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Research Article

Optimal Order Strategy in Uncertain Demands with Free Shipping Option

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Free shipping with conditions has become one of the most effective marketing tools; more and more companies especially e-business companies prefer to offer free shipping to buyers whenever their orders exceed the minimum quantity specified by them. But in practice, the demands of buyers are uncertain, which are affected by weather, season, and many other factors. Firstly, we model the centralization ordering problem of retailers who face stochastic demands when suppliers offer free shipping, in which limited distributional information such as known mean, support, and some deviation measures of the random data is needed only. Then, based on the linear decision rule mainly for stochastic programming, we analyze the optimal order strategies of retailers and discuss the approximate solution. Further, we present the core allocation between all retailers via dual and cooperative game theory. The existence of core shows that each retailer is pleased to cooperate with others in the centralization problem. Finally, a numerical example is implemented to discuss how uncertain data and parameters affect the optimal solution.

1. Introduction

With the rapid development of e-commerce and logistics industry, free shipping offered by e-business companies has become an effective means of attracting and keeping customers. Many business-to-consumer and business-to-business (B2B) companies now offer free shipping to buyers who spend more than a specific amount. More and more companies and businesses begin to take free shipping strategy, such as Amazon online bookstore; when the cost reaches \$25, the buyer can get free shipping service. Taobao and Dangdang also offer free shipping if the certain amount is satisfied. The growth and evolution of the e-commerce sector have highlighted the importance of shipping and handling (S&H) fees for business models. The supplier can effectively reduce order processing costs and implementation costs, if they can reduce the frequent small shipments. Otherwise, if many retailers are joined together to order, the cost may be saved for the satisfaction of free shipping. Therefore, free shipping schedules have become an interest study for both the supplier and demander. Survey evidence indicates that shipping fees are the main complaint of more than 50 percent of online shoppers and that more than 60 percent of shoppers have abandoned an order when shipping fees are added. Academic work has further confirmed that fulfillment issues are a key driver of customer satisfaction.

In this paper, we study the centralization ordering problem with uncertain demands considering free shipping. Considering a supply chain includes a supplier and a number of retailers whose demands for commodities are uncertain, and retailers focus on how free shipping schedules impact their ordering strategies. Lewis et al. [1] used an ordered probability model to account for the effects of nonlinear and discontinuous free shipping on purchasing decisions. It shows that the retailers are very sensitive to shipping charges, and promotions such as free shipping and free shipping for orders that exceed some size threshold are very effective in generating additional sales. Leng et al. [2, 3] considered the free shipping strategies of business-to-consumer and business-to-business environments companies by modeling the problem as a leader-follower game under complete information where the leader is the seller and the follower is the buyer. Yang et al. [4] analyzed the price-threshold relationship which was inverted-U shaped and explored how prices and the free shipping threshold interact to affect the optimal policy. The initial price level dictates the price dispersion for homogenous goods increases when the threshold is lowered. Zhou et al. [5] gave the management of stochastic inventory systems with free shipping option. Abad and Aggarwal [6] studied the pricing decisions with random demand in order to reduce transport costs, which is free shipping with condition. Hua et al. [7] studied the optimal order strategy of a retailer who faces deterministic or stochastic demand when suppliers offer free shipping. It analyzes the impacts of the transportation cost on the retailer's optimal order strategy based on EOQ model and newsvendor model.

One issue of the above study is the assumption of full distributional knowledge of the uncertain data. Because such information may rarely be available in practice, it has rekindled recent interests in robust optimization as an alternative perspective of data uncertainty. In robust optimization, compared with the full distributional knowledge which is hardly got, limited distributional information such as known mean, support, and some deviation measures of the random data is required only. So in this paper, we consider the uncertain demand in general and study the optimal ordering model with free shipping.

The paper is organized as follows. In Section 2, we give the stochastic programming model of optimal order strategy about the retailers, in which the demands are uncertain with free shipping option. In Section 3, based on the linear decision rules, we analyze the robust counterpart of the stochastic programming model and formulate a new equivalent determined model. In Section 4 we use cooperative game theory to get the core of all retailers. In Section 5, a numerical experiment confirms that order incidence is affected by free shipping option and varying interval of demand. Finally, Section 6 concludes this paper.

2. Problem Description

Assuming there are a supplier and a number of retailers, who just trade only a type of goods. The supplier offers the goods to retailers whose demands for commodities are uncertain. All retailers order goods uniformly and order price is constant. Only when the total ordering amount reaches a certain threshold, the supplier can offer free shipping for retailers. Here we consider how to maximize the benefits of all retailers by selecting their optimal order quantity. In this problem, we take the following assumptions. The retailers are all rational and their inventories are inadequate to meet the real demands, so they are willing to participate in group to order goods according to their actual situation. In addition, there is no competition among retailers and they are willing to participate in the group to pay the total minimum fee. So the problem aims to minimize the total cost of all retailers with constraint that their demands are met.

At first, we denote the following notions. The notions m and c are the retail price and order price of the goods separately and q is the known threshold of free shipping. The notion n is the number of retailers, and the random demand of the ith retailer is $d_i(\tilde{z})$ in which $\tilde{z}$ means random environment, and all $d_i(\tilde{z})$ are independent. L_i is the current inventory of the ith retailer. We denote the order quantity x_i by the decision variable. Only when the condition $\sum_{i=1}^{n} x_i \geq q$ holds, the supplier can offer free shipping for retailers; otherwise the retailers should cost the $f(\sum_{i=1}^{n} x_i)$. The symbol y is 0-1 variable, in which 1 means payoff for the transport and 0 is free shipping. The symbol $w_i(\tilde{z})$ is the amount of the shortage of goods of the ith retailer, which is caused by $d_i(\tilde{z})$ in uncertain environment. Then the model of optimal order strategy with free shipping option is given as follows (1):

$$\min \quad c \times \left(\sum_{i=1}^{n} x_i\right) + f\left(\sum_{i=1}^{n} x_i\right) \times y + m \times E\left(\sum_{i=1}^{n} w_i(\tilde{z})\right),$$
$$\text{s.t.} \quad \begin{cases} x_i + w_i(\tilde{z}) + L_i \geq d_i(\tilde{z}), & i = 1, 2, \ldots n, \\ x_i, w_i(\tilde{z}) \geq 0, & i = 1, 2, \ldots n, \\ y = 0, & \text{if} \sum_{i=1}^{n} x_i \geq q, \\ y = 1, & \text{if} \sum_{i=1}^{n} x_i < q, \end{cases} \tag{1}$$

where the objective function contains the ordering cost, transportation cost, and penalty cost incurred at the retailers if the demands are not satisfied. For the penalty cost is relevant to uncertain realization of $\tilde{z}$, the expectation is taken here. According to the objective function, we require that the order quantity of the goods be not too much to add inventory and at the same time the shortage be not too much to increase cost. The first constraint means that, for the ith retailer, the sum of ordering quantity, shortage, and inventory quantity be not less than their demand. The second constraint shows that the decision variables of ordering quantity and shortage are nonnegative. The third constraint ensures that when the ordering quantity is not less than the given threshold q, the supplier can offer free shipping for retailers; otherwise the retailers should pay for the transportation cost.

The slack variable $v_i(\tilde{z})$ is added in the first constraint, so model (1) can be rewritten to the model

$$\min \quad c \times \left(\sum_{i=1}^{n} x_i\right) + f\left(\sum_{i=1}^{n} x_i\right) \times y + m \times E\left(\sum_{i=1}^{n} w_i(\tilde{z})\right),$$
$$\text{s.t.} \quad \begin{cases} x_i + w_i(\tilde{z}) + L_i - v_i(\tilde{z}) = d_i(\tilde{z}), & i = 1, 2, \ldots n, \\ x_i, w_i(\tilde{z}),\ v_i(\tilde{z}) \geq 0, & i = 1, 2, \ldots n, \\ y = 0, & \text{if} \sum_{i=1}^{n} x_i \geq q, \\ y = 1, & \text{if} \sum_{i=1}^{n} x_i < q. \end{cases} \tag{2}$$

In this paper, we assume the random demand $d_i(\tilde{z})$ is generalized variable, whose only limited distributional information is known, but distributional function and other full knowledge are unknown. Then, we discuss the solvability approximately of the stochastic programming (2).

3. Approximation via Decision Rule

It is difficult to solve the general stochastic programming; besides this, the full distributional knowledge of the uncertain data is needed, which may rarely be available in practice. But assuming only limited distribution information is known such as mean, support, and some deviation measures of the random data, linear decision rule is the key enabling method that permits scalability to multistage models [8]. Interesting applications include designing supplier-retailer contracts, network design under uncertainty, and crashing projects with uncertain activity times. Even though linear decision rule allows us to derive tractable formulations in a variety of applications, it may lead to infeasible instances [9]. This fact motivates people to refine linear decision rule and improve it to a general linear decision rule, which improves the objective value. Chen et al. [10] gave the conclusion that when complete recourse exists, the general linear decision rule is equal to the linear decision rule. Because the recourse matrix in this paper is special and the support is bounded closed set, it is feasible to analyze the solvability of model (2) using the linear decision rule.

3.1. A Two-Stage Stochastic Linear Programming Model. The linear decision rule is used mainly to solve the multistage stochastic programming. For the stochastic linear programming (3), decision x has to be made before the actual value of $\tilde{z}$ is realized which consists the first stage. After applying the decision and after the uncertainty is realized, the subjects of (3) may be not satisfied and the optimal second-stage decisions or recourse decisions are carried out, in which $w(\tilde{z})$ is recourse variable and W is recourse matrix. So the subjects of (3) are satisfied and, at the same time, the cost that is aroused by the recourse in (4) is minimized. The problems (3) and (4) can be rewritten in model (5) equivalently:

$$\begin{aligned} &\min \quad c^T x + E\left(Q\left(x,\tilde{z}\right)\right), \\ &\text{s.t.} \quad \begin{cases} T\left(\tilde{z}\right) x = h\left(\tilde{z}\right), \\ \qquad x \geq 0, \end{cases} \end{aligned} \tag{3}$$

$$\begin{aligned} &Q\left(x,\tilde{z}\right) = \min \quad mw\left(\tilde{z}\right), \\ &\text{s.t. } T\left(\tilde{z}\right) x + Ww\left(\tilde{z}\right) = h\left(\tilde{z}\right), \end{aligned} \tag{4}$$

$$\begin{aligned} &\min \quad c^T x + mE\left(w\left(\tilde{z}\right)\right), \\ &\text{s.t.} \quad \begin{cases} T\left(\tilde{z}\right) x + Ww\left(\tilde{z}\right) = h\left(\tilde{z}\right), \\ \qquad\qquad x \geq 0. \end{cases} \end{aligned} \tag{5}$$

Comparing models (2) and (5), we know easily that (2) is also a two-stage stochastic linear program with the special recourse matrix.

3.2. Problem Analysis Based on Linear Decision Rule. According to the literature [10], assuming the stochastic programming with the relatively complete recourse, the second-stage problem is surely feasible for any choice of feasible first-stage decision vector x. The complete recourse is defined on the matrix W such that, for any t, there exists $w \geq 0$, satisfying $Ww = t$. Hence, the definition of complete recourse depends only on the structure of the matrix W, which makes the problem easier to solve. For the model (2), if we let $W_i = [-1, 1]$ for each retailer, there exists a simple case of complete recourse where the special matrix is

$$\mathbf{W} = \begin{bmatrix} -1 & 1 & & & & & \\ & & -1 & 1 & & & \\ & & & & \cdots & & \\ & & & & & -1 & 1 \end{bmatrix}. \tag{6}$$

So it is feasible to analyze the solvability of the problem (2) via the linear decision rule. Based on the linear decision rule given in [9], we assume both the recourse variable $w_i(\tilde{z})$ and the slack variable $v_i(\tilde{z})$ are the affine functions. For convenience, we describe them by using vector form below. Let $w(\tilde{z}) = w^{\mathbf{0}} + \sum_{k=1}^{N} w^k \tilde{z}_k$, where the coefficients w^k are unknown. At the same time, the demands of all retailers are in the same linear form $d(\tilde{z}) = d^0 + \sum_{k=1}^{N} d^k \tilde{z}_k$. Generally for the uncertain variable $\tilde{z}$, we may assume its mean is 0 and the support is $W_{\tilde{z}} = [-\underline{z}, \overline{z}]$, $\underline{z} > 0$, $\overline{z} > 0$, which is also the value set of $\tilde{z}$. So the model can be adjusted to the following problem:

$$\begin{aligned} &Z_{\text{STOC}} = \min \quad c^T x + f\left(x\right) y + me^T w^0, \\ &\text{s.t.} \quad \begin{cases} x + L + w^0 - v^0 = d^0, & \\ w^k - v^k = d^k, & k = 1, 2, \ldots, N, \\ x, w, v \geq 0, & \\ y = 0, & \text{if } \displaystyle\sum_{i=1}^{n} x_i \geq q, \\ y = 1, & \text{if } \displaystyle\sum_{i=1}^{n} x_i < q. \end{cases} \end{aligned} \tag{7}$$

Theorem 1. *For model* (7)*, the third subject $w(\tilde{z}) \geq 0$, for all $\tilde{z} \in W_{\tilde{z}} = [-\underline{z}, \overline{z}]$, holds if and only if there is $w^0 \geq \overline{z} \sum_{k=1}^{N} t^{1k} + \underline{z} \sum_{k=1}^{N} s^{1k}$.*

Proof. Let $w^k = s^{1k} - t^{1k}$; $s^{1k}, t^{1k} \in R^n$; $s^{1k}, t^{1k} \geq 0$, then the following equivalence relations are easy to get:

$$\begin{aligned} &w\left(\tilde{z}\right) = w^0 + \sum_{k=1}^{N} w^k \tilde{z}_k \geq 0 \\ &\Longleftrightarrow w^0 \geq -\sum_{k=1}^{N} w^k \tilde{z}_k = -\sum_{k=1}^{N} \left(s^{1k} - t^{1k}\right) \tilde{z}_k \\ &\qquad = \sum_{k=1}^{N} \left(t^{1k} \tilde{z}_k - s^{1k} \tilde{z}_k\right) \\ &\Longleftrightarrow w^0 \geq \max \sum_{k=1}^{N} \left(t^{1k} \tilde{z}_k - s^{1k} \tilde{z}_k\right) = \overline{z} \sum_{k=1}^{N} t^{1k} + \underline{z} \sum_{k=1}^{N} s^{1k}. \end{aligned} \tag{8}$$

□

Similar analysis is considered for vector v in model (7), and based on the assumption that the transportation cost is linear function, the uncertain model (7) is rewritten to the determinate model

$$Z_{\mathrm{LDR}} = \min \quad c^T x + re^T xy + me^T w^0,$$
$$\text{s.t.} \quad \begin{cases} x + L + w^0 - v^0 = d^0, & \\ w^k - v^k = d^k, & k = 1,2,\ldots N, \\ w^0 \geq \overline{z}\sum_{k=1}^{N} t^{1k} + \underline{z}\sum_{k=1}^{N} s^{1k}, & \\ v^0 \geq \overline{z}\sum_{k=1}^{N} t^{2k} + \underline{z}\sum_{k=1}^{N} s^{2k}, & \\ x, s^1, t^1, s^2, t^2 \geq 0, & \\ y = 0, & \text{if} \sum_{i=1}^{n} x_i \geq q, \\ y = 1, & \text{if} \sum_{i=1}^{n} x_i < q. \end{cases} \tag{9}$$

Because model (9) is the linear case of the model (7), the following conclusion is obtained.

Theorem 2 ($Z_{\mathrm{STOC}} \leq Z_{\mathrm{LDR}}$). *Hence, by introducing the linear decision rule for the primal model* (2)*, we get the robust model* (7)*, whose optimal value is approximate to the value of the determinate model* (9)*. Regarding the issue of the bound on the objective function, the literature [10] made detailed discussions.*

4. Cooperative Game of the Ordering Centralization Problem

In the ordering centralization problem considering free shipping, we only focus on the allocation of the expected cost. Whether it is possible to derive a stable allocation of the actual cost for each demand realization, the cost of each retailer is reduced. In this section, based on the dual theory of stochastic programming, we analyze the centralization problem with free shipping.

First we briefly introduce the concepts of cooperative game theory that will be used. Let $N = \{1 \cdots n\}$ be the set collection of players. A collection of players $S \subseteq N$ is called a coalition. A characteristic cost function is defined for each coalition $S \subseteq N$. A cooperative game is defined by the pair (N, C). Given a cooperative game, there are many ways to divide the cost allocation of the game among the players. The cost allocation has been extensively studied in the literature. We focus on the core allocation, which is defined below.

A vector $l = (l_1, l_2, \ldots, l_n)$ is called an imputation of the game (N, C) if $\sum_{j \in N} l_j = C(N)$ and $l_j \leq C(\{j\})$ for every $j \in N$. One can interpret an imputation as a division of $C(N)$ that charges every player at most as much as that they will play by themselves. When this idea is generalized to every coalition of players, the notion of core is given.

Definition 3. An allocation $l = (l_1, l_2, \ldots, l_n)$ is in the core of the game of (N, C), if $\sum_{j \in N} l_j = C(N)$ and $\sum_{j \in S} l_j \leq C(S)$ for every $S \subseteq N$.

Then, we consider the following two-stage stochastic linear programming:

$$\begin{aligned} \min \quad & v_1^T x_1 + E\left[v_2^T x_2\right], \\ \text{s.t.} \quad & A_{11} x_1 = b_1, \\ & A_{12} x_1 + A_{22} x_2 = b_2(\tilde{z}), \\ & x_1, x_2 \geq 0, \end{aligned} \tag{10}$$

where x_1 is the decision variable of the first stage, x_2 is the recourse variable of the second stage in which $A_{12}x_1 = b_2(\tilde{z})$ is not satisfied for the realization of $\tilde{z}$, and $v_2^T x_2$ is the recourse cost.

The dual problem of problem (10) is

$$\begin{aligned} \max \quad & E\left[b_1^T \pi_1 + b_2(\tilde{z})^T \pi_2(\tilde{z})\right], \\ \text{s.t.} \quad & A_{11}^T \pi_1 + E\left[A_{12}^T \pi_2(\tilde{z})\right] \leq v_1, \\ & A_{22}^T \pi_2(\tilde{z}) \leq v_2. \end{aligned} \tag{11}$$

For the realization of $\tilde{z} \in W$, if W is a finite set, then model (10) is a linear programming and the strict dual condition is satisfied. Otherwise, the strict dual condition may not be satisfied [11]. In the literature [12], there is a conclusion that the optimal value of (10) is equal to the optimal value of model (11) when model (10) is feasible and has relatively complete recourse matrix. So we can study the dual problem of the primal problem (1) to find the relationship of them.

Here, assuming that the condition of free-transportation cost holds, the problem (1) is equal to the problem (12). In the model (12), for every collaboration $S \subseteq N$, let the set S replace the set N; then the collaboration model (13) is given as follows:

$$C(S) = \min \quad c\left(\sum_{i=1}^{n} x_i\right) + mE\left(\sum_{i=1}^{n} w_i(\tilde{z})\right),$$
$$\text{s.t.} \quad \begin{cases} \sum_{i=1}^{n} x_i \geq q, & \\ x_i + w_i(\tilde{z}) \geq d_i(\tilde{z}) - L_i, & i = 1,2,\ldots n, \\ x_i, w_i(\tilde{z}) \geq 0, & i = 1,2,\ldots n, \end{cases} \tag{12}$$

$$\min \quad c\left(\sum_{i \in S} x_i\right) + mE\left(\sum_{i \in S} w_i(\tilde{z})\right),$$
$$\text{s.t.} \quad \begin{cases} \sum_{i \in S} x_i \geq q, & \\ x_i + w_i(\tilde{z}) \geq d_i(\tilde{z}) - L_i, & i \in S, \\ x_i, w_i(\tilde{z}) \geq 0, & i \in S. \end{cases} \tag{13}$$

It is easy to see that the dual problem of model (13) is

$$D(S) = \max q\alpha + \sum_{i=1}^{|S|} E\left(\left(d_i(\tilde{z}) - L_i\right)\alpha_i(\tilde{z})\right),$$
$$\text{s.t.} \begin{cases} E\left(\alpha_i(\tilde{z})\right) + \alpha \le c, & i \in S, \\ \alpha_i(\tilde{z}) \le m, & i \in S, \\ \alpha, \alpha_i(\tilde{z}) \ge 0, & i \in S. \end{cases} \quad (14)$$

In Section 3, we have given the example that the relatively complete recourse matrix exists in the inventory centralization problem considering free shipping. Hence we propose the following results.

Theorem 4. *For any collection of retailers $S \subseteq N$, the optimal value of* (13) *is equal to the optimal value of* (14).

Theorem 5. *The allocation in the core of retailers exists in model* (1) *of cooperative ordering game with free shipping option.*

Proof. For $S = N$, we denote by $(\alpha_j^*(\tilde{z}), \alpha^*)$ the optimal solution of problem Dual(N) and let

$$l_j = \varepsilon_j + E\left(\left(d_j(\tilde{z}) - L_j\right)\alpha_j^*(\tilde{z})\right), \quad (15)$$

where $\sum_{j=1}^{N} \varepsilon_j = q\alpha$, $\varepsilon_j \ge 0$. From Theorem 4, we know that

$$\sum_{j \in N} l_j = \sum_{j \in N} \left(\varepsilon_j + E\left(\left(d_j(\tilde{z}) - L_j\right)\alpha_j^*(\tilde{z})\right)\right) = C(N). \quad (16)$$

On the other hand, for every $S \subseteq N$, because $(\alpha_j^*(\tilde{z}), \alpha^*)$ is a feasible solution to Dual(S), we have the following inequality:

$$\sum_{j \in S} l_j = \sum_{j \in S} \left(\varepsilon_j + E\left(\left(d_j(\tilde{z}) - L_j\right)\alpha_j^*(\tilde{z})\right)\right) \le q\alpha + \sum_{j \in S} E\left(\left(d_j(\tilde{z}) - L_j\right)\alpha_j^*(\tilde{z})\right) \le C(S). \quad (17)$$

By Definition 3, the vector $(l_1 \cdots l_N)$ defined by (15) is an allocation in the core of the cooperative inventory game (N, C). □

In general, a core is the set of cost allocations under which no coalitions should be charged more than they would pay if they were to separate and follow an optimal strategy for themselves. That is, no coalition will be better off by deviation from the grand coalition. Since the core is nonempty, there is at least one allocation of the cost that is considered advantageous by all players. So for each retailer of the inventory centralization problem, he is pleased to cooperate with others based on the cost-vector $(l_1 \cdots l_N)$ defined by (15).

5. Numerical Experiment

Assuming that there are three retailers and they order goods from the same supplier. The retail price of the unit goods is $m = 70$ and the order price is $c = 40$. The transportation function is proportional to order quantity; that is, $r = 4$. The given threshold is $q = 60$. The current inventory of retailers is 5, 8, and 10, respectively, and their demands are all in the same form $d_i(\tilde{z}) = 30 + \tilde{z}$, where the mean of $\tilde{z}$ is zero and the support is $\tilde{z} \in [-1, 2]$.

Table 1: The relationship between support and the optimal value.

Support of $\tilde{z}$	$[-1,2]$	$[-1,1]$	$[-2,2]$	$[-3,3]$	$[-4,4]$	$[-5,5]$
The optimal value Z_{LDR}	2770	2796.6	2871.6	4017.5	3676.7	3728.7

Table 2: The relationship between threshold and the optimal value.

Threshold of q	45	50	55	60	65	70	75	80
The optimal value Z_{LDR}	3280	3159	2770	2770	2803	3026	3026	3026

Solving model (9) with the above data by MATALB solver, we get the optimal ordering strategy of three retailers being 24, 21, and 19, respectively and the optimal cost value is 2770, in which the free shipping is satisfied for the total ordering number which is 64. On the other hand, if the free shipping is not satisfied, the optimal ordering strategy is 20, 17, and 15, respectively, and the total ordering number is 52, and the optimal cost value is 3338. This numerical experiment shows that if all retailers participate in group to order goods, the total cost may reduce for more ordering quantity. So all rational retailers are willing to order jointly when their inventories are inadequate to meet the real demands with the consideration of free shipping option.

Then we consider how the support of $\tilde{z}$ affects the optimal value. For the normal demand is 30 with a little uncertain changes, we test other 5 kinds of support with the maximal interval being 5. There are global optimal solutions in the cases of $[-1, 2]$, $[-1, 1]$, and $[-2, 2]$, but in other cases MATALB solver shows the current solution may be nonoptimal and the local optimal solutions can be found after more than one hundred iterations. The experiment shows the support has obvious effect on the optimal value of model (9) which is given in Table 1.

Further, we consider how the threshold q influences the optimal value. At the beginning we take the value 5 times according to the current value that is 60, and the optimal function values are given in Table 2 (the current value may be local optimal according to the solver for the cases that the threshold is more than 70). It shows that the optimal cost value increases from $q = 45$ to $q = 55$; then the minimal value reaches the bottom from $q = 55$ to $q = 60$; afterwards it increases from $q = 60$ to $q = 80$. In order to find the relationship better, get 35 groups of the threshold and optimal value ($q = 45, 46, \ldots, 80$ resp.), which are displayed in Figure 1. The experiment shows that the optimal value obeys piecewise function with obvious character.

At last we try to compute the core of the primal problem above based on the discussion in Section 4. For the condition of free-transportation cost is satisfied, the total minimal cost is 2770. In (14), the total maximal payment is equal to 2770

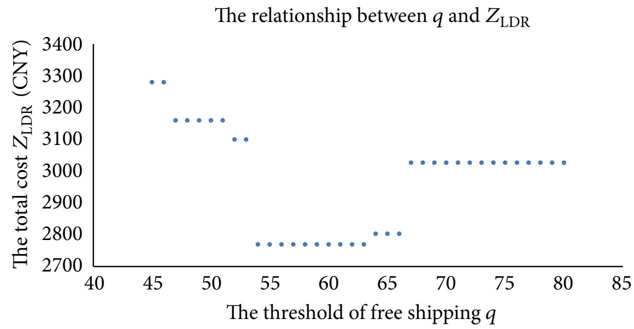

Figure 1: The detailed relationship between threshold and the optimal value.

also. According to dual theory, we have $\alpha^* = 0$, and the costs that three retailers should take are $l_1 = 25E(\alpha_1(\tilde{z}))$, $l_2 = 22E(\alpha_2(\tilde{z}))$, and $l_3 = 20E(\alpha_3(\tilde{z}))$ with constraint $E(\alpha_i(\tilde{z})) \leq 40$ and for $\alpha_i(\tilde{z}) \leq 70$ all i, where $\alpha_i(\tilde{z})$ is random decision variable. The allocations satisfying restrictions above are feasible.

6. Conclusions

It is a key research content of supply chain management. This research will provide effective effort for scientific decision making and e-business activity. This paper studied the optimal order problem under the uncertainty of demand and proposed the stochastic programming model, in which the objective function is to minimize the total cost. Considering the limited information of the uncertain variable in the model, we used the linear decision rule; one of the robust optimization methods to analyze this model and get the approximate model which is tractable. Afterwards, evidence attained from the numerical experiment strongly suggested its effectiveness and efficiency. Furthermore, this paper analyzed the core allocation between all retailers via dual and cooperative game theory. The results showed that the ordering centralization can save the cost of all retailers and it is economic for the society. We will study the application of robust optimization to supply management with other uncertainties or in the view of supplier.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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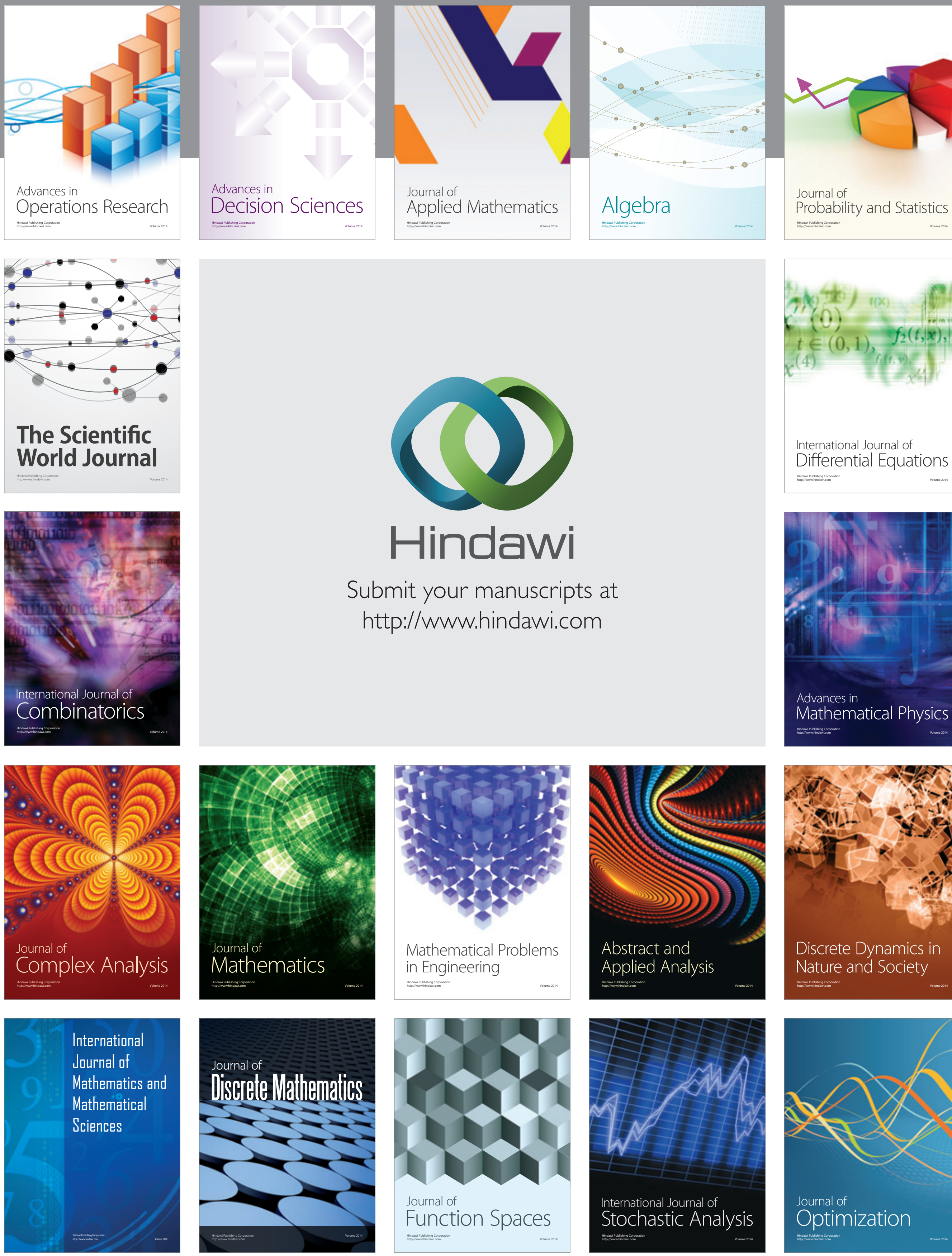
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