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Research Article

Note on Relation between Bottom-Up Holographic Models and Large- N_c QCD

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We discuss a derivation of the quadratic in fields part of action of bottom-up holographic models from some general properties of the large- N_c limit in QCD. The importance of rescaling of five-dimensional fields is emphasized using the soft wall model as an example.

1. Introduction

The physics of hadrons composed of light quarks are highly nonperturbative as these hadrons represent strongly coupled systems. Some time ago, an interesting proposal of analytical approach to the strongly coupled gauge theories came from the string theory, the hypothesis of AdS/CFT correspondence [1] (also referred to as gauge/gravity duality or holographic duality). It turns out that a description of certain strongly coupled gauge theories can be given in terms of weakly coupled higher-dimensional gravitational theories. The holographic ideas have penetrated into many branches of modern theoretical physics. In the hadron spectroscopy, these ideas are challenging, since the AdS/CFT correspondence provides a practical recipe for calculation of correlation functions in strongly coupled gauge theories. The calculation is performed via the semiclassical expansion of the action of higher-dimensional dual theories [2]. This can have a direct application to the hadron spectroscopy: the masses of hadron states appear as poles of correlation functions of QCD currents interpolating those states.

The concrete holographic prescription is as follows [2]. All correlation functions of a 4D field theory can be obtained from the generating functional of the connected correlators $W_{4D}[\varphi_0(x)]$ which depends on the sources $\varphi_0(x)$ for the 4D field theory operators. If we know the action S_{5D} of

dual theory, the holographic correspondence postulates the identification:

$$W_{4D}[\varphi_0(x)] = S_{5D}[\varphi(x, \epsilon)]. \quad (1)$$

By assumption, the 5D dual theory is in the weakly coupled regime. This implies an important consequence: as the first approximation, one may consider only the quadratic terms in S_{5D} and use the semiclassical analysis. The calculation of n -point correlators consists thus in evaluation of S_{5D} on a solution of equation of motion and subsequent differentiation n times with respect to boundary values of 5D fields.

The bottom-up holographic approach represents an attempt to apply the holographic ideas to QCD. The corresponding models try to describe the nonperturbative dynamics of strong interactions in terms of a putative semiclassical five-dimensional theory. The first versions of bottom-up models, the so-called Hard Wall (HW) models [3, 4], had a problem with description of linear Regge and radial spectrum ($m_n \sim n$ instead of the behavior $m_n^2 \sim n$ expected in the phenomenology and on some theoretical grounds). The construction of the soft wall (SW) holographic model [5] solved this problem.

There is still no consensus on why the bottom-up holographic models turned out to be unexpectedly successful in the phenomenology. This success looks surprising and a bit enigmatic. An inherent property of the holographic duality

is the large- N_c (planar) limit of gauge theories [6, 7]. It is believed therefore that all AdS/QCD models are models for the planar QCD, where mesons represent stable and freely propagating particles (their interactions and decays are suppressed by powers of $1/N_c$ [6, 7]). Within a hypothetical dual theory, the corresponding mass spectrum should emerge in the weak coupling regime; that is, in the first approximation, it must be determined by a quadratic in fields part of action.

The purpose of this work is to demonstrate explicitly that the quadratic in fields parts of actions of typical bottom-up holographic models follows directly from some general expectations about the behavior of correlation functions in the large- N_c limit of QCD. Partly, this analysis was done in [8]. But the derivation in [8] did not take into account the fact that, in general case, one must use rescaled (with respect to the holographic coordinate) fields in order to have finite sources. This is especially important for the scalar and tensor mesons, since it leads to different mass spectra. In the present work, we fix this drawback and provide more justified expressions for meson spectra of SW model. As a byproduct, we also demonstrate the importance of “weak gravity” approximation which allows replacing the covariant derivatives by the usual ones and using an important relation for the 5D mass as a function of spin.

2. The Large- N_c QCD as a Kaluza-Klein Reduction

We recall some general results from the analysis of QCD in the limit of large number of colors N_c [6, 7]. The meson masses scale as $m \sim N_c^0$ and their full decay widths do as $\Gamma \sim N_c^{-1}$. The decay widths thus disappear in the limit of large N_c . Since the meson masses vary slightly with N_c , this limit is often useful in the meson spectroscopy. In the extreme case $N_c \rightarrow \infty$, the mesons are infinitely narrow and noninteracting; in addition, an infinite number of states emerge for each set of quantum numbers. Such an infinite tower of resonance poles saturates completely the two-point correlation function of quark currents with quantum numbers $I^G(J^{PC})$ corresponding to the given tower:

$$\langle \mathcal{O}_J(q) \mathcal{O}_J(-q) \rangle \sim \sum_{n=0}^{\infty} \frac{(F_n^{(J)})^2}{q^2 - m_{n,J}^2 + i\varepsilon}. \quad (2)$$

With the help of functional integral formalism, this is described by the following action with source terms:

$$I_{[\mathcal{O}_J]} = (-1)^J \int d^4x \sum_{n=0}^{\infty} (\partial_\mu \phi_J^{(n)} \partial^\mu \phi_J^{(n)} - m_{n,J}^2 \phi_J^{(n)} \phi_J^{(n)}) + \dots + \phi_J^{(n)} \mathcal{O}_J^{(n)}. \quad (3)$$

Here, $\phi_J \doteq \phi_{\mu_1 \mu_2 \dots \mu_J}$, $\mu_i = 0, 1, 2, 3$ (the sign convention is $(+ - - -)$), corresponds to a meson field of spin J and the other quantum numbers are unspecified. The tensor ϕ_J is symmetric and traceless, $\phi_{\mu \dots}^\mu = 0$ (to provide the irreducible $(J/2, J/2)$ representation of the homogeneous Lorentz group), and satisfies the auxiliary condition

$\partial^\mu \phi_{\mu \dots} = 0$ (to give the required $2J + 1$ physical degrees of freedom). The dots denote additional derivative terms appearing in description of free higher spin fields. The auxiliary conditions can be chosen in such a way that they do not make a contribution (to be discussed below). The tensor structure in the right-hand side of representation (2) depends on the structure of derivative terms. The sources $\mathcal{O}_J^{(n)}$ can be represented as

$$\mathcal{O}_J^{(n)} = F_n^{(J)} \mathcal{O}_J, \quad (4)$$

where the constants $F_n^{(J)}$ are defined by

$$\langle 0 | \mathcal{O}_J^{(n)} | \phi_J^{(n)} \rangle = F_n^{(J)} \varepsilon_J \quad (5)$$

for a meson $\phi_J^{(n)}$ with “polarization” ε_J and \mathcal{O}_J is a common source to which the states $\phi_J^{(n)}$ are coupled with a “coupling” $F_n^{(J)}$. Representation (4) follows from the requirement to get the standard form for two-point correlators: integrating over the field ϕ_J in the generating functional

$$Z[\mathcal{O}_J] = \int D\phi_J e^{I[\phi_J]}, \quad (6)$$

and differentiating twice with respect to \mathcal{O}_J at $\mathcal{O}_J = 0$, one arrives at the sum over meson poles (2).

Our task is to rewrite expression (3) as a Kaluza-Klein reduction of action of some free 5D theory. We will look for the corresponding 5D actions in the form

$$S_{5D} = (-1)^J \int d^4x dz f_1(z) (\partial_M \varphi_J \partial^M \varphi^J - m_J^2 f_2(z) \varphi_J \varphi^J), \quad (7)$$

where $M = 0, 1, 2, 3, 4$, $\varphi_J = \varphi_{M_1 M_2 \dots M_J}(x, z)$, and $f_1(z)$ and $f_2(z)$ are yet unknown functions of the fifth (space-like) coordinate z . We start from a flat 5D space having boundaries in the fifth coordinate, $z_{\min} \leq z \leq z_{\max}$. Action (7) contains a z -dependent background that is different for the kinetic and mass terms.

We will imply the following condition for the physical components of 5D fields,

$$\varphi_{z \dots} = 0. \quad (8)$$

With this condition, no additional quadratic terms appear in description of free higher spin mesons [5, 9, 10].

The equation of motion for action (7) with condition (8) is

$$-\partial_z [f_1(z) \partial_z \varphi_n^{(J)}(z)] + f_1(z) f_2(z) m_J^2 \varphi_n^{(J)}(z) = m_{n,J}^2 f_1(z) \varphi_n^{(J)}(z), \quad (9)$$

where we used the usual plane-wave ansatz for the physical 4D particles carrying the 4D momentum q_n :

$$\varphi(x, z) = e^{iq_n x} \varphi_n(z), \quad (10)$$

$$q_n^2 = m_n^2.$$

After imposing boundary conditions on the fields, (9) represents a classical Sturm-Liouville (SL) problem. Consider the following mathematical problem: given a spectrum $m_{n,J}^2$ in (3), find the functions $f_1(z)$ and $f_2(z)$ which, being inserted in (9), would lead to eigenvalues $m_{n,J}^2$. The solution of this “inverse” SL problem constitutes the main intermediate step in rewriting action (3) with sources as some free 5D field theory.

First, we recall briefly the main results of the SL theory. The SL equation is

$$-\partial_z [p(z) \partial_z \varphi] + q(z) \varphi = \lambda \omega(z) \varphi. \quad (11)$$

Here the function $p(z) > 0$ has a continuous derivative; the functions $q(z) > 0$ and $\omega(z) > 0$ are continuous on the finite closed interval $[z_{\min}, z_{\max}]$. The SL problem consists in finding the values of λ for which there exists a nontrivial solution of (11) satisfying certain boundary conditions. Under the assumptions that $p(z)^{-1}$, $q(z)$, and $\omega(z)$ are real-valued integrable functions over the interval $[z_{\min}, z_{\max}]$, with the boundary conditions of the form

$$\varphi(z_{\min}) \cos \alpha - p(z_{\min}) \varphi'(z_{\min}) \sin \alpha = 0, \quad (12)$$

$$\varphi(z_{\max}) \cos \beta - p(z_{\max}) \varphi'(z_{\max}) \sin \beta = 0, \quad (13)$$

where $\alpha, \beta \in [0, \pi)$ and prime means derivative, the SL theorem states that (i) there is an infinite discrete set of real eigenvalues λ_n , $n = 0, 1, 2, \dots$, (ii) up to a normalization constant, there is a unique eigenfunction $\varphi_n(z)$ corresponding to each eigenvalue λ_n and this eigenfunction has exactly $n - 1$ zeros in $[z_{\min}, z_{\max}]$, and (iii) the normalized eigenfunctions form an orthonormal basis:

$$\int_{z_{\min}}^{z_{\max}} \varphi_m(z) \varphi_n(z) \omega(z) dz = \delta_{mn}. \quad (14)$$

Thus the solutions of the SL problem form a complete set of functions in the interval $[z_{\min}, z_{\max}]$ which can be used for expansion of arbitrary (but sufficiently smooth) functions in that interval.

Under these conditions, the SL problem (9) has solutions $\varphi_n^{(J)}(z)$ which are normalized as

$$\int_{z_{\min}}^{z_{\max}} f_1(z) \varphi_m^{(J)}(z) \varphi_n^{(J)}(z) dz = \delta_{mn} \quad (15)$$

and form a complete set of functions. The function $\varphi_J(x, z)$ in (7) can be expanded in the 4D harmonics:

$$\varphi_J(x, z) = \sum_{n=0}^{\infty} \varphi_J^{(n)}(x) \varphi_n^{(J)}(z). \quad (16)$$

Now we substitute expansion (16) into action (7):

$$\begin{aligned} S_{5D} = & (-1)^J \int d^4x dz f_1(z) \sum_{m,n=0}^{\infty} \{ \varphi_m^{(J)} \varphi_n^{(J)} \partial_\mu \varphi_J^{(m)} \partial_\mu \varphi_J^{(n)} \\ & - \varphi_J^{(m)} \varphi_J^{(n)} \partial_z \varphi_m^{(J)} \partial_z \varphi_n^{(J)} \\ & - m_J^2 f_2(z) \varphi_m^{(J)} \varphi_n^{(J)} \varphi_J^{(m)} \varphi_J^{(n)} \}. \end{aligned} \quad (17)$$

Integrating by parts and making use of (9), the second term in action (17) can be rewritten as (dropping the general factor $-\varphi_J^{(m)} \varphi_J^{(n)}$)

$$\begin{aligned} & \int_{z_{\min}}^{z_{\max}} dz f_1(z) \partial_z \varphi_m^{(J)} \partial_z \varphi_n^{(J)} \\ & = \varphi_m^{(J)} f_1(z) \partial_z \varphi_n^{(J)} \Big|_{z_{\min}}^{z_{\max}} \\ & + \int_{z_{\min}}^{z_{\max}} dz \varphi_m^{(J)} \varphi_n^{(J)} g_J(z) (m_{n,J}^2 - f_2(z) m_J^2). \end{aligned} \quad (18)$$

Integrating over z in action (17) with the help of (15) and (18), we obtain

$$\begin{aligned} S_{5D} = & (-1)^J \int d^4x \sum_{n=0}^{\infty} \{ (\partial^\mu \varphi_J^{(n)})^2 - m_{n,J}^2 (\varphi_J^{(n)})^2 \\ & - \varphi_J^{(n)} f_1(z) \partial_z \varphi_n^{(J)} \varphi_J(x, z) \Big|_{z_{\min}}^{z_{\max}} \}, \end{aligned} \quad (19)$$

with $\varphi_J(x, z)$ given by (16). Action (19) has the form of (3) if we identify

$$\mathcal{O}_J^{(n)} = -f_1(z) \partial_z \varphi_n^{(J)} \varphi_J(x, z) \Big|_{z_{\min}}^{z_{\max}}. \quad (20)$$

The classical Kaluza-Klein reduction corresponds to $\mathcal{O}_J^{(n)} = 0$. But the case $\mathcal{O}_J^{(n)} \neq 0$ is possible as well. And this very possibility turns out to be equivalent to the holographic prescription for obtaining 4D Green's functions.

Choosing $\beta = 0$ or $\beta = \pi/2$ in the boundary condition (13), we can nullify the term at $z = z_{\max}$ in (20). Note in passing that for the vector mesons the first possibility is actually realized in the SW holographic models [5], where $z_{\max} \rightarrow \infty$, while the second one is realized in the HW models [3, 4], where z_{\max} is the infrared cutoff. Thus,

$$\mathcal{O}_J^{(n)} = \lim_{z \rightarrow z_{\min}^+} f_1(z) \partial_z \varphi_n^{(J)}(z) \varphi_J(x, z). \quad (21)$$

3. From Large- N_c QCD to Bottom-up Holographic Models

Action (7) can be rewritten as a “weakly coupled” 5D gravitational theory of free field φ_J with the metric

$$ds^2 = f_2(z) (dx_\mu dx^\mu - dz^2), \quad (22)$$

which dictates the corresponding metric tensor G_{MN} and the background function $f_1(z)$:

$$f_1(z) = \sqrt{|\det G_{MN}|} = f_2^{5/2}(z). \quad (23)$$

We must impose the covariant rule for contraction of indices:

$$\begin{aligned} & \partial_M \varphi_J \partial^M \varphi^J \\ & = \partial_M \varphi_{M_1 \dots M_J} \partial^{M'} \varphi_{M'_1 \dots M'_J} G^{MM'} G^{M_1 M'_1} \dots G^{M_J M'_J}, \end{aligned} \quad (24)$$

where

$$G^{MN} = G_{MN}^{-1} = f_2^{-1}(z) \eta^{MN}. \quad (25)$$

This can be achieved only if the function $f_1(z)$ in (23) is replaced by the J -dependent background:

$$f_1^{(J)}(z) = f_2^{(3/2)-J}(z). \quad (26)$$

All this just rearranges the notations but does not change our discussions above.

The choice

$$f_2(z) = \frac{R^2}{z^2}, \quad 0 \leq z < \infty \quad (27)$$

in (22) corresponds to the AdS space of radius R . This choice will be exploited in what follows to find connections with the holographic approach.

Now we should identify the general source \mathcal{O}_J . The identification must provide finite and nonzero values for constants $F_n^{(J)}$ in representation (4). Our prescription is

$$\mathcal{O}_J = \frac{z f_1^{(J)} \varphi_J}{c} \sim \frac{z^{2(J-1)} \varphi_J}{c}, \quad (28)$$

$$F_n^{(J)} = \lim_{z \rightarrow +0} c \frac{\partial_z \varphi_n^{(J)}(z)}{z}. \quad (29)$$

The eigenfunctions $\varphi_n^{(J)}(z)$ in the AdS space behave as z^2 at small z for the twist-2 operators, that is, when relation (36) holds [8]. This ensures constant values for $F_n^{(J)}$ in (29). The appearance of factor c is due to freedom to redefine $F_n \rightarrow c F_n$ and $\mathcal{O} \rightarrow \mathcal{O}/c$, where the parameter c can be fixed by matching to the high energy asymptotics of the corresponding QCD correlators. For the case of vector mesons, our way of getting relation (29) is alternative (up to a general factor depending on normalization of fields) to the derivation of [3].

Relation (28) shows that if we want, following the AdS/CFT prescriptions, to interpret the source as a finite boundary value of a 5D field, $\varphi_J(x, +0) = \mathcal{O}_J$, we must rescale the higher spin ($J > 1$) fields:

$$\varphi_J = z^{2(1-J)} \tilde{\varphi}_J. \quad (30)$$

In this sense, the physical tensor field is the rescaled one, $\tilde{\varphi}_J$. This statement was obtained in [9] using a different approach.

Now we see that the holographic prescription for calculation of correlation functions in a 4D theory from a “dual” 5D theory is completely reconstructed on the level of quadratic in fields actions. The higher vertices (describing meson decays and scattering) are suppressed in the large- N_c limit and we therefore do not consider them.

Some clarifying remarks should be made. Since the AdS space is not flat, the usual derivatives must be replaced by the

covariant ones in the case of tensor mesons. However, the holographic duality for strongly coupled 4D theories works only if the dual 5D theories are weakly coupled. This means that we should use the “weak gravity” limit (very large AdS radius) as the first approximation. In this limit, the affine connections are neglected as the AdS space locally is almost flat. This justifies the use of usual derivatives for higher spin mesons in our analysis.

The second remark concerns the standard statement that the spectrum of the HW model depends on the boundary conditions. Indeed, the Dirichlet boundary conditions that have been used since the pioneering papers [3, 4] correspond to the choice $\beta = \pi/2$ in (13) and seemingly one could use any other choice for β . There is, however, a subtle point here. With other choices, generally speaking, the boundary contribution at $z = z_{\max}$ is not zero. This means the appearance of the second source that would look strange for the standard functional technics (the existence of two sources is not necessarily senseless in the real strong interactions, where the physical degrees of freedom at low and high energies are different (pseudo-Goldstone bosons versus quarks and gluons). For instance, the quark vector current $\bar{q} \vec{\tau} \gamma_\mu q$ interpolates the ρ -meson at high enough energies, where the current quarks are well defined. At low enough energies, the ρ -meson could be considered as a two-pion state interpolated by the current $\pi^a \vec{\tau} \overleftrightarrow{\partial}_\mu \pi^b$. The ultraviolet and infrared sources should be then different as long as the physical degrees of freedom are different. As far as we know, this possibility was not exploited in the bottom-up approach).

The last remark concerns the choice of background in the 5D action. It can be shown that if we want to reproduce the linear Regge spectrum and simultaneously the correct analytical structure of the operator product expansion of the two-point correlators, the background is fixed unambiguously; it corresponds to nothing but the SW model to be discussed in the next section. The corresponding analysis is presented in detail in [8].

4. Spectrum of the Soft Wall Model

An attractive feature of the SW holographic models is the Regge spectrum for meson masses [5]. The original derivation of [5] was based on gauge higher spin fields. Condition (8) was interpreted as a gauge choice. A residual invariance after imposing (8) (that canceled the additional derivative terms) allowed eliminating the mass term for the rescaled fields (30). Below we reproduce the Regge spectrum of [5] using the massive higher spin fields and our discussions above.

The action of SW model is a particular case of (7) written in the AdS space:

$$S_{5D} = (-1)^J \int d^4x dz \sqrt{|\det G|} e^{az^2} \left(\partial_M \varphi_J \partial^M \varphi^J - m_J^2 \varphi_J \varphi^J \right), \quad (31)$$

where the dilaton background e^{az^2} was introduced to get the Regge spectrum. Substituting the rescaled field (30) and lowering all indices, we find

$$\begin{aligned} S_{5D} = & (-1)^J R^{2J-1} \int d^4x dz e^{az^2} \left\{ (\partial_M \tilde{\varphi}_J)^2 z^{1-2J} \right. \\ & - \left. \left((2-2J)^2 + m_J^2 R^2 \right) z^{-1-2J} \tilde{\varphi}_J^2 \right. \\ & \left. - 4(1-J) z^{-2J} \tilde{\varphi}_J \partial_z \tilde{\varphi}_J \right\}. \end{aligned} \quad (32)$$

The corresponding equation of motion for physical modes (10) reads

$$\begin{aligned} -\partial_z \left[\frac{e^{az^2}}{z^{2J-1}} \partial_z \tilde{\varphi}_{n,J} \right] + \frac{m_J^2 R^2 - 4(J-1)}{z^{2J+1}} e^{az^2} \tilde{\varphi}_{n,J} \\ = m_{n,J}^2 \frac{e^{az^2}}{z^{2J-1}} \tilde{\varphi}_{n,J}. \end{aligned} \quad (33)$$

The SL equation (33) has the following discrete spectrum:

$$\begin{aligned} m_{n,J}^2 = & 2|a| \\ & \cdot \left[2n+1 + \frac{a}{|a|} (1-J) + \sqrt{J^2 + m_J^2 R^2 - 4(J-1)} \right]. \end{aligned} \quad (34)$$

This spectrum depends on the 5D mass m_J . The free p -form fields in the AdS_5 space have masses $m_p^2 R^2 = (\Delta - p)(\Delta + p - 4)$. As was shown in [10], condition (8) leads to decoupling of kinematical aspects and the symmetric tensors of rank p (describing mesons of spin p) obey the same equation as the p -form fields. In the ‘‘weak gravity’’ approximation, this results in the following 5D mass of higher spin fields propagating in the AdS_5 space [10, 11]:

$$m_J^2 R^2 = (\Delta - J)(\Delta + J - 4), \quad (35)$$

where Δ is the canonical dimension of operator in a 4D field theory. This operator is regarded as being dual to a corresponding 5D field on the AdS boundary.

The spectrum of the original SW model [5] corresponds to the operators of twist-2; that is, $\Delta = J + 2$. Substituting this dimension into (35), we have

$$m_J^2 R^2 \Big|_{\Delta=J+2} = 4(J-1). \quad (36)$$

Spectrum (34) reduces then to the spectrum of [5]:

$$m_{n,J}^2 = 2|a| \left[2n+J+1 + \frac{a}{|a|} (1-J) \right]. \quad (37)$$

As was noticed in [5], the spectrum does not depend on spin if $a > 0$. The standard SW model has $a < 0$ and this choice dictates a remarkably simple Regge spectrum $m_{n,J}^2 = 4|a|(n+J)$ which coincides with the spectrum of poles of the Veneziano amplitude.

In fact, only the 5D mass (36) is to be substituted into (34) because if the twist is different from 2, then prescriptions (28) and (29) must be changed since a constant value of the source

on the boundary $z = 0$ is provided by a different rescaling rule. This becomes especially important in the scalar sector. The case $J = 0$ in (37) formally corresponds to the spectrum of mesons interpolated by a scalar operator of canonical dimension $\Delta = 2$. But such a gauge-invariant operator cannot be constructed in QCD, where the simplest scalar operator is the twist-3 quark bilinear $\bar{q}q$. The corresponding 5D mass for $\Delta = 3$ is $m_J^2 R^2 = -3$ and the constant boundary value of the scalar source requires rescaling $\varphi \rightarrow z\tilde{\varphi}$ [3]. It can be easily checked that the scalar spectrum coincides then with the vector one in (37):

$$m_{n,0}^2 \Big|_{\Delta=3} = 4|a|(n+1). \quad (38)$$

A 5D field dual on the boundary $z = 0$ to a scalar operator of twist-4 (such as the gluon operator $\alpha_s G_{\mu\nu}^2$) does not require rescaling; the corresponding mass spectrum is known to be

$$m_{n,0}^2 \Big|_{\Delta=4} = 2|a| \left[2n+3 - \frac{a}{|a|} \right]. \quad (39)$$

The phenomenology again favors the choice of negative dilaton background: $a < 0$.

For the purposes of phenomenological applications, it can be interesting to construct an extension of the SW model which leads to a shifted Regge spectrum, $m_{n,J}^2 = 4|a|(n+J+b)$, where b is a phenomenological parameter regulating the intercept of Regge and radial trajectories. Such an extension was constructed in [12] for the vector case. The method of [12] can be generalized to higher spins. We will not display the derivation in detail and will just give the final answer that can be checked by a direct calculation: the dilaton background e^{az^2} must be replaced by

$$e^{az^2} \longrightarrow e^{az^2} U^2(b, -|J-1|; |a|z^2), \quad (40)$$

where U is Tricomi’s hypergeometric function.

5. Conclusion

In essence, we tried to demonstrate that the bottom-up holographic approach to QCD represents nothing but a five-dimensional language for expressing some general expectations from the large- N_c limit in QCD. This interpretation is valid only for holographic actions quadratic in fields.

We emphasized the importance of rescaling of five-dimensional fields for correct derivation of meson spectra. The simplest SW holographic model was used as a typical example. The action of SW model is purely phenomenological and is constructed with the aim of getting the linear Regge spectrum. It is not known which 5D dynamical model results in the background e^{az^2} as a solution of 5D Einstein equations. An ‘‘intermediate’’ dynamical model that leads to such a background could look as follows:

$$S = \int d^4x dz \sqrt{g} \left(\partial_M \phi \partial^M \phi - m^2 \phi^2 + e^\phi \mathcal{L} \right), \quad (41)$$

where the action is written in the AdS_5 space and \mathcal{L} represents a Lagrangian density for the meson sector. The

dilaton coupling in the form $e^\phi \mathcal{L}$ is typical in the string theory. If the 5D mass squared m^2 of the dilaton field ϕ takes the minimal value permitted in the AdS_5 space, $m^2 R^2 = -4$ (the Breitenlohner-Freedman stability bound [13]) and one neglects back-reaction from \mathcal{L} ; then the equation of motion for ϕ will have a solution $\phi = az^2$. This solution leads to the z -dependent background of SW model. One may speculate that the dilaton field ϕ in (41) mimics the nonperturbative gluodynamics. In the large- N_c limit, the gluon sector dominates. For instance, the probability of creation of glueballs scales as $\mathcal{O}(N_c^2)$, while that of mesons scales as $\mathcal{O}(N_c)$ [6, 7]. We do not see an abundant production of glueballs in the real world just because we cannot create experimentally the corresponding gluon currents. But theoretically the scaling $\mathcal{O}(N_c^2)$ of gluon sector justifies neglecting back-reaction from \mathcal{L} in the first approximation. In any case, the simplest SW model should be viewed as an approximation in which a dynamical dilaton field is replaced by a static dilaton background dictated by the phenomenology.


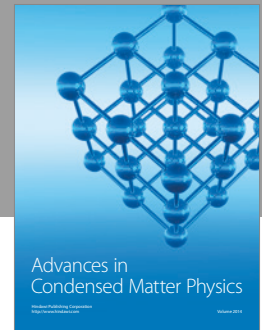
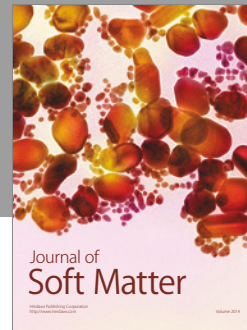
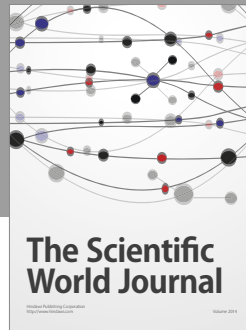
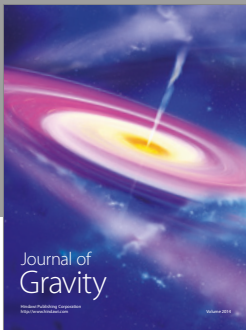
The use of the bottom-up models is much broader than just modelling the hadron spectra; it includes description of the spontaneous chiral symmetry breaking in QCD, hadron form factors, QCD phase diagram, and so forth. The corresponding applications usually contain certain extensions of simple bottom-up models that go beyond the large- N_c limit or have no clear connections with the planar QCD. Establishing these connections is an open problem.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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