THE CENTER OF TOPOLOGICALLY PRIMITIVE EXPONENTIALLY GALBED ALGEBRAS

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Let A be a unital sequentially complete topologically primitive exponentially galbed Hausdorff algebra over $\mathbb{C}$, in which all elements are bounded. It is shown that the center of A is topologically isomorphic to $\mathbb{C}$.

1. Introduction

(1) Let A be an associative topological algebra over the field of complex numbers $\mathbb{C}$ with separately continuous multiplication. Then A is an *exponentially galbed algebra* (see, e.g., [1–4, 19, 20]) if every neighbourhood O of zero in A defines another neighbourhood U of zero such that

$$\left\{\sum_{k=0}^{n}\frac{a_k}{2^k} : a_0,\dots,a_n \in U\right\} \subset O \tag{1.1}$$

for each $n \in \mathbb{N}$. Herewith, A is *locally pseudoconvex*, if it has a base $\{U_\lambda : \lambda \in \Lambda\}$ of neighbourhoods of zero consisting of *balanced* and *pseudoconvex* sets (i.e., of sets U for which $\mu U \subset U$, whenever $|\mu| \leqslant 1$, and $U + U \subset \rho U$ for a $\rho \geqslant 2$). In particular, when every U_λ in $\{U_\lambda : \lambda \in \Lambda\}$ is *idempotent* (i.e., $U_\lambda U_\lambda \subset U_\lambda$), then A is called a *locally m-pseudoconvex algebra*, and when every U_λ in $\{U_\lambda : \lambda \in \Lambda\}$ is *A-pseudoconvex* (i.e., for any $a \in A$ there is a $\mu > 0$ such that $aU_\lambda, U_\lambda a \subset \mu U_\lambda$), then A is called a *locally A-pseudoconvex algebra*. It is well known (see [21, page 4] or [6, page 189]) that the locally pseudoconvex topology on A is given by a family $\{p_\lambda : \lambda \in \Lambda\}$ of k_λ-homogeneous seminorms, where $k_\lambda \in (0,1]$ for each $\lambda \in \Lambda$. The topology of a locally m-pseudoconvex (A-pseudoconvex) algebra A is given by a family $\{p_\lambda : \lambda \in \Lambda\}$ of k_λ-homogeneous *submultiplicative* (i.e., $p_\lambda(ab) \leqslant p_\lambda(a)p_\lambda(b)$ for each $a,b \in A$ and $\lambda \in \Lambda$) (resp., *A-multiplicative* (i.e., for each $a \in A$ and each $\lambda \in \Lambda$ there are numbers $N(a,\lambda) > 0$ and $M(a,\lambda) > 0$ such that $p_\lambda(ab) \leqslant N(a,\lambda)p_\lambda(b)$ and $p_\lambda(ba) \leqslant M(a,\lambda)p_\lambda(b)$ for each $b \in A$)) seminorms, where $k_\lambda \in (0,1]$ for each $\lambda \in \Lambda$. In particular, when $k_\lambda = 1$ for each $\lambda \in \Lambda$, then A is a *locally convex* (resp., *locally m-convex* and *locally A-convex*) *algebra* and when the topology of A has been defined by only one

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k-homogeneous seminorm with $k \in (0,1]$, then A is a *locally bounded algebra*. It is easy to see that every locally pseudoconvex algebra is an exponentially galbed algebra.

Moreover, a complete locally bounded Hausdorff algebra A is a *p-Banach algebra*; a complete metrizable algebra A is a *Fréchet algebra*; a unital topological algebra A, in which the set of all invertible elements is open, is a *Q-algebra* (see, e.g., [14, page 43, Definition 6.2]) and a topological algebra A is a *topologically primitive algebra* (see [5]), if

$$\{a \in A : aA \subset M\} = \{\theta_A\} \qquad (\{a \in A : Aa \subset M\} = \{\theta_A\}) \tag{1.2}$$

for a closed maximal regular (or modular) left (resp., right) ideal M of A (here θ_A denotes the zero element of A).

An element a in a topological algebra A is *bounded*, if there exists an element $\lambda_a \in \mathbb{C}\setminus\{0\}$ such that the set

$$\left\{\left(\frac{a}{\lambda_a}\right)^n : n \in \mathbb{N}\right\} \tag{1.3}$$

is bounded in A and *nilpotent*, if $a^m = \theta_A$ for some $m \in \mathbb{N}$. If all elements in A are bounded (nilpotent), then A is a *topological algebra with bounded elements* (resp., a *nil algebra*).

(2) It is well known that the center of a primitive ring (a ring (in particular, algebra) R is *primitive* if it has a maximal left (right) regular ideal M such that $\{a \in R : aR \subset M\} = \{\theta_R\}$ (resp., $\{a \in R : Ra \subset M\} = \{\theta_R\}$)) is an integral domain (a ring R is an integral domain, if from $a, b \in R$ and $ab = \theta_R$ follows that $a = \theta_R$ or $b = \theta_R$) (see [12, Lemma 2.1.3]) and any commutative integral domain can be the center of a primitive ring (see [13, Chapter II.6, Example 3]). Herewith, every field is a commutative integral domain, but any commutative integral domain is not necessarily a field. In particular (see [5]), when R is a unital primitive locally A-pseudoconvex Hausdorff algebra over $\mathbb{C}$ or a unital locally pseudoconvex Fréchet Q-algebra over $\mathbb{C}$, then the center $Z(R)$ of R is topologically isomorphic to $\mathbb{C}$ (for Banach algebras a similar result has been given in [16, Corollary 2.4.5] (see also [8, page 127], [15, Theorem 4.2.11], and [9, Theorem 2.6.26 (ii)]); for k-Banach algebras in [6, Corollary 9.3.7]; for unital primitive locally m-convex Q-algebras in [17, Corollary 2], and for unital primitive locally A-convex algebras, in which all maximal ideals are closed, in [18, Theorem 3]). For topological algebras with all maximal regular one-sided or two-sided ideals closed see also [7, 10, 11, 14].

In the present paper we will show that a similar result will be true for any unital sequentially complete topologically primitive exponentially galbed Hausdorff algebra over $\mathbb{C}$ in which all elements are bounded.

2. Auxiliary results

For describing the center of primitive exponentially galbed algebras we need the following results.

PROPOSITION 2.1. *Let A be a unital exponentially galbed Hausdorff algebra over $\mathbb{C}$ with bounded elements, $\lambda_0 \in \mathbb{C}$ and $a_0 \in A$. If A is a sequentially complete or a nil algebra, then* *there exists a neighbourhood $O(\lambda_0)$ of λ_0 such that*

$$\sum_{k=0}^{\infty} (\lambda - \lambda_0)^k a_0^k \tag{2.1}$$

converges in A and

$$(e_A + (\lambda_0 - \lambda)a_0)^{-1} = \sum_{k=0}^{\infty} (\lambda - \lambda_0)^k a_0^k \tag{2.2}$$

for each $\lambda \in O(\lambda_0)$.

Proof. Let O be an arbitrary neighbourhood of zero in A. Then there is a closed and balanced neighbourhood O' of zero in A and a closed neighbourhood O'' of zero in $\mathbb{C}$ such that $O''O' \subset O$. Now O' defines a balanced neighbourhood V of zero in A such that

$$\left\{ \sum_{k=0}^{n} \frac{v_k}{2^k} : v_0, \dots, v_n \in V \right\} \subset O' \tag{2.3}$$

for each $n \in \mathbb{N}$. Since every element in A is bounded, then there is a number $\mu_0 = \mu_{a_0} \in \mathbb{C} \setminus \{0\}$ such that

$$\left\{ \left(\frac{a_0}{\mu_0} \right)^n : n \in \mathbb{N} \right\} \tag{2.4}$$

is bounded in A. Therefore, there exists a number $\rho_0 > 0$ such that

$$\left(\frac{a_0}{\mu_0} \right)^n \in \rho_0 V \tag{2.5}$$

for each $n \in \mathbb{N}$.

Let now $a_0 \in A$ and $\lambda_0 \in \mathbb{C}$ be fixed,

$$S_n(\lambda) = \sum_{k=0}^{n} (\lambda - \lambda_0)^k a_0^k \tag{2.6}$$

for each $n \in \mathbb{N}$ and $\lambda \in \mathbb{C}$,

$$U_{\mathbb{C}} = \left\{ \lambda \in \mathbb{C} : \; |\lambda| < \frac{1}{3|\mu_0|} \right\} \tag{2.7}$$

and $U(\lambda_0) = \lambda_0 + U_{\mathbb{C}}$. Then

$$S_m(\lambda) - S_n(\lambda) = \sum_{k=n+1}^{m} (\lambda - \lambda_0)^k a_0^k = \sum_{k=0}^{m-n-1} (\lambda - \lambda_0)^{n+k+1} a_0^{n+k+1} \tag{2.8}$$

for each $n, m \in \mathbb{N}$, whenever $m > n$ and $\lambda \in \mathbb{C}$. If we take

$$v_{n,k}(\lambda) = 2^k (\lambda - \lambda_0)^k \frac{a_0^{n+k+1}}{\rho_0 \mu_0^{n+1}} \tag{2.9}$$

for each $n,k \in \mathbb{N}$ and $\lambda \in \mathbb{C}$, then

$$S_m(\lambda) - S_n(\lambda) = (\lambda - \lambda_0)^{n+1} {\mu_0}^{n+1} \rho_0 \sum_{k=0}^{m-n-1} \frac{v_{n,k}(\lambda)}{2^k} \tag{2.10}$$

for each $n,m \in \mathbb{N}$, whenever $m > n$ and $\lambda \in \mathbb{C}$. Now,

$$v_{n,k}(\lambda) = \frac{1}{\rho_0}\big(2(\lambda - \lambda_0)\mu_0\big)^k \left(\frac{a_0}{\mu_0}\right)^{n+k+1} \in \frac{1}{\rho_0}\big(2\mu_0(\lambda - \lambda_0)\big)^k \rho_0 V \subset V \tag{2.11}$$

for each $n,k \in \mathbb{N}$ and $\lambda \in U(\lambda_0)$, because $|2\mu_0(\lambda - \lambda_0)| < 2/3 < 1$. Hence,

$$S_m(\lambda) - S_n(\lambda) \in \frac{\big(2\mu_0(\lambda - \lambda_0)\big)^{n+1}}{2^{n+1}} \rho_0 O', \tag{2.12}$$

whenever $m > n$ and $\lambda \in U(\lambda_0)$. Since again $|2\mu_0(\lambda - \lambda_0)| < 1$, then there exists a number $n_0 \in \mathbb{N}$ such that

$$\big(2\mu_0(\lambda - \lambda_0)\big)^{n+1} \in \frac{1}{\rho_0} O'' \tag{2.13}$$

for each $n > n_0$. Taking this into account,

$$S_m(\lambda) - S_n(\lambda) \in \frac{1}{2^{n+1}} \frac{1}{\rho_0} O'' \rho_0 O' \subset O'' O' \subset O, \tag{2.14}$$

whenever $m > n > n_0$ and $\lambda \in U(\lambda_0)$, since O' is balanced. It means that $(S_n(\lambda))$ is a Cauchy complete, the sequence in A for each $\lambda \in U(\lambda_0)$.

In the case when A is sequentially complete the sequence $(S_n(\lambda))$ converges in A. Suppose now that A is a nil algebra. Then $a_0^{m+1} = \theta_A$ for some $m \in \mathbb{N}$. Hence,

$$S_n(\lambda) = \sum_{k=0}^{m} (\lambda - \lambda_0)^k a_0^k \tag{2.15}$$

for each $\lambda \in \mathbb{C}$, whenever $n \geq m$. Consequently, $(S_n(\lambda))$ converges in A for each $\lambda \in O(\lambda_0)$ in both cases.

Since

$$(e_A + (\lambda_0 - \lambda)a_0) \sum_{k=0}^{\infty} (\lambda - \lambda_0)^k a_0^k = \sum_{k=0}^{\infty} (\lambda - \lambda_0)^k a_0^k (e_A + (\lambda_0 - \lambda)a_0) = e_A, \tag{2.16}$$

one gets

$$(e_A + (\lambda_0 - \lambda)a_0)^{-1} = \sum_{k=0}^{\infty} (\lambda - \lambda_0)^k a_0^k \tag{2.17}$$

for each $\lambda \in O(\lambda_0)$. □

Corollary 2.2. *Let A be a unital exponentially galbed algebra over* $\mathbb{C}$ *with bounded elements. If A is a sequentially complete or a nil algebra, then for each* $a_0 \in A$ *there exists a number* $R > 0$ *such that*

$$\sum_{k=0}^{\infty} \frac{a_0^k}{\mu^{k+1}} \tag{2.18}$$

converges in A, whenever $|\mu| > R$.

Proof. If we take $\lambda_0 = 0$ in the previous proposition, then we get that

$$\sum_{k=0}^{\infty} \lambda^k a_0^k \tag{2.19}$$

converges in A, whenever $|\lambda| < \delta$ for some $\delta > 0$. If now $\mu > R = \delta^{-1}$, then $|\mu^{-1}| < \delta$, which means that

$$\sum_{k=0}^{\infty} \frac{a_0^k}{\mu^k} \tag{2.20}$$

converges in A. Hence,

$$\sum_{k=0}^{\infty} \frac{a_0^k}{\mu^{k+1}} = \frac{1}{\mu} \sum_{k=0}^{\infty} \frac{a_0^k}{\mu^k} \tag{2.21}$$

converges in A, whenever $|\mu| > R$. □

3. Main result

Now, based on Proposition 2.1 and Corollary 2.2, we give a description of the center $Z(A)$ of such unital topologically primitive exponentially galbed Hausdorff algebras A over $\mathbb{C}$ in which all elements are bounded.

Theorem 3.1. *Let A be a unital sequentially complete topologically primitive exponentially galbed Hausdorff algebra over* $\mathbb{C}$ *with bounded elements. Then* $Z(A)$ *is topologically isomorphic to* $\mathbb{C}$.

Proof. Since A is a topologically primitive algebra, there is a closed maximal left ideal (if M is a closed maximal right ideal, then the proof is similar) M in A such that

$$\{a \in A : aA \subset M\} = \{\theta_A\} \tag{3.1}$$

(then $M \cap Z(A) = \{\theta_A\}$). Denote by π_M the canonical homomorphism from A onto the quotient space $A-M$ of A with respect to M. For each $z \in Z(A) \setminus \{\theta_A\}$ consider the left ideal

$$K_z = \{a \in A : az \in M\}. \tag{3.2}$$

Since $mz = zm \in M$ for each $m \in M$ and $e_A z = z \notin M$, $M \subset K_z \neq A$. Hence, K_z is a proper left ideal in A. Since the ideal M is maximal, $M = K_z$ for each $z \in Z(A) \setminus \{\theta_A\}$.

We will show that every $z \in Z(A)$ defines a number $\lambda_z \in \mathbb{C}$ such that $z = \lambda_z e_A$. If $z = \theta_A$, then we take $\lambda_z = 0$. Suppose now that there exists a $z \in Z(A) \setminus \{\theta_A\}$ such that $z(\lambda) = \lambda e_A - z \neq \theta_A$ for all $\lambda \in \mathbb{C}$. Then $z(\lambda) \in Z(A) \setminus \{\theta_A\}$ means that $z(\lambda) \notin M$ for each $\lambda \in \mathbb{C}$, $M + Az(\lambda)$ is a left ideal in A, $M \subset M + Az(\lambda)$ and

$$z(\lambda) = \theta_A + e_A z(\lambda) \in (M + Az(\lambda)) \setminus M \tag{3.3}$$

for each $\lambda \in \mathbb{C}$. Since M is a maximal left ideal in A, then $M + Az(\lambda) = A$ for each $\lambda \in \mathbb{C}$. Therefore, for each $\lambda \in \mathbb{C}$ there are elements $m(\lambda) \in M$ and $a(\lambda) \in A$ such that $e_A = m(\lambda) + a(\lambda)z(\lambda)$, because of which $a(\lambda)z(\lambda) - e_A \in M$.

Let $a'(\lambda) \in A$ be another element such that $a'(\lambda)z(\lambda) - e_A \in M$. Then from

$$[a(\lambda) - a'(\lambda)]z(\lambda) = a(\lambda)z(\lambda) - a'(\lambda)z(\lambda) \in M \tag{3.4}$$

it follows that $[a(\lambda) - a'(\lambda)] \in K_{z(\lambda)} = M$. Therefore, $\pi_M(a(\lambda)) = \pi_M(a'(\lambda))$ for each $\lambda \in \mathbb{C}$.

Let now $\lambda_0 \in \mathbb{C}$ and

$$d(\lambda) = e_A + (\lambda - \lambda_0)a(\lambda_0) \tag{3.5}$$

for each $\lambda \in \mathbb{C}$. Then there is (by Proposition 2.1) a neighbourhood $O(\lambda_0)$ of λ_0 such that

$$\sum_{k=0}^{\infty} (\lambda - \lambda_0)^k a(\lambda_0)^k \tag{3.6}$$

converges in A and

$$d(\lambda)^{-1} = \sum_{k=0}^{\infty} (\lambda - \lambda_0)^k a(\lambda_0)^k \tag{3.7}$$

for each $\lambda \in O(\lambda_0)$.

Now,

$$\begin{aligned}
& a(\lambda_0)d(\lambda)^{-1}z(\lambda) - e_A \\
&\quad = a(\lambda_0)d(\lambda)^{-1}z(\lambda) - [a(\lambda_0)z(\lambda_0) + m(\lambda_0)] \\
&\quad = -a(\lambda_0)d(\lambda)^{-1}[-z(\lambda) + d(\lambda)z(\lambda_0)] - m(\lambda_0) \\
&\quad = -a(\lambda_0)d(\lambda)^{-1}[(z - \lambda e_A) + (e_A + (\lambda - \lambda_0)a(\lambda_0))(\lambda_0 e_A - z)] - m(\lambda_0) \\
&\quad = -a(\lambda_0)d(\lambda)^{-1}[(\lambda_0 - \lambda)(e_A - a(\lambda_0)z(\lambda_0))] - m(\lambda_0) \\
&\quad = -a(\lambda_0)d(\lambda)^{-1}(\lambda_0 - \lambda)m(\lambda_0) - m(\lambda_0) \in M.
\end{aligned} \tag{3.8}$$

Therefore,

$$\pi_M(a(\lambda)) = \pi_M(a(\lambda_0)d(\lambda)^{-1}) \tag{3.9}$$

for each $\lambda \in O(\lambda_0)$.

Let now $\Psi(\lambda) = \pi_M(a(\lambda))$ for each $\lambda \in \mathbb{C}$. We will show that Ψ is an $(A-M)$-valued analytic function (i.e., if $\lambda_0 \in \mathbb{C}$, then there are a number $\delta > 0$ and a sequence (x_n) of elements of $A-M$ such that $\Psi(\lambda_0+\lambda) = \sum_{k=0}^{\infty}(x_k\lambda^k)$, whenever $|\lambda| < \delta$, and a number $R > 0$ and a sequence (y_n) of elements of $A-M$ such that $\Psi(\lambda) = \sum_{k=0}^{\infty}(y_k/\lambda^k)$, whenever $|\lambda| > R$) on $\mathbb{C}\cup\{\infty\}$. For it, let again $\lambda_0 \in \mathbb{C}$. Then $\Psi(\lambda) = \pi_M(a(\lambda_0)d(\lambda)^{-1})$ for each $\lambda \in O(\lambda_0)$ and there exists a number $\delta > 0$ such that $\lambda_0+\lambda \in O(\lambda_0)$, whenever $|\lambda| < \delta$. Now,

$$\begin{aligned}\Psi(\lambda_0+h) &= \pi_M\big(a(\lambda_0)d(\lambda_0+h)^{-1}\big)\\ &= \pi_M\left(a(\lambda_0)\sum_{k=0}^{\infty}h^k a(\lambda_0)^k\right) = \sum_{k=0}^{\infty}h^k\pi_M\big(a(\lambda_0)^{k+1}\big),\end{aligned} \tag{3.10}$$

if $|h| < \delta$, where $\pi_M(a(\lambda_0)^{k+1}) \in A-M$ for each $k \in \mathbb{N}$.

By Corollary 2.2, there is a number $R > 0$ such that

$$\sum_{k=0}^{\infty}\frac{z^k}{\lambda^{k+1}} \tag{3.11}$$

converges in A, if $|\lambda| > R$. Easy calculation shows that

$$z(\lambda)\sum_{k=0}^{\infty}\frac{z^k}{\lambda^{k+1}} = \sum_{k=0}^{\infty}\frac{z^k}{\lambda^{k+1}}z(\lambda) = e_A. \tag{3.12}$$

Therefore,

$$z(\lambda)^{-1} = \sum_{k=0}^{\infty}\frac{z^k}{\lambda^{k+1}}, \tag{3.13}$$

whenever $|\lambda| > R$. Since $z(\lambda)^{-1}z(\lambda) - e_A \in M$ for each λ with $|\lambda| > R$, then

$$\Psi(\lambda) = \pi_M\big(z(\lambda)^{-1}\big) = \pi_M\left(\sum_{k=0}^{\infty}\frac{z^k}{\lambda^{k+1}}\right) = \sum_{k=0}^{\infty}\frac{\pi_M(z^k)}{\lambda^{k+1}} \tag{3.14}$$

if $|\lambda| > R$, where $\pi_M(z^k) \in A-M$ for each $k \in \mathbb{N}$. Consequently, Ψ is an analytic $(A-M)$-valued function on $\mathbb{C}\cup\{\infty\}$. Since $A-M$ is an exponentially galbed Hausdorff space, Ψ is a constant map by Turpin's theorem (see [19, page 56]).

We show that $\Psi(\lambda) = \theta_{A-M}$ for each $\lambda \in \mathbb{C}$. So, if O is any neighbourhood of zero in A, then there exist in A a closed neighbourhood O' of zero and a neighbourhood V of zero such that $O' \subset O$ and

$$\left\{\sum_{k=0}^{n}\frac{v_k}{2^k} : v_1,\dots,v_n \in V\right\} \subset O' \tag{3.15}$$

for each $n \in \mathbb{N}$. Moreover, there are $\mu_z \in \mathbb{C}\backslash\{0\}$ and $\rho_V > 0$ such that

$$\left(\frac{z}{\mu_z}\right)^k \in \rho_V V \tag{3.16}$$

for each $k \in \mathbb{N}$. If now $|\lambda| > \max\{3|\mu_z|, \rho_V\}$, then

$$v_k(\lambda) = \frac{2^k z^k}{\lambda^{k+1}} = \frac{1}{\rho_V}\frac{\rho_V}{\lambda}\left(\frac{2\mu_z}{\lambda}\right)^k\left(\frac{z}{\mu_z}\right)^k \in \frac{1}{\rho_V}\left[\frac{\rho_V}{\lambda}\left(\frac{2\mu_z}{\lambda}\right)^k\right]\rho_V V \subset V \tag{3.17}$$

for each $k \in \mathbb{N}$. Therefore,

$$\sum_{k=0}^{n}\frac{z^k}{\lambda^{k+1}} = \sum_{k=0}^{n}\frac{v_k(\lambda)}{2^k} \in O' \tag{3.18}$$

for each $n \in \mathbb{N}$. Since O' is closed, then

$$z(\lambda)^{-1} = \sum_{k=0}^{\infty}\frac{z^k}{\lambda^{k+1}} = \lim_{n\to\infty}\sum_{k=0}^{n}\frac{v_k(\lambda)}{2^k} \in O' \subset O, \tag{3.19}$$

whenever $|\lambda| > \max\{3|\mu_z|, \rho_V, R\}$. Hence,

$$\begin{gathered}\lim_{|\lambda|\to\infty} z(\lambda)^{-1} = \theta_A,\\ \lim_{|\lambda|\to\infty}\Psi(\lambda) = \lim_{|\lambda|\to\infty}\pi_M\left(z(\lambda)^{-1}\right) = \pi_M\left(\lim_{|\lambda|\to\infty} z(\lambda)^{-1}\right) = \theta_{A-M}.\end{gathered} \tag{3.20}$$

Thus, $\Psi(\lambda) = \theta_{A-M}$ or $a(\lambda) \in M$ for each $\lambda \in \mathbb{C}$. Therefore,

$$e_A = -\left(a(\lambda)z(\lambda) - e_A\right) + a(\lambda)z(\lambda) \in M, \tag{3.21}$$

which is a contradiction. Consequently, every $z \in Z(A)$ defines a $\lambda_z \in \mathbb{C}$ such that $z = \lambda_z e_A$. Hence, $Z(A)$ is isomorphic to $\mathbb{C}$.

Moreover, the isomorphism ρ, defined by $\rho(z) = \lambda_z$ for each $z \in Z(A)$, is continuous. Indeed, if O is a neighbourhood of zero in $\mathbb{C}$, then there exists an $\epsilon > 0$ such that

$$O_\epsilon = \{\lambda \in \mathbb{C} : |\lambda| < \epsilon\} \subset O. \tag{3.22}$$

Let $\lambda_0 \in O_\epsilon \setminus \{0\}$. Since A is a Hausdorff space, there exists a balanced neighbourhood V of zero in A such that $\lambda_0 e_A \notin V$. But then we also have

$$\lambda_0 e_A \notin V' = V \cap Z(A). \tag{3.23}$$

If $|\lambda_z| \geqslant |\lambda_0|$, then $|\lambda_0\lambda_z^{-1}| \leqslant 1$ and $\lambda_0 e_A = (\lambda_0\lambda_z^{-1})z \in V'$ for each $z \in V'$, which is not possible. Hence, $\lambda_z \in O$ for each $z \in V'$. Thus, ρ is continuous (ρ^{-1} is continuous because $Z(A)$ is a topological linear space in the subspace topology). Consequently, $Z(A)$ is topologically isomorphic to $\mathbb{C}$. □

Remark 3.2. Using Theorem 3.1, it is possible to describe all closed maximal regular one-sided and two-sided ideals in sequentially complete exponentially galbed algebras with bounded elements.

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