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Research Article

Recursive Estimation for Dynamical Systems with Different Delay Rates Sensor Network and Autocorrelated Process Noises

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The recursive estimation problem is studied for a class of uncertain dynamical systems with different delay rates sensor network and autocorrelated process noises. The process noises are assumed to be autocorrelated across time and the autocorrelation property is described by the covariances between different time instants. The system model under consideration is subject to multiplicative noises or stochastic uncertainties. The sensor delay phenomenon occurs in a random way and each sensor in the sensor network has an individual delay rate which is characterized by a binary switching sequence obeying a conditional probability distribution. By using the orthogonal projection theorem and an innovation analysis approach, the desired recursive robust estimators including recursive robust filter, predictor, and smoother are obtained. Simulation results are provided to demonstrate the effectiveness of the proposed approaches.

1. Introduction

The Kalman filter is very popular for estimating the system states of a class of linear systems which are characterized by state-space models. Since its inception in the early 1960s, it has played an important role in the research fields of target tracking, communication, control engineering, and signal processing. An implied assumption of traditional Kalman filter is that the system model and measurement model are exactly known. Unfortunately, this assumption does not always hold due to the constrained knowledge and the variation of the system and environment. When the system model and measurement model under consideration are not exactly known, the performance of traditional Kalman filter can deteriorate appreciably [1–3]. Therefore, in the past decades, the recursive robust state-space estimation problem has become a hot topic of the estimation theory. There are many different ways to describe the model uncertainty. Multiplicative noise is an important stochastic uncertainty which is commonly encountered in aerospace systems [4], communication systems [5], and image processing systems [6, 7]. Different from the additive noise, the second-order statistics of the multiplicative noise are usually unknown and this property leads to more difficulties in the research. Up to now, there are several solutions to treat with the estimation and control problems for systems with multiplicative noises, including linear matrix inequality approach [8], Riccati equation approach [9, 10], and game-theoretic method [11], to name just a few.

In traditional state estimation theory, the process noises are usually assumed to be Gaussian and uncorrelated with each other. However, this assumption is not always realistic, correlated noises are commonly encountered in practical applications. For example, in a target tracking system, the system state is usually consecutive (i.e., the system state at time k is correlated with its neighbors); thus, when the process noises are dependent on the system state, the process noises are usually autocorrelated across time. So far, there have been several approaches to deal with the estimation problem for systems with correlated noises [12–16]. The optimal Kalman filtering fusion problem for dynamic systems with cross-correlated measurement noises has been dealt with in [13–15]. In [16], the state estimation for discrete-time systems with cross-correlated noises has been treated based on an optimal weighted matrix sequence, where the process noises and measurement noises are cross correlated. It should be pointed out that the estimators mentioned previously are only suited for the correlated noises at the same time instant. In [17, 18], a Kalman-type recursive filter has been proposed for dynamic systems with finite-step autocorrelated process noises, where the autocorrelation property is described by the covariances between different time instants. The filtering problem with finite-step cross-correlated process noises and measurement noises has been investigated in [19]. In [20], the optimal robust nonfragile Kalman-type recursive filter has been designed for a class of uncertain systems with finite-step autocorrelated noises.

On another research frontier, with the development of network technologies, the sensor network has attracted increasing attention from many researchers in different fields due to their wide scope applications in surveillance, environment monitoring, information collection, wireless networks, robotics, and so on. In the sensor network, the network-induced time-delay or/and packet dropouts cannot be avoided due to limited single-sensor energy and communication capability and these have brought us new challenges in the design of the desired state estimators. The binary switching sequence is a popular way to describe the network-induced time-delay or/and packet dropouts since the time-delay or/and packet dropouts in the sensor network are inherently random [21–24]. The least-mean-square filtering problem for one-step random sampling delay has been studied in [25, 26]. Unfortunately, the filters designed in [25, 26] are suboptimal since a colored noise due to augmentation has been treated as a white noise. The filtering problem for systems with random measurement delays and multiple packet dropouts has also been discussed in [24]. In [27], the problem of robust filtering for uncertain systems with missing measurements and finite-step correlated process noises has been investigated. It should be noted that, in all the aforementioned literature, sensors involved in the sensor network have the same delay characteristics. Recently, Hounkpevi and Yaz [28, 29] present minimum variance state estimators for multiple sensors with different delay or failure rates. The least-square filtering problem for systems with one- or two-step random delay has been studied in [30], where the algorithms are derived without requiring the knowledge of the state space model but only the means and covariance functions of the processes involved in the observation equations. The optimal unbiased filtering problem for uncertain systems with different delay rates sensor network and autocorrelated process noises has also been discussed in [31]. However, the estimator obtained in [31] is nonrecursive and a colored noise due to augmentation has been treated as white noise. Up to now, to the best of the authors' knowledge, the recursive robust estimation problem has not yet been addressed for uncertain systems with different delay rates sensor network and autocorrelated noises, and this situation motivates our current study.

Motivated by the above analysis, in this paper, we aim to investigate the recursive robust estimation problem for uncertain systems with different delay rates sensor network and autocorrelated noises. The system model and measurement model under consideration are both subject to stochastic uncertainties or multiplicative noises. Different sensors in the sensor network have different delay rates and different delay rates are described by different binary switching sequences. The process noises are assumed to be one-step autocorrelated across time and the autocorrelation property is described by the covariances between different time instants. Based on an innovation analysis approach (IAA) and the orthogonal projection theorem (OPT), recursive robust estimators including filter, predictor, and smoother are obtained. This paper extends the results in [31], in two directions: (1) the autocorrelated measurement noise due to augmentation leads to more difficulties in the design of the recursive robust estimators; however, in [31], the measurement noise is treated as a white noise; and (2) the filter obtained in [31] is actually a nonrecursive filter; however, in our current work, we do not only derive a recursive robust filter, but also derive a recursive robust predictor and a recursive robust smoother. Also, the current paper differs from [28, 30] for the model uncertainties considered and for the autocorrelated process noises considered to derive the desired recursive robust estimators.

The remainder of the paper is organized as follows. In Section 2, the recursive robust estimation problem is formulated for a class of uncertain systems with autocorrelated noises and different delay rates sensor network. The recursive robust estimators including filter, predictor, and smoother are derived in Section 3. In Section 4, a simulation example is provided to illustrate the usefulness of the theory developed in this paper. We end the paper with some concluding remarks in Section 5.

Notation 1. The notation used in the paper is fairly standard. The superscript "T" stands for matrix transposition, the notation $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, the notation $\mathbb{R}^{m\times n}$ is the set of all real matrices of dimension $m \times n$, and I and 0 represent the identity matrix and zero matrix, respectively. The notation $P > 0$ means that P is real symmetric and positive definite, and $\mathrm{diag}(\cdots)$ stands for block-diagonal matrix. The notation δ_{k-j} is the Kronecker delta function, which is equal to unity for $k = j$ and zero for $k \neq j$. In addition, $\mathscr{E}\{x\}$ means mathematical expectation of x and $\mathrm{Prob}\{\cdot\}$ represents the occurrence probability of the event "$\cdot$". Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulation

Consider the following system model and measurement model:

$$\begin{aligned}\breve{x}_{k+1} &= \left(\breve{A}_k + \breve{A}_{s,k}\mu_k\right)\breve{x}_k + \breve{B}_k\omega_k,\\ \breve{y}_k^i &= \left(\breve{C}_k^i + \breve{C}_{s,k}^i\eta_k^i\right)\breve{x}_k + \breve{v}_k^i,\\ y_k^i &= \left(1-\lambda_k^i\right)\breve{y}_k^i + \lambda_k^i\breve{y}_{k-1}^i, \quad i = 1, 2, \ldots, N,\end{aligned} \tag{1}$$

where $\breve{x}_k \in \mathbb{R}^n$ is the state to be estimated, the vector $\breve{y}_k^i \in \mathbb{R}$ is the actual output vector of the ith sensor, the vector $y_k^i \in \mathbb{R}$ is the measured output vector of the ith sensor, the vector $\omega_k \in \mathbb{R}^m$ is the process noise, the vectors $\mu_k \in \mathbb{R}$

and $\eta_k^i \in \mathbb{R}$ are multiplicative noises, the vector $\breve{v}_k^i \in \mathbb{R}$ is the measurement noise of the ith sensor, the matrices $\breve{A}_k$, $\breve{A}_{s,k}$, $\breve{B}_k$, $\breve{C}_k^i$, and $\breve{C}_{s,k}^i$ are known real time-varying matrices of appropriate dimensions, and the variable $\lambda_k^i \in \mathbb{R}$ is a mutually uncorrelated binary switching sequence (and uncorrelated with other random variables) taking values on 1 and 0 with

$$\begin{aligned}\text{Prob}\left\{\lambda_k^i = 1\right\} &= \mathscr{E}\left\{\lambda_k^i\right\} = \beta_k^i,\\ \text{Prob}\left\{\lambda_k^i = 0\right\} &= 1 - \mathscr{E}\left\{\lambda_k^i\right\} = 1 - \beta_k^i.\end{aligned} \tag{2}$$

Remark 1. The measurement model (1) is a popular way to model the random sensor delay. It can be seen that if $\lambda_k^i = 1$ then $y_k^i = \breve{y}_{k-1}^i$ which means that the measurement of the ith sensor is delayed; if $\lambda_k^i = 0$, then $y_k^i = \breve{y}_k^i$; that is to say, the measurement of the ith sensor is up to date.

The noise signals μ_k, $\breve{v}_k^i$, and η_k^i are all zero-mean Gaussian white noises. They, together with the initial state $\breve{x}_0$ and the process noise ω_k, have the following statistical properties:

$$\begin{gathered}\mathscr{E}\left\{\breve{x}_0\right\} = \bar{\bar{x}}_0, \qquad \mathscr{E}\left\{\left(\breve{x}_0 - \bar{\bar{x}}_0\right)\left(\breve{x}_0 - \bar{\bar{x}}_0\right)^T\right\} = \breve{P}_0,\\ \mathscr{E}\left\{\begin{bmatrix}\omega_k\\ \mu_k\\ \breve{v}_k^i\\ \eta_k^i\\ \breve{x}_0\end{bmatrix}\begin{bmatrix}\omega_l\\ \mu_l\\ \breve{v}_l^j\\ \eta_l^j\\ \breve{x}_0\end{bmatrix}^T\right\}\\ = \begin{bmatrix}Y_{k,l} & 0 & 0 & 0 & 0\\ 0 & \delta_{k-l} & 0 & 0 & 0\\ 0 & 0 & \breve{R}_k^i\delta_{k-l}\delta_{i-j} & 0 & 0\\ 0 & 0 & 0 & \delta_{k-l}\delta_{i-j} & 0\\ 0 & 0 & 0 & 0 & \breve{X}_0\end{bmatrix},\end{gathered} \tag{3}$$

where $Y_{k,l} = Q_k\delta_{k-l} + Q_{k,l}\delta_{k-l-1} + Q_{k,l}\delta_{k-l+1}$, $\breve{X}_0 = \breve{P}_0 + \bar{\bar{x}}_0\bar{\bar{x}}_0^T$.

By defining

$$\begin{gathered}x_k = \begin{bmatrix}\breve{x}_k\\ \breve{x}_{k-1}\end{bmatrix}, \qquad A_k = \begin{bmatrix}\breve{A}_k & 0\\ I & 0\end{bmatrix},\\ A_{s,k} = \begin{bmatrix}\breve{A}_{s,k} & 0\\ 0 & 0\end{bmatrix}, \qquad B_k = \begin{bmatrix}\breve{B}_k\\ 0\end{bmatrix},\\ V_k = \begin{bmatrix}\breve{v}_k\\ \breve{v}_{k-1}\end{bmatrix}, \qquad R_k = \begin{bmatrix}\breve{R}_k & 0\\ 0 & \breve{R}_{k-1}\end{bmatrix},\\ R_{k,k-1} = \begin{bmatrix}0 & 0\\ \breve{R}_{k-1} & 0\end{bmatrix}, \qquad R_{k,k+1} = \begin{bmatrix}0 & \breve{R}_k\\ 0 & 0\end{bmatrix},\\ y_k = \left[\left(y_k^1\right)^T \ \cdots \ \left(y_k^N\right)^T\right]^T,\\ D_k = \left[\left(I - J_k\right) \ J_k\right],\\ C_k = \left[\left(I - J_k\right)\breve{C}_k \ \ J_k\breve{C}_{k-1}\right],\\ C_{s,k} = \left[\left(I - J_k\right)\breve{C}_{s,k}\eta_k \ \ J_k\breve{C}_{s,k-1}\eta_{k-1}\right],\end{gathered} \tag{4}$$

where

$$\begin{gathered}\breve{C}_k = \left[\left(\breve{C}_k^1\right)^T \ \cdots \ \left(\breve{C}_k^N\right)^T\right]^T,\\ \breve{v}_k = \left[\left(\breve{v}_k^1\right)^T \ \cdots \ \left(\breve{v}_k^N\right)^T\right]^T,\\ J_k = \text{diag}\left(\lambda_k^1, \dots, \lambda_k^N\right),\\ \eta_k = \text{diag}\left(\eta_k^1, \dots, \eta_k^N\right), \qquad \breve{C}_{s,k} = \left[\left(\breve{C}_{s,k}^1\right)^T \ \cdots \ \left(\breve{C}_{s,k}^N\right)^T\right]^T,\\ \breve{R}_k = \text{diag}\left(\breve{R}_k^1, \dots, \breve{R}_k^N\right),\end{gathered} \tag{5}$$

a compact representation of (1) can be expressed as follows:

$$x_{k+1} = \left(A_k + A_{s,k}\mu_k\right)x_k + B_k\omega_k, \tag{6}$$

$$y_k = C_kx_k + C_{s,k}x_k + D_kV_k, \tag{7}$$

where V_k is the measurement noise of the newly obtained auxiliary system (6) and (7). It follows readily from (4) that V_k has the statistic properties as follows:

$$\begin{gathered}\mathscr{E}\left\{V_k\right\} = 0,\\ \mathscr{E}\left\{V_kV_t^T\right\} = R_k\delta_{k-t} + R_{k,k-1}\delta_{k-t-1} + R_{k,k+1}\delta_{k-t+1}.\end{gathered} \tag{8}$$

Remark 2. It can be seen from (3) and (8) that the process noise ω_k and the measurement noise V_k are both one-step autocorrelated across time. For example, the process noise at time k is correlated with the process noises at times $k-1$ and $k+1$ with covariances $Q_{k,k-1}$ and $Q_{k,k+1}$, respectively. The measurement noise at time k is correlated with the measurement noises at times $k-1$ and $k+1$ with covariances $R_{k,k-1}$ and $R_{k,k+1}$, respectively.

Remark 3. Observe that the system model and measurement model of system (6) and (7) are both subject to stochastic uncertainties and C_k, $C_{s,k}$, and D_k involve the stochastic variable λ_k^i. Thus, system (6) and (7) is actually a stochastic uncertain system. On the other hand, the process noise ω_k and the measurement noise V_k are both one-step autocorrelated across time. Therefore, the traditional recursive robust estimation approaches may not satisfy the performance requirements here.

Remark 4. A seemingly natural way of handling the auto-correlated noises is the augmentation of the system states. However, such a state augmentation approach gives rise to significant increase in the system dimension, which would inevitably lead to computational burden. In addition, in the state augmentation method, the noises are treated as components of the auxiliary system state, generally, it is difficult for an estimator to track noise signals, and this will affect the estimation of other components of the auxiliary system state. Without resorting to state augmentation, in our current work, we treat system (6) and (7) directly by using an IAA and the OPT.

3. The Main Results

For convenience of later development, let us introduce the following lemmas, which are very useful in establishing our main results.

Lemma 5. *For stochastic matrices J_k, C_k, D_k, and $C_{s,k}$, one has the following results:*

$$\begin{aligned}
\overline{J}_k &= \mathscr{E}\{J_k\} = \operatorname{diag}\left(\beta_k^1, \ldots, \beta_k^N\right), \qquad \widetilde{J}_k = J_k - \overline{J}_k, \\
\Sigma_k &= \mathscr{E}\left\{\widetilde{J}_k \widetilde{J}_k^T\right\} = \operatorname{diag}\left(\left(1-\beta_k^1\right)\beta_k^1, \ldots, \left(1-\beta_k^N\right)\beta_k^N\right), \\
\overline{C}_k &= \mathscr{E}\{C_k\} = \left[\left(I - \overline{J}_k\right)\check{C}_k \quad \overline{J}_k \check{C}_{k-1}\right], \\
\widetilde{C}_k &= C_k - \overline{C}_k = \left(J_k - \overline{J}_k\right)\left[-\check{C}_k \quad \check{C}_{k-1}\right] = \widetilde{J}_k C_{e,k}, \\
\overline{D}_k &= \mathscr{E}\{D_k\} = \left[\left(I - \overline{J}_k\right) \quad \overline{J}_k\right], \\
\widetilde{D}_k &= D_k - \overline{D}_k = \left(J_k - \overline{J}_k\right)\left[-I \quad I\right] = \widetilde{J}_k D_{e,k}, \\
C_{e,k} &= \left[-\check{C}_k \quad \check{C}_{k-1}\right], \qquad D_{e,k} = \left[-I \quad I\right], \\
\mathscr{E}\left\{\widetilde{J}_k\right\} &= \mathscr{E}\left\{\widetilde{C}_k\right\} = \mathscr{E}\left\{\widetilde{D}_k\right\} = \mathscr{E}\{C_{s,k}\} = 0.
\end{aligned} \tag{9}$$

Proof. Lemma 5 follows directly from (2), (4), and (5) and the fact that η_k is zero mean. □

Lemma 6. *For system state x_k and the process noise ω_k, one has the following result:*

$$\mathscr{E}\left\{x_k \omega_k^T\right\} = B_{k-1} Q_{k-1,k}. \tag{10}$$

Proof. Lemma 6 follows directly from (3) and (6). □

Lemma 7. *The state covariance matrix $X_k = \mathscr{E}\{x_k x_k^T\}$ has the following recursion:*

$$\begin{aligned}
X_{k+1} &= A_k X_k A_k^T + A_k B_{k-1} Q_{k-1,k} B_k^T + A_{s,k} X_k A_{s,k}^T \\
&\quad + B_k Q_{k,k-1} B_{k-1}^T A_k^T + B_k Q_k B_k^T.
\end{aligned} \tag{11}$$

Proof. Lemma 7 follows directly from (3), (6), and Lemma 6. □

Furthermore, defining $\check{X}_{k+1} = \mathscr{E}\{\check{x}_{k+1}\check{x}_{k+1}^T\}$ and $\check{X}_{k+1,k} = \mathscr{E}\{\check{x}_{k+1}\check{x}_k^T\}$, one has from (4) and Lemma 7 the following:

$$\begin{aligned}
\check{X}_{k+1} &= \check{A}_k \check{X}_k \check{A}_k^T + \check{A}_k \check{B}_{k-1} Q_{k-1,k} \check{B}_k^T + \check{A}_{s,k} \check{X}_k \check{A}_{s,k}^T \\
&\quad + \check{B}_k Q_{k,k-1} \check{B}_{k-1}^T \check{A}_k^T + \check{B}_k Q_k \check{B}_k^T, \\
\check{X}_{k+1,k} &= A_k \check{X}_k + \check{B}_k Q_{k,k-1} \check{B}_{k-1}^T.
\end{aligned} \tag{12}$$

Lemma 8 (see [32]). *If $A \in \mathscr{R}^{p\times p}$ is a real matrix and $B = \operatorname{diag}(b_1, \ldots, b_p)$ is a diagonal stochastic matrix, then*

$$\mathscr{E}\left\{\mathrm{BAB}^{\,T}\right\} = \begin{bmatrix} \mathscr{E}\left\{b_1^2\right\} & \cdots & \mathscr{E}\left\{b_1 b_p\right\} \\ \vdots & \cdots & \vdots \\ \mathscr{E}\left\{b_p b_1\right\} & \cdots & \mathscr{E}\left\{b_p^2\right\} \end{bmatrix} \otimes A, \tag{13}$$

where $\otimes$ is the Hadamard product (this product is defined as $[A \otimes B]_{i,j} = A_{i,j} \cdot B_{i,j}$).

3.1. Recursive Robust Filter

Theorem 9. *For the addressed system (6) and (7), one has the following recursive robust filter:*

$$\hat{x}_{k|k-1} = A_{k-1}\hat{x}_{k-1|k-1} + B_{k-1}Q_{k-1,k-2}B_{k-2}^T \overline{C}_{k-1}^T \Pi_{k-1}^{-1}\varepsilon_{k-1}, \tag{14}$$

$$\begin{aligned}
P_{k|k-1} &= A_{k-1}P_{k-1|k-1}A_{k-1}^T + B_{k-1}Q_{k-1}B_{k-1}^T \\
&\quad + A_{s,k-1}X_{k-1}A_{s,k-1}^T \\
&\quad + A_{k-1}\left(B_{k-2}Q_{k-2,k-1} - \digamma_{k-1,k-1}\right. \\
&\qquad \left. \times \Pi_{k-1}^{-1}\overline{C}_{k-1}B_{k-2}Q_{k-2,k-1}\right)B_{k-1}^T \\
&\quad + B_{k-1}\left(B_{k-2}Q_{k-2,k-1} - \digamma_{k-1,k-1}\right. \\
&\qquad \left. \times \Pi_{k-1}^{-1}\overline{C}_{k-1}B_{k-2}Q_{k-2,k-1}\right)^T A_{k-1}^T \\
&\quad - B_{k-1}Q_{k-1,k-2}B_{k-2}^T\overline{C}_{k-1}^T\Pi_{k-1}^{-1}\overline{C}_{k-1}B_{k-2} \\
&\quad \times Q_{k-1,k-2}^T B_{k-1}^T,
\end{aligned} \tag{15}$$

$$\varepsilon_k = y_k - \overline{C}_k\hat{x}_{k|k-1} - \overline{D}_k R_{k,k-1}\overline{D}_{k-1}^T\Pi_{k-1}^{-1}\varepsilon_{k-1}, \tag{16}$$

$$\begin{aligned}
\digamma_{k,k} &= P_{k|k-1}\overline{C}_k^T \\
&\quad - \left(A_{k-1}\digamma_{k-1,k-1} + B_{k-1}Q_{k-1,k-2}B_{k-2}^T\overline{C}_{k-1}^T\right) \\
&\quad \times \Pi_{k-1}^{-1}\overline{D}_{k-1}R_{k,k-1}\overline{D}_k^T,
\end{aligned} \tag{17}$$

$$
\begin{aligned}
\Pi_k = {} & \overline{C}_k P_{k|k-1} \overline{C}_k^T - \overline{C}_k \\
& \times \Big[\left(A_{k-1} \digamma_{k-1,k-1} + B_{k-1} Q_{k-1,k-2} B_{k-2}^T \overline{C}_{k-1}^T \right) \\
& \quad \times \Pi_{k-1}^{-1} \overline{D}_{k-1} R_{k-1,k} \Big] \overline{D}_k^T + \Sigma_k \otimes \left(C_{e,k} X_k C_{e,k}^T \right) \\
& + \left(I - \overline{J}_k \right) \check{C}_{s,k} \check{X}_k C_{s,k}^T \left(I - \overline{J}_k \right)^T + \Sigma_k \otimes \left(\check{C}_{s,k} \check{X}_k C_{s,k}^T \right) \\
& + \overline{J}_k \check{C}_{s,k-1} \check{X}_{k-1} \check{C}_{s,k-1}^T \overline{J}_k^T + \Sigma_k \otimes \left(\check{C}_{s,k-1} \check{X}_{k-1} C_{s,k-1}^T \right) \\
& - \overline{D}_k \Big[\left(A_{k-1} \digamma_{k-1,k-1} + B_{k-1} Q_{k-1,k-2} B_{k-2}^T \overline{C}_{k-1}^T \right) \\
& \quad \times \Pi_{k-1}^{-1} \overline{D}_{k-1} R_{k-1,k} \Big]^T \overline{C}_k^T + \overline{D}_k R_k \overline{D}_k^T \\
& + \Sigma_k \otimes \left(D_{e,k} R_k D_{e,k}^T \right) - \overline{D}_k R_{k,k-1} \overline{D}_{k-1}^T \\
& \times \Pi_{k-1}^{-1} \overline{D}_{k-1} R_{k,k-1}^T \overline{D}_k^T,
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
\widehat{x}_{k|k} &= \widehat{x}_{k|k-1} + \digamma_{k,k} \Pi_k^{-1} \varepsilon_k, \\
P_{k|k} &= P_{k|k-1} - \digamma_{k,k} \Pi_k^{-1} \digamma_{k,k}^T,
\end{aligned}
\tag{19}
$$

where ε_k is the innovation with covariance Π_k, the matrix $\digamma_{k,k}$ is the covariance between x_k and ε_k, the vectors $\widehat{x}_{k|k}$ and $\widehat{x}_{k|k-1}$ are the filter and one-step predictor, and the matrices $P_{k|k}$ and $P_{k|k-1}$ are the filter error covariance and one-step prediction error covariance. The initial values are $\widehat{x}_{0|0} = [\overline{\check{x}}_0^T \ \ 0]^T$, $P_{0|0} =$ $\mathrm{diag}(\check{P}_0, 0)$*, and $\varepsilon_1 = y_1 - \overline{C}_1 \widehat{x}_{1|0}$.*

Proof. Please see Appendix A. □

Remark 10. In the traditional recursive estimation problem, the innovation is calculated as $\varepsilon_k = y_k - C_k \widehat{x}_{k|k-1}$. However, due to possible sensor delay which occurs in a random way, this is not true for the problem at hand; thus, we have to recalculate the innovation as in (16). Furthermore, it can be seen that the second term on the right-hand side of (14), the last four terms on the right-hand side of (15), the second term of the right-hand side of (17), and the last ten terms on the right-hand side of (18) are caused by the random delays, the stochastic uncertainties, and autocorrelated noises. These terms constitute the main differences between our work and the traditional Kalman filter.

Next, we will derive the recursive robust predictor and recursive robust smoother based on Theorem 9.

3.2. Recursive Robust Predictor

Theorem 11. *For the addressed system* (6) *and* (7)*, one has the following L-step* ($L \geq 2$) *recursive robust predictor:*

$$
\begin{aligned}
\widehat{x}_{k+L|k} = {} & A_{k+L-1} \widehat{x}_{k+L-1|k}, \\
P_{k+L|k} = {} & A_{k+L-1} P_{k+L-1|k} A_{k+L-1}^T \\
& + A_{k+L-1} B_{k+L-2} Q_{k+L-2,k+L-1} B_{k+L-1}^T + A_{s,k+L-1} \\
& \times X_{k+L-1} A_{s,k+L-1}^T + B_{k+L-1} Q_{k+L-1,k+L-2} \\
& \times B_{k+L-2}^T A_{k+L-1}^T + B_{k+L-1} Q_{k+L-1} B_{k+L-1}^T,
\end{aligned}
\tag{20}
$$

where the initial values $\widehat{x}_{k+1|k}$ and $P_{k+1|k}$ can be calculated as in Theorem 9.

Proof. Please see Appendix B. □

3.3. Recursive Smoother

Theorem 12. *For the addressed system* (6) *and* (7)*, one has the following robust recursive L-step* ($L > 0$) *fixed-lag smoother:*

$$
\begin{aligned}
\widehat{x}_{k|k+L} &= \widehat{x}_{k|k+L-1} + \digamma_{k,k+L} \Pi_{k+L}^{-1} \varepsilon_{k+L}, \\
\digamma_{k,k+L} &= \Psi_{k+L} \overline{C}_{k+L}^T - \digamma_{k,k+L-1} \Pi_{k+L-1}^{-1} \\
& \quad \times \overline{D}_{k+L-1} R_{k+L,k+L-1}^T \overline{D}_{k+L}^T, \\
\Psi_{k+1} &= P_{k|k-1} A_k^T - \digamma_{k,k} \Pi_k^{-1} \digamma_{k,k}^T A_k^T + B_{k-1} Q_{k-1,k} B_k^T \\
& \quad - \digamma_{k,k} \Pi_k^{-1} \overline{C}_k B_{k-1} Q_{k,k-1}^T B_k^T, \\
\Psi_{k+L} &= \Psi_{k+L-1} A_{k+L-1}^T - \digamma_{k,k+L-1} \Pi_{k+L-1}^{-1} \digamma_{k+L-1,k+L-1} \\
& \quad - \digamma_{k,k+L-1} \Pi_{k+L-1}^{-1} \overline{C}_{k+L-1} B_{k+L-2} \\
& \quad \times Q_{k+L-1,k+L-2}^T B_{k+L-1}^T, \quad (L > 1), \\
P_{k|k+L} &= P_{k|k+L-1} - \digamma_{k,k+L} \Pi_{k+L}^{-1} \digamma_{k,k+L}^T,
\end{aligned}
\tag{21}
$$

where the initial values $\widehat{x}_{k|k}$, $P_{k|k}$, and $\digamma_{k,k}$ are supplied by Theorem 9.

Proof. Please see Appendix C. □

4. An Illustrative Example

Consider the following uncertain system with different delay rates sensor network and autocorrelated process noises:

$$
\check{x}_{k+1} = \left(\begin{bmatrix} 0.95 & 0.1 \\ 0 & 0.95 \end{bmatrix} + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \mu_k \right) \check{x}_k + \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix} \omega_k, \tag{22}
$$

$$
\omega_k = \zeta_k + \zeta_{k-1}, \tag{23}
$$

$$
\check{y}_k^i = \left(\check{C}_k^i + \check{C}_{k,s}^i \eta_k^i \right) \check{x}_k + \check{v}_k^i, \quad i = 1, 2, \tag{24}
$$

$$
y_k^i = \left(1 - \lambda_k^i \right) \check{y}_k^i + \lambda_k^i \check{y}_{k-1}^i, \quad i = 1, 2, \tag{25}
$$

where $\check{x}_k \in \mathbb{R}^2$ is the state to be estimated. The vectors $\zeta_k \in \mathbb{R}$, $\mu_k \in \mathbb{R}$, $\eta_k^i \in \mathbb{R}$, and $v_k^i \in \mathbb{R}$, $i = 1, 2$ are zero-mean Gaussian white noises with covariances 0.5, 1, 1, and 1, respectively.

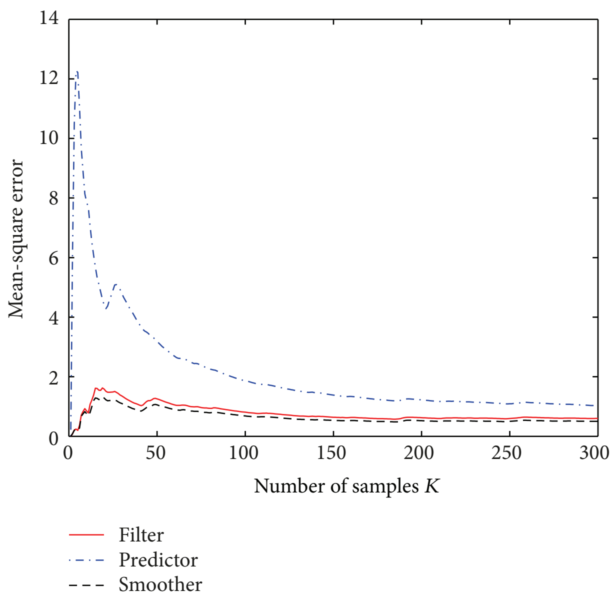

Figure 1: MSE1 filter, predictor, and smoother.

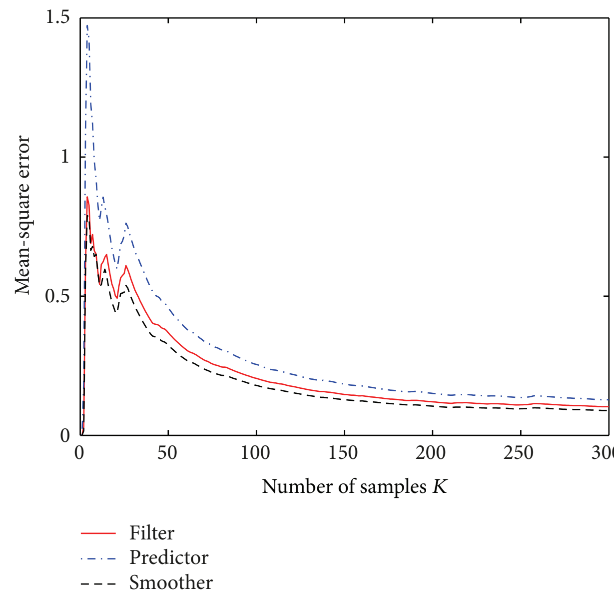

Figure 2: MSE2 filter, predictor, and smoother.

Without loss of generality, the process noise ω_k is chosen to be as defined in (23).

In the simulation, the initial value $\breve{x}_0$ has mean $\mathscr{E}\{\breve{x}_0\} = \bar{\breve{x}}_0^T = [100\ \ 10]^T$ and covariance $\breve{P}_0 = \mathrm{diag}(20, 1)$. The variables $\lambda_k^i \in \mathbb{R}, i = 1, 2$ are binary switching sequences taking values on 1 with $\mathrm{Prob}\{\lambda_k^1 = 1\} = \mathscr{E}\{\lambda_k^1\} = \beta_k^1 = 0.15$ and $\mathrm{Prob}\{\lambda_k^2 = 1\} = \mathscr{E}\{\lambda_k^2\} = \beta_k^2 = 0.25$, respectively, and the matrices are set as $\breve{C}_k^1 = [0\ \ 1]$, $\breve{C}_k^2 = [1\ \ 0]$, $\breve{C}_{k,s}^1 = [0\ \ 0.1]$, and $\breve{C}_{k,s}^2 = [0.1\ \ 0]$. The newly obtained recursive robust estimators and the filter of Zeng et al. [31] are compared in the simulation. Let MSE1 denotes the mean-square error for estimation of the first component of $\breve{x}_k$; that is, $(1/K)\sum_{k=1}^{K}\{[1\ \ 0](\breve{x}_k - \hat{\breve{x}}_{k|k})\}$, where K is the number of the samples. Similarly, MSE2 denotes the mean-square error for estimation of the second component of $\breve{x}_k$; that is, $(1/K)\sum_{k=1}^{K}\{[0\ \ 1](\breve{x}_k - \hat{\breve{x}}_{k|k})\}$.

From Figures 1 and 2, we can see that the smoother has the best performance and the predictor has the worst performance. This is due to the fact that smoother uses the most measurement information and the predictor uses the least measurement information.

From Figures 3 and 4, we can see that the filter developed in this work has better performance than the filter of Zeng et al. [31]. This is due to the fact that the autocorrelated measurement noise V_k was treated as zero-mean Gaussian white noise in the filter of Zeng et al. [31].

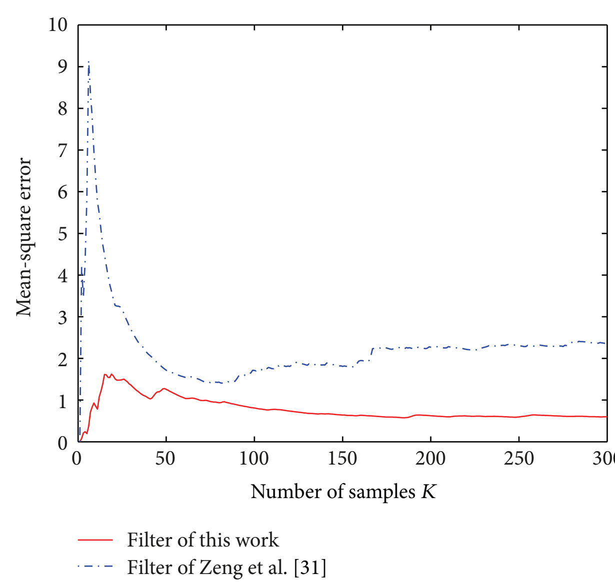

Figure 3: MSE1 filter of this work and Zeng et al. [31].

5. Conclusions

In this paper, we have studied the recursive robust estimation problem for a class of uncertain systems with autocorrelated process noises and different delay rates sensor network. The system model and measurement model are both subject to stochastic uncertainties. The process noises are one-step autocorrelated across time. Each sensor in the sensor network has a different delay rate and the delay rate has been described by an individual binary switching sequence obeying a conditional probability distributed. Based on an IAA and the OPT, recursive robust estimators including filter, predictor, and smoother have been obtained. Simulation results have indicated that the smoother has the best performance and the predictor has the worst performance, and the filter obtained in this work has better performance than the filter of Zeng et al. [31].

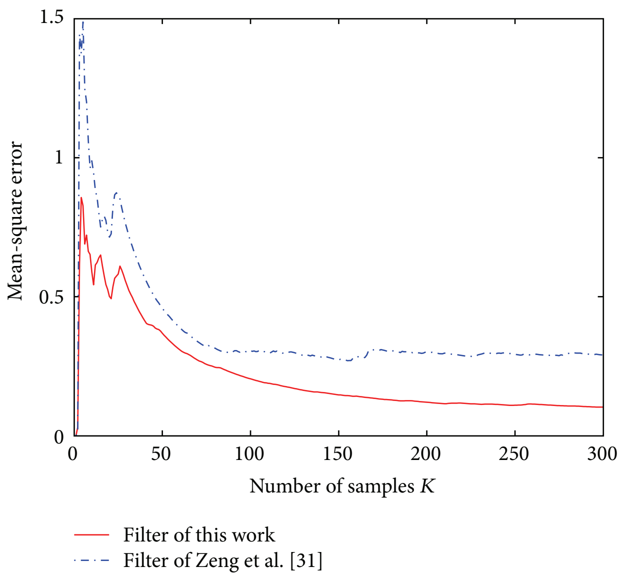

FIGURE 4: MSE2 filter of this work and Zeng et al. [31].

Appendices

A. The Proof of Theorem 9

Proof. Using the OPT, the one-step measurement prediction $\widehat{y}_{k|k-1}$ can be calculated as follows:

$$\begin{aligned}\widehat{y}_{k|k-1} &= \sum_{i=1}^{k-1} \mathscr{E}\left\{y_k \varepsilon_i^T\right\} \Pi_i^{-1} \varepsilon_i \\ &= \sum_{i=1}^{k-1} \mathscr{E}\left\{C_k x_k \varepsilon_i^T\right\} \Pi_i^{-1} \varepsilon_i + \sum_{i=1}^{k-1} \mathscr{E}\left\{D_k V_k \varepsilon_i^T\right\} \Pi_i^{-1} \varepsilon_i \\ &= \overline{C}_k \widehat{x}_{k|k-1} + \sum_{i=1}^{k-1} \mathscr{E}\left\{D_k V_k \varepsilon_i^T\right\} \Pi_i^{-1} \varepsilon_i.\end{aligned} \quad (A.1)$$

Taking into account the fact that V_k is one-step autocorrelated, we have from (4), (8), and (9) the following:

$$\begin{aligned}\mathscr{E}\left\{D_k V_k \varepsilon_i^T\right\} &= 0, \quad i \le k-2, \\ \mathscr{E}\left\{D_k V_k \varepsilon_{k-1}^T\right\} &= \mathscr{E}\left\{D_k V_k \left(y_{k-1} - \widehat{y}_{k-1|k-2}\right)^T\right\} \\ &= \mathscr{E}\left\{D_k V_k V_{k-1}^T D_{k-1}^T\right\} \\ &= \overline{D}_k R_{k,k-1} \overline{D}_{k-1}^T.\end{aligned} \quad (A.2)$$

Substituting (A.2) into (A.1), we have

$$\widehat{y}_{k|k-1} = \overline{C}_k \widehat{x}_{k|k-1} + \overline{D}_k R_{k,k-1} \overline{D}_{k-1}^T \Pi_{k-1}^{-1} \varepsilon_{k-1}. \quad (A.3)$$

Therefore, the innovation ε_k can be calculated as follows:

$$\begin{aligned}\varepsilon_k &= y_k - \widehat{y}_{k|k-1} \\ &= y_k - \overline{C}_k \widehat{x}_{k|k-1} - \overline{D}_k R_{k,k-1} \overline{D}_{k-1}^T \Pi_{k-1}^{-1} \varepsilon_{k-1} \\ &= \left(\overline{C}_k + \widetilde{C}_k\right) x_k + C_{s,k} x_k + D_k V_k - \overline{C}_k \widehat{x}_{k|k-1} \\ &\quad - \overline{D}_k R_{k,k-1} \overline{D}_{k-1}^T \Pi_{k-1}^{-1} \varepsilon_{k-1} \\ &= \overline{C}_k \widetilde{x}_{k|k-1} + \widetilde{C}_k x_k + C_{s,k} x_k + D_k V_k \\ &\quad - \overline{D}_k R_{k,k-1} \overline{D}_{k-1}^T \Pi_{k-1}^{-1} \varepsilon_{k-1},\end{aligned} \quad (A.4)$$

where $\widetilde{x}_{k|k-1} = x_k - \widehat{x}_{k|k-1}$.

Again, according to the OPT, the state prediction $\widehat{x}_{k|k-1}$ can be obtained as follows:

$$\begin{aligned}\widehat{x}_{k|k-1} &= \sum_{i=1}^{k-1} \mathscr{E}\left\{x_k \varepsilon_i^T\right\} \Pi_i^{-1} \varepsilon_i, \\ &= \sum_{i=1}^{k-1} \mathscr{E}\left\{\left(A_{k-1} x_{k-1} + A_{s,k-1} \mu_{k-1} x_{k-1}\right.\right. \\ &\qquad \left.\left. + B_{k-1} \omega_{k-1}\right) \varepsilon_i^T\right\} \Pi_i^{-1} \varepsilon_i \\ &= A_{k-1} \widehat{x}_{k-1|k-1} + B_{k-1} \sum_{i=1}^{k-1} \mathscr{E}\left\{\omega_{k-1} \varepsilon_i^T\right\} \Pi_i^{-1} \varepsilon_i.\end{aligned} \quad (A.5)$$

Taking (3) into consideration, the expectation $\mathscr{E}\{\omega_{k-1}\varepsilon_i^T\}$ can be calculated as follows:

$$\begin{aligned}\mathscr{E}\left\{\omega_{k-1} \varepsilon_i^T\right\} &= 0, \quad i \le k-2, \\ \mathscr{E}\left\{\omega_{k-1} \varepsilon_{k-1}^T\right\} &= \mathscr{E}\left\{\omega_{k-1} \left(y_{k-1} - \widehat{y}_{k-1|k-2}\right)^T\right\} \\ &= \mathscr{E}\left\{\omega_{k-1} x_{k-1}^T C_{k-1}^T\right\}, \\ &= \mathscr{E}\left\{\omega_{k-1} \left(A_{k-2} x_{k-2} + A_{s,k-2} \mu_{k-2} x_{k-2}\right.\right. \\ &\qquad \left.\left. + B_{k-2} \omega_{k-2}\right)^T\right\} \overline{C}_{k-1}^T \\ &= Q_{k-1,k-2} B_{k-2}^T \overline{C}_{k-1}^T.\end{aligned} \quad (A.6)$$

Substituting (A.6) into (A.5), we have

$$\begin{aligned}\widehat{x}_{k|k-1} &= A_{k-1} \widehat{x}_{k-1|k-1} + B_{k-1} Q_{k-1,k-2} \\ &\quad \times B_{k-2}^T \overline{C}_{k-1}^T \Pi_{k-1}^{-1} \varepsilon_{k-1}.\end{aligned} \quad (A.7)$$

Therefore, the one-step prediction error $\widetilde{x}_{k|k-1}$ can be calculated as follows:

$$\begin{aligned}\widetilde{x}_{k|k-1} &= x_k - \widehat{x}_{k|k-1} \\ &= \left(A_{k-1} + A_{s,k-1}\mu_{k-1}\right) x_{k-1} \\ &\quad + B_{k-1}\omega_{k-1} - A_{k-1}\widehat{x}_{k-1|k-1} \\ &\quad - B_{k-1}Q_{k-1,k-2}B_{k-2}^{T}\overline{C}_{k-1}^{T}\Pi_{k-1}^{-1}\varepsilon_{k-1}, \\ &= A_{k-1}\widetilde{x}_{k-1|k-1} + A_{s,k-1}\mu_{k-1}x_{k-1} \\ &\quad + B_{k-1}\omega_{k-1} - B_{k-1}Q_{k-1,k-2}B_{k-2}^{T}\overline{C}_{k-1}^{T}\Pi_{k-1}^{-1}\varepsilon_{k-1},\end{aligned} \tag{A.8}$$

where $\widetilde{x}_{k-1|k-1}$ is the filter error at time instant $k-1$. Taking into account the fact that ω_k is one-step autocorrelated across time and μ_k is uncorrelated with other signals, the one-step prediction error covariance $P_{k|k-1}$ can be calculated as follows:

$$\begin{aligned}P_{k|k-1} &= \mathscr{E}\left\{\widetilde{x}_{k|k-1}\widetilde{x}_{k|k-1}^{T}\right\} \\ &= A_{k-1}P_{k-1|k-1}A_{k-1}^{T} + A_{k-1}\mathscr{E}\left\{\widetilde{x}_{k-1|k-1}\omega_{k-1}^{T}\right\}B_{k-1}^{T} \\ &\quad + A_{s,k-1}X_{k-1}A_{s,k-1}^{T} + B_{k-1}\mathscr{E}\left\{\omega_{k-1}\widetilde{x}_{k-1|k-1}^{T}\right\}A_{k-1}^{T} \\ &\quad + B_{k-1}Q_{k-1}B_{k-1}^{T} - B_{k-1}\mathscr{E}\left\{\omega_{k-1}\varepsilon_{k-1}^{T}\right\}\Pi_{k-1}^{-1}\overline{C}_{k-1} \\ &\quad \times B_{k-2}Q_{k-1|k-2}^{T}B_{k-1}^{T} \\ &\quad - B_{k-1}Q_{k-1,k-2}B_{k-2}^{T}\overline{C}_{k-1}^{T}\Pi_{k-1}^{-1}\mathscr{E}\left\{\varepsilon_{k-1}\omega_{k-1}^{T}\right\}B_{k-1}^{T} \\ &\quad + B_{k-1}Q_{k-1,k-2}B_{k-2}^{T}\overline{C}_{k-1}^{T}\Pi_{k-1}^{-1}\overline{C}_{k-1} \\ &\quad \times B_{k-2}Q_{k-1|k-2}^{T}B_{k-1}^{T},\end{aligned} \tag{A.9}$$

where the expectation $\mathscr{E}\{\omega_{k-1}\varepsilon_{k-1}^{T}\}$ can be calculated as in (A.6) and expectation $\mathscr{E}\{\widetilde{x}_{k-1|k-1}\omega_{k-1}^{T}\}$ can be obtained as follows:

$$\begin{aligned}\mathscr{E}\left\{\widetilde{x}_{k-1|k-1}\omega_{k-1}^{T}\right\} &= \mathscr{E}\left\{x_{k-1}\omega_{k-1}^{T}\right\} - \mathscr{E}\left\{\widehat{x}_{k-1|k-1}\omega_{k-1}^{T}\right\} \\ &= B_{k-2}\mathscr{E}\left\{\omega_{k-2}\omega_{k-1}^{T}\right\} \\ &\quad - \mathscr{E}\left\{\left(\sum_{i=1}^{k-1}\mathscr{E}\left\{x_{k-1}\varepsilon_{i}^{T}\right\}\Pi_{i}^{-1}\varepsilon_{i}\right)\omega_{k-1}^{T}\right\} \\ &= B_{k-2}Q_{k-2,k-1} - \mathscr{E}\left\{x_{k-1}\varepsilon_{k-1}^{T}\right\}\Pi_{k-1}^{-1} \\ &\quad \times \mathscr{E}\left\{\varepsilon_{k-1}\omega_{k-1}^{T}\right\} \\ &= B_{k-2}Q_{k-2,k-1} - F_{k-1,k-1}\Pi_{k-1}^{-1}\overline{C}_{k-1} \\ &\quad \times B_{k-2}Q_{k-2,k-1},\end{aligned} \tag{A.10}$$

where the third equality in (A.10) holds since ω_k is one-step autocorrelated across time. Substituting (A.10) into (A.9) yields (15).

Noting the fact that $\widetilde{x}_{k|k-1}$ is orthogonal to $\widehat{x}_{k|k-1}$, we have from (9) and (A.4) the following:

$$\begin{aligned}F_{k,k} &= \mathscr{E}\left\{x_k\varepsilon_k^{T}\right\} \\ &= \mathscr{E}\left\{x_k\left(\overline{C}_k\widetilde{x}_{k|k-1} + \widetilde{C}_kx_k + C_{s,k}x_k + D_kV_k\right.\right. \\ &\qquad \left.\left. -\overline{D}_kR_{k,k-1}\overline{D}_{k-1}^{T}\Pi_{k-1}^{-1}\varepsilon_{k-1}\right)^{T}\right\} \\ &= P_{k|k-1}\overline{C}_k^{T} - \mathscr{E}\left\{x_k\varepsilon_{k-1}^{T}\right\}\Pi_{k-1}^{-1}\overline{D}_{k-1}R_{k,k-1}\overline{D}_k^{T} \\ &= P_{k|k-1}\overline{C}_k^{T} - \left(A_{k-1}\mathscr{E}\left\{x_{k-1}\varepsilon_{k-1}^{T}\right\} + B_{k-1}\mathscr{E}\left\{\omega_{k-1}\varepsilon_{k-1}^{T}\right\}\right) \\ &\quad \times \Pi_{k-1}^{-1}\overline{D}_{k-1}R_{k,k-1}\overline{D}_k^{T} \\ &= P_{k|k-1}\overline{C}_k^{T} - \left(A_{k-1}F_{k-1,k-1} + B_{k-1}Q_{k-1,k-2}\right. \\ &\qquad \left.\times B_{k-2}^{T}\overline{C}_{k-1}^{T}\right)\Pi_{k-1}^{-1}\overline{D}_{k-1}R_{k,k-1}\overline{D}_k^{T}.\end{aligned} \tag{A.11}$$

It implies from (9), (A.4), and Lemmas 5 and 8 that the expectation Π_k can be obtained as follows:

$$\begin{aligned}\Pi_k &= \mathscr{E}\left\{\varepsilon_k\varepsilon_k^{T}\right\} \\ &= \mathscr{E}\left\{\left(\overline{C}_k\widetilde{x}_{k|k-1} + \widetilde{C}_kx_k + C_{s,k}x_k + D_kV_k\right.\right. \\ &\qquad \left. -\overline{D}_kR_{k,k-1}\overline{D}_{k-1}^{T}\Pi_{k-1}^{-1}\varepsilon_{k-1}\right) \\ &\qquad \times\left(\overline{C}_k\widetilde{x}_{k|k-1} + \widetilde{C}_kx_k + C_{s,k}x_k + D_kV_k\right. \\ &\qquad \left.\left. -\overline{D}_kR_{k,k-1}\overline{D}_{k-1}^{T}\Pi_{k-1}^{-1}\varepsilon_{k-1}\right)^{T}\right\} \\ &= \overline{C}_kP_{k|k-1}\overline{C}_k^{T} + \overline{C}_k\mathscr{E}\left\{\widetilde{x}_{k|k-1}V_k^{T}\right\}\overline{D}_k^{T} \\ &\quad + \mathscr{E}\left\{\widetilde{C}_kx_kx_k^{T}\widetilde{C}_k^{T}\right\} + \mathscr{E}\left\{C_{s,k}x_kx_k^{T}C_{s,k}^{T}\right\} \\ &\quad + \overline{D}_k\mathscr{E}\left\{V_k\widetilde{x}_{k|k-1}^{T}\right\}\overline{C}_k^{T} + \mathscr{E}\left\{D_kV_kV_k^{T}D_k^{T}\right\} \\ &\quad - \mathscr{E}\left\{D_kV_k\varepsilon_{k-1}^{T}\right\}\Pi_{k-1}^{-1}\overline{D}_{k-1}R_{k,k-1}^{T}\overline{D}_k^{T} \\ &\quad - \overline{D}_kR_{k,k-1}\overline{D}_{k-1}^{T}\Pi_{k-1}^{-1}\mathscr{E}\left\{\varepsilon_{k-1}V_k^{T}D_k^{T}\right\} \\ &\quad + \overline{D}_kR_{k,k-1}\overline{D}_{k-1}^{T}\Pi_{k-1}^{-1}\overline{D}_{k-1}R_{k,k-1}^{T}\overline{D}_k^{T},\end{aligned} \tag{A.12}$$

where the remaining expectations can be obtained as follows:

$$\begin{aligned}&\mathscr{E}\left\{\widetilde{x}_{k|k-1}V_k^{T}\right\} \\ &\quad = \mathscr{E}\left\{x_kV_k^{T}\right\} - \mathscr{E}\left\{\widehat{x}_{k|k-1}V_k^{T}\right\} \\ &\quad = 0 - \mathscr{E}\left\{\left(\sum_{i=1}^{k-1}\mathscr{E}\left\{x_k\varepsilon_i^{T}\right\}\Pi_i^{-1}\varepsilon_i\right)V_k^{T}\right\}\end{aligned}$$

$$
\begin{aligned}
&= -\mathscr{E}\left\{x_k\varepsilon_{k-1}^T\right\}\Pi_i^{-1}\mathscr{E}\left\{\varepsilon_{k-1}V_k^T\right\} \\
&= -\left(A_{k-1}\mathscr{E}\left\{x_{k-1}\varepsilon_{k-1}^T\right\}\right. \\
&\qquad \left. + B_{k-1}\mathscr{E}\left\{\omega_{k-1}\varepsilon_{k-1}^T\right\}\right) \\
&\quad \times \Pi_{k-1}^{-1}\overline{D}_{k-1}R_{k-1,k} \\
&= -\left(A_{k-1}F_{k-1,k-1}\right. \\
&\qquad \left. + B_{k-1}Q_{k-1,k-2}B_{k-2}^T\overline{C}_{k-1}^T\right) \\
&\quad \times \Pi_{k-1}^{-1}\overline{D}_{k-1}R_{k-1,k},
\end{aligned}
$$

$$
\begin{aligned}
&\mathscr{E}\left\{\widetilde{C}_k x_k x_k^T \widetilde{C}_k^T\right\} \\
&\quad = \mathscr{E}\left\{\widetilde{J}_k C_{e,k} x_k x_k^T C_{e,k}^T \widetilde{J}_k^T\right\} \\
&\quad = \Sigma_k \otimes \left(C_{e,k} X_k C_{e,k}^T\right),
\end{aligned}
$$

$$
\begin{aligned}
&\mathscr{E}\left\{C_{s,k} x_k x_k^T C_{s,k}^T\right\} \\
&\quad = \mathscr{E}\left\{\left[(I-J_k)\check{C}_{s,k}\eta_k \quad J_k\check{C}_{s,k-1}\eta_{k-1}\right]\right. \\
&\qquad \times \begin{bmatrix} \check{x}_k\check{x}_k^T & \check{x}_k\check{x}_{k-1}^T \\ \check{x}_{k-1}\check{x}_k^T & \check{x}_{k-1}\check{x}_{k-1}^T \end{bmatrix} \\
&\qquad \left. \times\left[(I-J_k)\check{C}_{s,k}\eta_k \quad J_k\check{C}_{s,k-1}\eta_{k-1}\right]^T\right\} \\
&\quad = \mathscr{E}\left\{(I-J_k)\check{C}_{s,k}\mathscr{E}\left\{\eta_k\check{x}_k\check{x}_k^T\eta_k^T\right\}\right. \\
&\qquad \left. \times C_{s,k}^T(I-J_k)^T\right\} \\
&\qquad + \mathscr{E}\left\{(I-J_k)\check{C}_{s,k}\mathscr{E}\left\{\eta_k\right\}\check{X}_{k,k-1}\right. \\
&\qquad\quad \left. \times \mathscr{E}\left\{\eta_{k-1}^T\right\}\check{C}_{s,k-1}^T J_k^T\right\} \\
&\qquad + \mathscr{E}\left\{J_k\check{C}_{s,k-1}\mathscr{E}\left\{\eta_{k-1}\right\}\check{X}_{k-1,k}\mathscr{E}\{\eta_k\}^T\right. \\
&\qquad\quad \left. \times \check{C}_{s,k}^T(1-J_k)^T\right\} \\
&\qquad + \mathscr{E}\left\{J_k\check{C}_{s,k-1}\mathscr{E}\left\{\eta_{k-1}\check{x}_{k-1}\check{x}_{k-1}^T\eta_{k-1}^T\right\}\right. \\
&\qquad\quad \left. \times \check{C}_{s,k-1}^T J_k^T\right\} \\
&\quad = \mathscr{E}\left\{(I-J_k)\check{C}_{s,k}\check{X}_k C_{s,k}^T(I-J_k)^T\right\} \\
&\qquad + \mathscr{E}\left\{J_k\check{C}_{s,k-1}\check{X}_{k-1}\check{C}_{s,k-1}^T J_k^T\right\} \\
&\quad = \mathscr{E}\left\{\left[\left(I-\overline{J}_k\right)-\widetilde{J}_k\right]\check{C}_{s,k}\check{X}_k C_{s,k}^T\right. \\
&\qquad \left. \times\left[\left(I-\overline{J}_k\right)-\widetilde{J}_k\right]^T\right\} \\
&\qquad + \mathscr{E}\left\{\left(\overline{J}_k+\widetilde{J}_k\right)\check{C}_{s,k-1}\check{X}_{k-1}\right. \\
&\qquad\quad \left. \times \check{C}_{s,k-1}^T\left(\widetilde{J}_k+\overline{J}_k\right)^T\right\} \\
&\quad = \left(I-\overline{J}_k\right)\check{C}_{s,k}\check{X}_k C_{s,k}^T\left(I-\overline{J}_k\right)^T \\
&\qquad + \Sigma_k\otimes\left(\check{C}_{s,k}\check{X}_k C_{s,k}^T\right) \\
&\qquad + \overline{J}_k\check{C}_{s,k-1}\check{X}_{k-1}\check{C}_{s,k-1}^T\overline{J}_k^T \\
&\qquad + \Sigma_k\otimes\left(\check{C}_{s,k-1}\check{X}_{k-1}C_{s,k-1}^T\right),
\end{aligned}
$$

$$
\begin{aligned}
&\mathscr{E}\left\{D_k V_k V_k^T D_k^T\right\} \\
&\quad = \mathscr{E}\left\{\left(\overline{D}_k+\widetilde{D}_k\right)V_k V_k^T\left(\overline{D}_k+\widetilde{D}_k\right)^T\right\} \\
&\quad = \mathscr{E}\left\{\overline{D}_k V_k V_k^T\overline{D}_k^T\right\} \\
&\qquad + \mathscr{E}\left\{\widetilde{J}_k D_{e,k}V_k V_k^T D_{e,k}^T\widetilde{J}_k^T\right\} \\
&\quad = \overline{D}_k R_k\overline{D}_k^T + \Sigma_k\otimes\left(D_{e,k}R_k D_{e,k}^T\right),
\end{aligned} \tag{A.13}
$$

where Lemmas 5–8 have been used. Substituting (A.13) into (A.12) yields to (18).

Again, by using the OPT, the state estimation $\widehat{x}_{k|k}$ can be calculated as follows:

$$
\begin{aligned}
\widehat{x}_{k|k} &= \sum_{i=1}^{k}\mathscr{E}\left\{x_k\varepsilon_i^T\right\}\Pi_i^{-1}\varepsilon_i \\
&= \sum_{i=1}^{k-1}\mathscr{E}\left\{x_k\varepsilon_i^T\right\}\Pi_i^{-1}\varepsilon_i + \mathscr{E}\left\{x_k\varepsilon_k^T\right\}\Pi_k^{-1}\varepsilon_k \\
&= \widehat{x}_{k|k-1} + F_{k,k}\Pi_k^{-1}\varepsilon_k.
\end{aligned} \tag{A.14}
$$

Therefore, the estimation error $\widetilde{x}_{k|k}$ can be obtained as follows:

$$
\begin{aligned}
\widetilde{x}_{k|k} &= x_k - \widehat{x}_{k|k} \\
&= x_k - \widehat{x}_{k|k-1} - F_{k,k}\Pi_k^{-1}\varepsilon_k \\
&= \widetilde{x}_{k|k-1} - F_{k,k}\Pi_k^{-1}\varepsilon_k.
\end{aligned} \tag{A.15}
$$

From (A.15), the estimation error covariance $P_{k|k}$ can be calculated as follows:

$$
\begin{aligned}
P_{k|k} &= \mathscr{E}\left\{\widetilde{x}_{k|k}\widetilde{x}_{k|k}^T\right\} \\
&= P_{k|k-1} - \mathscr{E}\left\{\widetilde{x}_{k|k-1}\varepsilon_k^T\right\}\Pi_k^{-1}F_{k,k}^T \\
&\quad - F_{k,k}\Pi_k^{-1}\mathscr{E}\left\{\varepsilon_k\widetilde{x}_{k|k-1}^T\right\} + F_{k,k}\Pi_k^{-1}F_{k,k}^T,
\end{aligned} \tag{A.16}
$$

where the remaining expectation $\mathscr{E}\{\tilde{x}_{k|k-1}\varepsilon_k^T\}$ can be calculated as follows:

$$\begin{aligned}\mathscr{E}\left\{\tilde{x}_{k|k-1}\varepsilon_k^T\right\} &= \mathscr{E}\left\{x_k\varepsilon_k^T\right\} - \mathscr{E}\left\{\hat{x}_{k|k-1}\varepsilon_k^T\right\} \\ &= F_{k,k} - \mathscr{E}\left\{\sum_{i=1}^{k-1}\mathscr{E}\left\{x_k\varepsilon_i^T\right\}\Pi_i^{-1}\varepsilon_i\varepsilon_k^T\right\} \\ &= F_{k,k}.\end{aligned} \tag{A.17}$$

Substituting (A.17) into (A.16), we have

$$P_{k|k} = P_{k|k-1} - F_{k,k}\Pi_k^{-1}F_{k,k}^T, \tag{A.18}$$

which completes the proof of Theorem 9. □

B. The Proof of Theorem 11

Proof. Taking into account the fact that the process noise ω_k is one-step autocorrelated across time, the L-step prediction $\hat{x}_{k+L|k}$ can be calculated as follows:

$$\begin{aligned}\hat{x}_{k+L|k} &= \sum_{i=1}^{k}\mathscr{E}\left\{x_{k+L}\varepsilon_i^T\right\}\Pi_i^{-1}\varepsilon_i \\ &= \sum_{i=1}^{k}\mathscr{E}\left\{\left[\left(A_{k+L-1} + A_{s,k+L-1}\mu_{k+L-1}\right)x_{k+L-1}\right.\right. \\ &\qquad \left.\left.+B_{k+L-1}\omega_{k+L-1}\right]\varepsilon_i^T\right\}\Pi_i^{-1}\varepsilon_i \\ &= A_{k+L-1}\hat{x}_{k+L-1|k}.\end{aligned} \tag{B.1}$$

Therefore, the L-step prediction error $\tilde{x}_{k+L|k}$ can be obtained as follows:

$$\begin{aligned}\tilde{x}_{k+L|k} &= x_{k+L} - \hat{x}_{k+L|k} \\ &= \left(A_{k+L-1} + A_{s,k+L-1}\mu_{k+L-1}\right)x_{k+L-1} \\ &\quad + B_{k+L-1}\omega_{k+L-1} - A_{k+L-1}\hat{x}_{k+L-1|k} \\ &= A_{k+L-1}\tilde{x}_{k+L-1|k} + A_{s,k+L-1}\mu_{k+L-1}x_{k+L-1} \\ &\quad + B_{k+L-1}\omega_{k+L-1}.\end{aligned} \tag{B.2}$$

Thus, the L-step prediction error covariance $P_{k+L-1|k}$ can be calculated as follows:

$$\begin{aligned}P_{k+L|k} &= \mathscr{E}\left\{\tilde{x}_{k+L|k}\tilde{x}_{k+L|k}^T\right\} \\ &= A_{k+L-1}P_{k+L-1|k}A_{k+L-1}^T + A_{k+L-1} \\ &\quad \times \mathscr{E}\left\{\tilde{x}_{k+L-1|k}\omega_{k+L-1}^T\right\}B_{k+L-1}^T + A_{s,k+L-1} \\ &\quad \times X_{k+L-1}A_{s,k+L-1}^T + B_{k+L-1}\mathscr{E}\left\{\omega_{k+L-1}\tilde{x}_{k+L-1|k}^T\right\} \\ &\quad \times A_{k+L-1}^T + B_{k+L-1}Q_{k+L-1}B_{k+L-1}^T \\ &= A_{k+L-1}P_{k+L-1|k}A_{k+L-1}^T \\ &\quad + A_{k+L-1}B_{k+L-2}Q_{k+L-2,k+L-1}B_{k+L-1}^T + A_{s,k+L-1} \\ &\quad \times X_{k+L-1}A_{s,k+L-1}^T + B_{k+L-1}Q_{k+L-1,k+L-2} \\ &\quad \times B_{k+L-2}^T A_{k+L-1}^T + B_{k+L-1}Q_{k+L-1}B_{k+L-1}^T,\end{aligned} \tag{B.3}$$

which completes the proof of Theorem 11. □

C. The Proof of Theorem 12

Proof. According to the OPT, the L-step fixed-lag smoother $\hat{x}_{k|k+L}$ can be calculated as follows:

$$\begin{aligned}\hat{x}_{k|k+L} &= \sum_{i=1}^{k+L}\mathscr{E}\left\{x_k\varepsilon_i^T\right\}\Pi_i^{-1}\varepsilon_i \\ &= \sum_{i=1}^{k+L-1}\mathscr{E}\left\{x_k\varepsilon_i^T\right\}\Pi_i^{-1}\varepsilon_i + \mathscr{E}\left\{x_k\varepsilon_{k+L}^T\right\}\Pi_{k+L}^{-1}\varepsilon_{k+L} \\ &= \hat{x}_{k|k+L-1} + F_{k,k+L}\Pi_{k+L}^{-1}\varepsilon_{k+L},\end{aligned} \tag{C.1}$$

where $F_{k,k+L}$ can be calculated as follows:

$$\begin{aligned}F_{k,k+L} &= \mathscr{E}\left\{x_k\varepsilon_{k+L}^T\right\} \\ &= \mathscr{E}\left\{x_k\left(\overline{C}_{k+L}\tilde{x}_{k+L|k+L-1} + \widetilde{C}_{k+L}x_{k+L} + C_{s,k+L}x_{k+L}\right.\right. \\ &\qquad + D_{k+L}V_{k+L} - \overline{D}_{k+L}R_{k+L,k+L-1}\overline{D}_{k+L-1}^T \\ &\qquad \left.\left.\times\Pi_{k+L-1}^{-1}\varepsilon_{k+L-1}\right)^T\right\} \\ &= \mathscr{E}\left\{x_k\left(\overline{C}_{k+L}\tilde{x}_{k+L|k+L-1} - \overline{D}_{k+L}R_{k+L,k+L-1}\right.\right. \\ &\qquad \left.\left.\times\overline{D}_{k+L-1}^T\Pi_{k+L-1}^{-1}\varepsilon_{k+L-1}\right)^T\right\} \\ &= \Psi_{k+L}\overline{C}_{k+L}^T - F_{k,k+L-1}\Pi_{k+L-1}^{-1} \\ &\quad \times\overline{D}_{k+L-1}R_{k+L,k+L-1}^T\overline{D}_{k+L}^T,\end{aligned} \tag{C.2}$$

where the third equality holds since $\widetilde{C}_{k+L}$, $C_{s,k+L}$, and V_{k+L} are zero-mean stochastic matrices and they are all uncorrelated with x_k. From (A.8) the expectation $\Psi_{k+L} = \mathscr{E}\{x_k\tilde{x}_{k+L|k+L-1}^T\}$ can be obtained as follows:

$$\begin{aligned}\Psi_{k+1} &= \mathscr{E}\left\{x_k\tilde{x}_{k+1|k}^T\right\} \\ &= \mathscr{E}\left\{x_k\left(A_k\tilde{x}_{k|k} + A_{s,k}\mu_k x_k + B_k\omega_k\right.\right. \\ &\qquad \left.\left.-B_kQ_{k,k-1}B_{k-1}^T\overline{C}_k^T\Pi_k^{-1}\varepsilon_k\right)^T\right\} \\ &= \mathscr{E}\left\{x_k\tilde{x}_{k|k}^T\right\}A_k^T + \mathscr{E}\left\{x_k\omega_k^T\right\}B_k^T \\ &\quad - \mathscr{E}\left\{x_k\varepsilon_k^T\right\}\Pi_k^{-1}\overline{C}_kB_{k-1}Q_{k,k-1}^TB_k^T\end{aligned}$$

$$
\begin{aligned}
&= \mathscr{E}\left\{x_k\left(\tilde{x}_{k|k-1} - F_{k,k}\Pi_k^{-1}\varepsilon_k\right)^T\right\} A_k^T \\
&\quad + B_{k-1}Q_{k-1,k}B_k^T - F_{k,k}\Pi_k^{-1}\overline{C}_k B_{k-1}Q_{k,k-1}^T B_k^T \\
&= P_{k|k-1}A_k^T - F_{k,k}\Pi_k^{-1}F_{k,k}^T A_k^T + B_{k-1}Q_{k-1,k}B_k^T \\
&\quad - F_{k,k}\Pi_k^{-1}\overline{C}_k B_{k-1}Q_{k,k-1}^T B_k^T.
\end{aligned}
\tag{C.3}
$$

Similarly, when $L \geq 2$, the expectation Ψ_{k+L} can be calculated as follows:

$$
\begin{aligned}
\Psi_{k+L} &= \Psi_{k+L-1}A_{k+L-1}^T - F_{k,k+L-1}\Pi_{k+L-1}^{-1}F_{k+L-1,k+L-1} \\
&\quad - F_{k,k+L-1}\Pi_{k+L-1}^{-1}\overline{C}_{k+L-1}B_{k+L-2} \\
&\quad \times Q_{k+L-1,k+L-2}^T B_{k+L-1}^T.
\end{aligned}
\tag{C.4}
$$

From (6) and (C.1), the smoother error can be obtained as follows:

$$
\begin{aligned}
\tilde{x}_{k|k+L} &= x_k - \hat{x}_{k|k+L-1} - F_{k,k+L}\Pi_{k+L}^{-1}\varepsilon_{k+L} \\
&= \tilde{x}_{k|k+L-1} - F_{k,k+L}\Pi_{k+L}^{-1}\varepsilon_{k+L}.
\end{aligned}
\tag{C.5}
$$

Therefore, the smoother error covariance can be obtained as follows:

$$
\begin{aligned}
P_{k|k+L} &= \mathscr{E}\left\{\tilde{x}_{k|k+L}\tilde{x}_{k|k+L}^T\right\} \\
&= \mathscr{E}\left\{\left(\tilde{x}_{k|k+L-1} - F_{k,k+L}\Pi_{k+L}^{-1}\varepsilon_{k+L}\right)\right. \\
&\qquad \left. \times \left(\tilde{x}_{k|k+L-1} - F_{k,k+L}\Pi_{k+L}^{-1}\varepsilon_{k+L}\right)^T\right\} \\
&= P_{k|k+L-1} - \mathscr{E}\left\{\tilde{x}_{k|k+L-1}\varepsilon_{k+L}^T\right\}\Pi_{k+L}^{-1}F_{k,k+L}^T \\
&\quad - F_{k,k+L}\Pi_{k+L}^{-1}\mathscr{E}\left\{\varepsilon_{k+L}\tilde{x}_{k|k+L-1}^T\right\} \\
&\quad + F_{k,k+L}\Pi_{k+L}^{-1}F_{k,k+L}^T \\
&= P_{k|k+L-1} - F_{k,k+L}\Pi_{k+L}^{-1}F_{k,k+L}^T,
\end{aligned}
\tag{C.6}
$$

which completes the proof of Theorem 12. □

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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