

Research Article

Stagnation Point Flow of a Nanofluid toward an Exponentially Stretching Sheet with Nonuniform Heat Generation/Absorption

A. Malvandi,¹ F. Hedayati,² and G. Domairry³

¹ Department of Mechanical Engineering, Amirkabir University of Technology (Tehran Polytechnic), 424 Hafez Avenue, P.O. Box 15875-4413, Tehran, Iran

² Department of Mechanical Engineering, Islamic Azad University, Sari Branch, Sari, Iran

³ Department of Mechanical Engineering, Babol Noshirvani University of Technology, Babol 47148-71167, Iran

Correspondence should be addressed to A. Malvandi; amirmalvandi@aut.ac.ir

Received 1 May 2013; Accepted 8 July 2013

Academic Editor: Mohammad Al-Nimr

Copyright © 2013 A. Malvandi et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper deals with the steady two-dimensional stagnation point flow of nanofluid toward an exponentially stretching sheet with nonuniform heat generation/absorption. The employed model for nanofluid includes two-component four-equation nonhomogeneous equilibrium model that incorporates the effects of Brownian diffusion and thermophoresis simultaneously. The basic partial boundary layer equations have been reduced to a two-point boundary value problem via similarity variables and solved analytically via HAM. Effects of governing parameters such as heat generation/absorption λ , stretching parameter ϵ , thermophoresis N_t , Lewis number Le , Brownian motion N_b , and Prandtl number Pr on heat transfer and concentration rates are investigated. The obtained results indicate that in contrast with heat transfer rate, concentration rate is very sensitive to the abovementioned parameters. Also, in the case of heat generation $\lambda > 0$, despite concentration rate, heat transfer rate decreases. Moreover, increasing in stretching parameter leads to a gentle rise in both heat transfer and concentration rates.

1. Introduction

For years, many researchers have paid much attention to viscous fluid motion near the stagnation region of a solid body, where “body” corresponds to either fixed or moving surfaces in a fluid. This multidisciplinary flow has frequent applications in high speed flows, thrust bearings, and thermal oil recovery. Hiemenz [1] developed the first investigation in this field. He applied similarity transformation to collapse two-dimensional Navier-Stokes equations to a nonlinear ordinary differential one and then presented its exact solution. Extension of this study was carried out with a similarity solution by Homann [2] to the case of axisymmetric three-dimensional stagnation point flow. After these original studies, many researchers have put their attention on this subject [3–9]. Besides stagnation point flow, stretching surfaces have a wide range of applications in engineering and several technical purposes particularly in metallurgy and

polymer industry, for instance, gradual cooling of continuous stretched metal or plastic strips which have multiple applications in mass production. Crane [10] was the first to present a similarity solution in the closed analytical form for steady two-dimensional incompressible boundary layer flow caused by the stretching plate whose velocity varies linearly with the distance from a fixed point on the sheet. The combination of stretching surface and stagnation point flow was analyzed by Yao et al. [11]. Different types fluids such as viscoelastic [12] or micropolar ones [13] past a stretching sheet have been studied later. The popularity of stretching surfaces can be gauged from the researches done by scientists for its frequent applications and can be found in the literature, for example, [14–19]. Recently, Nadeem and Lee [20] have investigated the boundary layer over exponentially stretching surfaces analytically.

Improving the technology, limit in enhancing the performance of conventional heat transfer is a main issue [21]

owing to low thermal conductivity of the most common fluids such as water, oil, and ethylene-glycol mixture. Since the thermal conductivity of solids is often higher than that of liquids, the idea of adding particles to a conventional fluid to enhance its heat transfer characteristics emerged. Among all dimensions of particles such as macro, micro, and nano, due to some obstacles in the pressure drop through the system or keeping the mixture homogenous, nanoscaled particles have attracted more attention. These tiny particles are fairly close in size to the molecules of the base fluid and thus can realize extremely stable suspensions with slight gravitational settling over long periods of time. Coined term “nanofluid” was proposed by Choi [22] to point out engineered colloids composed of nanoparticles dispersed in a base fluid. Following the seminal study of this concept by Masuda et al. [23], a considerable amount of research in this field has risen exponentially. Meanwhile, theoretical studies emerged to model the nanofluids behaviors for which the proposed models are twofold: homogeneous flow models and dispersion models. Buongiorno [24] indicated that the homogeneous models tend to underpredict the nanofluid heat transfer coefficient, and due to nanoparticle size, the dispersion effect is completely negligible. Hence, Buongiorno developed an alternative model to explain the abnormal convective heat transfer enhancement in nanofluids and eliminate the shortcomings of the homogenous and dispersion models. He considers seven slip mechanisms, including inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus, fluid drainage, and gravity, and claimed that, of these seven, only Brownian diffusion and thermophoresis are important slip mechanisms in nanofluids. Moreover, Buongiorno concluded that turbulence is not affected by nanoparticles. Based on this finding, he proposed a two-component four-equation nonhomogeneous equilibrium model for convective transport in nanofluids. The aforementioned model has recently been used by Kuznetsov and Nield [25] to study the influence of nanoparticles on natural convection boundary layer flow past a vertical plate. Then, a comprehensive survey of convective transport of nanofluids in the boundary layer flow was conducted by Khan and Pop [26], Bachok et al. [27], Alsaedi et al. [28], and Rana and Bhargava [29]. Some very recent review papers are written by Daungthongsuk and Wongwises [30], Wang and Mujumdar [31], and Kakaç and Pramuanjaroenkij [32].

In order to study the aforementioned issues, there is a certain need to model them mathematically and solve them with an appropriate technique. For years, numerical approaches have been developed, but due to some restrictions [33], analytical solutions have been considered as alternative ways by scientists. Perturbation technique is one of the most common methods in this field which is widely applied in science and engineering [34]. A drawback of perturbation techniques is that they strongly depend upon small/large physical parameters, so they cannot be applied to strongly nonlinear problems. Hence, nonperturbation techniques such as Adomian decomposition method [35] and variational iteration method [36–38] appear in order to omit the dependency on small/large parameters. It must be noted that these methods cannot ensure the convergence of

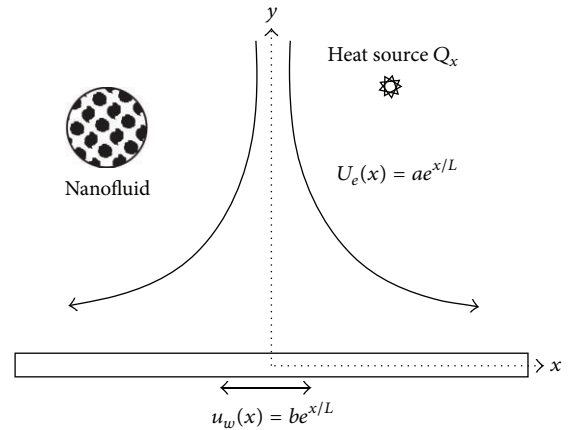


FIGURE 1: Geometry of physical model and coordinates system.

series solution. On the other hand, the homotopy analysis method (HAM) proposed by Liao [39–41] is a general analytical approach to get series solutions of strongly nonlinear equations [42–44] with great freedom options to ensure the convergence of solutions series. Moreover, in contrast with numerical methods, it can be implemented with far field boundary conditions. Needless to say that for boundary layer problems, the physical domain is unbounded, whereas the computational domain has to be finite, so similarity variable at infinity must be evaluated with the aid of previous studies [45]. Newly, Liao [46] has presented the optimal HAM to guarantee the convergence of solution. He defined a new kind of averaged residual error to find the optimal convergence-control parameters which can accelerate the convergence of series.

This paper deals with the analytical study of boundary layer stagnation point flow of nanofluid toward an exponentially stretching surface with nonuniform heat generation/absorption which is the extension of Hassani and coworkers’ study [47] and the mentioned Nadeem and Lees’ one [20]. The studied model incorporates the effects of suction injection parameter α , Lewis number Le , the Brownian motion parameter N_b , and thermophoresis parameter N_t . It is hoped that the obtained results will not only present useful information for applications, but also serve as a complement to the previous studies.

2. Governing Equations

Consider the steady laminar two-dimensional flow of nanofluids near the stagnation point at a stretching sheet in the presence of heat generation/absorption as shown in Figure 1. The coordinates x and y are taken with the origin O at the stagnation point. Two opposite forces are applied along the x -axis similarly so that the wall is stretched whilst keeping the position of the origin fixed. The free stream fluid’s velocity and the stretching one are assumed to vary nonlinearly, which corresponded to $U_e(x) = ae^{x/L}$ and $u_w(x) = be^{x/L}$, respectively. It is to be said that a and b are constants and always positive; that is, $a, b > 0$. It is also assumed that the

temperature and concentration at the surface have constant values of T_w and C_w , respectively, while the ambient temperature and concentration beyond boundary layer have constant values T_∞ and C_∞ , respectively. The continuity, momentum, and energy equations in the Cartesian coordinates for this flow can be expressed as [48]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_e \frac{dU_e}{dx} + \nu \frac{\partial^2 u}{\partial y^2}, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{Q_x}{\rho c_p} (T - T_\infty), \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}, \quad (4)$$

subject to the boundary conditions

$$\begin{aligned} v = 0, \quad u = be^{x/L}, \quad T = T_w, \quad C = C_\infty \quad \text{at } y = 0, \\ u = u_e(x) = ae^{x/L}, \quad T = T_\infty, \quad C = C_\infty \quad \text{as } y \rightarrow \infty. \end{aligned} \quad (5)$$

Here, u and v are the velocity components along the x - and y -directions, respectively, and T is the temperature. D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, τ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid, μ_{nf} is the viscosity of nanofluid, $Q_x = \lambda(b/2L)e^{x/L}$ is the nonuniform heat generation/absorption where $\lambda > 0$ and $\lambda < 0$ stand for heat generation and absorption, respectively, ρ_{nf} is the density of nanofluid, and α_{nf} is the thermal diffusivity of nanofluid. In order to find a similarity solution of (1)–(4), we employed the following dimensionless parameters:

$$\begin{aligned} \eta = \sqrt{\frac{b}{2\nu L}} e^{x/2L} y, \quad \psi = \sqrt{2b\nu L} e^{x/2L} f(\eta), \\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad C(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \end{aligned} \quad (6)$$

Here, η is the similarity variable and ψ is the usual stream function; that is, $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. The governing equations (1)–(3) and the boundary conditions of (5) collapse into

$$f''' + ff'' + 2(\varepsilon^2 - f'^2) = 0, \quad (7)$$

$$\theta'' + \text{Pr}\theta'(N_b\phi' + N_t\theta' + f) + \text{Pr}\lambda\theta = 0, \quad (8)$$

$$\phi'' + \text{Le}f\theta' + \frac{N_t}{N_b}\theta'' = 0. \quad (9)$$

The appropriate dimensionless forms of boundary condition of (5) are

$$\begin{aligned} \text{At } \eta = 0: \quad f = 0, \quad f' = 1, \quad \theta = 1, \quad \phi = 1 \\ \text{At } \eta = \infty: \quad f'(\eta) = \varepsilon, \quad \theta(\eta) = 0, \quad \phi(\eta) = 0, \end{aligned} \quad (10)$$

where prime denotes differentiation with respect to η and Pr , Le , N_b , N_t , and ε denote Prandtl number, Lewis number, Brownian motion, thermophoresis, and stretching parameter, respectively. The physical quantities of interest in this study are

$$\begin{aligned} \text{Cf}_x &= \frac{\tau_w|_{y=0}}{\rho U_e^2}, \\ \text{Nu}_x &= -\frac{x}{(T_w - T_\infty)} \frac{\partial T}{\partial y} \Big|_{y=0}, \\ \text{Sh}_x &= -\frac{x}{(C_w - C_\infty)} \frac{\partial C}{\partial y} \Big|_{y=0} \end{aligned} \quad (11)$$

and can be expressed as

$$\begin{aligned} \sqrt{2\text{Re}_x} \text{Cf}_x &= f''(0), \\ \frac{\text{Nu}_x}{\sqrt{2\text{Re}_x}} &= -\sqrt{\frac{x}{2L}} \theta'(0), \\ \frac{\text{Sh}_x}{\sqrt{2\text{Re}_x}} &= -\sqrt{\frac{x}{2L}} \phi'(0), \end{aligned} \quad (12)$$

where $\text{Re}_x = u_w x / \nu$ is the local Reynolds number.

3. Analytical Solution by Homotopy Analysis Method

In order to solve the equations by means of HAM, we have to choose the initial guesses and auxiliary linear operators which are assumed as follows:

$$\begin{aligned} f_0(\eta) &= \varepsilon\eta + (1 - \varepsilon)(1 - e^{-\eta}), \\ \theta_0(\eta) &= e^{-\eta}, \\ \phi_0(\eta) &= e^{-\eta}, \\ L(f) &= f''' - f', \\ L(\theta) &= \theta'' - \theta, \\ L(\phi) &= \phi'' - \phi. \end{aligned} \quad (13)$$

Then, we have proceeded the solution of HAM for which, for the sake of brevity, the details are skipped here and can be found in [49–51]. As pointed out by Liao [40], the convergence rate of approximation for the HAM solution strongly depends on the value of auxiliary parameter, c_i ($i = 1, 2, 3$). In order to seek the permissible values of c_i , the considered functions have to be plotted for a specific physical point at an appropriate order of approximations. Obtaining the suitable

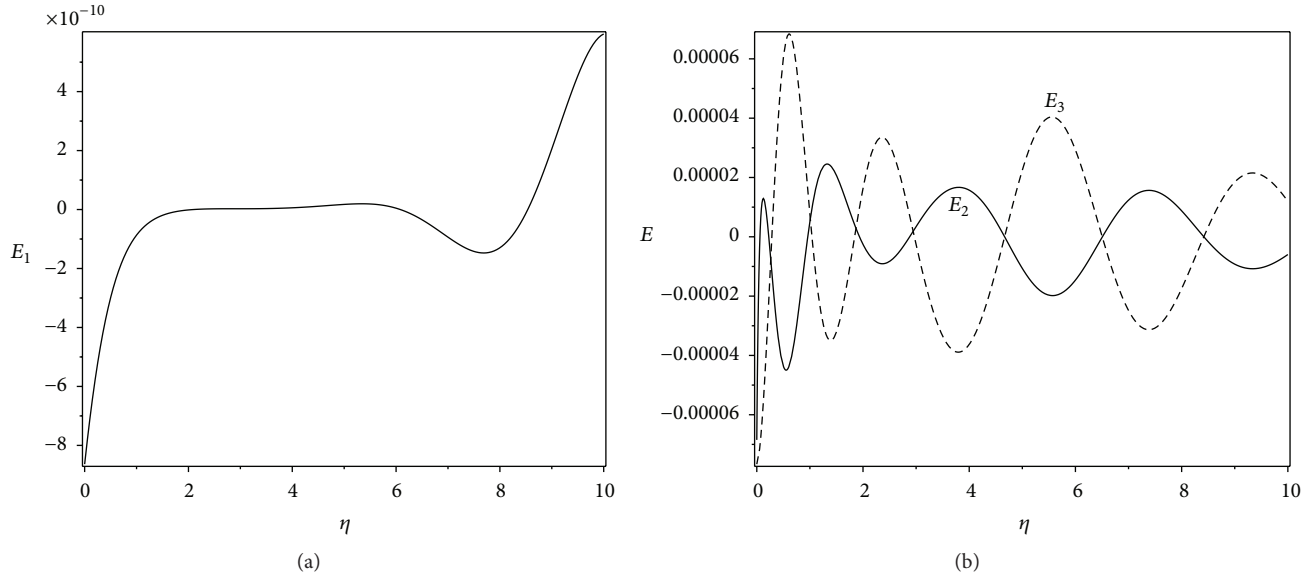


FIGURE 2: Average residual errors which are defined by (15).

c_i , we followed Liao and defined the averaged residual errors to find the optimal convergence-control parameters

$$\begin{aligned}
 E_1 &= \int_0^\infty \left(N \left[\sum_{i=0}^m F_i(\xi) \right] \right)^2 d\xi, \\
 E_2 &= \int_0^\infty \left(N \left[\sum_{i=0}^m \theta_i(\xi) \right] \right)^2 d\xi, \\
 E_3 &= \int_0^\infty \left(N \left[\sum_{i=0}^m \phi_i(\xi) \right] \right)^2 d\xi.
 \end{aligned} \tag{15}$$

The more quickly E_i ($i = 1, 2, 3$) decreases to zero, the faster the corresponding homotopy-series solution converges. So, at a proper order of approximation m , the corresponding optimal values of the convergence-control parameter will be obtained by the minimum values of E_i which can be estimated as

$$\begin{aligned}
 \frac{\partial E_1}{\partial c_1} &= 0, \\
 \frac{\partial E_2}{\partial c_2} &= 0, \\
 \frac{\partial E_3}{\partial c_3} &= 0.
 \end{aligned} \tag{16}$$

It is worth mentioning that since (8) and (9) are coupled, for minimizing the E_2 and E_3 , we should apply either a least square technique or the method introduced by Yabushita et al. [52], where, following [53], we have applied the second one.

4. Result and Discussion

The system of (7)–(9) with boundary conditions of (10) has been solved analytically via homotopy analysis method (HAM). The best accuracy of our results has been obtained and shown in Figure 2 where residual values of (16) are plotted in the solution domain. In addition, for selective parameters, we have compared our analytical outcomes with numerical ones which are cited in Table 1. In comparison with regular fluids, the present study involves three more parameters: Le , N_b , and N_t ; hence, contour plots have been presented instead of regular diagrams. The advantage of contour lines is that they illustrate more physical interpretation rather than common plots and are appropriate for problems with many effective parameters. Here, in all contours, the negative values have been shown with dashed lines. Needless to say that higher density of the contour lines demonstrates wide range of variations of the understudied parameter, and for sparse sets of contours, it is vice versa.

N_t - N_b contour lines of the physical interests, that is, the heat transfer and concentration rates for different values of unsteadiness parameter (A), stretching parameter (ε), Lewis number (Le), heat generation/absorption parameter (λ), and Prandtl number (Pr), are shown in Figures 3 and 6. As the eye sees, the behavior of heat transfer rate $-\theta'(0)$ reveals its straightforward dependency on almost all parameters except for higher values of Pr for which variations of N_t and N_b have stronger effects and should be considered in depth. Clearly, increase in the values of N_t and N_b leads to the decrease in the values of heat transfer rate, that is, $-\theta'(0)$. In contrast to heat transfer, we can observe that the concentration rate $-\phi'(0)$ is very sensitive to all parameters and its variations are not predictable at a glance. A rise in N_t at lower values of Le leads to a drop in concentration rate, while for higher Lewis numbers, concentration rate takes an increasing trend except for some values of low thermophoresis and Brownian

TABLE I: Comparison of our analytical results with numerical results via shooting method.

N_b	Le	$-\theta'(0)$			$-\phi'(0)$		
		HAM	Numerical	Error %	HAM	Numerical	Error %
0.1	1	0.644835	0.645023	-0.029148	-1.52771	-1.52922583	-0.099355
	2	0.589582	0.5883122	0.2153212	-0.65237	-0.64663798	0.886434
0.5	1	0.458667	0.4586645	0.0005066	0.312874	0.31278339	0.0289599
	2	0.373846	0.3735257	0.0857389	0.752468	0.75300072	-0.070857
1	1	0.284093	0.2838612	0.0815335	0.514955	0.51509636	-0.027359
	2	0.197857	0.1985839	-0.367558	0.886216	0.88559894	0.0696118

motion. Furthermore, as N_b increases, the $-\phi'(0)$'s variation is dwindled down which is more suppressed as Lewis number increases.

Considering Figure 3, we can observe that the increase in stretching parameter ε causes a gentle rise in both heat transfer and concentration rates. Additionally, in the case of heat absorption, that is, $\lambda < 0$, the values of the heat transfer rate increase while concentration rate decreases; a reversed trend can be observed for the case of heat generation which is supported by Figure 4. According to Figure 5, it can be observed that despite the concentration rate, a rise in Le number decreases the heat transfer rate which is more obvious for lower values of Le. An interesting feature that can be observed in Figure 5 is that with lower Lewis number and Brownian motion, $-\phi'(0)$ is negative; that is, reverse concentration rate occurs. Effects of Pr have been considered in Figure 6 which shows that the increase in Pr leads to a rise in the values of heat transfer rate at lower values of Brownian motion; however, for higher values of Brownian motion, increasing in Prandtl number decreases the heat transfer rate. In contrast, a rise Prandtl number increases the concentration rate especially with higher value of N_t .

Finally, sample profiles of boundary layer including velocity, temperature, and concentration for different values of ε have been presented in Figures 7, 8, and 9. It is noteworthy that these profiles have the same form with regular fluids. Increasing the stretching parameter, due to less difference between the sheet and free stream velocities, the momentum boundary layer gets thinner. This causes an increase of the momentum at the surface. So, it is not surprising that heat transfer and concentration rates climb up both (Figure 3).

5. Conclusion

This paper deals with an analytical study of boundary layer stagnation point flow of nanofluid toward an exponentially stretching surface with nonuniform heat generation/absorption. The governing PDE equations including continuity, momentum, and energy have been transformed into ODE ones with similarity solution and are solved with HAM. The main outcomes of the paper can be summarized as follows.

- (i) Heat transfer rate has simple dependency on almost all parameters except for higher values of Pr. Increasing the values of thermophoresis (N_t), Brownian

motion (N_b), and Lewis number (Le) results in a reduction in heat transfer rate.

- (ii) In contrast to heat transfer rate, concentration rate is sensitive to the parameters of thermophoresis (N_t), Brownian motion (N_b), Lewis (Le), and Prandtl numbers (Pr). A rise in N_t at lower values of Le leads to a drop in concentration rate, while for higher Lewis numbers, concentration rate takes an increasing trend except for some values of low thermophoresis and Brownian motion parameters.
- (iii) Heat generation $\lambda > 0$ reduces the heat transfer rate while increasing the amounts of concentration rate. A reversed behavior can be observed for heat absorption $\lambda < 0$. Also, we can see that increasing the stretching parameter ε causes a gentle rise in both heat transfer and concentration rates.
- (iv) Increasing Pr leads to a rise in the values of heat transfer rate at lower values of Brownian motion; however, for higher values of Brownian motion, increasing Prandtl number decreases the heat transfer rate.

Nomenclature

- a, b : Positive constants
- C : Nanoparticle volume fraction
- Cf_x : Local skin friction coefficient
- D_B : Brownian diffusion coefficient
- D_T : Thermophoresis diffusion coefficient
- f : Dimensionless stream function
- L : Characteristic length
- Le: Lewis number
- N_b : Brownian motion parameter
- N_t : Thermophoresis parameter
- Nu_x : Local Nusselt number
- p : Pressure
- Pr: Prandtl number
- Q_x : Nonuniform heat generation/absorption
- Re_x : Local Reynolds number
- Sh_x : Local Sherwood number
- T : Temperature
- u, v : Velocity components along the x - and y -directions, respectively
- x, y : Cartesian coordinates system.

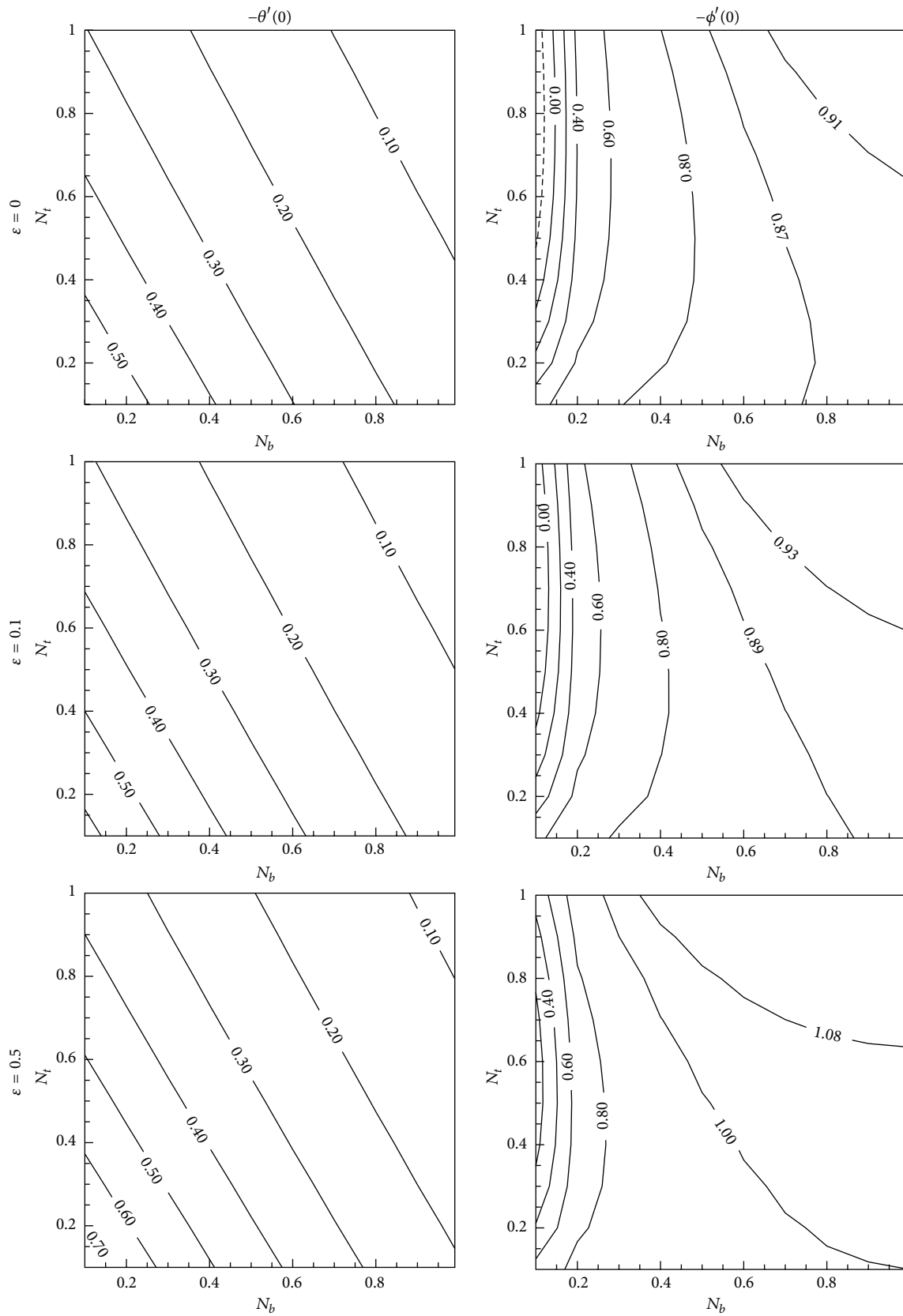


FIGURE 3: Contour lines of the heat transfer and concentration rates for different values of ε when $Le = Pr = 2$, $\lambda = 0.1$.

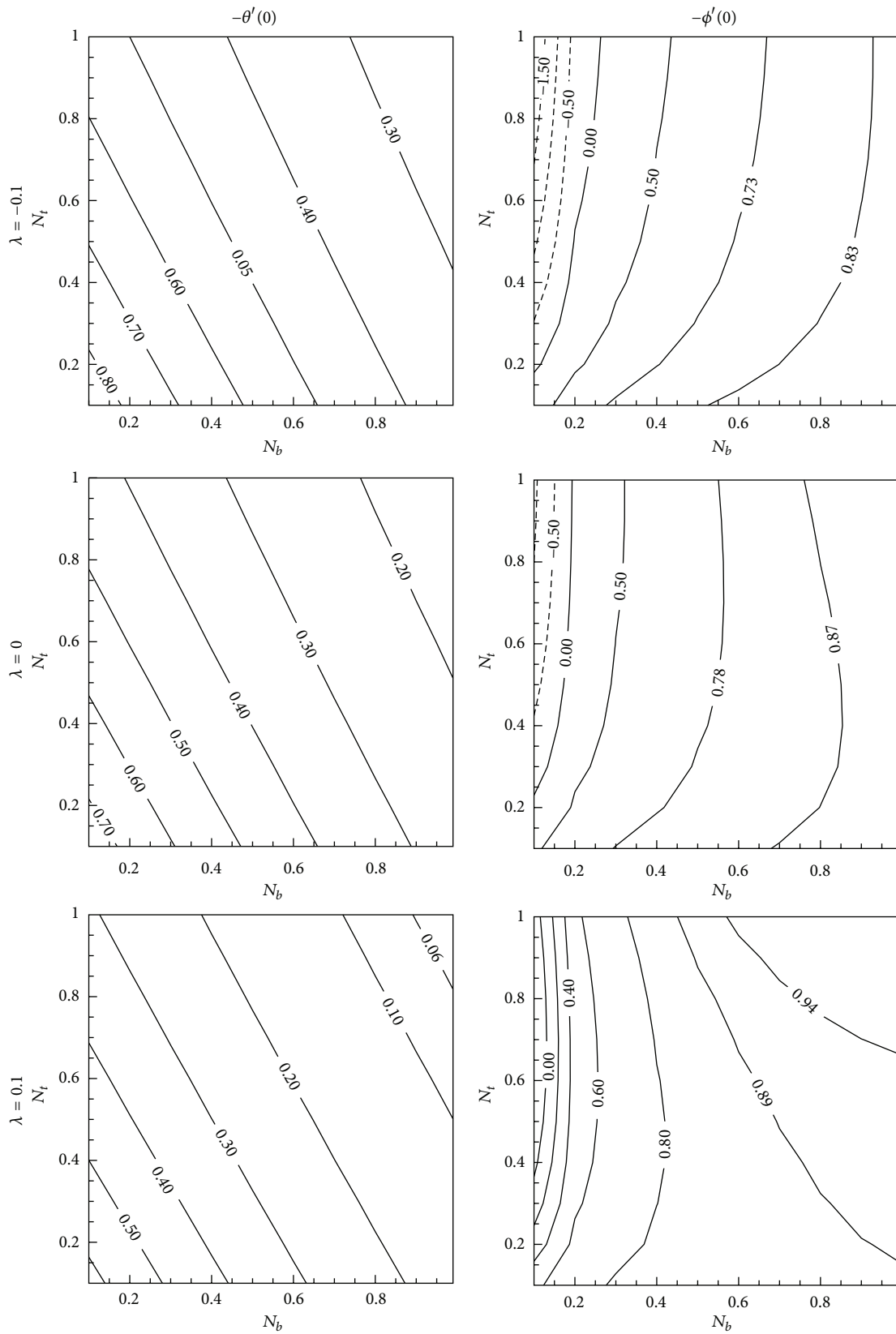


FIGURE 4: Contour lines of heat transfer and concentration rates for different values of λ when $Le = Pr = 2$, $\varepsilon = 0.1$.

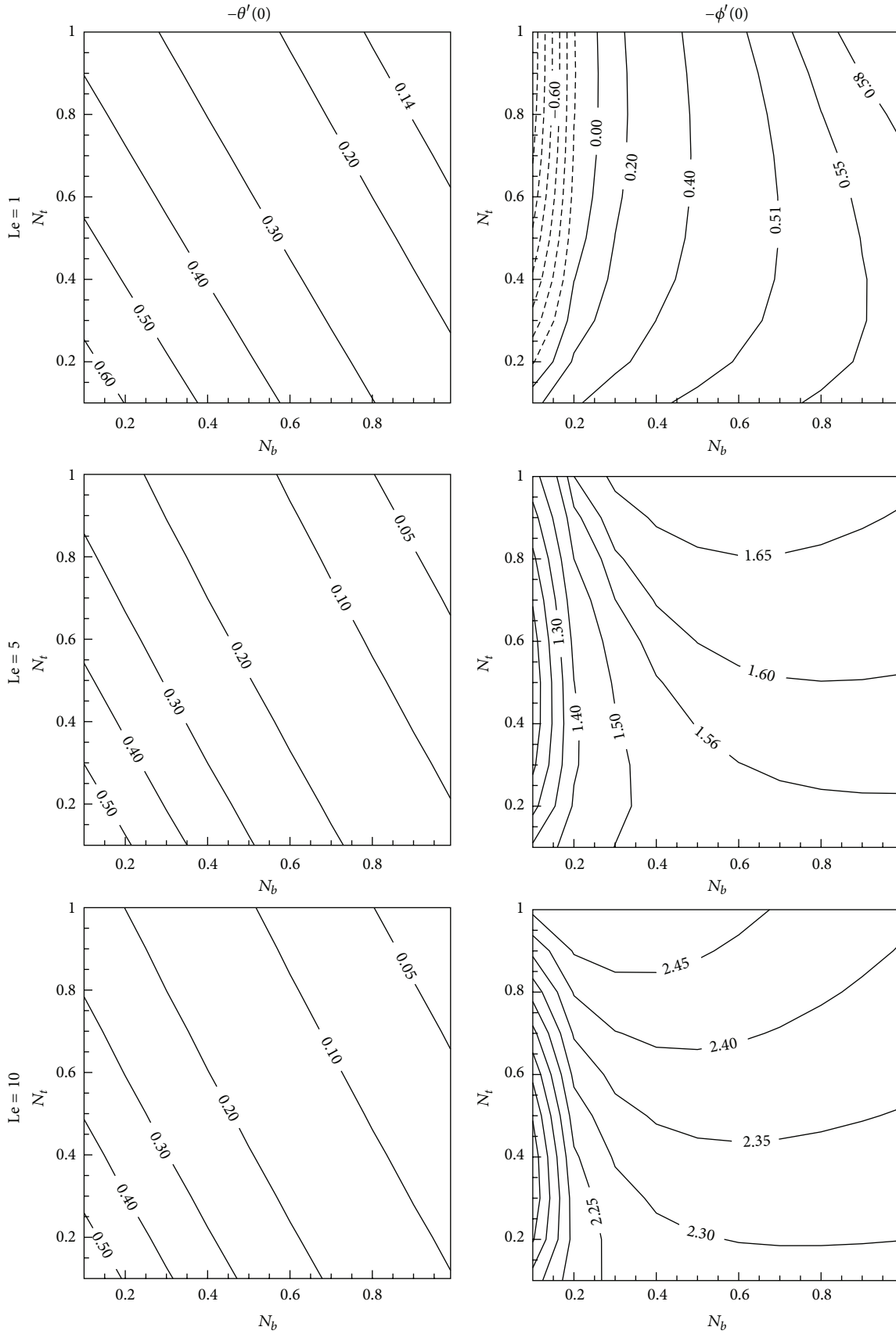


FIGURE 5: Contour lines of heat transfer and concentration rates for different values of Le when $Pr = 2$, $\lambda = \varepsilon = 0.1$.

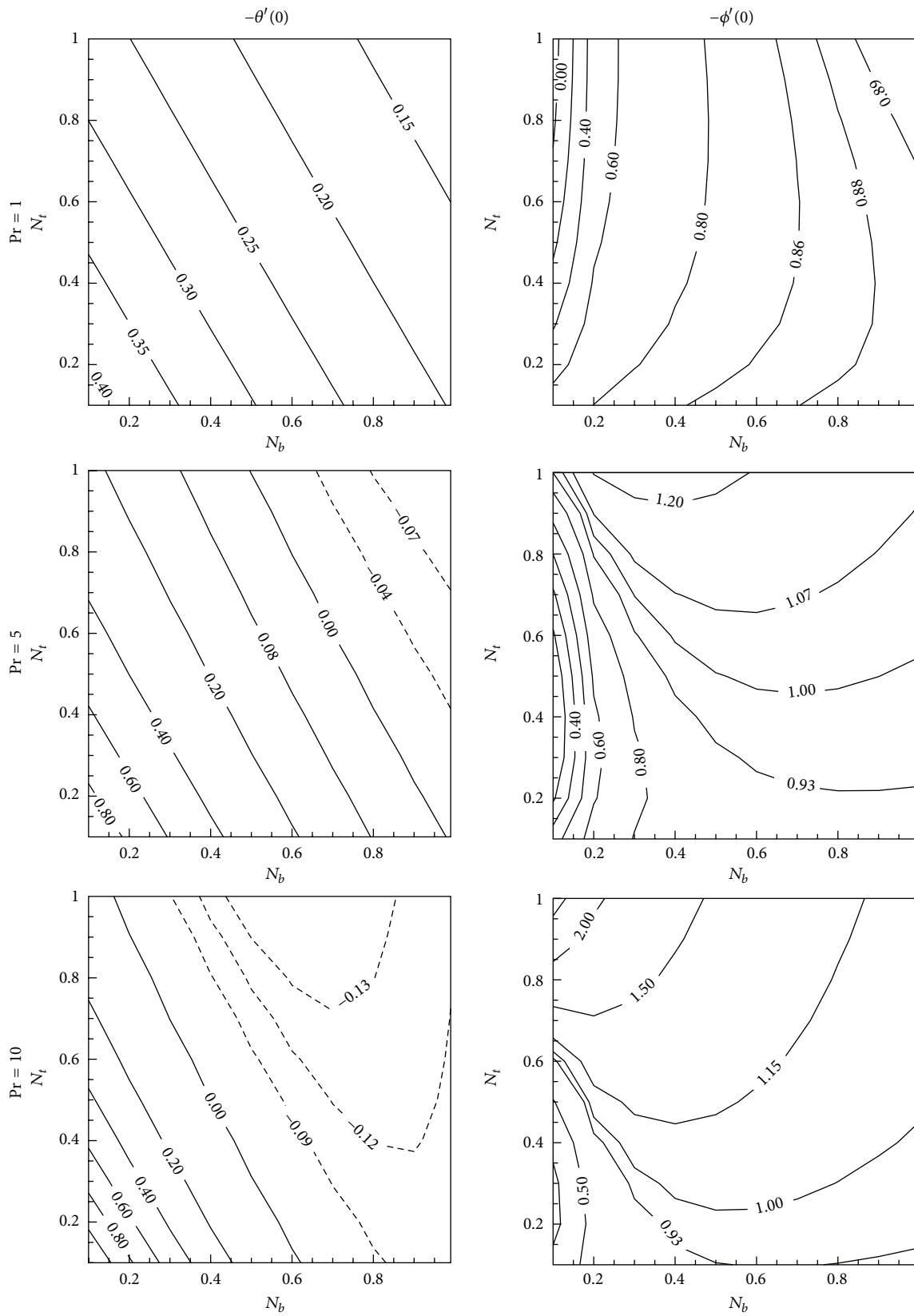


FIGURE 6: Contour lines of heat transfer and concentration rates for different values of Pr when $Le = 2$, $\lambda = \epsilon = 0.1$.

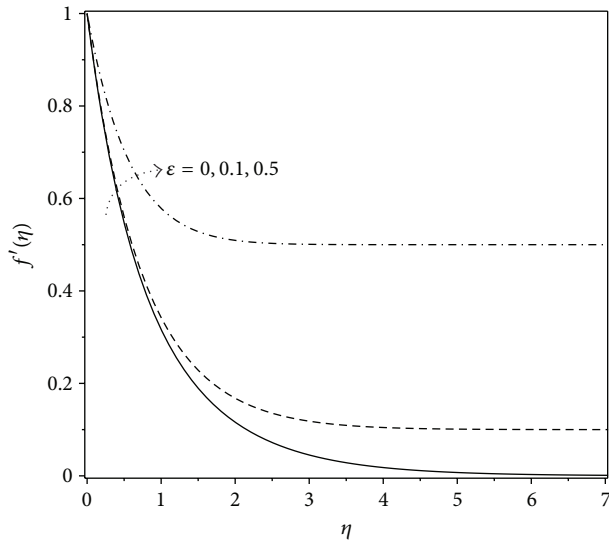


FIGURE 7: Hydrodynamic boundary layer for different values of stretching parameter.

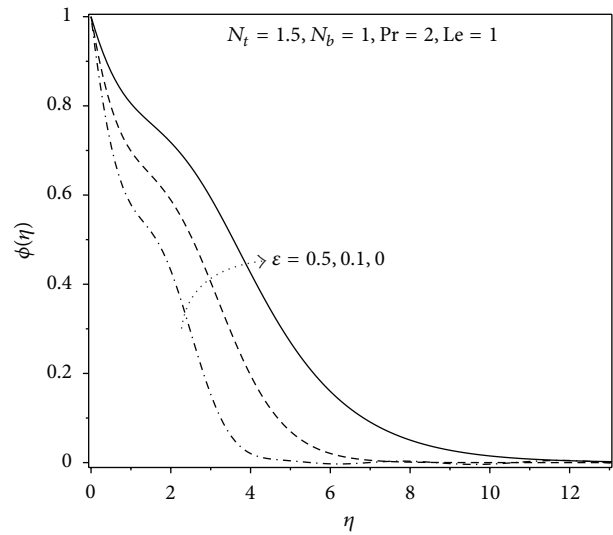


FIGURE 9: Concentration profiles for different values of stretching parameter.

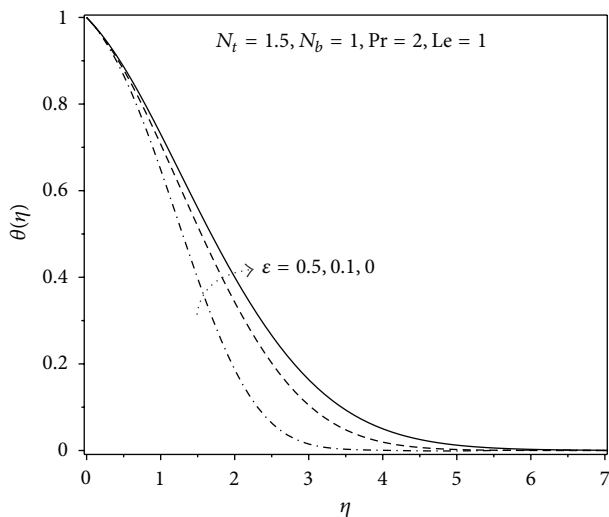


FIGURE 8: Temperature profiles for different values of stretching parameter.

Greek Symbols

α :	Thermal diffusivity
ε :	Stretching sheet parameter
ϕ :	Rescaled nanoparticle volume fraction
η :	Similarity variable
μ :	Dynamic viscosity
ν :	Kinematic viscosity
ρ :	Density
$(\rho c_p)_f$:	Heat capacity of the fluid
$(\rho c_p)_p$:	Effective heat capacity of the nanoparticle
λ :	Dimensionless heat generation/absorption
τ :	Parameter defined by $(\rho c_p)_p / (\rho c_p)_f$
ψ :	Stream function.

Subscripts

nf:	Nanofluid
f:	Fluid
w:	Condition on the sheet
∞ :	Ambient conditions.

References

- [1] K. Hiemenz, "Die Grenzschicht an einem in den gleichförmigen Flüssigkeitsstrom eingetauchten geraden Kreiszyylinder," *Dingler's Polytechnic Journal*, vol. 326, pp. 321–324, 1911.
- [2] F. Homann, "Der Einfluß großer Zähigkeit bei der Strömung um den Zylinder und um die Kugel," *Zeitschrift für Angewandte Mathematik und Mechanik*, vol. 16, no. 3, pp. 153–164, 1936.
- [3] H. A. Attia, "Homann magnetic flow and heat transfer with uniform suction or injection," *Canadian Journal of Physics*, vol. 81, no. 10, pp. 1223–1230, 2003.
- [4] K. Bhattacharyya and K. Vajravelu, "Stagnation-point flow and heat transfer over an exponentially shrinking sheet," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 7, pp. 2728–2734, 2012.
- [5] A. Malvandi, "The unsteady flow of a nanofluid in the stagnation point region of a time-dependent rotating sphere," *THERMAL SCIENCE*, 2013.
- [6] V. Kumaran, R. Tamizharasi, and K. Vajravelu, "Approximate analytic solutions of stagnation point flow in a porous medium," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 6, pp. 2677–2688, 2009.
- [7] M. A. A. Hamad and M. Ferdows, "Similarity solution of boundary layer stagnation-point flow towards a heated porous stretching sheet saturated with a nanofluid with heat absorption/generation and suction/blowing: a Lie group analysis," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 1, pp. 132–140, 2012.
- [8] Z. Ziabakhsh, G. Domairry, and H. Bararnia, "Analytical solution of non-Newtonian micropolar fluid flow with uniform

- suction/blowing and heat generation," *Journal of the Taiwan Institute of Chemical Engineers*, vol. 40, no. 4, pp. 443–451, 2009.
- [9] Z. Ziabakhsh, G. Domairry, and H. R. Ghazizadeh, "Analytical solution of the stagnation-point flow in a porous medium by using the homotopy analysis method," *Journal of the Taiwan Institute of Chemical Engineers*, vol. 40, no. 1, pp. 91–97, 2009.
- [10] L. J. Crane, "Flow past a stretching plate," *Zeitschrift für Angewandte Mathematik und Physik*, vol. 21, no. 4, pp. 645–647, 1970.
- [11] S. Yao, T. Fang, and Y. Zhong, "Heat transfer of a generalized stretching/shrinking wall problem with convective boundary conditions," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 2, pp. 752–760, 2011.
- [12] K. R. Rajagopal, T. Y. Na, and A. S. Gupta, "Flow of a viscoelastic fluid over a stretching sheet," *Rheologica Acta*, vol. 23, no. 2, pp. 213–215, 1984.
- [13] R. Nazar, N. Amin, D. Filip, and I. Pop, "Stagnation point flow of a micropolar fluid towards a stretching sheet," *International Journal of Non-Linear Mechanics*, vol. 39, no. 7, pp. 1227–1235, 2004.
- [14] M. A. A. Hamad, "Analytical solution of natural convection flow of a nanofluid over a linearly stretching sheet in the presence of magnetic field," *International Communications in Heat and Mass Transfer*, vol. 38, no. 4, pp. 487–492, 2011.
- [15] Z. Ziabakhsh, G. Domairry, M. Mozaffari, and M. Mahbobifar, "Analytical solution of heat transfer over an unsteady stretching permeable surface with prescribed wall temperature," *Journal of the Taiwan Institute of Chemical Engineers*, vol. 41, no. 2, pp. 169–177, 2010.
- [16] S. Nadeem, A. Hussain, and M. Khan, "HAM solutions for boundary layer flow in the region of the stagnation point towards a stretching sheet," *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 3, pp. 475–481, 2010.
- [17] M. M. Nandeppanavar, K. Vajravelu, M. Subhas Abel, S. Ravi, and H. Jyoti, "Heat transfer in a liquid film over an unsteady stretching sheet," *International Journal of Heat and Mass Transfer*, vol. 55, no. 4, pp. 1316–1324, 2012.
- [18] A. Malvandi, F. Hedayati, and D. D. Ganji, "Thermodynamic optimization of fluid flow over an isothermal moving plate," *Alexandria Engineering Journal*, 2013.
- [19] A. K. Singh, "Heat source and radiation effects on magneto-convection flow of a viscoelastic fluid past a stretching sheet: analysis with Kummer's functions," *International Communications in Heat and Mass Transfer*, vol. 35, no. 5, pp. 637–642, 2008.
- [20] S. Nadeem and C. Lee, "Boundary layer flow of nanofluid over an exponentially stretching surface," *Nanoscale Research Letters*, vol. 7, article 94, pp. 1–15, 2012.
- [21] M. R. H. Nobari and A. Malvandi, "Torsion and curvature effects on fluid flow in a helical annulus," *International Journal of Non-Linear Mechanics*, vol. 57, pp. 90–101, 2013.
- [22] S. U. S. Choi, "Enhancing thermal conductivity of fluids with nanoparticles," in *Developments and Applications of Non-Newtonian Flows*, D. A. Siginer and H. P. Wang, Eds., pp. 99–105, ASME, 1995.
- [23] H. Masuda, A. Ebata, K. Teramae, and N. Hishinuma, "Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles. Dispersion of Al_2O_3 , SiO_2 and TiO_2 ultra-fine particles," *Netsu Bussei*, vol. 7, no. 4, pp. 227–233, 1993.
- [24] J. Buongiorno, "Convective transport in nanofluids," *Journal of Heat Transfer*, vol. 128, no. 3, pp. 240–250, 2006.
- [25] A. V. Kuznetsov and D. A. Nield, "Natural convective boundary-layer flow of a nanofluid past a vertical plate," *International Journal of Thermal Sciences*, vol. 49, no. 2, pp. 243–247, 2010.
- [26] W. A. Khan and I. Pop, "Boundary-layer flow of a nanofluid past a stretching sheet," *International Journal of Heat and Mass Transfer*, vol. 53, no. 11–12, pp. 2477–2483, 2010.
- [27] N. Bachok, A. Ishak, and I. Pop, "Boundary-layer flow of nanofluids over a moving surface in a flowing fluid," *International Journal of Thermal Sciences*, vol. 49, no. 9, pp. 1663–1668, 2010.
- [28] A. Alsaedi, M. Awais, and T. Hayat, "Effects of heat generation/absorption on stagnation point flow of nanofluid over a surface with convective boundary conditions," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 11, pp. 4210–4223, 2012.
- [29] P. Rana and R. Bhargava, "Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet: a numerical study," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 1, pp. 212–226, 2012.
- [30] W. Daungthongsuk and S. Wongwises, "A critical review of convective heat transfer of nanofluids," *Renewable and Sustainable Energy Reviews*, vol. 11, no. 5, pp. 797–817, 2007.
- [31] X.-Q. Wang and A. S. Mujumdar, "Heat transfer characteristics of nanofluids: a review," *International Journal of Thermal Sciences*, vol. 46, no. 1, pp. 1–19, 2007.
- [32] S. Kakaç and A. Pramuanjaroenkij, "Review of convective heat transfer enhancement with nanofluids," *International Journal of Heat and Mass Transfer*, vol. 52, no. 13–14, pp. 3187–3196, 2009.
- [33] S. Liao, "On the homotopy analysis method for nonlinear problems," *Applied Mathematics and Computation*, vol. 147, no. 2, pp. 499–513, 2004.
- [34] B. Wu and H. Zhong, "Summation of perturbation solutions to nonlinear oscillations," *Acta Mechanica*, vol. 154, no. 1–4, pp. 121–127, 2002.
- [35] G. Adomian, "A review of the decomposition method in applied mathematics," *Journal of Mathematical Analysis and Applications*, vol. 135, no. 2, pp. 501–544, 1988.
- [36] J.-H. He, "Variational iteration method—a kind of non-linear analytical technique: some examples," *International Journal of Non-Linear Mechanics*, vol. 34, no. 4, pp. 699–708, 1999.
- [37] F. Hedayati, D. Ganji, S. Hamidi, and A. Malvandi, "An analytical study on a model describing heat conduction in rectangular radial fin with temperature-dependent thermal conductivity," *International Journal of Thermophysics*, vol. 33, no. 6, pp. 1042–1054, 2012.
- [38] A. Malvandi, D. D. Ganji, F. Hedayati, M. H. Kaffash, and M. Jamshidi, "Series solution of entropy generation toward an isothermal flat plate," *Thermal Science*, vol. 16, no. 5, pp. 1289–1295, 2012.
- [39] S. Liao, "On the relationship between the homotopy analysis method and Euler transform," *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 6, pp. 1421–1431, 2010.
- [40] S. Liao, "Homotopy analysis method: a new analytical technique for nonlinear problems," *Communications in Nonlinear Science and Numerical Simulation*, vol. 2, no. 2, pp. 95–100, 1997.
- [41] S.-I. Liao, "A short review on the homotopy analysis method in fluid mechanics," *Journal of Hydrodynamics*, vol. 22, no. 5, pp. 839–841, 2010.
- [42] D. G. Domairry, A. Mohsenzadeh, and M. Famouri, "The application of homotopy analysis method to solve nonlinear

- differential equation governing Jeffery-Hamel flow,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 1, pp. 85–95, 2009.
- [43] G. Domairry and M. Fazeli, “Homotopy analysis method to determine the fin efficiency of convective straight fins with temperature-dependent thermal conductivity,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 2, pp. 489–499, 2009.
- [44] G. Domairry and N. Nadim, “Assessment of homotopy analysis method and homotopy perturbation method in non-linear heat transfer equation,” *International Communications in Heat and Mass Transfer*, vol. 35, no. 1, pp. 93–102, 2008.
- [45] S.-J. Liao, “An explicit, totally analytic approximate solution for Blasius’ viscous flow problems,” *International Journal of Non-Linear Mechanics*, vol. 34, no. 4, pp. 759–778, 1999.
- [46] S. Liao, “An optimal homotopy-analysis approach for strongly nonlinear differential equations,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 8, pp. 2003–2016, 2010.
- [47] M. Hassani, M. Mohammad Tabar, H. Nemati, G. Domairry, and F. Noori, “An analytical solution for boundary layer flow of a nanofluid past a stretching sheet,” *International Journal of Thermal Sciences*, vol. 50, no. 11, pp. 2256–2263, 2011.
- [48] N. Bachok, A. Ishak, and I. Pop, “On the stagnation-point flow towards a stretching sheet with homogeneous-heterogeneous reactions effects,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 11, pp. 4296–4302, 2011.
- [49] G. Domairry and Z. Ziabakhsh, “Solution of boundary layer flow and heat transfer of an electrically conducting micropolar fluid in a non-Darcian porous medium,” *Meccanica*, vol. 47, no. 1, pp. 195–202, 2012.
- [50] A. A. Joneidi, G. Domairry, and M. Babaelahi, “Analytical treatment of MHD free convective flow and mass transfer over a stretching sheet with chemical reaction,” *Journal of the Taiwan Institute of Chemical Engineers*, vol. 41, no. 1, pp. 35–43, 2010.
- [51] Z. Ziabakhsh, G. Domairry, H. Bararnia, and H. Babazadeh, “Analytical solution of flow and diffusion of chemically reactive species over a nonlinearly stretching sheet immersed in a porous medium,” *Journal of the Taiwan Institute of Chemical Engineers*, vol. 41, no. 1, pp. 22–28, 2010.
- [52] K. Yabushita, M. Yamashita, and K. Tsuboi, “An analytic solution of projectile motion with the quadratic resistance law using the homotopy analysis method,” *Journal of Physics A*, vol. 40, no. 29, pp. 8403–8416, 2007.
- [53] Z. Niu and C. Wang, “A one-step optimal homotopy analysis method for nonlinear differential equations,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 8, pp. 2026–2036, 2010.

