Research Article

Reducing Transmitted Vibration Using Delayed Hysteretic Suspension

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Previous numerical and experimental works show that time delay technique is efficient to reduce transmissibility of vibration in a single pneumatic chamber by controlling the pressure in the chamber. The present work develops an analytical study to demonstrate the effectiveness of such a technique in reducing transmitted vibrations. A quarter-car model is considered and delayed hysteretic suspension is introduced in the system. Analytical predictions based on perturbation analysis show that a delayed hysteretic suspension enhances vibration isolation comparing to the case where the nonlinear damping is delay-independent.

1. Introduction

Various strategies for reducing transmitted vibrations have been proposed; see, for instance, [1–5] and references therein. A standard technique using linear viscous damping in the vibration isolation device reduces the transmissibility near the resonance but increases it elsewhere. To enhance vibration isolation in the whole frequency range, cubic nonlinear viscous damping has been successfully introduced [3]. It was shown, in the case of a single degree of freedom (sdof) spring damper system, that cubic nonlinear damping can effectively produce an ideal vibration isolation in the whole frequency range [4]. Recently, a strategy based on adding a nonlinear parametric time-dependent viscous damping to the basic cubic nonlinear damper has been proposed [5]. This method significantly enhances vibration isolation comparing to the case where the nonlinear damping is time-independent [3, 4]. Specifically, it was reported that increasing the amplitude of the parametric damping enhances substantially the vibration isolation over the whole frequency range.

On the other hand, time delay control (TDC) [6, 7] was applied to a pneumatic isolator to enhance the isolation performance by controlling the pressure in chamber in the low frequency range [8]. Effectiveness of this active control technique in enhancement of transmissibility performance was demonstrated using simulation as well as experiments testing in the case of a single pneumatic chamber.

In the present paper, we use a delayed hysteretic suspension in the system and we examine its influence on vibration isolation. In this TDC technique, the values of all the state variables and their first derivatives have to be provided by some means.

To achieve our analysis, we implement the multiple scales method [9] on the equation of motion to derive the corresponding modulation equations and we examine the steady state solutions of this modulation equation to obtain indications on transmissibility (TR) versus the system parameters.

2. Equation of Motion

A representative model of a suspension system with nonlinear stiffness, nonlinear viscous damping, and delayed hysteretic suspension under external excitation is proposed.
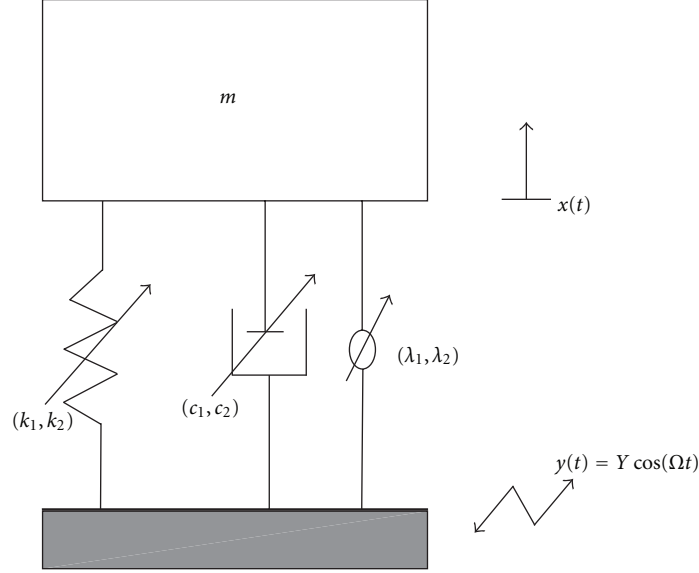
FIGURE 1: Schematic diagram of pneumatic vibration isolator under ground excitation.

It consists of a sdof model, as shown in Figure 1, described by

$$\ddot{X} + \omega^2 X + B_1 X^3 + (B_2 \dot{X} + B_3 \dot{X}^3)$$
$$= -g + Y\Omega^2 \cos\Omega t + \lambda_1 \dot{X}(t - \tau) \qquad (1)$$
$$+ \lambda_2 (\dot{X}(t - \tau))^3,$$

where $\omega^2 = (k_1/m)$, $B_1 = (k_2/m)$, $B_2 = (c_1/m)$, and $B_3 = (c_2/m)$. In the case of a single pneumatic chamber, as shown in Figure 1, m is the body mass, k_1 and k_2 are the linear and the nonlinear stiffness of vehicle suspension, c_1 and c_2 are the linear and the nonlinear equivalent viscous damping, g is the acceleration gravity, and X is the relative vertical displacement of the mass. The parameters Y and Ω denote, respectively, the amplitude and the frequency of the external excitation, while λ_1 and λ_2 denote the gains of the linear- and the nonlinear-delayed viscous damping, and τ is the time delay. This delayed viscous damping can be practically implemented using delayed nonlinear dampers based on magnetorheological fluid.

It is worthy to notice that the particular case of linear stiffness ($B_1 = 0$) and undelayed state feedback ($\lambda_1 = \lambda_2 = 0$) was studied in [3], while the case of $B_1 \neq 0$, $\lambda_1 = \lambda_2 = 0$, and time-dependent (parametric) damping was treated in [5]. It was demonstrated that adding nonlinear parametric damping to the basic nonlinear damping enhances significantly the vibration isolation [5]. The purpose of the present work is to study the effect of delayed nonlinear damping on vibration isolation of system (1). Note also that a similar delayed nonlinear system to (1) was investigated near primary resonances [10]. Attention was focused on performing an approach to analyze the dynamic of the system with arbitrarily large gains. Note also that a delayed feedback was used to quench undesirable vibrations in a van der Pol type system [11].

3. Frequency Response and Transmissibility

To obtain the frequency-response equation and TR, we perform a perturbation method. Introducing a bookkeeping parameter ϵ and scaling $Y = \epsilon \tilde{Y}$, $B_1 = \epsilon \tilde{B}_1$, $B_2 = \epsilon \tilde{B}_2$, $B_3 = \epsilon \tilde{B}_3$, $\lambda_1 = \epsilon \tilde{\lambda}_1$, and $\lambda_2 = \epsilon \tilde{\lambda}_2$, (1) reads

$$\ddot{z} + \omega^2 z = -g + \epsilon \Big[\tilde{Y}\Omega^2 \cos\Omega t - \tilde{B}_1 z^3 - \tilde{B}_2 \dot{z} - \tilde{B}_3 \dot{z}^3$$
$$+ \tilde{\lambda}_1 \dot{z}(t - \tau) + \tilde{\lambda}_2 (\dot{z}(t - \tau))^3 \Big]. \qquad (2)$$

Using the multiple scales technique [9], a two-scale expansion of the solution is sought in the form

$$X(t) = z_0(T_0, T_1) + \epsilon z_1(T_0, T_1) + O(\epsilon^2), \qquad (3)$$

where $T_i = \epsilon^i t$. In terms of the variables T_i, the time derivatives become $d/dt = D_0 + \epsilon D_1 + O(\epsilon^2)$ and $d^2/dt^2 = D_0^2 + 2\epsilon D_0 D_1 + O(\epsilon^2)$, where $D_i^j = (\partial^j/\partial^j T_i)$. Substituting (3) into (2), we obtain

$$(D_0^2 + 2\epsilon D_0 D_1)(z_0 + \epsilon z_1) + \omega^2(z_0 + \epsilon z_1)$$
$$= -g + \epsilon \Big[\tilde{Y}\Omega^2 \cos(\Omega t) - \tilde{B}_2 (D_0 + \epsilon D_1)(z_0 + \epsilon z_1)$$
$$- \tilde{B}_1 (z_0 + \epsilon z_1)^3 - \tilde{B}_3 ((D_0 + \epsilon D_1)(z_0 + \epsilon z_1))^3$$
$$+ \tilde{\lambda}_1 (D_0 + \epsilon D_1)(z_0(t - \tau) + \epsilon z_1(t - \tau))$$
$$+ \tilde{\lambda}_2 ((D_0 + \epsilon D_1)(z_0(t - \tau) + \epsilon z_1(t - \tau)))^3 \Big], \qquad (4)$$

and equating coefficients of the same power of ϵ, we obtain at different orders

$$D_0^2 z_0 + \omega^2 z_0 = -g,$$

$$D_0^2 z_1 + \omega^2 z_1 + 2D_1 D_0 z_0$$

$$= \tilde{Y}\Omega^2 \cos(\Omega t) \tag{5}$$

$$- \tilde{B}_1 z_0^3 - \tilde{B}_2 D_0 z_0 - \tilde{B}_3 (D_0 z_0)^3$$

$$+ \tilde{\lambda}_1 D_0 z_0 (t - \tau) + \tilde{\lambda}_2 (D_0 z_0 (t - \tau))^3.$$

In the case of the principal resonance, that is, $\Omega = \omega + \epsilon\sigma$, where σ is a detuning parameter, standard calculations yield the first-order solution:

$$z(t) = -\frac{g}{\omega^2} + a\cos(\Omega t - \gamma) + O(\epsilon), \tag{6}$$

where the amplitude a and the phase γ are given by the modulation equations:

$$\dot{a} = \frac{\tilde{Y}\Omega^2}{2\omega}\sin(\gamma) - s_1 a - s_2 a^3,$$

$$a\dot{\gamma} = \frac{\tilde{Y}\Omega^2}{2\omega}\cos(\gamma) - s_3 a - s_4 a^3. \tag{7}$$

Here $s_1 = (\tilde{B}_2/2) - (\tilde{\lambda}_1 \cos(\omega\tau)/2)$, $s_2 = (3\tilde{B}_3\omega^2/8) - (3\tilde{\lambda}_2\omega^2\cos(\omega\tau)/8)$, $s_3 = (3\tilde{B}_1/2)(g^2/\omega^5) - \sigma - (\tilde{\lambda}_1\sin(\omega\tau)/2)$, and $s_4 = (3\tilde{B}_1/8\omega) - (3\tilde{\lambda}_2\omega^2\sin(\omega\tau)/8)$. Periodic solutions of (2) corresponding to stationary regimes ($\dot{a} = \dot{\gamma} = 0$) of the modulation equations (7) are given by the algebraic equation:

$$(s_2^2 + s_4^2)a^6 + (2s_1 s_2 + 2s_3 s_4)a^4 + (s_1^2 + s_3^2)a^2 - \left(\frac{Y\Omega^2}{2\omega}\right)^2 = 0. \tag{8}$$

On the other hand, the relationship between displacement transmissibility and the system parameters is defined by

$$\text{TR} = \frac{X}{Y} = \sqrt{\left(1 + \frac{a}{Y}\cos(\gamma)\right)^2 + \left(\frac{a}{Y}\right)^2 \sin^2(\gamma)}. \tag{9}$$

4. Influence of Delayed Damping

The model we consider consists in quarter-car model with softening spring in which $m = 240\,\text{kg}$, $k_1 = 160000\,\text{N/m}$, $k_2 = -30000\,\text{N/m}^3$, $c_1 = 250\,\text{N}\cdot\text{s/m}$, and $c_2 = 25\,\text{N}\cdot\text{s}^3/\text{m}^3$. The amplitude of the excitation frequency is fixed as $Y = 0.11$.

Figure 2 illustrates the relative amplitude of motion a versus the frequency Ω, as given by (8) (solid line), and for validation we plot the result obtained by numerical integration using a Rung-Kutta method (circles).

In Figure 3(a) is shown the TR versus $r = \Omega/\omega$ for various feedback gain λ_1 and for $\lambda_2 = 0$. It can be seen in this figure that as λ_1 increases from 0.01 to 15, the TR reduces. The effect of the feedback gain λ_2 (with $\lambda_1 = 0$) on the TR is also
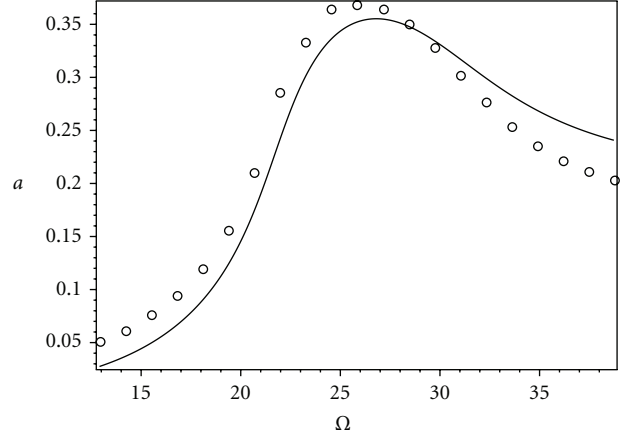

FIGURE 2: Amplitude a versus Ω, for $\lambda_1 = 0.01$, $\lambda_2 = 0.01$, and $\tau = 0.1$. Analytical prediction: solid line; numerical simulation: circles.

illustrated in Figure 3(b) showing also a decrease of TR as λ_2 increases from 0.01 to 4. The plots in the figures indicate that to ensure a significant decrease of TR, a small increase in the nonlinear gain λ_2 is sufficient while a larger value of the linear gain λ_1 is necessary to have an equivalent effect.

Figure 4 shows the variation of TR with respect to time delay τ for different values of the gains λ_1 and λ_2. These plots indicate that increasing the gains causes TR to reduce in repeated periodic intervals of τ. Also, a small increase of λ_2 ($\lambda_2 = 0.5$) produces this reduction (Figure 4(b)), while a larger value of λ_1 ($\lambda_1 = 10$) should be introduced to obtain a comparable effect (Figure 4(a)). To validate the analytical predictions (solid lines), we show in Figures 5 and 6 comparisons with the numerical simulations (circles).

5. Conclusions

In this work, a strategy based on adding hysteretic nonlinear suspension with time delay to control transmitted vibration is presented. The analytical prediction, based on perturbation method, shows clearly that increasing the amplitude gains of the delayed damping reduces transmitted vibrations to a support structure isolation. This analytical prediction confirms previous numerical and experimental works obtained in the case of a single pneumatic chamber [8]. The results revealed that the case where only the nonlinear gain is acting improves greatly vibration isolation comparing to the case where only the linear gain is applied. Vibration isolation enhancement can be obtained for a small increase of the nonlinear gain λ_2, while a larger increase of the linear gain λ_1 is required to obtain a comparable effect. It is also shown that for small values of the nonlinear gain, vibration isolation can be reduced in repeated periodic intervals of time delay, whereas larger values of the linear gain is needed to obtain a similar result. The method performed in this work provides an approximate expression of transmissibility relating the parameters of the system, thereby showing the explicit dependence between transmissibility and control parameters which is important from monitoring view point.
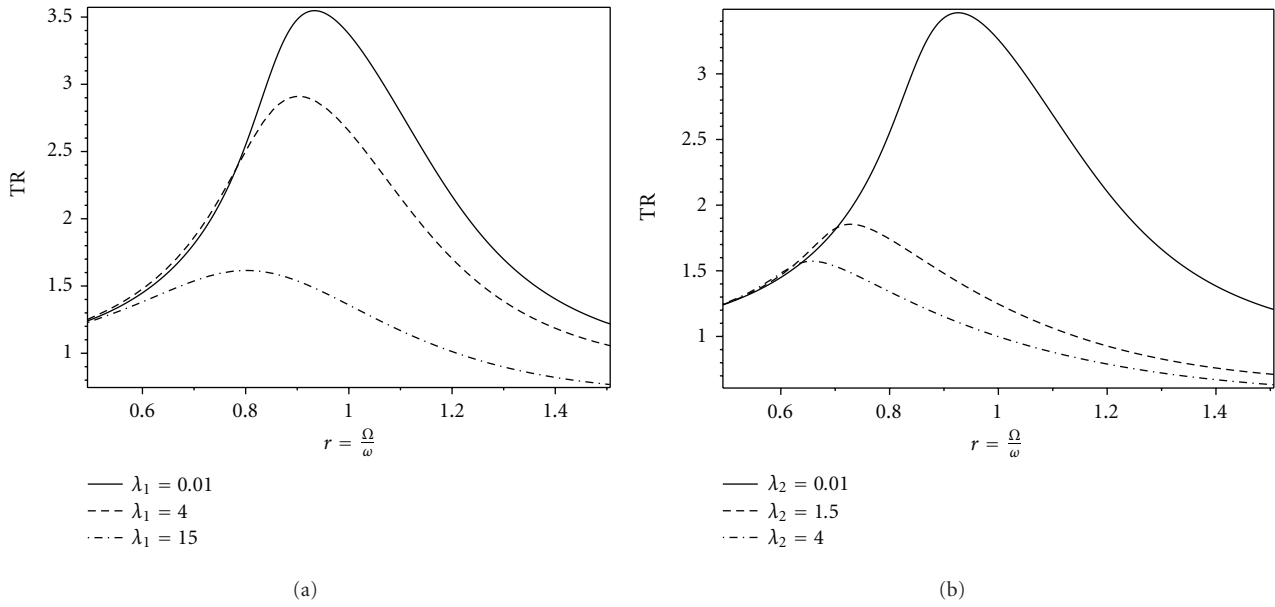
(a)

(b)

Figure 3: Transmissibility versus r for $\tau = 0.1$. (a) $\lambda_2 = 0$, and (b) $\lambda_1 = 0$.
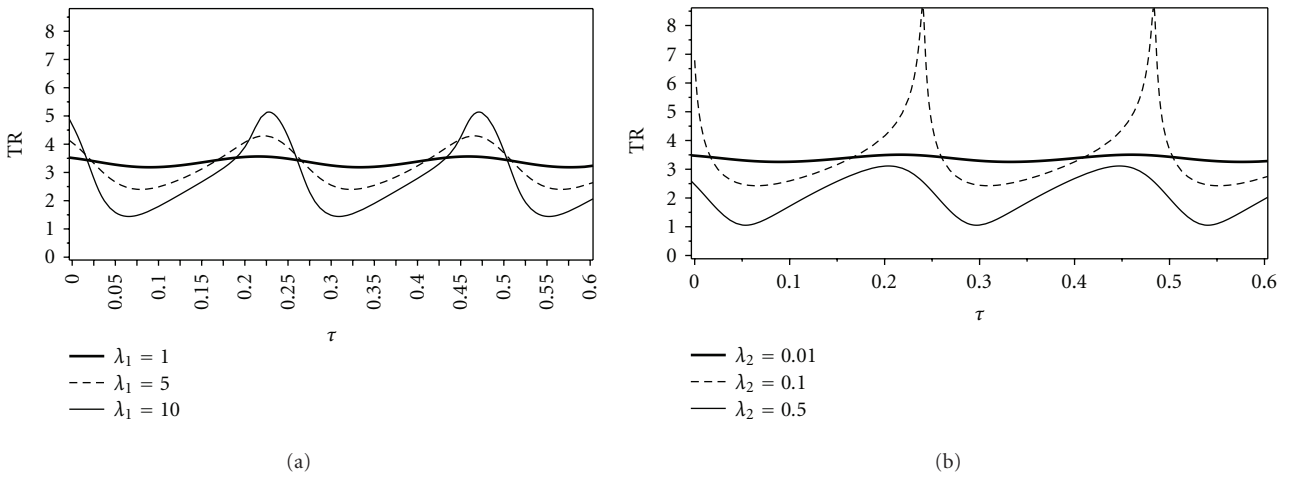

(a)

(b)

Figure 4: Transmissibility versus τ for $r = 1$. (a) $\lambda_2 = 0$, and (b) $\lambda_1 = 0$.
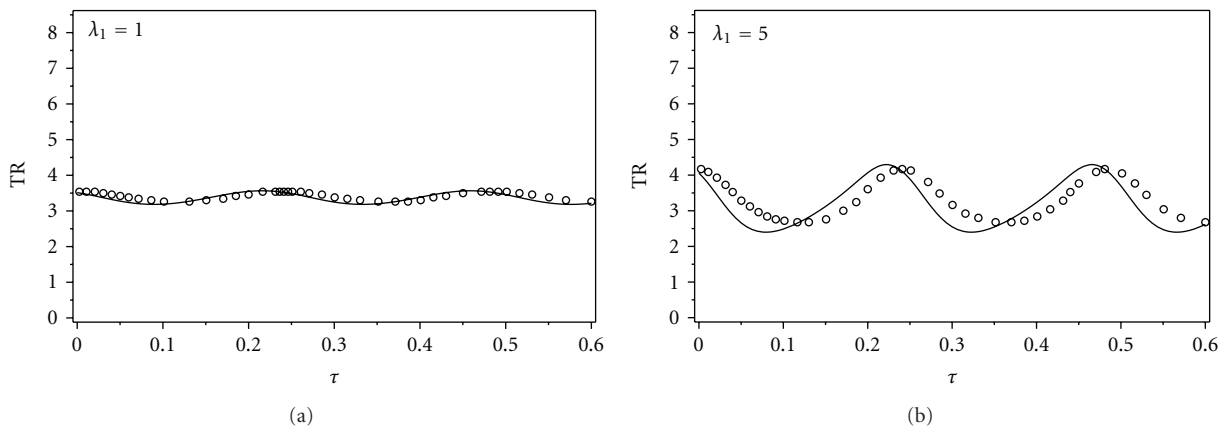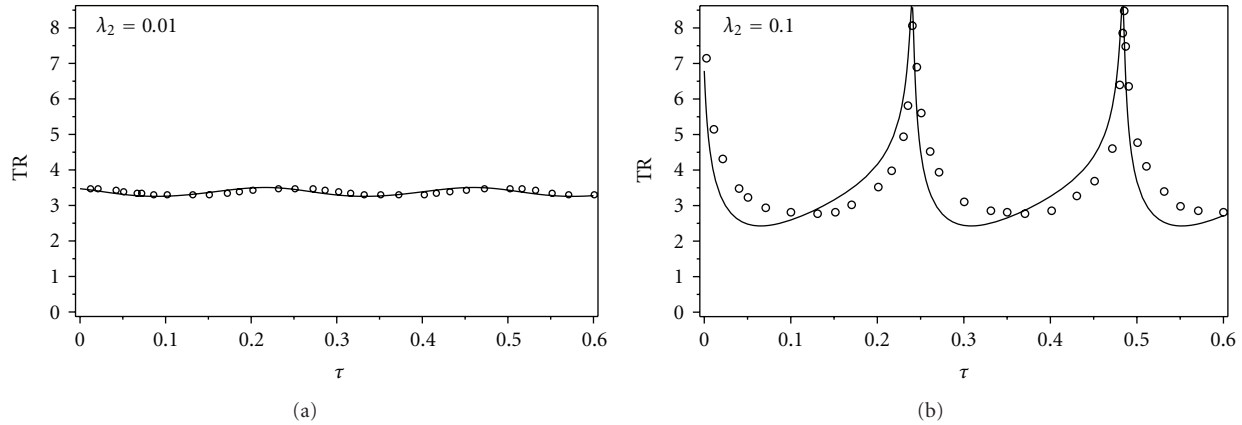

(a)

(b)

Figure 5: Transmissibility versus τ for $r = 1$ and $\lambda_2 = 0$.

FIGURE 6: Transmissibility versus τ for $r = 1$ and $\lambda_1 = 0$.

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