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Research Article

On Quasi-ω-Confluent Mappings

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We introduce a new class of mappings called quasi-ω-confluent maps, and we study the relation between these mappings, and some other forms of confluent maps. Moreover, we prove several results about some operations on quasi-ω-confluent mappings such as: composition, factorization, pullbacks, and products.

1. Introduction

A generalization of the notion of the classical open sets which has received much attention lately is the so-called ω-open sets. These sets are characterized as follows [1]: a subset W of a topological space (X, τ) is an ω-open set if and only if for each $x \in W$, there exists $U \in \tau$ such that $x \in U$ and $U - W$ is countable. One can then show that the family of all ω-open subsets of a space (X, τ), denoted by τ_w, forms a topology on X finer than τ. Using this notion of ω-open sets, one can then define notions such as ω-compact and ω-connected sets whose definitions follow closely the definitions of the related classical notions. For example, a space X is called ω-connected provided that X is not the union of two disjoint nonempty ω-open sets. And X is said to be ω-compact if every ω-open cover of X has a finite subcover. For more information regarding these notions and some recent related results, see [2–4].

Recall that a subset K of a space X is said to be a continuum if K is connected and compact. Using this idea of a continuum, Charatonik introduced and studied the idea of a confluent mapping between topological spaces [5] as follow: A mapping $f : X \to Y$ is said to be confluent provided that for each continuum K of Y and for each component C of $f^{-1}(K)$, we have $f(C) = K$.

In [6], motivated by Charatonik's work, we have introduced the notion of ω-confluent mappings and studied its basic properties. In particular, we say a space X is an ω-continuum if it is ω-connected and ω-compact at the same time, and a subset K of a space X is said to be ω-continuum if K is ω-connected and ω-compact as a subspace of X. Moreover, a mapping $f : X \to Y$ is said to be ω-confluent provided that for each ω-continuum K of Y and for each component C of $f^{-1}(K)$, we have $f(C) = K$.

In this paper, we are interested in the further generalizations of the work of Charatonik in the context of ω-open sets and the idea of quasicomponents. Recall that a quasicomponent of space X containing a point $p \in X$ is the intersection of all nonempty clopen sets in X containing p [7]. In particular, we will introduce the notion of quasi-ω-confluent maps and study its relation with the classical mappings such as confluent, ω-confluent, and quasiconfluent maps. We also study operations on such mappings like compositions, pullback of quasi-ω-confluent, factorizations, and products.

2. Quasi-ω-Confluent Mappings

In this section, we introduce and study a new form of ω-confluent mapping, which is a quasi-ω-confluent mapping. Throughout this paper, all mappings are assumed to be continuous.

Now, we introduce the following notion.

Definition 2.1. A mapping $f : X \to Y$ is said to be quasi-ω-confluent (resp., quasiconfluent) if for each ω-continuum (resp., continuum) K in Y and for each quasicomponent QC of $f^{-1}(K)$, we have $f(QC) = K$.

First, we need the following theorem.

Theorem 2.2 (see [6]). *Let X be a topological space. Then,*

(1) *every ω-connected subset K of X is connected,*

(2) *every ω-compact subset K of X is compact,*

(3) *every ω-continuum subset K of X is a continuum.*

Proposition 2.3. *(1) Every ω-confluent mapping is quasi-ω-confluent.*
(2) Every quasiconfluent mapping is quasi-ω-confluent.

Proof. (1) Suppose that $f : X \to Y$ be an ω-confluent mapping, let K be any ω-continuum in Y, and let x be any point in $f^{-1}(K)$ and QC_x be the quasicomponent of x in $f^{-1}(K)$. Then, any component C_x of x in $f^{-1}(K)$ contained in the quasicomponents QC_x, or $C_x \subset QC_x$. Thus, $f(C_x) \subset f(QC_x)$. Since f is an ω-confluent, then $f(C_x) = K$. This implies, $K \subset f(QC_x)$. But we have $QC_x \subset f^{-1}(K)$. So, $f(QC_x) \subset K$. Thus, $f(QC_x) = K$. Therefore, f is quasi-ω-confluent mapping.

(2) Let K be any ω-continuum in Y and QC be any quasicomponent of $f^{-1}(K)$. Then, K is a continuum in Y by the Theorem 2.2(3). Since, f is quasiconfluent. So that, $f(QC) = K$. Thus, f is quasi-ω-confluent mapping. □

Remark 2.4. It is clear that every ω-confluent (confluent or quasiconfluent) mapping is quasi-ω-confluent, but the converses are not necessarily true, as shown by the following examples.

Example 2.5. Let $K = \{1/n : n \text{ is a positive integer}\}$, $D = K \times [0, 1]$.

(a) Let $X = D \cup \{(0,0),(0,1)\}$ subspaces of $\mathbb{R}^2$ under the usual topology τ_u, and $Y = \{0,1\}$, with the topology $\tau_Y = \{\phi, Y\}$. Let $f : X \to Y$ be the mapping defined by

$$f(x,y) = \begin{cases} 0, & \text{for } (x,y) \in \{(0,0),(0,1)\}, \\ 1, & \text{for } (x,y) \in \{k\} \times [0,1], \text{ for each } k \in K. \end{cases} \tag{2.1}$$

Then, f is quasi-ω-confluent but not quasiconfluent. Since, if we take the continuum $K = \{0,1\}$ in Y, then the quasicomponents of $f^{-1}(K)$ are $\{(0,0),(0,1)\}$ and D. So, $f(\{(0,0),(0,1)\}) \neq K$, and $f(D) \neq K$.

(b) Let $X = D \cup \{(0,0),(0,1)\} \cup ([0,1] \times \{0\})$ subspaces of $\mathbb{R}^2$ under the usual topology τ_u, and $Y = \{0,1\}$, with the topology $\tau_Y = \{\phi, Y\}$. Let $f : X \to Y$ be the mapping defined by

$$f(x,y) = \begin{cases} 0, & \text{for } (x,y) = (0,1), \\ 1, & \text{otherwise.} \end{cases} \tag{2.2}$$

Then, f is quasi-ω-confluent, but not confluent. Since if we take the continuum $K = \{0,1\}$ in Y, then the components of $f^{-1}(K)$ are $\{(0,1)\}$ and $X \setminus \{(0,1)\}$. So, $f(\{(0,1)\}) \neq K$, and $f(X \setminus \{(0,1)\} \neq K$.

Example 2.6. Let $X = \{p,q,r\}$ and $Y = \{a,b,c\}$ with topologies $\tau = \{\phi, X, \{p\}, \{q\}, \{p,q\}\}$ and $\sigma = \{\phi, Y, \{a\}\}$ defined on X and Y, respectively. Let $f : X \to Y$ be a mapping defined by $f(p) = a$, $f(q) = b$, $f(r) = c$. Then, f is quasi-ω- confluent, but it is not confluent.

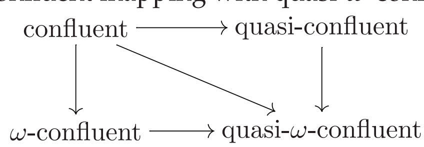
Remark 2.7. Quasi-ω-confluent does not imply ω-confluent in general, since the quasicomponent containing p, $QC(X,p)$ may be different from the component containing p, $C(X,p)$, as the following example shows.

Example 2.8. Let $X = K \times [0,1] \cup \{(0,0),(0,1)\} \cup ([0,1] \times \{0\})$ be a subspaces of $\mathbb{R}^2$ under the usual topology τ_u, where K be as in Example 2.5, and let $Y = [0,1]$ with the topology $\tau_{\text{ind}} = \{\phi, Y, \}$. Let $f : X \to Y$ be the mapping defined by

$$f(x,y) = x, \quad \forall (x,y) \in X. \tag{2.3}$$

Then, f is quasi-ω-confluent, but f is not ω-confluent. Note that if we take the ω-continuum $K = [0,1]$, then the components of $f^{-1}(K)$ are $C_1 = \{(0,1)\}$ and $C_2 = X - \{(0,1)\}$. Thus, $f(C_1) \neq K$ and $f(C_2) = K$.

The following diagram summarizes the relations between confluent mapping, quasi-confluent mapping, and ω-confluent mapping with quasi-ω-confluent mapping.



The following theorem shows that under certain conditions, quasi-ω-confluent mapping will be ω-confluent.

Theorem 2.9. *Every quasi-ω-confluent mapping $f : X \to Y$ of a compact Hausdorff space X into a Hausdorff space Y is ω-confluent.*

Proof. Let K be any ω-continuum in Y and C any component of $f^{-1}(K)$. Then, by the Theorem 2.2, K is continuum subset of Y. Since Y is Hausdorff, then K is closed in Y and since f is continuous, then $f^{-1}(K)$ is closed in X, since X is compact Hausdorff space, so that $f^{-1}(K)$ is compact Hausdorff space. Thus, the quasicomponents are connected and coincide with components of $f^{-1}(K)$. Thus, $f(C) = K$. Therefore, f is ω-confluent. □

Proposition 2.10. *If X is hereditarily locally connected, then any quasi-ω-confluent mapping $f : X \to Y$ is ω-confluent.*

Proof. It follows that from the fact that in locally connected space, the components and quasicomponents are the same. □

Definition 2.11 (see [2]). A space (X, τ) is said to be ω-space if every ω-open set is open in X.

It is easy to see that in an ω-space that the continuum and ω-continuum sets coincide.

Proposition 2.12. *If Y is an ω-space and if $f : X \to Y$ is a mapping of a compact Hausdorff space X into a Hausdorff space Y, then the following are equivalent:*

(1) *f is confluent,*

(2) *f is ω-confluent,*

(3) *f is quasiconfluent,*

(4) *f is quasi-ω-confluent.*

Proof. (1) $\Rightarrow$ (2). Obvious.

(2) $\Rightarrow$ (3). Let f be an ω-confluent mapping, K any continuum in Y, and QC any quasicomponent of $f^{-1}(K)$. Since Y is an ω-space, then K is an ω-continuum, since Y is Hausdorff and X is compact Hausdorff, so that the components and quasicomponents of $f^{-1}(K)$ are the same. Hence, $f(QC) = K$ by assumption. Thus, f is quasiconfluent mapping.

(3) $\Rightarrow$ (4). It follows from Proposition 2.3(2).

(4) $\Rightarrow$ (1). Let f is quasi-ω-confluent mapping, $K \subseteq Y$ any continuum, and C be an arbitrary component of $f^{-1}(K)$, since Y is an ω-space, then K is an ω-continuum in Y, since X is a compact Hausdorff and Y is a Hausdorff. Then, C is a quasicomponent of $f^{-1}(K)$. Thus, $f(C) = K$. Therefore, f is confluent mapping. □

Theorem 2.13. *Let $f : X \to Y$ be a mapping of zero-dimensional space X into space Y. Then, the following are equivalent:*

(1) *f is an ω-confluent,*

(2) *f is quasi-ω-confluent.*

Proof. (1) $\Rightarrow$ (2). Obvious.

(2) $\Rightarrow$ (1). Let f be quasi-ω-confluent mapping, $K \subseteq Y$ any ω-continuum, and C any component of $f^{-1}(K)$. Since X is a *zero*-dimensional space, then it is totally disconnected. Then the components of $f^{-1}(K)$ are coincide with quasicomponents. Thus, C is a quasicomponent of $f^{-1}(K)$. Then, $f(C) = K$, by the assumption. Therefore, f is an ω-confluent. □

Proposition 2.14. *Let $f : X \to Y$ be a mapping of space X into zero-dimensional space Y. Then, the following are equivalent:*

(1) *f is quasiconfluent,*

(2) *f is quasi-ω-confluent.*

Proof. (1) $\Rightarrow$ (2). It follows immediately from the Proposition 2.3(2).

(2) $\Rightarrow$ (1). Let K be any ω-continuum, and let QC be any quasicomponent of $f^{-1}(K)$. Since Y is *zero*-dimensional space. Then, the connected subsets of Y are precisely the singleton sets. Thus, the ω-continuum are coincide with continuum sets in Y, therefore, K is a continuum in Y, so that $f(QC) = K$. Hence, f is quasiconfluent mapping. □

Proposition 2.15. *Let $f : X \to Y$ be any mapping. If X is a hereditarily locally connected space, then the following conditions (1) and (2) are equivalent, and the conditions (3) and (4) are equivalent:*

(1) *f is ω-confluent mapping,*

(2) *f quasi-ω-confluent mapping,*

(3) *f is confluent mapping,*

(4) *f quasiconfluent mapping.*

Proof. Similar to the proof of Proposition 2.10. □

3. Composition and Factorization of Quasi-ω-Confluent Mappings

In this section, we study the composition and factorization of quasi-ω-confluent mapping. So, we need to recall the following theorem.

Theorem 3.1 (see [6]). *Let $f : X \to Y$ and $g : Y \to Z$ be two ω-confluent mappings, where f is a surjective. Then, $h = g \circ f$ is an ω-confluent mapping.*

Theorem 3.2. *Let $f : X \to Y$ be a surjective quasi-ω-confluent of compact Hausdorff space X into space Y and $g : Y \to Z$ a quasi-ω-confluent of space Y into Hausdorff space Z. Then, $h = g \circ f$ is quasi-ω-confluent mapping.*

Proof. Since X and Y are two compact Hausdorff spaces and since f and g are two quasi-ω-confluent mappings, then f and g are ω-confluent mappings by Theorem 2.9. Therefore, $h = g \circ f$ is an ω-confluent mapping by Theorem 3.1. Then, from Proposition 2.3, $h = g \circ f$ is quasi-ω-confluent mapping. □

Proposition 3.3. *If X is hereditarily locally connected space and if $f : X \to Y$ and $g : Y \to Z$ are two quasi-ω-confluent mapping such that f is onto closed or open map, then $h = g \circ f$ is quasi-ω-confluent mapping.*

Proof. The proof follows immediately from Proposition 2.10 and Theorem 3.1. □

Theorem 3.4. *Let $f : X \to Y$ be a mapping of strongly connected space X into Hausdorff space Y, and let f be a canonical decomposition ($f = \mathrm{inc} \circ f' \circ p_{R_f}$) of the following mappings:*

$$f' : \frac{X}{R_f} \longrightarrow f(X), \quad \mathrm{inc} : f(X) \longrightarrow Y, \quad \text{and } p_{Rf} : X \longrightarrow \frac{X}{R_f}, \tag{3.1}$$

where p_{R_f} is the quotient surjection map, inc is the inclusion map, and f' is the bijection mapping, where X/R_f denote to quotients space over the kernel relation $R_f = \{(x,x') : f(x) = f(x')\}$. Then, f is a canonical decomposition of ω-confluent mappings.

Proof. We have to prove that these mappings p_{R_f}, i, and f' are ω-confluent mappings. Let K be any arbitrary ω-continuum in the quotients space X/R_f and C any component of $p_{R_f}^{-1}(K)$. Since p_{R_f} is continuous mapping, then X/R_f is a Hausdorff, so that K is closed in X/R_f. Then, by the continuity of p_{R_f}, we have $p_{R_f}^{-1}(K)$ is closed in X. But X is strongly connected. Therefore, $p_{R_f}^{-1}(K)$ is connected. This means $p_{R_f}^{-1}(K) = C$. So, $p_{R_f}(C) = K$. Thus, p_{R_f} is an ω-confluent mapping.

It is clearly that f' and inc are ω-confluent mappings, since Y is a Hausdorff, then the subspace $f(X)$ is Hausdorff, and since X is strongly connected, then X/R_f is strongly connected and also Hausdorff. Thus, f' and the inclusion map inc are ω-confluent. Hence, f is canonical decomposition of ω-confluent mappings. □

Remark 3.5. In the above theorem, if X is strongly connected compact Hausdorff space, then the mapping f is the canonical decomposition of quasi-ω-confluent mappings.

Corollary 3.6. *If X, Y, and Z are Hausdorff spaces, X is a compact space, and if $f : X \to Y$ is a surjective ω-confluent mapping and $g : Y \to Z$ is a quasi-ω-confluent mapping, then $h = g \circ f$ is ω-confluent mapping.*

Corollary 3.7. *If X, Y, and Z are Hausdorff spaces, X is a compact space, and if $f : X \to Y$ is a surjective quasi-ω-confluent mapping and $g : Y \to Z$ is ω-confluent mapping, then $h = g \circ f$ is a quasi-ω- confluent mapping.*

Now, we study Whyburn's factorization theorem for quasi-ω-confluent mappings. Thus, we recall the definition of a factorable mapping.

Definition 3.8 (see [8]). If $f : X \to Y$ be a mapping, any representation of f in the form $f = f_2 \circ f_1$, where $f_1 : X \to Z$ and $f_2 : Z \to Y$ are two mappings and Z is a certain space, will said to be factorization of f, and f is said be a factorable mapping and Z a middle space.

Before we study the factorization property, we state the following theorem.

Theorem 3.9 (see [6]). *If $f : X \to Y$ is an ω-confluent of strongly connected compact space X into Hausdorff space Y, then there exists a unique factorization for f into two ω-confluent mappings*

$$f(x) = f_2 \circ f_1(x), \quad \forall x \in X, \tag{3.2}$$

such that f_1 is confluent mapping.

Now, we can get the factorization of a quasi-ω-confluent mapping in the following proposition.

Proposition 3.10. *If $f : X \to Y$ be a quasi-ω-confluent of strongly connected compact Hausdorff space X into Hausdorff space Y, then there exists a unique factorization for f into two quasi-ω-confluent mappings in the form $f = f_2 \circ f_1$.*

Proof. Since f and g are two quasi-ω-confluent mappings and since X is strongly connected compact Hausdorff space and Y is a Hausdorff space, then from Theorem 2.9 f and g are ω-confluent mappings. Thus, f has unique factorization in the form $f = f_2 \circ f_1$ by Theorem 3.9. □

Next, we study the product property of quasiconfluent mappings.

Let $\{X_i\}_{i\in I}$ and $\{Y_i\}_{i\in I}$ be any two families of topological spaces. The product space of $\{X_i\}_{i\in I}$ and $\{Y_i\}_{i\in I}$ is denoted by $\prod_{i\in I} X_i$ and $\prod_{i\in I} Y_i$, respectively. Let $f_i : X_i \to Y_i$ be a mapping for each $i \in I$. Let $f : \prod_{i\in I} X_i \to \prod_{i\in I} Y_i$ be the product mappings as follows: $f((x_i)) = (f_i(x_i))$ for each $(x_i) \in \prod_{i\in I} X_i$. The projection of $\prod_{i\in I} X_i$ and $\prod_{i\in I} Y_i$ onto X_i and Y_i, respectively, is denoted by p_i and q_i.

Before we get the following result, we need to state the following theorem.

Theorem 3.11 (see [6]). *Let $f_i : X_i \to Y_i$ be an ω-confluent mapping, for each $i \in I$ of space X_i into Hausdorff space Y_i. Then,*

$$f : \prod_{i\in I} X_i \longrightarrow \prod_{i\in I} Y_i \tag{3.3}$$

is an ω-confluent mapping if the following equality holds:

$$\left(\prod_{i\in I} \tau_i\right)_\omega = \prod_{i\in I} (\tau_i)_\omega, \quad \forall i \in I. \tag{3.4}$$

As immediate consequence of the above theorem, we get the following corollary.

Corollary 3.12. *Let $f_i : X_i \to Y_i$ be a quasi-ω-confluent mapping of compact Hausdorff space X into Hausdorff space Y for each $i \in I$, then*

$$f : \prod_{i\in I} X_i \longrightarrow \prod_{i\in I} Y_i \tag{3.5}$$

is quasi-ω-confluent mapping if the following equality holds:

$$\left(\prod_{i\in I} \tau_i\right)_\omega = \prod_{i\in I} (\tau_i)_\omega, \quad \forall i \in I. \tag{3.6}$$

Proof. Since, X_i is compact Hausdorff, then the product space $\prod_{i\in I} X_i$ is compact Hausdorff, and since Y_i is Hausdorff, then the product space $\prod_{i\in I} Y_i$ is also Hausdorff. From Theorem 2.9, we infer that $f_i : X_i \to Y_i$ is an ω-confluent for each $i \in I$. Then, by Theorem 3.11, $f : \prod_{i\in I} X_i \to \prod_{i\in I} Y_i$ is an ω-confluent mapping. Therefore, f is quasi-ω-confluent by Proposition 2.3. □

4. Pullback of Quasi-ω-Confluent Mappings

In this section, we study the pullback of quasi-ω-confluent mappings. So, we recall the following definitions.

Definition 4.1 (see [9]). A fiber structure is a triple (X, f, Y) consisting of two spaces X and Y and a mapping $f : X \to Y$. The space X is said to be the fibered (or, total) space, f is termed the projection, and Y is the base space. Next, we recall the definition of the pullback.

Definition 4.2 (see [9]). Let (X, f, Y) be a fiber structure. Let Z be any space, and let $g : Z \to Y$ be any mapping into the base Y. Let E_f be a subspace of the cartesian product $X \times Z$, where $E_f = \{(x, z) : f(x) = g(z)\}$, and let $p : E_f \to Z$ be the projection of E_f onto Z such that $p(x, z) = z$, $\forall(x, z) \in E_f$. The fiber structure (E_f, p, Z) is said to be the fiber structure over Z induced by the mapping g, and the projection p is said to be the pullback of f by g.

Now, let $\gamma : E_f \to X$ be the projection such that $\gamma(x, z) = x$, $\forall(x, z) \in E_f$.
We observe that the following diagram is commutative.

$$\begin{array}{ccc} E_f & \xrightarrow{\gamma} & X \\ {\scriptstyle p}\downarrow & & \downarrow{\scriptstyle f} \\ Z & \xrightarrow[g]{} & Y \end{array}$$

Before we prove the main results in this section, we state the following lemma.

Lemma 4.3 (see [6]). *Let $f : X \to Y$ be a mapping, let Z be any space, and let $g : Z \to Y$ be any mapping, and if $K \subseteq Z$, then $p^{-1}(K) = f^{-1}(g(K)) \times K$, where p is the pullback of f by g.*

Theorem 4.4. *The pullback of a quasiconfluent mapping is quasi-ω-confluent.*

Proof. Let $f : X \to Y$ be a quasiconfluent mapping, let Z be any space, and let $g : Z \to Y$ be any mapping. Let $K \subseteq Z$ be any ω-continuum and QC any quasicomponent of $p^{-1}(K)$. Then, QC is a quasicomponent of $f^{-1}(g(K)) \times K$ by Lemma 4.3. Since every ω-continuum is continuum, then K is a continuum by Theorem 2.2. Thus, $g(K)$ is continuum in Y. Since f is quasiconfluent mapping, then $f(QC') = g(K)$ for each quasicomponent QC' of $f^{-1}(g(K))$. Since $p^{-1}(K) = f^{-1}(g(K)) \times K$, so $K = p(f^{-1}(g(K)) \times K) = p(QC' \times K)$ such that $QC = QC' \times K$ for some quasicomponent QC' of $f^{-1}(g(K))$. Thus, $p(QC) = P(QC' \times K) = K$. Therefore, p is quasi-ω-confluent. □

The pullback of quasi-ω-confluent mapping is not necessarily quasi-ω-confluent as shown by the following example.

Example 4.5. Let $X = \mathbb{R}$ be the real number with upper limit topology, $Y = \{a, b\}$ with the topology $\tau_Y = \{\phi, \{a\}, Y\}$, and $Z = \mathbb{R}$ with topology $\tau_Z = \{\phi, \mathbb{R}, \mathbb{R} - \{1\}, \mathbb{R} - \{2\}, \mathbb{R} - \{1, 2\}\}$.

Let $f : X \to Y$ be a mapping defined by

$$f(x) = \begin{cases} a, & \text{if } x > 0, \\ b, & \text{if } x \leq 0, \end{cases} \tag{4.1}$$

and let $g : Z \to Y$ be a mapping defined by

$$g(z) = \begin{cases} a, & \text{if } z \in \mathbb{R} - \{1, 2\}, \\ b, & \text{if } z \in \{1, 2\}. \end{cases} \tag{4.2}$$

Let E_f be a subspace of the cartesian product $X \times Z$, where

$$E_f = \{(x, z) : f(x) = g(z)\}. \quad (4.3)$$

Then, the pullback of f by g is the projection $p : E_f \to Z$ which is defined by

$$p(x, z) = z, \quad \forall(x, z) \in E_f. \quad (4.4)$$

We note that f is quasi-ω-confluent mapping, but p is not quasi-ω-confluent mapping. Since if we take the ω-continuum $K = [0, \infty) \subset Z$, then by Lemma 4.3, we get $p^{-1}(K) = f^{-1}(g(K)) \times K$. But $g(K) = \{a, b\}$ is not ω-continuum in Y.

Under certain condition, the pullback p of quasi-ω-confluent mapping f will be quasi-ω-confluent as shown by the following theorem.

Theorem 4.6. *If Y is a zero -dimensional space and if $f : X \to Y$ is a quasi-ω-confluent mapping, then the pullback p of f is quasi-ω-confluent.*

Proof. Let $f : X \to Y$ be a quasi-ω-confluent mapping, let Z be any space, and let $g : Z \to Y$ be an mapping. Let K be any ω-continuum in Z, and let QC be any quasicomponent of $p^{-1}(K)$, where p is the pullback of f by g. Then QC is the quasicomponent of $f^{-1}(g(K)) \times K$ by Lemma 4.3. By Theorem 2.2, K is continuum. Thus, $g(K)$ is continuum in Y by the continuity of g. Since Y is $zero$-dimensional space, then the quasiconfluent mapping equivalent to the quasi-ω-confluent by Proposition 2.14. This implies the continuum and ω-continuum sets coincide in Y. Thus, $g(K)$ is an ω-continuum in Y. Since f is a quasi-ω-confluent, then $f(QC') = g(K)$ for each quasicomponents QC' of $f^{-1}(g(K))$, and since $p^{-1}(K) = f^{-1}(g(K)) \times K$. $K = p(f^{-1}(g(K)) \times K) = p(QC' \times K)$ such that $QC = QC' \times K$ for some quasicomponent of $f^{-1}(g(K))$. Thus, $p(QC) = P(QC' \times K) = K$. Therefore, p is a quasi-ω-confluent. □

Corollary 4.7. *If $f : X \to Y$ is a quasi- ω-confluent mapping of space X into ω-space Y, then the pullback of f is quasi-ω-confluent mapping.*

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