Hindawi Publishing Corporation
International Journal of Mathematics and Mathematical Sciences
Volume 2011, Article ID 853870, 10 pages
doi:10.1155/2011/853870

Research Article

$W\theta g$-Closed and $W\delta g$-Closed in $[0,1]$-Topological Spaces

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Received 8 June 2010; Accepted 14 September 2010

Academic Editor: Ayman Badawi

We investigate various classes of generalized closed fuzzy sets in $[0,1]$-topological spaces, namely, $W\theta g$-closed fuzzy sets and $W\delta g$-closed fuzzy sets. Also, we introduce a new separation axiom $FT^*_{3/4}$ of the $[0,1]$-topological spaces, and we prove that every $FT^*_{3/4}$-space is a $FT_{3/4}$-space. Furthermore, we using the new generalized closed fuzzy sets to construct new types of fuzzy mappings.

1. Introduction

In 1970, Levine [1] introduced the notion of generalized closed sets in topological spaces as a generalization of closed sets. Since then, many concepts related to generalized closed sets were defined and investigated. In 1997, Balasubramanian and Sundaram [2] introduced the concepts of generalized closed sets in fuzzy setting. Also, they studied various generalizations fuzzy continues mappings.

Recently, El-Shafei and Zakari [3–5] introduced new types of generalized closed fuzzy sets in $[0,1]$-topological spaces and studied many of their properties. Also, they studied various generalizations fuzzy continues mappings.

In the present paper, we introduce the concepts of $W\theta g$-closed fuzzy sets and $W\delta g$-closed fuzzy sets and study some of their properties. Also, we introduce the concept of $FT^*_{3/4}$-space. Moreover, we introduce and study the concepts of two new classes of fuzzy mappings, namely, fuzzy $W\theta g$-continuous mappings and fuzzy $W\delta g$-irresolute mappings.

2. Preliminaries

Let X be a set and I the unit interval. A fuzzy set in X is an element of the set of all functions from X into I. The family of all fuzzy sets in X is denoted by I^X. A fuzzy singleton x_t is a fuzzy set in X defined by $x_t(x) = t$, $x_t(y) = 0$ for all $y \neq x$, $t \in (0,1]$. The set of all fuzzy singletons in X is denoted by $S(X)$. For every $x_t \in S(X)$ and $\mu \in I^X$, we define $x_t \in \mu$ if and only if $t \leq \mu(x)$. A fuzzy set μ is called quasicoincident with a fuzzy set ρ, denoted by $\mu q \rho$, if and only if there exists $x \in X$ such that $\mu(x) + \rho(x) > 1$. If μ is not quasicoincident with ρ, then we write $\mu \bar{q} \rho$. By $\mathrm{cl}(\mu)$, $\mathrm{int}(\mu)$, μ^c, $N(x_t, \tau)$, and $N_Q(x_t, \tau)$, we mean the fuzzy closure of μ, the fuzzy interior of μ, the complement of μ, the class of all open neighborhoods of x_t, and the class of all open Q-neighborhoods of x_t, respectively.

Definition 2.1 (see [6, 7]). A fuzzy subset μ of a $[0,1]$-topological space (X, τ) is called

(i) regular open if and only if $\mu = \mathrm{int}(\mathrm{cl}(\mu))$,

(ii) preopen if and only if $\mu \leq \mathrm{int}(\mathrm{cl}(\mu))$.

The complement of a regular open (resp. preopen) fuzzy set is called a regular closed (resp. preclosed).

Definition 2.2 (see [8, 9]). Let (X, τ) be a $[0,1]$-topological space, $x_t \in S(X)$, and $\mu \in I^X$. Then,

(i) the θ-closure of μ, denoted by $\mathrm{cl}_\theta(\mu)$, is defined by

$x_t \in \mathrm{cl}_\theta(\mu)$ if and only if $\mathrm{cl}(\eta) q \mu$ for each $\eta \in N_Q(x_t, \tau)$,

(ii) the δ-closure of μ denoted by $\mathrm{cl}_\delta(\mu)$, is defined by

$x_t \in \mathrm{cl}_\delta(\mu)$ if and only if $\mathrm{int}(\mathrm{cl}(\eta)) \mathrm{q} \mu$ for each $\eta \in N_Q(x_t, \tau)$,

(iii) μ is called θ-closed (resp. δ-closed) if and only if $\mu = \mathrm{cl}_\theta(\mu)$ (resp. $\mu = \mathrm{cl}_\delta(\mu)$).

Definition 2.3 (see [9]). Let (X, τ) be a $[0,1]$-topological space and $\mu \in I^X$. Then,

(i) the family $\gamma = \{\eta_j : j \in J\} \subseteq \tau$ is called an open P-cover of μ if and only if for every $x_t \in \mu$, there exists $j_0 \in J$ such that $x_t \in \eta_{j_0}$,

(ii) μ is called a C-set if and only if every open P-cover of μ has a finite subcover.

Definition 2.4 (see [2–4]). Let (X, τ) be a $[0,1]$-topological space. A fuzzy set $\mu \in I^X$ is called

(i) a generalized closed (g-closed, for short) if and only if $\mathrm{cl}(\mu) \leq \eta$ whenever $\mu \leq \eta$ and η is open fuzzy set,

(ii) a θ-generalized closed (θg-closed, for short) if and only if $\mathrm{cl}_\theta(\mu) \leq \eta$ whenever $\mu \leq \eta$ and η is open fuzzy set,

(iii) a δ-generalized closed (δg-closed, for short) if and only if $\mathrm{cl}_\delta(\mu) \leq \eta$ whenever $\mu \leq \eta$ and η is open fuzzy set.

Definition 2.5 (see [2, 4, 6, 10]). A $[0,1]$-topological space (X, τ) is called

(i) FR_1 if and only if $x_t \bar{q}\, \mathrm{cl}(y_r)$ implies that there exist $\eta \in N(x_t, \tau)$ and $\upsilon \in N(y_r, \tau)$ such that $\eta \bar{q} \upsilon$,

(ii) FR_2 or F-regular if and only if $x_t \bar{q} \lambda$, λ is closed fuzzy set implies that there exist $\eta \in N(x_t, \tau)$ and $\upsilon \in \tau$, $\lambda \leq \upsilon$ such that $\eta \bar{q} \upsilon$,

(iii) $FT_{1/2}$ if and only if every g-closed fuzzy set in X is closed,

(iv) $FT_{3/4}$ if and only if every δg-closed fuzzy set in X is δ-closed,

(v) fuzzy weakly Hausdorff (FWT_2, for short) if $x_t\overline{q}y_r$ implies that there exists regular open fuzzy set $\eta \in N(x_t, \tau)$ such that $y_r\overline{q}\eta$,

(vi) fuzzy semiregular if and only if the collection of all regular open fuzzy sets in X forms a base for the $[0,1]$-topology τ,

(vii) a fuzzy partition space if and only if every open fuzzy subset is closed.

Definition 2.6 (see [2–4, 11]). A fuzzy mapping $f : (X, \tau) \to (Y, \Delta)$ is called

(i) fuzzy generalized continuous (fuzzy g-continuous, for short) if and only if $f^{-1}(\eta)$ is g-closed in X for any closed fuzzy set η in Y,

(ii) fuzzy θ-generalized continuous (fuzzy θg-continuous, for short) if and only if $f^{-1}(\eta)$ is θg-closed in X for any closed fuzzy set η in Y,

(iii) fuzzy δ-generalized continuous (fuzzy δg-continuous, for short) if and only if $f^{-1}(\eta)$ is δg-closed in X for any closed fuzzy set η in Y,

(iv) fuzzy δ-continuous if the inverse image of every δ-open fuzzy set in Y is δ-open in X,

(v) fuzzy δ-open (fuzzy δ-open, for short) if and only if $f(\eta)$ is δ-open in Y for any δ-open fuzzy set η in X,

(vi) fuzzy δ-closed (fuzzy δ-closed, for short) if and only if $f(\eta)$ is δ-closed in Y for any δ-closed fuzzy set η in X.

Theorem 2.7 (see [3]). *A fuzzy subset μ of an FR_2-fts (X, τ) is θg-closed if and only if it is g-closed.*

Theorem 2.8 (see [3]). *Let (X, τ) be a $[0,1]$-topological space. Then, the following conditions are equivalent:*

(i) *(X, τ) is FR_1-space,*

(ii) *for each C-set $\mu \in I^X$, $\mathrm{cl}(\mu) = \mathrm{cl}_\theta(\mu)$,*

(iii) *for each $x_t \in S(X)$, $\mathrm{cl}(x_t) = \mathrm{cl}_\theta(x_t)$.*

Theorem 2.9 (see [3, 4]). *Let (X, τ) be a $[0,1]$-topological space and $\mu \in I^X$ be a preopen. Then, μ is θg-closed (resp. δg-closed) if and only if it is g-closed.*

Theorem 2.10 (see [3]). *Let (X, τ) be a $[0,1]$-topological space and $\mu, \eta \in I^X$. Then,*

(i) $\mathrm{cl}_\theta(\mu \vee \eta) = \mathrm{cl}_\theta(\mu) \vee \mathrm{cl}_\theta(\eta)$,

(ii) $\mathrm{cl}_\theta(\mu \wedge \eta) \leq \mathrm{cl}_\theta(\mu) \wedge \mathrm{cl}_\theta(\eta)$.

Theorem 2.11 (see [4]). *Let (X, τ) be a fuzzy semiregular space and $\mu \in I^X$. Then,*

(i) *μ is δg-closed if and only if μ is g-closed,*

(ii) *If, in addition, (X, τ) is $FT_{1/2}$, then μ is δg-closed if and only if μ is closed.*

Theorem 2.12 (see [4]). *Let (X, τ) be an FR_1-space and $\mu \in I^X$ be a C-set. Then, μ is δg-closed if and only if it is g-closed.*

Theorem 2.13 (see [4]). *Let (X, τ) be a fuzzy partition space and $\mu \in I^X$. Then, μ is δg-closed if and only if it is g-closed.*

Theorem 2.14 (see [4]). *Let (X, τ) be a $[0,1]$-topological space and $\mu, \eta \in I^X$. Then,*

(i) $\mathrm{cl}_\delta(\mu \vee \eta) = \mathrm{cl}_\delta(\mu) \vee \mathrm{cl}_\delta(\eta)$,

(ii) $\mathrm{cl}_\delta(\mu \wedge \eta) \leq \mathrm{cl}_\delta(\mu) \wedge \mathrm{cl}_\delta(\eta)$.

Theorem 2.15 (see [4]). *A $[0,1]$-topological space (X, τ) is $FT_{3/4}$-space if for every $x_t \in S(X)$ either x_t is δ-open or x_t is closed.*

Theorem 2.16 (see [4]). *Let (X, τ) be a $[0,1]$-topological space. Then, the following conditions are equivalent:*

(i) *X is an FWT_2-space,*

(ii) *$x_t = \mathrm{cl}_\delta(x_t)$ for each $x_t \in S(X)$.*

3. $W\theta g$-Closed Fuzzy Sets

In this section, we introduce the concept of weakly θ-generalized closed fuzzy sets, and we study some of their properties.

Definition 3.1. A fuzzy subset μ of a $[0,1]$-topological space (X, τ) is said to be weakly θ-generalized closed ($W\theta g$-closed, for short) if and only if $\mathrm{cl}_\theta(\mu) \leq \eta$ whenever $\mu \leq \eta$ and η is θ-open fuzzy set.

The complement of a $W\theta g$-closed fuzzy set is called $W\theta g$-open.

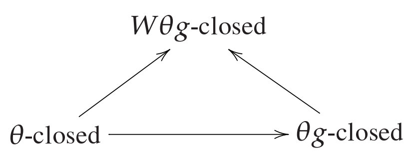
Theorem 3.2. *Let (X, τ) be a $[0,1]$-topological space. Then,*

(i) *Every θ-closed fuzzy set is $W\theta g$-closed,*

(ii) *Every θg-closed fuzzy set is $W\theta g$-closed.*

Proof. Obvious. □

From the above discussion, we introduce the following diagram.



(3.1)

None of these implications is reversible as the following examples show.

Example 3.3. Let $X = \{x, y\}$ and $\tau = \{0_X, y_{0.7}, 1_X\}$. If $\mu = x_{0.5} \vee y_{0.6}$, then μ is $W\theta g$-closed fuzzy set but not θ-closed.

Example 3.4. Let $X = \{x\}$ and $\tau = \{0_X, x_{0.6}, 1_X\}$. If $\mu = x_{0.5}$, then μ is $W\theta g$-closed, since the only θ-open superset of μ is 1_X. But μ is not θg-closed, since $\mu \leq x_{0.6}$ and $\mathrm{cl}_\theta(\mu) = 1_X \nleq x_{0.6}$.

Theorem 3.5. *A fuzzy subset μ of a $[0,1]$-topological space (X,τ) is $W\theta g$-closed if for every $x_t \in S(X)$ such that $x_t q \mathrm{cl}_\theta(\mu)$, one has $\mathrm{cl}_\theta(x_t) q\mu$.*

Proof. Let η be θ-open and $\mu \leq \eta$. If $x_t q \mathrm{cl}_\theta(\mu)$, then by assumption, $\mathrm{cl}_\theta(x_t) q\mu$. Hence, there exists $y \in X$ such that $\mathrm{cl}_\theta(x_t)(y) + \mu(y) > 1$. Put $\mathrm{cl}_\theta(x_t)(y) = \varepsilon$. Then, $y_\varepsilon \in \mathrm{cl}_\theta(x_t)$ and $y_\varepsilon q\mu$. Thus, $\rho q x_t$ for each $\rho \in N_Q(y_\varepsilon, \tau_\theta)$. Since $y_\varepsilon q\eta$, then $\eta q x_t$ and so $\mathrm{cl}_\theta(\mu) \leq \eta$. Thus, μ is $W\theta g$-closed. □

Theorem 3.6. *Let (X,τ) be a $[0,1]$-topological space and $\mu \in I^X$. Then, μ is $W\theta g$-closed if there is not any θ-closed fuzzy set λ such that $\lambda \; \overline{q} \; \mu$ and $\lambda q \mathrm{cl}_\theta(\mu)$.*

Proof. Suppose that μ is not $W\theta g$-closed. Then, there exists θ-open fuzzy set η such that $\mu \leq \eta$ and $\mathrm{cl}_\theta(\mu) \nleq \eta$. Put $\lambda = \eta^c$. Then, there exists θ-closed fuzzy set λ such that $\lambda \; \overline{q} \; \mu$ and $\lambda q \mathrm{cl}_\theta(\mu)$. This is a contradiction. □

Theorem 3.7. *Let (X,τ) be an FR_1-space and $\mu \in I^X$ be a C-set and g-closed. Then, μ is $W\theta g$-closed.*

Proof. Suppose that (X,τ) is an FR_1-space and μ is a C-set in X. If μ is g-closed, then by Theorem 2.8 μ is θg-closed and hence $W\theta g$-closed. □

Theorem 3.8. *Let (X,τ) be a $[0,1]$-topological space and $\mu \in I^X$ be a preopen and g-closed. Then, μ is $W\theta g$-closed.*

Proof. It is an immediate consequence of Theorems 2.9 and 3.2. □

Theorem 3.9. *Let (X,τ) be an FR_2-space and $\mu \in I^X$ be a g-closed. Then, μ is $W\theta g$-closed.*

Proof. It is an immediate consequence of Theorems 2.7 and 3.2. □

Theorem 3.10. *A finite union of $W\theta g$-closed fuzzy sets, is always $W\theta g$-closed fuzzy set.*

Proof. Suppose that $\mu, \eta \in I^X$ are $W\theta g$-closed fuzzy sets and let $\upsilon \in \tau_\theta$ such that $\mu \vee \eta \leq \upsilon$. Since μ and η are $W\theta g$-closed, then we have $\mathrm{cl}_\theta(\mu) \vee \mathrm{cl}_\theta(\eta) \leq \upsilon$ and by Theorem 2.10(i) $\mathrm{cl}_\theta(\mu \vee \eta) \leq \upsilon$. Hence, $\mu \vee \eta$ is $W\theta g$-closed. □

4. $W\delta g$-Closed Fuzzy Sets

In this section, we introduce the concept of weakly δ-generalized closed fuzzy sets, and we study some of their properties. Also, we introduce the notion of $FT^*_{3/4}$-space, and we prove that every $FT^*_{3/4}$-space is a $FT_{3/4}$-space.

Definition 4.1. A fuzzy subset μ of $[0,1]$-topological space (X,τ) is said to be weakly δ-generalized closed ($W\delta g$-closed, for short) if and only if $\mathrm{cl}_\delta(\mu) \leq \eta$ whenever $\mu \leq \eta$ and η is δ-open fuzzy set.

The complement of a $W\delta g$-closed fuzzy set is called $W\delta g$-open.

Theorem 4.2. *Let (X,τ) be a $[0,1]$-topological space. Then,*

(i) *Every δ-closed fuzzy set is $W\delta g$-closed,*

(ii) *Every δg-closed fuzzy set is $W\delta g$-closed.*

Proof. Obvious. □

From the above discussion, we introduce the following diagram.

$$\begin{array}{ccc} & W\delta g\text{-closed} & \\ \nearrow & & \nwarrow \\ \delta\text{-closed} & \longrightarrow & \delta g\text{-closed} \end{array} \tag{4.1}$$

None of these implications is reversible as the following examples show.

Example 4.3. Let $X = \{x\}$ and $\tau = \{0_X, x_{0.8}, 1_X\}$. If $\mu = x_{0.7}$, then μ is $W\delta g$-closed, since the only δ-open superset of μ is 1_X. But μ is not δg-closed, since $\mu \leq x_{0.8}$ and $\mathrm{cl}_\delta(\mu) = 1_X \not\leq x_{0.8}$.

Example 4.4. Let $X = \{x, y\}$ and $\tau = \{0_X, y_{0..8}, 1_X\}$. A fuzzy subset $\mu = x_{0.2} \vee y_{0.3}$ is δg-closed and hence $W\delta g$-closed, but it is not δ-closed.

Theorem 4.5. *A fuzzy subset μ of a $[0,1]$-topological space (X,τ) is $W\delta g$-closed if and only if for every $x_t \in S(X)$ such that $x_t q \mathrm{cl}_\delta(\mu)$ one has $\mathrm{cl}_\delta(x_t) q\mu$.*

Proof. Let $x_t q \mathrm{cl}_\delta(\mu)$ and suppose that $\mathrm{cl}_\delta(x_t)\overline{q}\mu$. Since μ is $W\delta g$-closed, then it is easy to observe that $\mathrm{cl}_\delta(x_\delta)\overline{q}\mathrm{cl}_\delta(\mu)$ which implies that $x_t \overline{q}\mathrm{cl}_\delta(\mu)$. This is a contradiction.

The converse is similar to the proof of Theorem 3.5. □

Theorem 4.6. *Let (X,τ) be a $[0,1]$-topological space and $\mu \in I^X$. Then, μ is $W\delta g$-closed if and only if there is not any δ-closed fuzzy set λ such that $\lambda\overline{q}\mu$ and $\lambda q \mathrm{cl}_\delta(\mu)$.*

Proof. Suppose that there is a δ-closed fuzzy set λ such that $\lambda\overline{q}\mu$ and $\lambda q\mathrm{cl}_\delta(\mu)$. Then, there exists some $x_t \in \lambda$ such that $x_t q\mathrm{cl}_\delta(\mu)$. Since μ is $W\delta g$-closed, then by using Theorem 4.5, $\mathrm{cl}_\delta(x_t) q\mu$ and hence $\mathrm{cl}_\delta(\lambda) q\mu$. Since λ is δ-closed, then we have $\lambda q \mu$. This is a contradiction.

The converse is similar to the proof of Theorem 3.6. □

Theorem 4.7. *Let (X,τ) be a fuzzy semiregular space and $\mu \in I^X$. Then,*

(i) *μ is $W\delta g$-closed if and only if it is δg-closed,*

(ii) *If, in addition, (X,τ) is $FT_{1/2}$-space, then μ is $W\delta g$-closed if and only if it is closed.*

Proof. (i) Since (X,τ) is semiregular space, then $\tau = \tau_\delta$, and so μ is $W\delta g$-closed if and only if it is δg-closed.

(ii) From (i), Theorem 2.11, and by $FT_{1/2}$-ness, the result is given. □

Theorem 4.8. *Let (X,τ) be an FR_1-space and $\mu \in I^X$ be a C-set and g-closed. Then, μ is $W\delta g$-closed.*

Proof. Suppose that (X,τ) is an FR_1-space and μ is a C-set in X. If μ is g-closed, then by Theorem 2.12 μ is δg-closed and hence $W\delta g$-closed. □

Theorem 4.9. *Let (X,τ) be a $[0,1]$-topological space and $\mu \in I^X$ be a preopen and g-closed. Then, μ is $W\delta g$-closed.*

Proof. It is an immediate consequence of Theorems 2.9 and 4.2. □

Theorem 4.10. *Every fuzzy subset of a fuzzy partition space* (X, τ) *is* $W\delta g$*-closed.*

Proof. Let (X, τ) be a fuzzy partition space, and let μ be a fuzzy subset of X. Then, by Theorem 2.13, μ is δg-closed and hence, by Theorem 4.2, μ is $W\delta g$-closed. □

Theorem 4.11. *A finite union of* $W\delta g$*-closed fuzzy sets is always* $W\delta g$*-closed fuzzy set.*

Proof. Similar to the proof of Theorem 3.10. □

The following example shows that the finite intersection of $W\delta g$-closed fuzzy set may fail to be $W\delta g$-closed fuzzy set.

Example 4.12. Let $X = \{a, b, c, d, e\}$. Define $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 : X \to [0,1]$ as follows:

$$\begin{array}{lllll} \lambda_1(a) = 1, & \lambda_1(b) = 1, & \lambda_1(c) = 0, & \lambda_1(d) = 0, & \lambda_1(e) = 0, \\ \lambda_2(a) = 0, & \lambda_2(b) = 0, & \lambda_2(c) = 1, & \lambda_2(d) = 0, & \lambda_2(e) = 0, \\ \lambda_3(a) = 1, & \lambda_3(b) = 1, & \lambda_3(c) = 1, & \lambda_3(d) = 0, & \lambda_3(e) = 0, \\ \lambda_4(a) = 1, & \lambda_4(b) = 0, & \lambda_4(c) = 1, & \lambda_4(d) = 1, & \lambda_4(e) = 0, \\ \lambda_5(a) = 0, & \lambda_5(b) = 1, & \lambda_5(c) = 1, & \lambda_5(d) = 0, & \lambda_5(e) = 1. \end{array} \tag{4.2}$$

Consider the $[0,1]$-topology $\tau = \{0_X, \lambda_1, \lambda_2, \lambda_3, 1_X\}$. It is clear that λ_4 and λ_5 are $W\delta g$-closed fuzzy sets. But $\lambda_4 \wedge \lambda_5 = \lambda_2$ is not $W\delta g$-closed.

Definition 4.13. A $[0,1]$-topological space (X, τ) is called $FT^*_{3/4}$-space if and only if every $W\delta g$-closed fuzzy set is δ-closed.

Theorem 4.14. *Every* $FT^*_{3/4}$*-space is* $FT_{3/4}$*-space.*

Proof. It is an immediate consequence of Theorem 4.2(ii). □

Theorem 4.15. *A* $[0,1]$*-topological space* (X, τ) *is* $FT^*_{3/4}$*-space if for every* $x_t \in S(X)$ *either* x_t *is* δ*-open or* δ*-closed.*

Proof. Let $\mu \in I^X$ be $W\delta g$-closed, and let $x_t \overline{q} \mu$. We consider the following two cases.

Case 1. x_t is δ-open. Then, x_t^c is δ-closed. Since $x_t \overline{q} \mu$, then $\mu \le x_t^c$. But x_t^c is δ-closed. Then, $\mathrm{cl}_\delta(\mu) \le x_t^c$. This shows that $x_t \overline{q} \mathrm{cl}_\delta(\mu)$.

Case 2. x_t is δ-closed. Then, x_t^c is δ-open. Since $x_t \overline{q} \mu$, then $\mu \le x_t^c$. But μ is $W\delta g$-closed. Then, $\mathrm{cl}_\delta(\mu) \le x_t^c$ and hence $x_t \overline{q} \mathrm{cl}_\delta(\mu)$. □

Corollary 4.16. *Every* FWT_2*-space is* $FT^*_{3/4}$*-space.*

Proof. This is an immediate consequence of Theorems 2.16 and 4.15.

The converse of Corollary 4.16 need not be true, in general, and as a sample, we give the following example. □

Example 4.17. Let $X = \{a, b, c\}$. Define $\lambda_1, \lambda_2, \lambda_3 : X \to [0,1]$ as follows:

$$\begin{aligned} &\lambda_1(a) = 1, \quad &\lambda_1(b) = 0, \quad &\lambda_1(c) = 0, \\ &\lambda_2(a) = 0, \quad &\lambda_2(b) = 1, \quad &\lambda_2(c) = 0, \\ &\lambda_3(a) = 1, \quad &\lambda_3(b) = 1, \quad &\lambda_3(c) = 0. \end{aligned} \tag{4.3}$$

Consider the $[0,1]$-topology $\tau = \{0_X, \lambda_1, \lambda_2, \lambda_3, 1_X\}$. Then, X is $FT^*_{3/4}$-space but not FWT_2-space.

Theorem 4.18. *Let (X, τ) be a $[0,1]$-topological space. Then, the following conditions are equivalent:*

(i) *X is FWT_2-space,*

(ii) *X is $FT^*_{3/4}$ and each $x_t \in S(X)$ is $W\delta g$-closed.*

Proof. Obvious. □

5. $W\theta g$-Continuous and $W\delta g$-Continuous Mappings

Definition 5.1. A fuzzy mapping $f : (X, \tau) \to (Y, \Delta)$ is called

(i) fuzzy $W\theta$g-continuous if the inverse image of every closed fuzzy set in Y is $W\theta$g-closed fuzzy set in X,

(ii) fuzzy $W\delta g$-continuous if the inverse image of every closed fuzzy set in Y is $W\delta g$-closed fuzzy set in X.

Theorem 5.2. *Every fuzzy θg-continuous (resp. δg-continuous) mapping is fuzzy $W\theta g$-continuous (resp. $W\delta g$-continuous).*

Proof. Obvious. □

The converse of the above Theorem may not be true, in general, by the following example.

Example 5.3. Let $X = Y = \{x\}$ and consider a $[0,1]$-topology τ of Example 4.3, $\Delta = \{0_Y, x_{0.3}, 1_Y\}$. If $f : (X, \tau) \to (Y, \Delta)$ is the identity fuzzy mapping, then f is fuzzy $W\theta$g-continuous but not fuzzy θg-continuous, since $x_{0.7} \in \Delta'$ and $f^{-1}(x_{0.7}) = x_{0.7} \leq x_{0.8} \in \tau$ but $\mathrm{cl}_\theta(x_{0.7}) = 1_X \not\leq x_{0.8}$. Also, f is fuzzy $W\delta g$-continuous but not fuzzy δg-continuous.

Theorem 5.4. *Let $f : (X, \tau) \to (Y, \Delta)$ be fuzzy mapping and (X, τ) be fuzzy semiregular space. Then, the following conditions are equivalent:*

(i) *f is fuzzy $W\delta g$-continuous,*

(ii) *f is fuzzy δg-continuous,*

(iii) *f is fuzzy g-continuous.*

Proof. It follows directly from Theorems 2.11 and 4.7(i). □

Definition 5.5. A fuzzy mapping $f : (X,\tau) \to (Y,\Delta)$ is called

(i) fuzzy $W\theta$g-irresolute if the inverse image of every $W\theta$g-closed fuzzy set in Y is $W\theta$g-closed fuzzy set in X,

(ii) fuzzy $W\delta g$-irresolute if the inverse image of every $W\delta g$-closed fuzzy set in Y is $W\delta g$-closed fuzzy set in X.

Theorem 5.6. *Let $f : (X,\tau) \to (Y,\Delta)$ and $g : (Y,\Delta) \to (Z,\Omega)$ be two fuzzy mappings. Then,*

(i) *$g \circ f$ is fuzzy $W\theta g$-continuous if g is fuzzy continuous and f is fuzzy $W\theta g$-continuous,*

(ii) *$g \circ f$ is fuzzy $W\theta g$-irresolute if g is fuzzy $W\theta g$-irresolute and f is fuzzy $W\theta g$-irresolute,*

(iii) *$g \circ f$ is fuzzy $W\theta g$-continuous if g is fuzzy $W\theta g$-continuous and f is fuzzy $W\theta g$-irresolute.*

Proof. Obvious. □

Theorem 5.7. *Let $f : (X,\tau) \to (Y,\Delta)$ and $g : (Y,\Delta) \to (Z,\Omega)$ be two fuzzy mappings. Then,*

(i) *$g \circ f$ is fuzzy $W\delta g$-continuous if g is fuzzy continuous and f is fuzzy $W\delta g$-continuous,*

(ii) *$g \circ f$ is fuzzy $W\delta g$-irresolute if g is fuzzy $W\delta g$-irresolute and f is fuzzy $W\delta g$-irresolute,*

(iii) *$g \circ f$ is fuzzy $W\delta g$-continuous if g is fuzzy $W\delta g$-continuous and f is fuzzy $W\delta g$-irresolute,*

(iv) *Let (Y,Δ) be $FT^*_{3/4}$-space. Then, $g \circ f$ is fuzzy $W\delta g$-continuous if g is fuzzy $W\delta g$-continuous and f is fuzzy $W\delta g$-continuous,*

(v) *Let (Y,Δ) be a fuzzy semiregular space. Then, $g \circ f$ is fuzzy $W\delta g$-continuous if g is fuzzy g-continuous and f is fuzzy $W\delta g$-irresolute.*

Proof. Obvious. □

Definition 5.8. A fuzzy mapping $f : (X,\tau) \to (Y,\Delta)$ is called fuzzy θ-open if and only if $f(\eta)$ is θ-open in Y for any θ-open fuzzy set η in X.

Theorem 5.9. *If a fuzzy mapping $f : (X,\tau) \to (Y,\Delta)$ is bijective, fuzzy θ-open, and fuzzy $W\theta g$-continuous, then f is fuzzy $W\theta g$-irresolute.*

Proof. Let λ be $W\theta$g-closed fuzzy set in Y, and let $f^{-1}(\lambda) \le \eta$, where $\eta \in \tau_\theta$. Clearly, $\lambda \le f(\eta)$. Since $f(\eta) \in \Delta_\theta$ and λ is $W\theta$g-closed in Y, then $\mathrm{cl}_\theta(\lambda) \le f(\eta)$ and thus $f^{-1}(\mathrm{cl}_\theta(\lambda)) \le \eta$. Since f is fuzzy $W\theta$g-continuous and $\mathrm{cl}_\theta(\lambda)$ is closed in Y, then $f^{-1}(\mathrm{cl}_\theta(\lambda))$ is $W\theta$g-closed in X and hence $\mathrm{cl}_\theta(f^{-1}(\mathrm{cl}_\theta(\lambda))) \le \eta$. Thus, $\mathrm{cl}_\theta(f^{-1}(\lambda)) \le \eta$ and so $f^{-1}(\lambda)$ is $W\theta$g-closed in X. This shows that f is fuzzy $W\theta$g-irresolute. □

Theorem 5.10. *If a fuzzy mapping $f : (X,\tau) \to (Y,\Delta)$ is bijective, fuzzy δ-open, and fuzzy $W\delta g$-continuous, then f is fuzzy $W\delta g$-irresolute.*

Proof. Similar to the proof of Theorem 5.9. □

Theorem 5.11. *If a fuzzy mapping $f : (X,\tau) \to (Y,\Delta)$ is fuzzy $W\delta g$-irresolute and (X,τ) is $FT^*_{3/4}$-space, then f is fuzzy δ-continuous.*

Proof. Let μ be a δ-closed fuzzy set in Y. By using Theorem 4.2, μ is $W\delta g$-closed. Since f is fuzzy $W\delta g$-irresolute, then $f^{-1}(\mu)$ is $W\delta g$-closed in X. Since X is $FT^*_{3/4}$-space, then $f^{-1}(\mu)$ is δ-closed in X. Thus, f is fuzzy δ-continuous. □

Theorem 5.12. *If a mapping $f : (X, \tau) \to (Y, \Delta)$ is fuzzy δ-continuous and fuzzy δ-closed, and μ is $W\delta g$-closed fuzzy set in X, then $f(\mu)n$ is $W\delta g$-closed in X.*

Proof. Let μ be $W\delta g$-closed in X, and let $f(\mu) \leq \eta$, where η is δ-open fuzzy set in Y. Since $\mu \leq f^{-1}(\eta)$, μ is $W\delta g$-closed fuzzy set in X and since $f^{-1}(\eta)$ is δ-open in X, then $\mathrm{cl}_\delta(\mu) \leq f^{-1}(\eta)$. Thus $f(\mathrm{cl}_\delta(\mu)\,) \leq \eta$. Hence, $\mathrm{cl}_\delta(f(\mu)) \leq \mathrm{cl}_\delta(f(\mathrm{cl}_\delta(\mu))) = f(\mathrm{cl}_\delta(\mu)) \leq \eta$, since f is fuzzy δ-closed. Hence, $f(\mu)$ is $W\delta g$-closed in Y. □

Theorem 5.13. *Let (X, τ) be an $FT^*_{3/4}$-space. If a fuzzy mapping $f : (X, \tau) \to (Y, \Delta)$ be surjective, fuzzy $W\delta g$-irresolute, and fuzzy δ-closed, then (Y, Δ) is also $FT^*_{3/4}$-space.*

Proof. Let μ be $W\delta g$-closed fuzzy set in Y. Since f is fuzzy $W\delta g$-irresolute, then $f^{-1}(\mu)$ is $W\delta g$-closed in X. Since X is $FT^*_{3/4}$-space, then $f^{-1}(\mu)$ is δ-closed in X. Thus, μ is δ-closed in Y, since f is surjective and fuzzy δ-closed. Hence, (Y, Δ) is $FT^*_{3/4}$-space. □

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