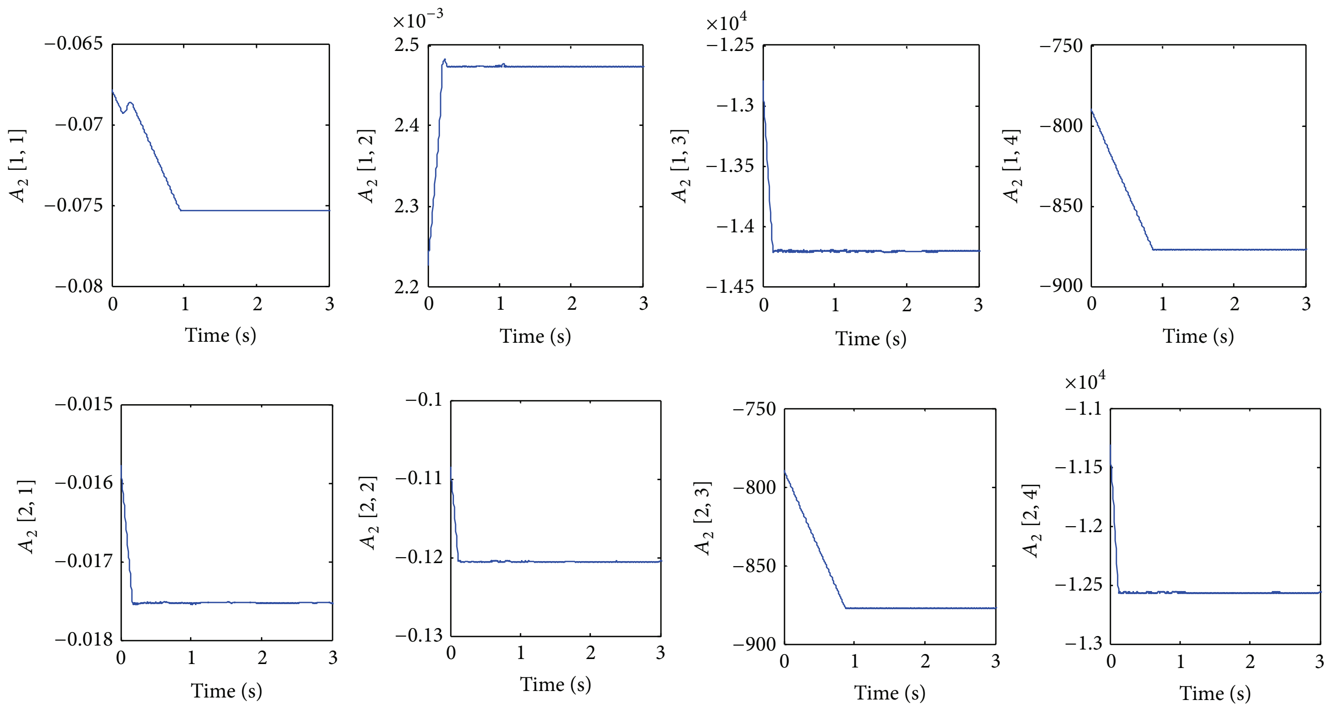

FIGURE 4: The change of plant parameters in A_2.

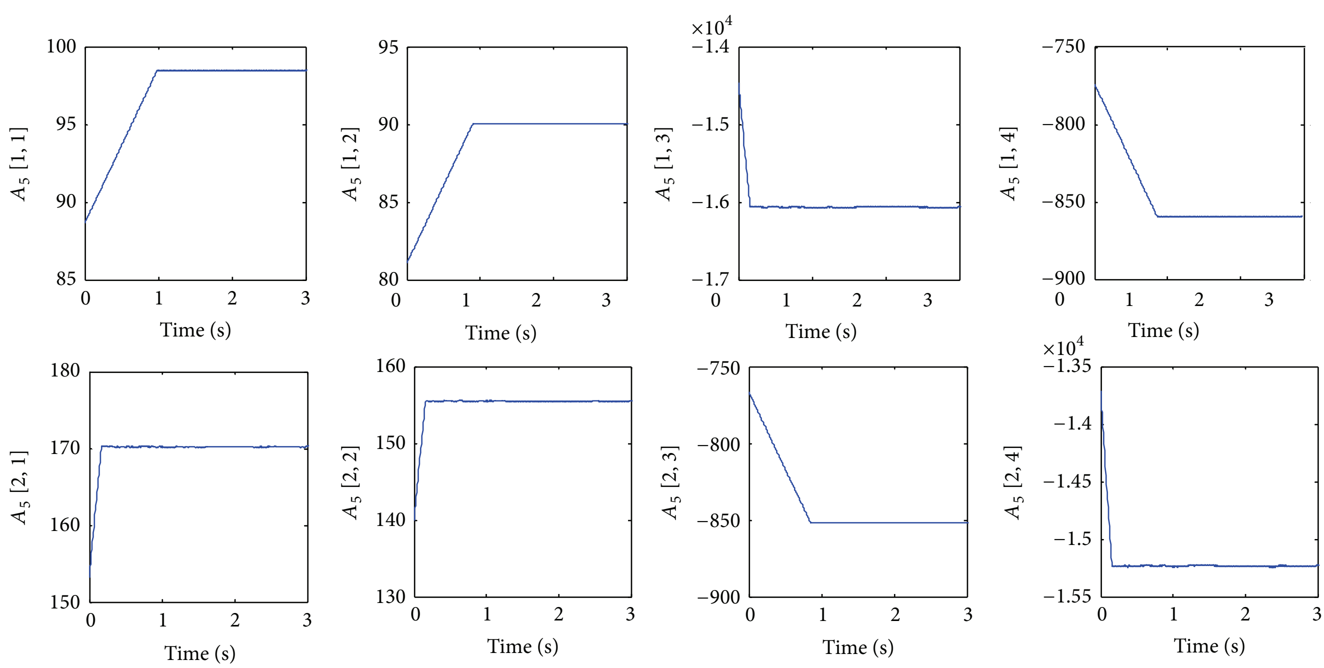

FIGURE 5: The change of plant parameters in A_5.

to ensure that each parameter of first and second column in P is nonzero values as $\begin{bmatrix} 0.5 & -0.25 \\ -0.25 & 0.75 \\ -0.25 & 0.25 \\ -0.25 & 0.25 \end{bmatrix}$.

The fuzzy rules for T-S fuzzy model for the system can be obtained by linearizing the nonlinear model (5) at the points $x \in \{-1 \ \ 0 \ \ 1\}$, $\dot{x} \in \{-A_x\omega_x \ \ 0 \ \ A_x\omega_x\}$, $y \in \{-1.2 \ \ 0 \ \ 1.2\}$, and $\dot{y} \in \{-A_y\omega_y \ \ 0 \ \ A_y\omega_y\}$ and then the membership functions of states x, y, $\dot{x}$, and $\dot{y}$ with respect to fuzzy set M_{i1}, M_{i2}, M_{i3}, and M_{i4} used in the T-S fuzzy model are shown as in Figure 3.

In this simulation, it is assumed that the physical parameters in the T-S fuzzy model are not known exactly. Hence, the

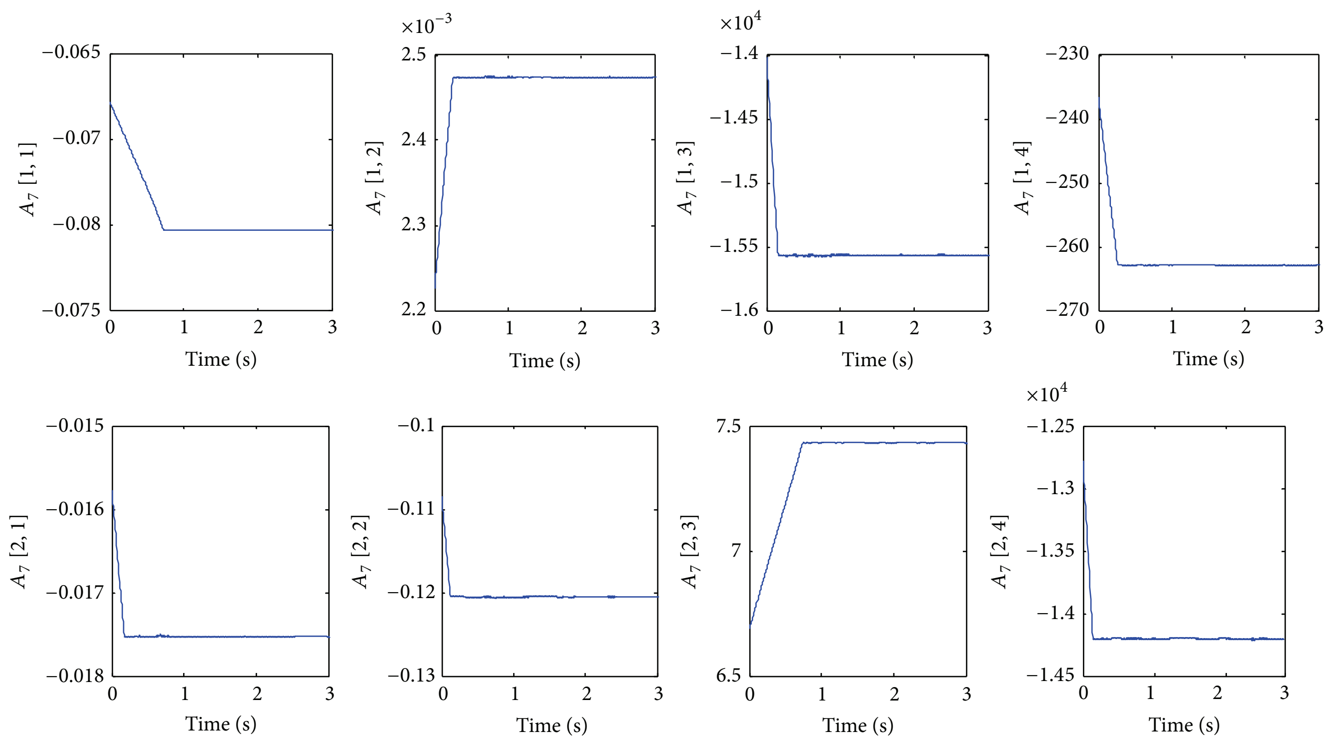

FIGURE 6: The change of plant parameters in A_7.

parameters are tuned by the proposed adaptive law. The true values of A_i^* of fuzzy T-S model are as shown in Table 1. The initial value $A_i = 0.9 * A_i^*$. The variations of each parameter in A_2, A_5, and A_7 are shown in Figures 4, 5, and 6. Here only parts of parameters are shown in table for examples. In Figures 4, 5, and 6 we can see each parameter converges to its true value asymptotically.

Figure 7 depicts the tracking error of x- and y-axis between the reference model and T-S model. It can be observed from Figure 7 that the position of x and y in the T-S model can track the position of reference model in very short time and tracking errors converge to zero asymptotically. The tracking error $e_{\text{T-S}}$ converges to zero asymptotically rather than shocks near zero because the new adaptive law is chosen to be the previous adaptive law $\gamma_i(\mu_i(\eta)/\sum_{i=1}^{9}\mu_i(\eta))\left[P_1^T \quad P_2^T\right]e(t)q^T(t)$ plus a robust term $\rho\,\text{sgn}(\tilde{\alpha}_i)$ to guarantee that the derivative of the Lyapunov function is negative rather than nonpositive. Figure 8 plots the tracking error of x- and y-axis between the reference model and the nonlinear model. We can notice that the performance of the tracking errors in Figure 8 is worse than that in Figure 7.

The tracking error $e_{\text{T-S}}$ shocks near zero because the controller is designed based on the T-S model rather than the NON model. From Figures 7 and 8, we can see the validity of the adaptive fuzzy control with parameter estimation scheme on the T-S model and the feasibility of the proposed controller on the nonlinear model of gyroscope. Then it can be concluded that the MEMS gyroscope can maintain

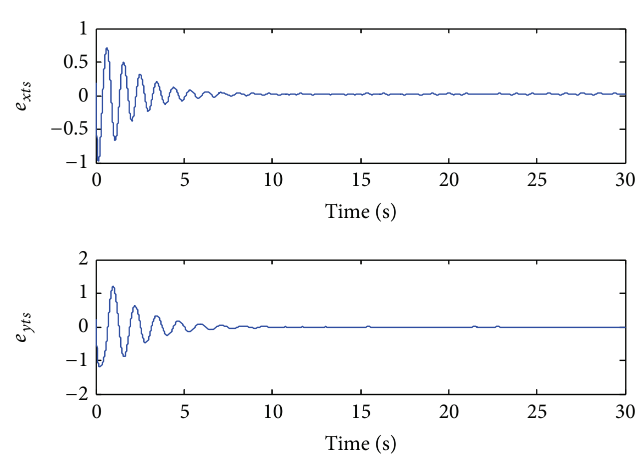

FIGURE 7: The tracking error $e_{\text{T-S}}$ between the reference model and T-S model.

the proof mass oscillating in the x and y direction at given frequency and amplitude by using adaptive T-S fuzzy controller. The proposed T-S modeling method can provide a possibility for developing a systematic analysis and design method for complex nonlinear control systems, which can approximate the nonlinear system and compensate model uncertainties and external disturbances. It can be observed from Figure 9 that the controller input of the adaptive fuzzy controller is stable.

Table 1: Parameters of T-S fuzzy model.

$A_1^* = \begin{bmatrix} -0.0753 & -0.0175 & 1 & 0 \\ 0.0025 & -0.1205 & 0 & 1 \\ -14207 & -877 & 0 & 0 \\ -877 & -12564 & 0 & 0 \end{bmatrix}^T$	$A_2^* = \begin{bmatrix} -0.0753 & -0.0175 & 1 & 0 \\ 0.0025 & -0.1205 & 0 & 1 \\ -14207 & -877 & 0 & 0 \\ -877 & -12564 & 0 & 0 \end{bmatrix}^T$	$A_3^* = \begin{bmatrix} -0.0753 & -0.0175 & 1 & 0 \\ 0.0025 & -0.1205 & 0 & 1 \\ -14207 & -877 & 0 & 0 \\ -877 & -12564 & 0 & 0 \end{bmatrix}^T$
$A_4^* = \begin{bmatrix} -0.0753 & -0.0175 & 1 & 0 \\ 0.0025 & -0.1205 & 0 & 1 \\ -15568 & 7.432 & 0 & 0 \\ -262 & -14201 & 0 & 0 \end{bmatrix}^T$	$A_5^* = \begin{bmatrix} 98 & 170 & 1 & 0 \\ 90 & 155 & 0 & 1 \\ -16066 & -851 & 0 & 0 \\ -859 & -15232 & 0 & 0 \end{bmatrix}^T$	$A_6^* = \begin{bmatrix} -98 & -170 & 1 & 0 \\ -90 & -155 & 0 & 1 \\ -16066 & -851 & 0 & 0 \\ -859 & -15232 & 0 & 0 \end{bmatrix}^T$
$A_7^* = \begin{bmatrix} -0.0753 & -0.0175 & 1 & 0 \\ 0.0025 & -0.1205 & 0 & 1 \\ -15568 & 7.432 & 0 & 0 \\ -262 & -14201 & 0 & 0 \end{bmatrix}^T$	$A_8^* = \begin{bmatrix} -98 & -170 & 1 & 0 \\ -90 & -155 & 0 & 1 \\ -16066 & -851 & 0 & 0 \\ -859 & -15232 & 0 & 0 \end{bmatrix}^T$	$A_9^* = \begin{bmatrix} 98 & 170 & 1 & 0 \\ 90 & 155 & 0 & 1 \\ -16066 & -851 & 0 & 0 \\ -859 & -15232 & 0 & 0 \end{bmatrix}^T$

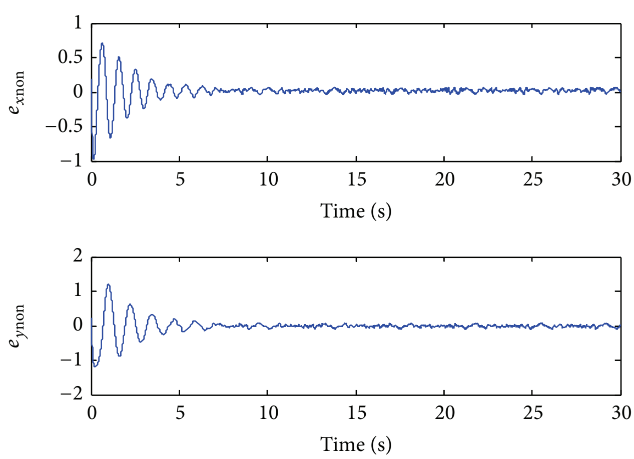

Figure 8: The tracking error e_{NON} between the reference model and nonlinear model.

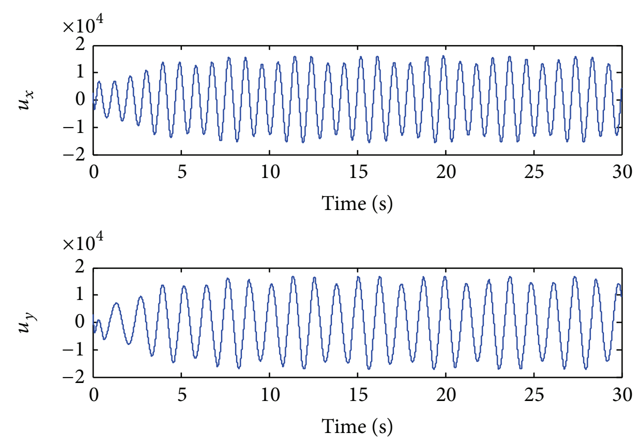

Figure 9: Control input with the proposed controller.

6. Conclusions

A T-S fuzzy model based adaptive controller for angular velocity sensor is presented in this paper. An adaptive T-S fuzzy compensator is used to approximate the model uncertainties and external disturbances. The output of the fuzzy controller is used as compensator to reduce the effects of the system nonlinearities. The proposed new adaptive law can guarantee that the derivative of the Lyapunov function is negative rather than nonpositive. The proposed estimator for the existing fuzzy state feedback controller can achieve a good robust performance against parameter uncertainties. Simulation studies are implemented to verify the effectiveness of the proposed adaptive fuzzy control and the correctness of the T-S fuzzy model. However, experimental demonstration should be investigated to verify the validity of the proposed approach in the next research step.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The authors thank the anonymous reviewers for useful comments that improved the quality of the paper. This work is partially supported by National Science Foundation of China under Grant no. 61374100 and Natural Science Foundation of Jiangsu Province under Grant no. BK20131136.

References

[1] R. P. Leland, "Adaptive control of a MEMS gyroscope using lyapunov methods," *IEEE Transactions on Control Systems Technology*, vol. 14, no. 2, pp. 278–283, 2006.

[2] S. Sung, W.-T. Sung, C. Kim, S. Yun, and Y. J. Lee, "On the mode-matched control of MEMS vibratory gyroscope via phase-domain analysis and design," *IEEE/ASME Transactions on Mechatronics*, vol. 14, no. 4, pp. 446–455, 2009.

[3] R. Antonello, R. Oboe, L. Prandi, and F. Biganzoli, "Automatic mode matching in MEMS vibrating gyroscopes using extremum-seeking control," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 10, pp. 3880–3891, 2009.

[4] S. Park, R. Horowitz, S. K. Hong, and Y. Nam, "Trajectory-switching algorithm for a MEMS gyroscope," *IEEE Transactions on Instrumentation and Measurement*, vol. 56, no. 6, pp. 2561–2569, 2007.

[5] J. D. John and T. Vinay, "Novel concept of a single-mass adaptively controlled triaxial angular rate sensor," *IEEE Sensors Journal*, vol. 6, no. 3, pp. 588–595, 2006.

[6] J. Fei and C. Batur, "A novel adaptive sliding mode control with application to MEMS gyroscope," *ISA Transactions*, vol. 48, no. 1, pp. 73–78, 2009.

[7] J. Fei and F. Chowdhury, "Robust adaptive controller for triaxial angular velocity sensor," *International Journal of Innovative Computing, Information and Control*, vol. 7, no. 6, pp. 2439–2448, 2010.

[8] L. Wang, *Adaptive Fuzzy Systems and Control-Design and Stability Analysis*, Prentice Hall, Upper Saddle River, NJ, USA, 1994.

[9] Y. Guo and P.-Y. Woo, "An adaptive fuzzy sliding mode controller for robotic manipulators," *IEEE Transactions on Systems, Man, and Cybernetics Part A: Systems and Humans.*, vol. 33, no. 2, pp. 149–159, 2003.

[10] B. K. Yoo and W. C. Ham, "Adaptive control of robot manipulator using fuzzy compensator," *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 2, pp. 186–199, 2000.

[11] R.-J. Wai, "Fuzzy sliding-mode control using adaptive tuning technique," *IEEE Transactions on Industrial Electronics*, vol. 54, no. 1, pp. 586–594, 2007.

[12] Y.-H. Chang, C.-W. Chang, C.-L. Chen, and C.-W. Tao, "Fuzzy sliding-mode formation control for multirobot systems: design and implementation," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 42, no. 2, pp. 444–457, 2012.

[13] Y.-H. Chien, W.-Y. Wang, Y.-G. Leu, and T.-T. Lee, "Robust adaptive controller design for a class of uncertain nonlinear systems using online T-S fuzzy-neural modeling approach," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 41, no. 2, pp. 542–552, 2011.

[14] M. C. M. Teixeira and S. H. Zak, "Stabilizing controller design for uncertain nonlinear systems using fuzzy models," *IEEE Transactions on Fuzzy Systems*, vol. 7, no. 2, pp. 133–142, 1999.

[15] C.-W. Park and Y.-W. Cho, "T-S model based indirect adaptive fuzzy control using online parameter estimation," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 34, no. 6, pp. 2293–2302, 2004.

[16] S. F. Asokanthan and T. Wang, "Nonlinear instabilities in a vibratory gyroscope subjected to angular speed fluctuations," *Nonlinear Dynamics*, vol. 54, no. 1-2, pp. 69–78, 2008.

[17] T. Wang, *Nonlinear and stochastic dynamics of MEMS-based angular rate sensing and switching systems [Ph.D. thesis]*, The University of Western of Ontario, Ontario, Canada, 2009.

[18] Z. Chi, M. Jia, and Q. Xu, "Fuzzy PID feedback control of piezoelectric actuator with feedforward compensation," *Mathematical Problems in Engineering*, vol. 2014, Article ID 107184, 14 pages, 2014.

[19] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Transactions on Systems, Man and Cybernetics Part B*, vol. 15, no. 1, pp. 116–132, 1985.

[20] P. Ioannou and J. Sun, *Robust Adaptive Control*, Prentice Hall, Upper Saddle River, NJ, USA, 1996.