Research Article

Reliability Evaluation for the Running State of the Manufacturing System Based on Poor Information

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The output performance of the manufacturing system has a direct impact on the mechanical product quality. For guaranteeing product quality and production cost, many firms try to research the crucial issues on reliability of the manufacturing system with small sample data, to evaluate whether the manufacturing system is capable or not. The existing reliability methods depend on a known probability distribution or vast test data. However, the population performances of complex systems become uncertain as processing time; namely, their probability distributions are unknown, if the existing methods are still taken into account; it is ineffective. This paper proposes a novel evaluation method based on poor information to settle the problems of reliability of the running state of a manufacturing system under the condition of small sample sizes with a known or unknown probability distribution. Via grey bootstrap method, maximum entropy principle, and Poisson process, the experimental investigation on reliability evaluation for the running state of the manufacturing system shows that, under the best confidence level $P = 0.95$, if the reliability degree of achieving running quality is $r > 0.65$, the intersection area between the inspection data and the intrinsic data is $A(T) > 0.3$ and the variation probability of the inspection data is $P_B(T) \leq 0.7$, and the running state of the manufacturing system is reliable; otherwise, it is not reliable. And the sensitivity analysis regarding the size of the samples can show that the size of the samples has no effect on the evaluation results obtained by the evaluation method. The evaluation method proposed provides the scientific decision and suggestion for judging the running state of the manufacturing system reasonably, which is efficient, profitable, and organized.

1. Introduction

Mechanical manufacturing process is an important link in the quality forming process of mechanical products. The good running state of the manufacturing system is a key aspect of the industrial production since it contributes to ensuring the manufacturing process to be reliable and further guarantees the quality of the products. Therefore, for a long time, the evaluation for the running state of the mechanical manufacturing system has been the important subject of much research that has been devoted to the theory and practice of the mechanical manufacturing, and many effort achievements have been performed on reliability investigations of the manufacturing system in recent years.

For example, considering the impact of various factors on the reliability of mechanical products, Schuh et al. [1] investigated achieving product design reliability and then taking measures to control to ensure the reliability of mechanical

products; Zhou [2] analyzed comprehensively the process reliability of machinery manufacturing from the aspects of design, manufacturing, and management, to ensure the reliability of mechanical products; based on Weibull analysis technology, Ma et al. [3] studied a reliability assessment method of how to assess the product reliability in the manufacturing process and identify the manufacturing bottle neck which should be focused attentively by the reliability engineer in manufacturing process; as the reliability design and analysis are proposed in the course of process design to solve the process defects, Zhang et al. [4] reported the structure of top technology and the basic idea about the process of how to realize the inhered reliability; by Monte Carlo simulation, Samadani et al. [5] put forward a systematic framework for realistic reliability assessment of an electrohydraulic servo system, in order to select to the best manufacturing process for each servo valve component; based on the analytic network process, Dai et al. [6] performed reliability modeling

and verification of manufacturing processes using a novel modeling method that draws upon a knowledge network of process scenarios; based on the reliability theory and the forming limit diagram, Li et al. [7] proposed a new method to assess the probability of failure of the tube hydroforming process; avoiding the traditional research method relying on the experiment, Li et al. [8] researched a rationality analytical method of the reliability on the mechanism system wear simulation under the small-scale sample; using nonlinear programming approach, Komal [9] conducted on fuzzy reliability analysis of plastic-pipe manufacturing system to achieve some suggestions in maintenance planning; Lin and Chang [10] proposed a predecessor-set technique for reliability evaluation of a stochastic manufacturing system with multiple production lines in parallel; and based on the recommended maximum value of the incapability index, Lin [11] implemented process reliability assessment with a Bayesian approach that can judge whether the process satisfies the preset quality reliability requirement.

Poor information means incomplete information, which indicates the characteristic information presented in the subject investigated is incomplete and insufficient and even the lack of a priori knowledge, such as, in the system analysis, a known probability distribution with only a small sample, an unknown probability distribution with few data only, and trends without any prior information. Poor information theory mainly includes the grey system theory, Bayesian theory, the fuzzy set theory, the bootstrap method, the maximum entropy method, and the chaos theory.

Viewing the existing research on poor information, the research and application of the problems involving poor information have drawn much attention and made remarkable progresses. For instance, based on poor information, Wang et al. [12] proposed a dynamic bootstrap grey method to estimate multisensor measurement results with small data samples and an unknown data distribution, having a lower relative estimation error of the measurement results compared to the grey bootstrap method and the Monte Carlo method; considering unknown probability distribution and very small sample data, He et al. [13] undertook performance analysis for material and structure using fuzzy norm method in uncertainty metric with poor information; on account of data rich but information poor, Ferraro [14] considered adopting procedures for efficient data sharing as a low-cost way to shorten development cycles; Xia [15] proposed the grey bootstrap method in the information poor theory for the reliability analysis of zero-failure data under the condition of a known or unknown probability distribution of lifetime; based on the grey theory, Zhang et al. [16] introduced the reliability assessment method to analyze the mechanism motion.

Poisson process is a kind of the most basic independent increment processes with cumulative number of random events, belonging to a relatively simple stochastic process owning continuous time and discrete state. It plays an important role in the theory and application of stochastic processes. At present, Poisson process has been widely applied in the fields of physics, geology, biology, medicine, astronomy, automation system, service system, the reliability theory,

and so forth [17–21]. In terms of reliability theory, Şenol [17] proposed the Poisson process approach to determine the occurrence degree for failure mode and effect reliability analysis methodology. Huang and Chen [18] recommended a time-dependent reliability model of deteriorating structures that considers both aging effects and random shocks based on stochastic Poisson processes and Bayesian inference which is a reasonable method for evaluating the reliability of deteriorating structures containing model uncertainties. Iervolino et al. [20] formulated such a model with reference to simple elastic-perfectly-plastic single degree of freedom systems, to assess reliability of structures to earthquake clusters.

Specific to the evaluation of the running state of the manufacturing system, poor information situations presented in the workpiece quality inspection process and in the manufacturing system adjustment process need to be concerned urgently.

There are many factors that interfere with the manufacturing process, which may lead to reducing the reliability of the manufacturing system. In order to ensure the high reliability of the running state of the manufacturing system, it is necessary to do the workpiece quality inspection regularly. Once the running state of the manufacturing system turns into or begins growing unreliable, it must terminate the manufacturing process and the manufacturing system should be conducted on adjustment or maintenance.

Workpiece quality detection is usually accomplished by sampling a few workpieces. It is performed to estimate the true value and confidence interval of the quality data by sampling workpieces, which can be used to timely assess that the running state of the manufacturing system is reliable or not.

Adjustment of the manufacturing system is usually done via trial cut several workpieces. It is to estimate the true value and confidence interval of the quality data by trial cut workpieces, so as to predict that the future running state of the manufacturing system is reliable or not.

In the manufacturing process that is either workpiece quality detection or adjustment of manufacturing system, the workpieces under investigation are very few and usually are only 4~10, and the quality data obtained belongs to the category of a small sample data. In addition, in the existing research, in order to realize to estimate the true value and confidence interval, the quality data are usually assumed as a set of data obeying the normal distribution. However, in practical production, the probability distribution function of mass quality data, such as roundness, parallelism, perpendicularity, concentricity, run-out, burns, and crack, conforms to the nonnormal distribution or unknown distribution. Therefore, it is difficult to solve this problem of reliability evaluation of the running state of manufacturing system using the existing achievements.

The reliability evaluation for the running state of the manufacturing system can be analyzed based on the output workpiece quality data in the manufacturing process. However, the workpiece quality data are a dynamic random process with unknown probability distribution, along with the trend of known or unknown disturbance, which belongs to a poor information system with uncertainty. At present,

in the application of poor information theory, the bootstrap method [22, 23] and the grey prediction model [24] in the grey system theory are two popular methods to evaluate the system reliability. The bootstrap method is good at simulating the unknown distribution, but the evaluation error may be infinite [25]. And the grey prediction model has a good forecast function, which can make up for the defect of the bootstrap method. For this end, synthesizing the advantages of the bootstrap method and the grey prediction model, this paper puts forward the grey bootstrap method to analyze the running state of the manufacturing system, which lays the foundation for reliability evaluation for the running state of the manufacturing system.

The maximum entropy principle [26] is proposed as an inference in 1957 by Jaynes. According to the maximum entropy principle, the reasonable state that should conform to the constraints and the maximum entropy value can be applied to infer the system state, which is the only impartial choice in the case of only poor information. And the probability of appearing the probability distribution with the maximum entropy is the largest in the maximum entropy principle [26–28]. Because of only small workpiece quality data with unknown probability distribution for the running state of the manufacturing system, there is no sufficient reason to choose other analytic function, and the maximum entropy principle should be preferred to determine the probability distribution function and the characteristic parameter of the running state of the manufacturing system.

This paper recommends a new evaluation method based on poor information to solve the problem of reliability evaluation of the running state of manufacturing system under the condition of small sample size and unknown probability distribution. Using the grey bootstrap method, the maximum entropy principle, and Poisson process, it aims to realize reliability evaluation for the running state of manufacturing system with no variation and variation. Via the computer simulations and actual cases, the evaluation results of reliability of the running state of manufacturing system can be obtained. And the research method proposed in the paper provides scientific decisions and recommendations on how to decide properly on the running state of the manufacturing system, which can ensure product quality to be stable and reliable and realize the low manufacturing costs.

2. Mathematical Model

2.1. Collection of Small Sample Data.
Suppose that the workpiece quality data is a random variable x in the adjustment process of the manufacturing system. Via a measurement system, the raw intrinsic data, namely, small sample data meeting the quality requirements, are collected to constitute a raw intrinsic data sequence that is expressed as a vector $\mathbf{X}$, as follows:

$$\mathbf{X} = (x(1), x(2), \ldots, x(n), \ldots, x(N));$$
$$n = 1, 2, \ldots, N, \qquad (1)$$

where $\mathbf{X}$ stands for the raw intrinsic data sequence of the workpiece quality data; $x(n)$ stands for the nth data in $\mathbf{X}$; n

stands for the sequence number of the raw intrinsic data; N stands for the number of the data in $\mathbf{X}$. Here it should be noted that N is a small integer and its value range is [4, 10] generally.

The intrinsic data sequence characterizes the data sequence obtained in the optimal running state of the manufacturing system, which can satisfy the characteristics demands of population distribution of the workpiece quality parameter.

Based on small sample data in the raw intrinsic data sequence $\mathbf{X}$, the intrinsic generated data of large size can be generated by the grey bootstrap, which lay the foundation for establishment of the probability density function to characterize the running state of the manufacturing system.

2.2. Prediction of the Intrinsic Generated Data.
The grey bootstrap method consists of the bootstrap method and the grey prediction model.

The bootstrap method can simulate a large number of bootstrap resampling samples via small sample data under investigation, and the grey prediction model can predict a large number of generated data via bootstrap resampling samples.

According to the bootstrap method, via an equiprobable sampling with replacement from the raw intrinsic data sequence $\mathbf{X}$, B steps of bootstrap resampling samples can be conducted in the case of each step of extraction N and each extraction of size 1. Then, B simulation samples of size N can be obtained and the result is expressed as a vector Θ, as follows:

$$\Theta = (\Theta_1, \Theta_2, \ldots, \Theta_b, \ldots, \Theta_B); \quad b = 1, 2, \ldots, B, \qquad (2)$$

where B is the number of Θ and B is usually a large integer; namely, $B \geq 1000$; Θ_b is the bth bootstrap resampling sample and can be written as

$$\Theta_b = (\theta_b(1), \theta_b(2), \ldots, \theta_b(n), \ldots, \theta_b(N)), \qquad (3)$$

where $\theta_b(n)$ is the nth simulation sample in Θ_b and N is the number of the data in Θ_b.

According to the grey system theory, suppose that the first-order accumulated generating operation (1-AGO) of Θ_b is a vector Φ_b, as follows:

$$\Phi_b = (\varphi_b(1), \varphi_b(2), \ldots, \varphi_b(n), \ldots, \varphi_b(N)) \qquad (4)$$

with

$$\varphi_b(n) = \sum_{j=1}^{n} \theta_b(j). \qquad (5)$$

Via the grey prediction model, Φ_b can be defined as a grey differential formula, as follows:

$$\frac{d\varphi_b(n)}{dn} + c_{b1}\varphi_b(n) = c_{b2}, \qquad (6)$$

where n is considered as a continuous variable and c_{b1} and c_{b2} are the coefficients to be estimated.

Suppose that the generated mean vector is

$$\mathbf{Z}_b = (z_b(2), z_b(3), \ldots, z_b(n), \ldots, z_b(N)); \\ n = 2, 3, \ldots, N \tag{7}$$

with

$$z_b(n) = (0.5\varphi_b(n) + 0.5\varphi_b(n-1)). \tag{8}$$

Under the initial condition that $\varphi_b(1) = \theta_b(1)$, the least-squares solution to (6) is

$$\eta_b(N+1) = \left(\theta_b(1) - \frac{c_{b2}}{c_{b1}}\right) \exp(-c_{b1}N) + \frac{c_{b2}}{c_{b1}}, \tag{9}$$

where $\eta_b(N+1)$ represents the bth first-order accumulated generated data predicted by $\mathbf{\Theta}_b$; and the coefficients c_{b1} and c_{b2} are given by

$$(c_{b1}, c_{b2})^{\mathrm{T}} = (\mathbf{D}^{\mathrm{T}}\mathbf{D})^{-1}\mathbf{D}^{\mathrm{T}}\mathbf{\Theta}_b^{\mathrm{T}}; \quad n = 2, 3, \ldots, N \tag{10}$$

with

$$\mathbf{D} = (-\mathbf{Z}_b\mathbf{I})^{\mathrm{T}}, \tag{11}$$

where $\mathbf{I}$ is a unit vector of dimension $N - 1$.

According to the inverse AGO in the grey system theory, the bth intrinsic generated data x_b can be predicted, as follows:

$$x_b = \eta_b(N+1) - \eta_b(N). \tag{12}$$

A large number of generated data obtained by (12) can constitute an intrinsic generated data sequence, which is given by $\mathbf{X}_{GB}$, as follows:

$$\mathbf{X}_{GB} = (x_1, x_2, \ldots, x_b, \ldots, x_B); \quad b = 1, 2, \ldots, B, \tag{13}$$

where $\mathbf{X}_{GB}$ indicates the intrinsic generated data sequence which is from the raw intrinsic data sequence using the grey bootstrap method.

Based on the intrinsic generated data sequence $\mathbf{X}_{GB}$ in (13), the probability density function of characterizing the running state of the manufacturing system can be simulated using the maximum entropy principle.

2.3. Establishment for the Probability Density Function of the Manufacturing System.

According to the maximum entropy principle in information theory, in all the feasible solutions, it is necessary to solve a problem; namely, the probability density function that maximizes the information entropy is the most unbiased estimation of the information source in the running state of the manufacturing system.

In the information theory, the information entropy H can be defined as

$$H = -\int_\Omega f(x) \ln f(x) \, \mathrm{d}x, \tag{14}$$

where x represents a continuous random variable for describing x_b in (13); Ω represents the feasible region for x; $f(x)$

represents the probability density function of the running state of the manufacturing system.

Let the information entropy maximum, namely,

$$H \longrightarrow \max \tag{15}$$

satisfy the constraint conditions

$$\int_\Omega f(x) \, \mathrm{d}x = 1, \\ \int_\Omega x^i f(x) \, \mathrm{d}x = m_i; \quad i = 1, 2, \ldots, M, \tag{16}$$

where m_i stands for the ith order origin moment of x, i stands for the origin moment order, and M stands for the highest origin moment order.

According to the statistical theory, the estimated value of the ith order origin moment is written as

$$m_i = \frac{1}{B} \sum_{b=1}^{B} x_b^i. \tag{17}$$

Hence, using the Lagrange multiplier method, the probability density function which stratifies (14) can be defined according to (15) and (16), as follows:

$$f(x) = \exp\left(\lambda_0 + \sum_{i=1}^{M} \lambda_i x^i\right) \tag{18}$$

and the Lagrange multiplier λ_0 is given by

$$\lambda_0 = -\ln\left(\int_\Omega \exp\left(\sum_{i=1}^{M} \lambda_i x^i\right) \mathrm{d}x\right), \tag{19}$$

where λ_i is the ith Lagrange multiplier and can be solved according to (20), as follows:

$$m_i \int_\Omega \exp\left(\sum_{i=1}^{M} \lambda_i x^i\right) \mathrm{d}x - \int_\Omega x^i \exp\left(\sum_{i=1}^{M} \lambda_i x^i\right) \mathrm{d}x \\ = 0; \quad i = 1, 2, \ldots, M. \tag{20}$$

Equation (18) obtained by the maximum entropy principle can accurately characterize the probability density function $f(x)$ of the running state of the manufacturing system.

With the help of the principle of statistics, the true value estimate and the confidence interval estimate of the running state of the manufacturing system can be implemented using (18).

2.4. Estimate of the True Value and the Confidence Interval of the Manufacturing System.

According to the principle of statistics, the estimated true value is given by

$$X_0 = \int_\Omega x f(x) \, \mathrm{d}x, \tag{21}$$

where X_0 is the estimated true value of the running state of the manufacturing system.

The estimated true value is a characteristic index to evaluate the running state of the manufacturing system, namely, an estimate of the size of the workpiece quality parameter.

Suppose that the significance level is $\alpha \in [0, 1]$; then, the confidence level P is expressed as

$$P = (1 - \alpha) \times 100\%. \qquad (22)$$

The confidence level P represents the accuracy of the sample statistic values, which is regarded as the probability that the sample statistic values fall within a certain range of the parameter value.

Under the confidence level P, the confidence interval of the running state of the manufacturing system is given by

$$[X_L, X_U] = [X_{\alpha/2}, X_{1-\alpha/2}], \qquad (23)$$

where $X_{\alpha/2}$ is the value of the variable x corresponding to the probability $\alpha/2$; $X_{1-\alpha/2}$ is the value of the variable x corresponding to the probability $1 - \alpha/2$; X_L is the lower boundary of the estimated interval; X_U is the upper boundary of the estimated interval.

The confidence interval $[X_L, X_U]$ is a characteristic index to evaluate the running state of the manufacturing system, namely, an estimate of the value range of the size of the workpiece quality parameter.

According to (23), the expanded uncertainty U is defined as

$$U = \frac{1}{2}(X_U - X_L). \qquad (24)$$

The expanded uncertainty U can be used to evaluate the uncertainty of the running state of the manufacturing system. The smaller the expanded uncertainty is, the better the running state of the manufacturing system is, the higher the running quality of the manufacturing system is, and the smaller the fluctuation of the workpiece quality is; otherwise, the worse the running state of the manufacturing system is, the lower the running quality of the manufacturing system is, and the larger the fluctuation of the workpiece quality is. Accordingly, in the evaluation for the running state of the manufacturing system, the reliability of the running state of the manufacturing system is expressed by 2 times the expanded uncertainty $2U$ which can quantitatively describe the fluctuation range of the workpiece quality data.

In the end, the workpiece quality can be characterized as the estimated true value X_0 and the confidence interval $[X_L, X_U]$ under the confidence level P.

2.5. Reliability Evaluation for the Running State of the Manufacturing System

2.5.1. Application of Poisson Process.
Suppose that the output workpiece quality data in the manufacturing process falling out of the fluctuation range of the workpiece quality data in the reliable running state of the manufacturing system is defined as the event A. $N(t)$ stands for the occurrence frequency of the event A until a certain time t. The occurrence frequency $N(t)$ of the event A is considered as a random variable, and the stochastic process $\{N(t), t \geq 0\}$ can be called the counting process.

In the actual manufacturing process, the number of the workpiece quality data out of the fluctuation range in the reliable running state of the manufacturing system is always greater than or equal to zero, and they are all the integers. When $t = 0$, the number of the workpiece quality data out of the fluctuation range in the reliable running state of the manufacturing system is zero; namely, $N(0) = 0$. And $N(t)$ grows larger and larger with the extension of the time t; namely, if $t_1 < t_2$, it is obtained that $N(t_1) \leq N(t_2)$, and $N(t_2) - N(t_1)$ can be equal to the occurrence frequency of the event A in the interval $[t_1, t_2]$.

For the counting process $\{N(t), t \geq 0\}$, the occurrence frequencies of the event A in nonoverlapping time intervals are independent of each other; namely, if $t_1 < t_2 \leq t_3 < t_4$, the occurrence frequency $N(t_2) - N(t_1)$ of the event A in the interval $[t_1, t_2]$ is independent of the occurrence frequency $N(t_4) - N(t_3)$ of the event A in the interval $[t_3, t_4]$, and vice versa. And the counting process $\{N(t), t \geq 0\}$ is only related to the time difference and has nothing to do with a certain moment. Thus, the counting process $\{N(t), t \geq 0\}$ is an independent increment process smoothly, which can be described by Poisson process.

2.5.2. The Reliability Analysis Based on Poisson Process.
According to Poisson process, the occurrence frequency of the event A in an interval with length t obeys Poisson distribution with the parameter $\lambda > 0$. The concept of Poisson process with zero-failure probability is applied in this section; for this end, the failure distribution function regarding Poisson process is defined as

$$P(l, t) = P\{N(t + w) - N(w) = l\}$$
$$= \exp(-\lambda t)\frac{(\lambda t)^l}{l!}; \quad l = 0, 1, 2, \ldots, L; \ t \geq 0, \qquad (25)$$

where $P(l, t)$ stands for the failure distribution function regarding Poisson process; t stands for the time variable; l stands for a discrete random variable recording the number of event occurrences; L stands for the number of event occurrences; λ is the frequency of event occurrences and is regarded as the variation intensity of the subject under investigation, which can describe variation characteristics of the running state of the manufacturing system; w stands for a random moment.

For real time evaluation for the reliability of the manufacturing system in the normal run of the manufacturing system after being adjusted, it is essential to continuously inspect the workpiece quality. Suppose that the raw inspection data of the workpiece quality are obtained by inspecting a few workpieces and constitute the raw inspection data sequence, which is expressed as by $\mathbf{X}_A$, as follows:

$$\mathbf{X}_A = (x_A(1), x_A(2), \ldots, x_A(s), \ldots, x_A(S));$$
$$s = 1, 2, \ldots, S, \qquad (26)$$

where $\mathbf{X}_A$ is the raw inspection data sequence; $x_A(s)$ is the sth data in $\mathbf{X}_A$; S is the number of the data in $\mathbf{X}_A$.

Generally, there are a few data in the raw inspection data sequence $\mathbf{X}_A$; in general, $S = 4 \sim 10$, which is difficult to conduct on count analysis in statistical significance precisely. To solve the problem, mass inspection data are generated using the grey bootstrap method mentioned in Section 2.2 and constitute the inspection generated data sequence, which is given by $\mathbf{X}_{GA}$, as follows:

$$\mathbf{X}_{GA} = \left(x_{GA}(1), x_{GA}(2), \ldots, x_{GA}(a), \ldots, x_{GA}(A) \right);$$
$$a = 1, 2, \ldots, A, \tag{27}$$

where $\mathbf{X}_{GA}$ is the inspection generated data sequence; $x_{GA}(a)$ is the ath data in $\mathbf{X}_{GA}$; A is the number of the data in $\mathbf{X}_{GA}$ and A is usually a large integer; namely, $A \geq 1000$.

If many raw inspection data in $\mathbf{X}_A$ can be obtained via workpiece quality inspection, and it need not use (27), let $\mathbf{X}_{GA} = \mathbf{X}_A$ and $A = S$ directly; namely, $\mathbf{X}_{GA}$ is considered as the raw inspection data sequence $\mathbf{X}_A$. Then the probability density function of the running state of the manufacturing system can be obtained using the maximum entropy principle based on the inspection data sequence $\mathbf{X}_{GA}$. It is worth noting that if there are many inspection data in $\mathbf{X}_A$, make $\mathbf{X}_{GA} = \mathbf{X}_A$ and $A = S$; namely, $\mathbf{X}_{GA}$ is replaced with $\mathbf{X}_A$ at this time. If there are a few inspection data in $\mathbf{X}_A$, $\mathbf{X}_{GA}$ is generated by $\mathbf{X}_A$ according to the grey bootstrap method which belong to a large data vector. And $\mathbf{X}_{GA}$ is not equal to $\mathbf{X}_A$ at this time.

Via using the maximum entropy principle mentioned in Section 2.3, the probability density function of the inspection generated data sequence $\mathbf{X}_{GA}$ can be written as

$$p = p(x). \tag{28}$$

Suppose there are Q data within $\mathbf{X}_{GA}$ but outside the confidence interval $[X_L, X_U]$ by counting; hence, the variation intensity λ under investigation can be computed as

$$\lambda = \frac{Q}{A} \times 100\%. \tag{29}$$

According to Poisson process, the variation intensity λ represents the frequency of event occurrences, and in terms of the reliability evaluation for the running state of the manufacturing system, the variation intensity λ represents the frequency that the inspection data of the workpiece quality falls outside the confidence interval $[X_L, X_U]$. It obviously belongs to the Poisson counting process. In the counting process, the smaller the variation intensity λ is, the more reliable the running state of the manufacturing system is, and vice versa.

When $l = 0$, the occurrence frequency of the event A in the interval with time length t is zero, which can be utilized to characterize that the running state of the manufacturing system in this stage is reliable; namely, the manufacturing system has no failure. With the help of Poisson process with zero-failure probability ($l = 0$), namely, only considering $l = 0$ for (25), (25) can be further studied to become (30),

and the reliability function $R(t)$ of achieving running quality of the manufacturing system can be given by

$$R(t) = P(0, t) = P\{N(t + w) - N(w) = 0\}$$
$$= \exp(-\lambda t); \quad t \in [0, +\infty), \tag{30}$$

where $R(t)$ stands for the reliability function of achieving running quality of the manufacturing system.

In (30), if the time variable is that $t = t_0$, the reliability degree r of achieving running quality of the manufacturing system under the moment $t = t_0$ can be obtained as

$$r = R(t_0), \tag{31}$$

where r is the reliability degree of achieving running quality of the manufacturing system under the moment $t = t_0$. The reliability degree r can be used to evaluate the possibility of achieving running quality of the manufacturing system. Namely, the larger the reliability degree r is, the larger the possibility of achieving running quality of the manufacturing system is, and vice versa.

2.6. Reliability Evaluation for the Running State Variation Process of the Manufacturing System. In the actual production, with the continuous accumulation of processing time, the running state of the manufacturing system becomes unknown; meanwhile the variation process of the manufacturing system becomes uncertain. However, on the whole, the manufacturing system itself follows the variation law from good to bad in the long-term manufacturing process. To ensure products meet the quality requirements, the dynamic analysis of the running state variation process of the manufacturing system is a demanding task.

According to Poisson process mentioned in Section 2.5, suppose that t is a continuous time variable and its value range is $t \in [0, +\infty)$ in (30), and it can be performed to take the derivative of the reliability function $R(t)$ of achieving running quality of the manufacturing system, namely, the derivative of (30). Thus the probability density function $p(0, t)$ of the reliability of the running state variation process of the manufacturing system as the time variable t changes can be written as

$$p(0, t) = \frac{dR(t)}{dt} = \frac{dP(0, t)}{dt} = -\lambda \exp(-\lambda t);$$
$$t \in [0, +\infty), \tag{32}$$

where $p(0, t)$ is the probability density function of the reliability of the running variation process of the manufacturing system as $t \in [0, +\infty)$.

The experimental data under investigation are grouped to analyze the reliability of the running state variation process of the manufacturing system. The inspection data sequence $\mathbf{X}_A$ is divided into M equal portions as the time interval and M groups of the inspection data subsequence X_{Am} ($m = 1, 2, \ldots, M$) are obtained. Each group of inspection data subsequence X_{Am} contains N inspection data and N is the

number of the raw intrinsic data sequence $\mathbf{X}$. The grouping result can be given by

$$\mathbf{X}_A = (X_{A1}, X_{A2}, \ldots, X_{Am}, \ldots, X_{AM});$$
$$m = 1, 2, \ldots, M, \tag{33}$$

where X_{Am} is the mth inspection data subsequence in $\mathbf{X}_A$ and M is the number of groups in $\mathbf{X}_A$.

The inspection data subsequence X_{Am} is defined as

$$X_{Am}$$
$$= (x_{Am}(1), x_{Am}(2), \ldots, x_{Am}(k), \ldots, x_{Am}(N)); \tag{34}$$
$$k = 1, 2, \ldots, N; \ S = M \times N,$$

where $x_{Am}(k)$ is the kth data in X_{Am}; N is the number of the data in X_{Am}; S is the number of the data in $\mathbf{X}_A$.

According to the grey bootstrap method and the maximum entropy principle and Poisson process, namely, (28)–(32), the inspection data subsequence X_{Am} can be analyzed to obtain M probability density functions with different variation degree of the running variation process of the manufacturing system.

On the basis of the raw intrinsic data sequence $\mathbf{X}$, the inspection data subsequence X_{Am} is performed by contrastive analysis in contrast to the raw intrinsic data sequence $\mathbf{X}$, respectively. The probability density functions are processed by subsection integral to (32), to solve the intersection area $A(T)$ of the probability density functions of the inspection data subsequence X_{Am} and the intrinsic data sequence $\mathbf{X}$, which is expressed as

$$A(T) = \int_0^T p(0, t) \, dt + \int_T^{+\infty} p(0, t) \, dt; \tag{35}$$
$$t \in [0, +\infty),$$

where T is the abscissa value of the intersection point of the probability density functions of the inspection data sequence and the intrinsic data sequence.

Define the variation probability $P_B(T)$ of the inspection data sequence relative to the intrinsic data sequence as

$$P_B(T) = 1 - A(T), \tag{36}$$

where $P_B(T)$ is the variation probability of the inspection data sequence.

The variation probability $P_B(T)$ can be used to evaluate the reliability of the running state variation process of the manufacturing system. Via comparing the inspection data sequence with the intrinsic data sequence, it can be obtained that the larger the intersection area $A(T)$ of the probability density functions of inspection data sequence and the intrinsic data sequence is, the smaller the variation probability $P_B(T)$ of the inspection data sequence is, and the smaller the variation intensity λ of the running variation process of the manufacturing system is; namely, the larger the possibility of achieving running quality of the manufacturing system is, the more reliable the running state of the manufacturing system is, and vice versa.
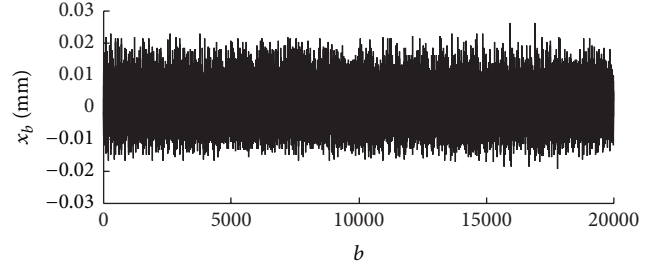

Figure 1: The intrinsic generated data sequence $\mathbf{X}_{GB}$ (Case 1).

3. Case Studies and Discussions

There are a total of five cases studies involving two types of the evaluation issues on the running process of the manufacturing system. The former three cases are cases of evaluations for the running state of the manufacturing system, and the latter two cases are cases of evaluations for the running state variation process of the manufacturing system. It is worth mentioning that the problems of prior information with known or unknown probability distributions and trends are taken into account in the case studies.

3.1. Cases of Evaluation for the Running State of the Manufacturing System

Case 1. This is a simulation case of evaluation for the running state of the manufacturing system which obeys a normal distribution. Based on the simulation data with respect to a processing quality parameter Q_1, there is an evaluation for the running state of the manufacturing system with a normal distribution in the case.

Suppose that the mathematical expectation $E = 0$ mm and the standard deviation $s = 0.01$ mm are the known parameters, and 10 ($N = 10$) simulation datasets obeying the normal distribution are generated by Monte Carlo simulation method and are assumed as the measured data with respect to a processing quality parameter Q_1 in an adjusted manufacturing system with a good state running. Then 10 simulation data can constitute a raw intrinsic data sequence $\mathbf{X} = (-0.00418, 0.00493, -0.00271, 0.0029, -0.00824, 0.01387, -0.01377, 0.00184, -0.00231, 0.0198)$. Processing the raw intrinsic data sequence $\mathbf{X}$, 20000 ($B = 20000$) generated datasets are obtained using the grey bootstrap method, which can constitute an intrinsic generated data sequence $\mathbf{X}_{GB}$, as shown in Figure 1. Then, processing the intrinsic generated data sequence $\mathbf{X}_{GB}$, the probability density function $f(x)$ of the running state of the manufacturing system is obtained using the maximum entropy principle, as shown in Figure 2. Let the confidence level be P, let the confidence interval be $[X_L, X_U]$, and let the expanded uncertainty be U of the running state of the manufacturing system.

Suppose that the mathematical expectation $E = 0$ mm and the standard deviation $s = 0.01$ mm are the known parameters, and 20000 ($S = 20000$) simulation datasets obeying the normal distribution are generated by Monte
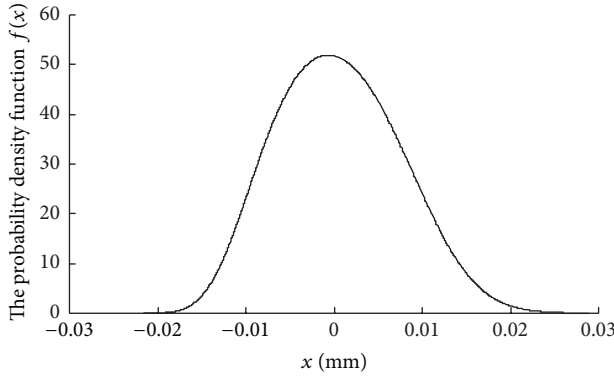
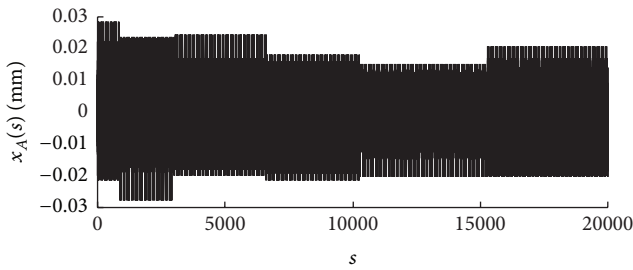Figure 2: The probability density function $f(x)$ (Case 1).

Figure 3: The inspection data sequence $\mathbf{X}_A$ (Case 1).

Figure 4: The relationship between $2U$ and r (Case 1).

Figure 5: The intrinsic generated data sequence $\mathbf{X}_{GB}$ (Case 2).

Carlo simulation method and are assumed as the measured data which is obtained by inspecting the processing quality parameter Q_1 in the normal manufacturing process, namely, the inspection data sequence $\mathbf{X}_A$, as shown in Figure 3.

Let $\mathbf{X}_{GA} = \mathbf{X}_A$ and $A = S$, and the variation intensity λ of the running state of the manufacturing system is obtained by counting process. Using Poisson process with zero-failure probability, the reliability function $R(t)$ of achieving running quality of the manufacturing system is obtained. Then let $t = 1$; the reliability degree r of achieving running quality of the manufacturing system under the moment $t = 1$ is obtained. For the convenience of research, it shows that the relationship between 2 times the expanded uncertainty $2U$ and the reliability degree r is given as shown in Figure 4.

In Figure 4, $r \in [0.3679, 0.9921]$ and $2U \in [0\,\mu\text{m}, 56.82\,\mu\text{m}]$. On the whole, there is the nonlinear relationship between $2U$ and r. The nonlinear relationship can be divided into 3 regions with I, II, and III, corresponding to 3 approximate straight lines with different slopes, respectively.

In I region, $r \in [0.3679, 0.7315]$ and $2U \in [0\,\mu\text{m}, 19.28\,\mu\text{m}]$. It is obvious that the expanded uncertainty and the reliability degree are both small and the linear relationship between $2U$ and r is characterized by the small slope. It shows that the possibility of achieving high quality running state for the manufacturing system is small in I region.

In III region, $r \in [0.9233, 0.9921]$ and $2U \in [35.88\,\mu\text{m}, 56.82\,\mu\text{m}]$. It can be seen that the expanded uncertainty and the reliability degree are both large and the linear relationship between $2U$ and r is characterized by the large slope. It shows that the possibility of achieving low quality running state for the manufacturing system is large in III region.
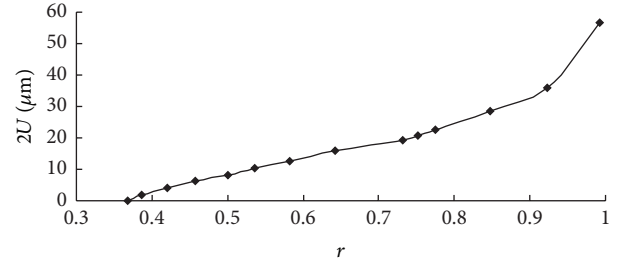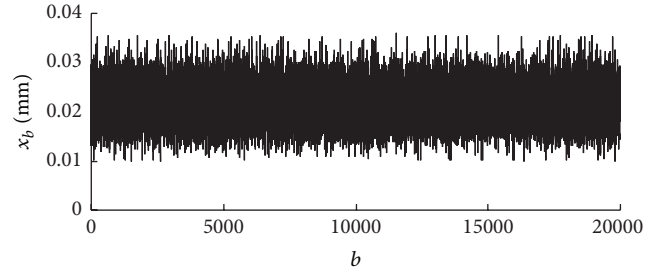
In II region, $r \in [0.7315, 0.9233]$ and $2U \in [19.28\,\mu\text{m}, 35.88\,\mu\text{m}]$. Obviously, the expanded uncertainty $2U$ and the reliability degree r and the slope of the linear relationship between $2U$ and r are all moderate in comparison with the above two regions. It shows that the running quality of achieving the running state is in line with the possibility of achieving the quality of the running state, and both are moderate in II region.

Case 2. This is a simulation case of evaluation for the running state of the manufacturing system which obeys a Rayleigh distribution. Based on the simulation data with respect to a processing quality parameter Q_2, there is an evaluation for the running state of the manufacturing system of a Rayleigh distribution in the case.

Suppose that the mathematical expectation $E = 0.0215$ mm and the standard deviation $s = 0.01$ mm are the known parameters, and 10 ($N = 10$) simulation datasets obeying the Rayleigh distribution are generated by Monte Carlo simulation method and are assumed as the measured data with respect to a processing quality parameter Q_2 in an adjusted manufacturing system with a good state running. Then 10 simulation datasets can constitute a raw intrinsic data sequence $\mathbf{X} = (0.01664, 0.01412, 0.02564, 0.01899, 0.02301, 0.01957, 0.01747, 0.02203, 0.03343, 0.02131)$. Processing the intrinsic data sequence $\mathbf{X}$, 20000 ($B = 20000$) generated datasets are obtained using the grey bootstrap method, which can constitute an intrinsic generated data sequence $\mathbf{X}_{GB}$, as shown in Figure 5. Thus, processing the intrinsic generated data sequence $\mathbf{X}_{GB}$, the probability density function $f(x)$ of the running state of the manufacturing system is obtained using the maximum entropy principle, as shown in Figure 6. Let the confidence level be P, let the confidence interval be $[X_L, X_U]$, and let the expanded
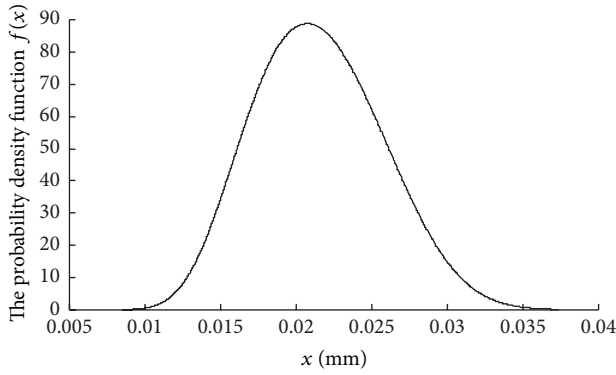
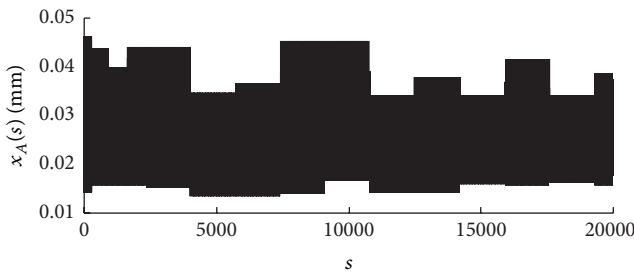FIGURE 6: The probability density function $f(x)$ (Case 2).

FIGURE 7: The inspection data sequence $\mathbf{X}_A$ (Case 2).

uncertainty be U for the running state of the manufacturing system.

Suppose that the mathematical expectation E = 0.0215 mm and the standard deviation s = 0.01 mm are the known parameters, and 20000 (S = 20000) simulation datasets obeying the Rayleigh distribution are generated by Monte Carlo simulation method and are assumed as the measured data, which is obtained by inspecting the processing quality parameter Q_2 in the normal manufacturing process, namely, the inspection data sequence $\mathbf{X}_A$, as shown in Figure 7.

Let $\mathbf{X}_{GA} = \mathbf{X}_A$ and $A = S$, and the variation intensity λ of the running state of the manufacturing system is obtained by counting process. Using Poisson process with zero-failure probability, the reliability function $R(t)$ of achieving running

quality of the manufacturing system is obtained. Then let t = 1; the reliability degree r of achieving running quality of the manufacturing system under the moment t = 1 is obtained. For the convenience of research, it shows that the relationship between 2 times the expanded uncertainty $2U$ and the reliability value r is given as shown in Figure 8.

In Figure 8, $r \in [0.3679, 0.9817]$ and $2U \in [0\,\mu\text{m}, 34.09\,\mu\text{m}]$. On the whole, there is the nonlinear relationship between $2U$ and r. The nonlinear relationship can be divided into 3 regions with I, II, and III, corresponding to 3 approximate straight lines with different slopes, respectively.

In I region, $r \in [0.3679, 0.7072]$ and $2U \in [0\,\mu\text{m}, 14.47\,\mu\text{m}]$. It is obvious that the expanded uncertainty and the reliability degree are both small and the linear relationship between $2U$ and r is characterized by the small slope. It shows that the possibility of achieving high quality running state for the manufacturing system is small in I region.

In III region, $r \in [0.8490, 0.9817]$ and $2U \in [22.43\,\mu\text{m}, 34.09\,\mu\text{m}]$. It is easy to see that the expanded uncertainty and the reliability value are both large and the linear relationship between $2U$ and r is characterized by the large slope. It shows that the possibility of achieving low quality running state for the manufacturing system is large in III region.

In II region, $r \in [0.7072, 0.8490]$ and $2U \in [14.47\,\mu\text{m}, 22.43\,\mu\text{m}]$. Obviously, the expanded uncertainty $2U$ and the reliability degree r and the slope of the linear relationship between $2U$ and r are all moderate in comparison with the above two regions. It shows that the running quality of achieving the running state is in line with the possibility of achieving the quality of the running state, and both are moderate in II region.

Case 3. This is an actual case of evaluation for the running state of the manufacturing system with unknown probability distribution. A rolling bearing inner raceway grinding machine is involved to grind the inner raceway of the tapered rolling bearing with 30204 in the case. By grinding and measurement after regulation of the machine, the 30 measured datasets of the inner raceway roundness of the bearing, in μm, are collected and the result is as follows:

1.08	0.90	1.06	1.28	0.88	1.87	1.16	1.06	0.97	1.01
0.70	1.15	0.72	1.08	0.67	1.10	0.98	1.15	1.14	1.64
0.73	0.87	1.91	1.95	1.19	0.78	1.51	1.39	1.39	3.28

Based on the above 30 measured datasets of the inner raceway roundness in regard to a processing quality parameter, namely, roundness, there is an evaluation for the running state of the grinding machine in the case.

The former 5 datasets of the above 30 measured datasets are selected as the elements of the raw intrinsic data sequence $\mathbf{X}$ (N = 5) and the latter 25 datasets of the above 30 measured datasets are selected as the elements of the inspection data sequence $\mathbf{X}_A$ (S = 25). The intrinsic generated data sequence

$\mathbf{X}_{GB}$ is obtained by the grey bootstrap method (B = 20000), as shown in Figure 9. And the probability density function $f(x)$ of the running state of the manufacturing system is obtained using the maximum entropy principle, as shown in Figure 10.

Let $\mathbf{X}_{GA} = \mathbf{X}_A$ and $A = S$, and the inspection data sequence $\mathbf{X}_A$ is as shown in Figure 11. The variation intensity λ of the running state of the manufacturing system is obtained by counting process. Using Poisson process with zero-failure probability, the reliability function $R(t)$ of achieving running
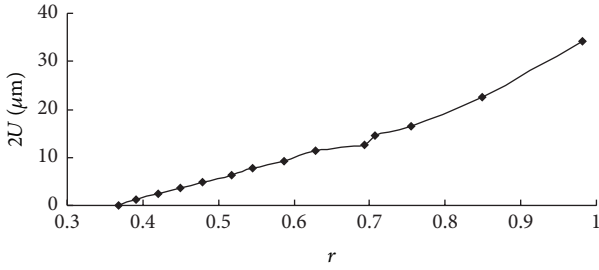
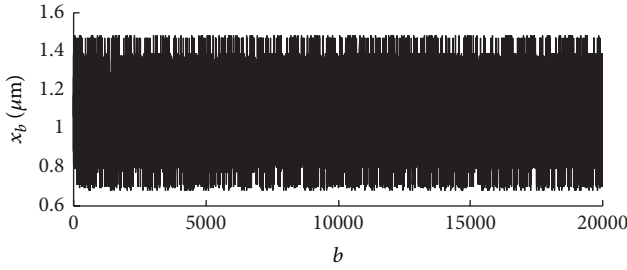FIGURE 8: The relationship between $2U$ and r (Case 2).
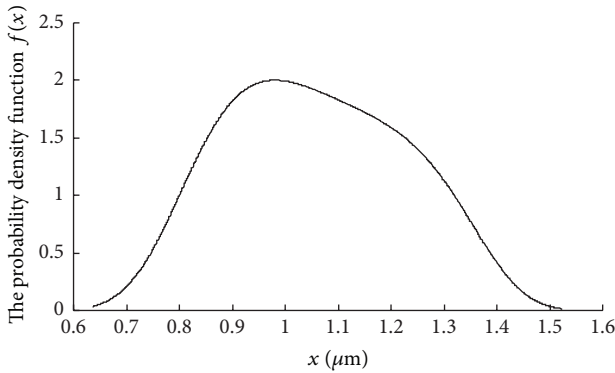

FIGURE 9: The intrinsic generated data sequence $\mathbf{X}_{GB}$ (Case 3).

FIGURE 10: The probability density function $f(x)$ (Case 3).

FIGURE 11: The inspection data sequence $\mathbf{X}_A$ (Case 3).
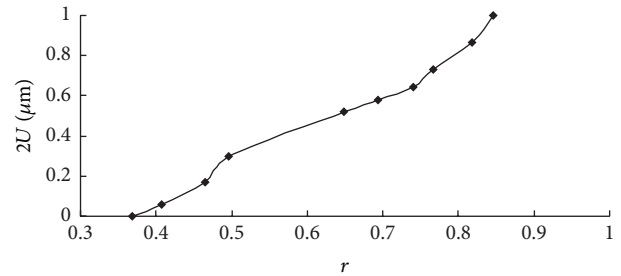
FIGURE 12: The relationship between $2U$ and r (Case 3).

quality of the manufacturing system is obtained. Then let $t = 1$; the reliability degree r of achieving running quality of the manufacturing system under the moment $t = 1$ is obtained. For the convenience of research, the relationship between 2 times the expanded uncertainty $2U$ and the reliability value r is as shown in Figure 12.

In Figure 12, $r \in [0.3679, 0.8465]$ and $2U \in [0\,\mu m, 1\,\mu m]$. On the whole, there is the nonlinear relationship between $2U$ and r. The nonlinear relationship can be divided into 3 regions with I, II, and III, corresponding to 3 approximate straight lines with different slopes, respectively.

In I region, $r \in [0.3679, 0.7408]$ and $2U \in [0\,\mu m, 0.65\,\mu m]$. It is obvious that the expanded uncertainty and the reliability degree are both small and the linear relationship between $2U$ and r is characterized by the small slope. It shows that the possibility of achieving high quality running state for the manufacturing system is small in I region.

In III region, $r \in [0.8187, 0.8465]$ and $2U \in [0.87\,\mu m, 1\,\mu m]$. It can be seen that the expanded uncertainty and the reliabi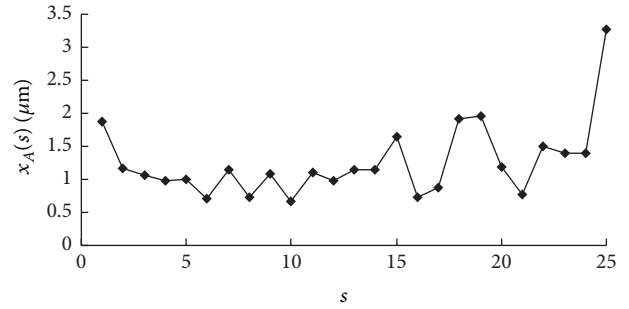lity degree are both large and the linear relationship between $2U$ and r is characterized by the large slope. It shows that the possibility of achieving low quality running state for the manufacturing system is large in III region.

In II region, $r \in [0.7408, 0.8187]$ and $2U \in [0.65\,\mu m, 0.87\,\mu m]$. It is easy to see that the expanded uncertainty $2U$, the reliability degree r, and the slope of the linear relationship between $2U$ and r are all moderate in comparison with the above two regions. It shows that the running quality of achieving the running state is in line with the possibility of achieving the quality of the running state, and both are moderate in II region.

3.2. Discussions on the Cases of Evaluation for the Running State of the Manufacturing System.
According to the evaluation results of the above 3 cases with respect to the running state of the manufacturing system, the comparative analysis of the above 3 cases is performed via the discussion and analysis of Figures 4, 8, and 12, to realize the comprehensive evaluation of the running state of the manufacturing system.

Cases 1 and 2 are the simulation experiments obeying a normal distribution and a Rayleigh distribution, respectively. The evaluation results of two simulation cases are compared and discussed to gain valuable knowledge of reliability evaluation for the running state of the manufacturing system.

Via comparing Figures 4 and 8, the following conclusions can be drawn.

On the whole, both can present the nonlinear linear relationship between $2U$ and r.

In I region, it is easy to see that both of the linear relationships between $2U$ and r are characterized by the small slope; namely, the variation trends of the approximate straight lines are pretty smooth.

In III region, both of the linear relationships between $2U$ and r are characterized by the large slope; namely the variation trends of the approximate straight lines are relatively steep.

In II region, both of the linear relationships between $2U$ and r are characterized by the moderate slope; namely the variation trends of the approximate straight lines are moderate.

That is, there are the roughly similar characteristic laws of $2U$ and r in Cases 1 and 2 as a whole. It shows that the running quality of achieving the running state is in line with the possibility of achieving the quality of the running state, and both are moderate in II region.

Case 3 is an actual case of evaluation for the running state of the manufacturing system with unknown probability distribution. By means of Cases 1 and 2, the evaluation results of the actual case and the simulation cases are compared and discussed. Via comparing Figure 12 to Figures 4 and 8, there are mass influence factors of the manufacturing system in practical processing in Figure 12, leading to decreasing slightly the maximum of the reliability degree r; namely, $r \in [0.3679, 0.8465]$. Hence the value range of the reliability degree r in Figure 12 becomes slightly smaller than $r \in [0.3679, 0.9921]$ in Figure 4 and $r \in [0.3679, 0.9921]$ in Figure 8. Additionally, it can be seen that the characteristic law of $2U$ and r in Figure 12 is roughly in conformity with Figures 4 and 8. It shows that the running quality of achieving

the running state of the manufacturing system is in line with the possibility of achieving the quality of the running state, and both are moderate in II region.

The simulation cases and the actual case show that the research method of the reliability evaluation for the running state of the manufacturing system is feasible, and the research results are of theoretical value and practical significance.

3.3. Cases of Evaluation for the Running State Variation Process of the Manufacturing System. With the increase of the processing time, the running state of the manufacturing system may emerge as random variation, which is regarded as the running state variation process of the manufacturing system. Because the variation trend is uncertain and the variation time is unknown, the reliability evaluation for the running state variation process of the manufacturing system is a demanding task, to ensure that the quality meets the requirements.

Case 4. This is a practical case of evaluation for the running state variation process of the manufacturing system with unknown probability distribution. A rolling bearing roller diameter grinding machine is involved to grind the roller diameter of the tapered rolling bearing with 30204 in the case. The 30 raw datasets of the average diameter deviation of the roller are collected in turn according to the processing sequence, in μm, and the result is as follows:

0.0118	0.0116	0.0102	0.0108	0.0106	0.0114	0.0057	0.0108	0.0100	0.0114
0.0114	0.0118	0.0115	0.0112	0.0112	0.0114	0.0130	0.0114	0.0114	0.0122
0.0121	0.0121	0.0115	0.0116	0.0109	0.0124	0.0117	0.0123	0.0114	0.0131

And it is easy to see the internal characteristic law of 30 raw datasets in Figure 13.

On the basis of the 30 raw datasets in Figure 13, this case is of real time evaluation for the running state variation process of the manufacturing system.

The former 6 raw datasets in Figure 13 are selected as the elements of the raw intrinsic data sequence $\mathbf{X}$ ($N = 6$). Based on the raw intrinsic data sequence $\mathbf{X}$, 30000 generated datasets are obtained using the grey bootstrap method and can constitute an intrinsic generated data sequence $\mathbf{X}_{GB}$ ($B = 30000$), as shown in Figure 14. Based on the intrinsic generated data sequence $\mathbf{X}_{GB}$, the probability density function $f(x)$ of the running state of the manufacturing system is obtained using the maximum entropy principle, as shown in Figure 15.

Let the confidence level $P = 95\%$, the confidence interval is obtained that $[X_L, X_U] = [0.0099 \,\mu\text{m}, 0.0121 \,\mu\text{m}]$, and 2 times the expanded uncertainty $2U$ is obtained that $2U = 0.0031 \,\mu\text{m}$ of the running state of the manufacturing system.

In order to evaluate the running state variation process of the manufacturing system in real time, the latter 24 raw datasets in Figure 13 are selected as the elements of the inspection data sequence $\mathbf{X}_A$ ($S = 24$), which are divided into 4 groups as the time interval to obtain the inspection data subsequence X_{Am} ($M = 4$) in the 4 time intervals in proper

order, respectively, namely, $X_{A1} \sim X_{A4}$. Each inspection data subsequence X_{Am} contains 6 raw data ($S = 6$). In view of $S = 6$, the inspection generated data sequences X_{GAm} ($M = 4$) can be obtained by the grey bootstrap method, respectively, namely, $X_{GA1} \sim X_{GA4}$ ($A = 30000$).

Based on the maximum entropy principle and Poisson process, by counting and calculating, the variation intensity λ of the running state of the manufacturing system is obtained by counting process, as shown in Table 1. Then the reliability function $R(t)$ of the running state of the manufacturing system is obtained using Poisson process with zero-failure probability. The reliability degree r ($t = 1$) of the running state of the manufacturing system is obtained according to (31), as shown in Table 1.

For further evaluating the running state variation process of the manufacturing system in real time, the inspection generated data sequences X_{GAm} is analyzed. Via taking the derivative of the reliability function $R(t)$ of the running state of the manufacturing system, namely, the derivative of (30), and making that the continuous time variable $t \in [0, +\infty)$, the probability density function $p(0, t)$ of the reliability of the running variation process of the manufacturing system as $t \in [0, +\infty)$, namely, (32), can be obtained. The probability density functions of the intrinsic data sequence $\mathbf{X}$ and the
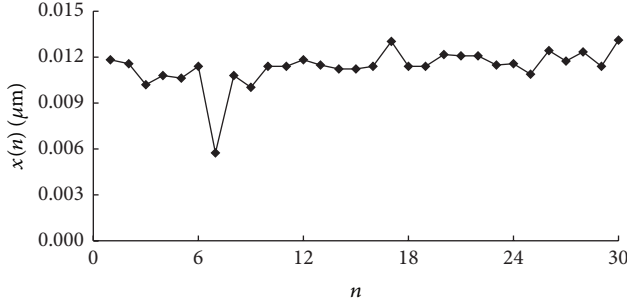
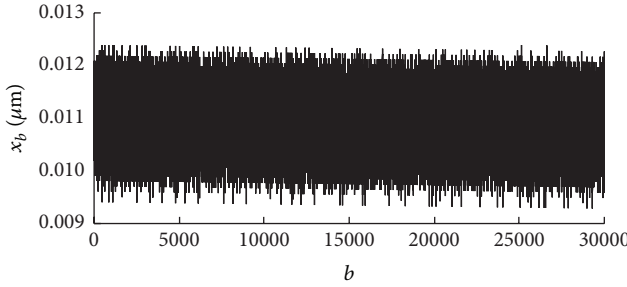Figure 13: The 30 raw datasets of the roller average diameter deviation (Case 4).

Figure 14: The intrinsic generated data sequence $\mathbf{X}_{GB}$ (Case 4).

Table 1: The variation intensity λ and the reliability degree r (Case 4).

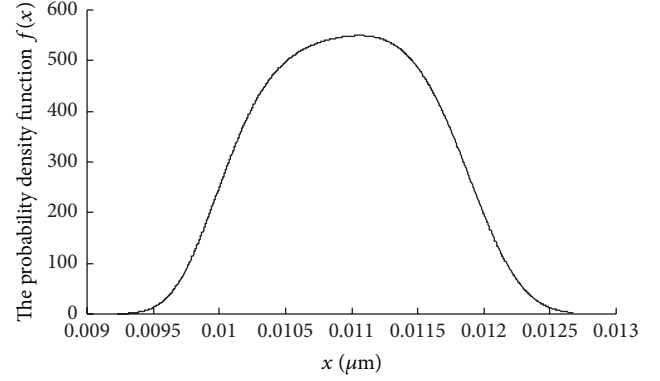| Number	Inspection data sequence	Variation intensity λ	Reliability degree r
1	X_{A1}	0.36287	0.695679
2	X_{A2}	0.26563	0.7667
3	X_{A3}	0.2536	0.7760
4	X_{A4}	0.4184	0.6581

Figure 15: The probability density function $f(x)$ (Case 4).
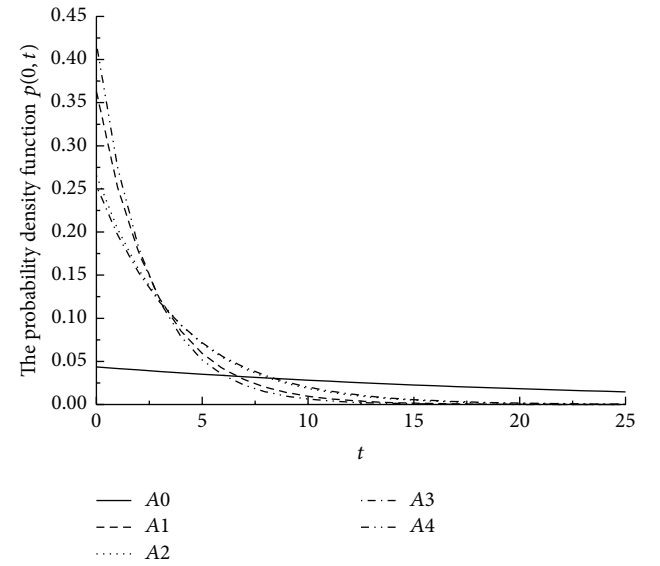
Figure 16: The variation of the reliability of the manufacturing system ($X_{A1} \sim X_{A4}$) (Case 4).

inspection data sequences $X_{A1} \sim X_{A4}$ are chosen to draw the graph as Figure 16, which can clearly reflect the running variation process of the manufacturing system. In Figure 16, the probability density function of the intrinsic data sequence $\mathbf{X}$ is described as the curve $A0$, and the probability density functions of the inspection data sequences $X_{A1} \sim X_{A4}$ are described as the curves $A1 \sim A4$, respectively.

In Figure 16, it is easy to see an intersection of the probability density function of the intrinsic data sequence and each inspection data sequence, namely, $X_{A1} \sim X_{A4}$, respectively, and there is only an intersection point of the curve $A0$ and the curves $A1 \sim A4$, respectively. Four intersection points are clearly observed in Figure 16 and are denoted as p_1, p_2, p_3, and p_4, corresponding to the curves $A1 \sim A4$, respectively.

Let the probability density function of the intrinsic data sequence be equal to the probability density function of the inspection data sequence, and the abscissa values (T_1, T_2, T_3, T_4) of the intersection points (p_1, p_2, p_3, p_4) of the intrinsic data sequence and the inspection data sequences are obtained according to (32) by solving equations, respectively; namely, $(T_1, T_2, T_3, T_4) = (6.6334, 8.1341, 8.3794, 6.0307)$.

Via putting the abscissa values (T_1, T_2, T_3, T_4) into (35), the intersection areas $A(T)$ of the probability density functions of the inspection data sequences and the intrinsic data sequence are solved by subsection integral to (32), respectively; namely, $\{A(T_1), A(T_2), A(T_3), A(T_4)\} = \{0.3416, 0.4142, 0.4259, 0.3117\}$, as shown in Figure 17. Using (36), the variation probability $P_B(T)$ of the inspection data sequence can be computed, respectively; namely, $\{P_B(T_1), P_B(T_2), P_B(T_3), P_B(T_4)\} = \{0.6584, 0.5858, 0.5741, 0.6883\}$, as shown in Figure 18. The calculation results are shown in Table 2.

In Figure 17, based on the intrinsic data sequence $\mathbf{X}$, it shows that the intersection area $A(T_1)$ of the probability density functions of X_{A1} and $\mathbf{X}$ is slightly smaller than $A(T_2)$ of the probability density functions of X_{A2} and $\mathbf{X}$, and $A(T_2)$ is rather smaller relative to $A(T_3)$ of the probability density functions of X_{A3} and $\mathbf{X}$. The main reason of the above phenomenon is that 0.0057 is a significant value less than other data in X_{A1}, as shown in Figure 13. From the point of view of the overall trend, the intersection area $A(T)$

TABLE 2: The running state variation process of the manufacturing system (Case 4).

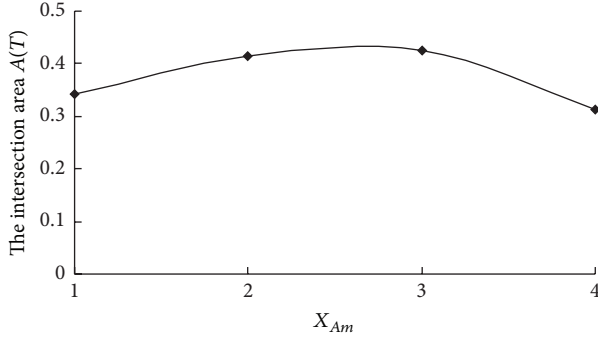| Number	Inspection data sequence	Abscissa value T	Intersection area $A(T)$	Variation probability $P_B(T)$
1	X_{A1}	6.6334	0.3416	0.6584
2	X_{A2}	8.1341	0.4142	0.5858
3	X_{A3}	8.3794	0.4259	0.5741
4	X_{A4}	6.0307	0.3117	0.6883

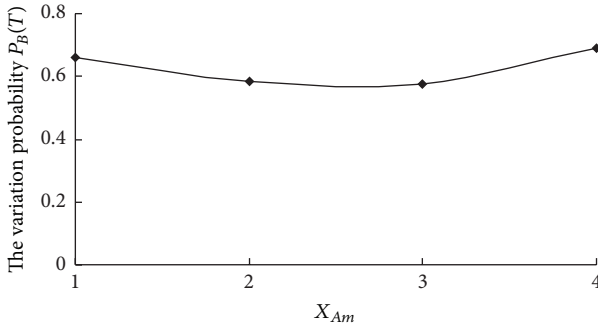FIGURE 17: The variation process of the intersection area $A(T)$ (Case 4).

FIGURE 18: The variation process of the variation probability $P_B(T)$ (Case 4).

FIGURE 19: The raw data sequence with a linear trend (Case 5).

appears as a downtrend as time accumulation. Figure 17 can reflect the law; namely, the smaller the intersection area $A(T)$ is, the larger the variation intensity λ of the running state variation process of the manufacturing system is and the less reliable the running state of the manufacturing system is. What is more, the law conforms that the running quality of the manufacturing system presents the evolution of the declining trend as the accumulation of processing time in the practical manufacturing process.

In Figure 18, based on the intrinsic data sequence **X**, it shows that the variation probability $P_B(T_1)$ of the inspection data sequence X_{A1} is slightly larger than the variation probability $P_B(T_2)$ of the inspection data sequence X_{A2}, and $P_B(T_2)$ is rather larger relative to the variation probability $P_B(T_3)$ of the inspection data sequence X_{A3}. The main reason of the above phenomenon is that 0.0057 is a significant value less than other data in X_{A1}, as shown in Figure 13. From the point of view of the overall trend, the variation probability $P_B(T)$ of the inspection data sequence appears as a rising
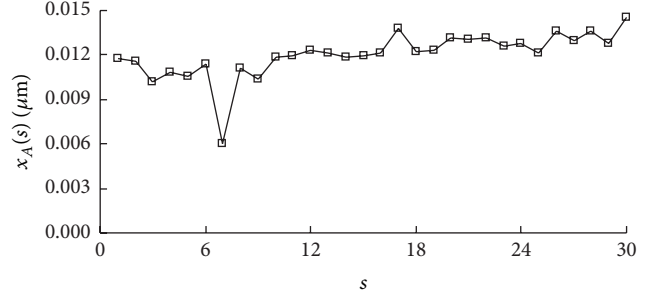
trend as time accumulation. Figure 18 can reflect the law; namely, the larger the variation probability $P_B(T)$ is, the larger the variation intensity λ of the running variation process of the manufacturing system is and the less reliable the running state of the manufacturing system is. What is more, the law conforms to the practical case that the running quality of the manufacturing system presents the evolution of the declining trend as the accumulation of processing time in the actual manufacturing process.

Case 5. This is a simulation case of evaluation for the running state variation process of the manufacturing system with unknown probability distribution. The case is further simulated as a manufacturing system with variation to assess the running state variation process based on Case 4.

Based on 30 raw datasets in Figure 13 of Case 4, a manufacturing system with a linear trend and unknown probability distribution is simulated to evaluate the running state variation process of a manufacturing system with variation in real time in Case 5. Then the contrastive analysis of the results in Cases 4 and 5 is implemented in the latter part in Case 5. The raw data sequence with a linear trend is simulated as shown in Figure 19.

For 30 raw datasets in Figure 19 of Case 5, the former 6 raw datasets in Figure 19 are the same as the former 6 raw datasets in Figure 13. Via artificially adding the trace linear component $y = y(s)$ to the latter 24 raw datasets in Figure 13, the latter 24 raw datasets with the linear component in Figure 19 are obtained to simulate a linear system. The trace linear component is shown as in Figure 20.

The former 6 raw datasets in Figure 19 are selected as the elements of the raw intrinsic data sequence **X** ($N = 6$). Based on the intrinsic data sequence **X**, the latter 24 raw datasets with $y = y(s)$ in Figure 19 are regarded as the inspection data sequence $\mathbf{X}_A$ of the running state variation process of the manufacturing system with variation.
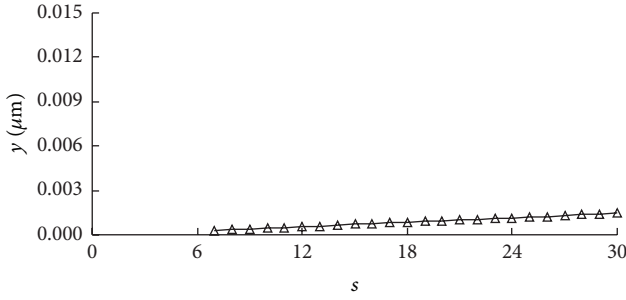
Figure 20: The simulation trace linear component (Case 5).

Table 3: The variation intensity λ and the reliability degree r (Case 5).

Number	Inspection data sequence	Variation intensity λ	Reliability degree r
1	X_{A1}	0.4914	0.6118
2	X_{A2}	0.5352	0.5856
3	X_{A3}	0.9860	0.3731
4	X_{A4}	0.9341	0.3929

In accordance with Case 4, let the confidence level $P =$ 95% and $B = 30000$. According to the grey bootstrap method and the maximum entropy principle, the confidence interval is obtained that $[X_L, X_U] = [0.0099\,\mu m, 0.0121\,\mu m]$ and 2 times the expanded uncertainty $2U$ is obtained that $2U = 0.0031\,\mu m$ for the running state of the manufacturing system.

In order to evaluate the running state variation process of the manufacturing system with variation in real time, referring to Case 4, the latter 24 datasets in Figure 19 are selected as the elements of the inspection data sequence $\mathbf{X}_A$, which are divided into 4 groups as the time interval to obtain the inspection data subsequence X_{Am} ($M = 4$) in the 4 time intervals in proper order, respectively, namely, $X_{A1} \sim X_{A4}$. Each inspection data subsequence X_{Am} contains 6 raw datasets ($S = 6$). In view of $S = 6$, the inspection generated data sequences X_{GAm} ($M = 4$) can be obtained by the grey bootstrap method, respectively, namely, $X_{GA1} \sim X_{GA4}$ ($A = 30000$).

Based on the maximum entropy principle and Poisson process, by counting and calculating, the variation intensity λ of the running state of the manufacturing system is obtained by counting process, as shown in Table 3. Then the reliability function $R(t)$ of the running state of the manufacturing system is obtained using Poisson process with zero-failure probability. The reliability degree r ($t = 1$) of the running state of the manufacturing system is obtained according to (31), as shown in Table 3.

In order to further evaluate the running state variation process of the manufacturing system in real time, the inspection generated data sequences X_{GAm} is analyzed. Via taking the derivative of the reliability function $R(t)$ of the running state of the manufacturing system, namely, the derivative of (30), and making that the continuous time variable $t \in [0, +\infty)$, the probability density function $p(0, t)$ of the reliability of the running variation process of the manufacturing
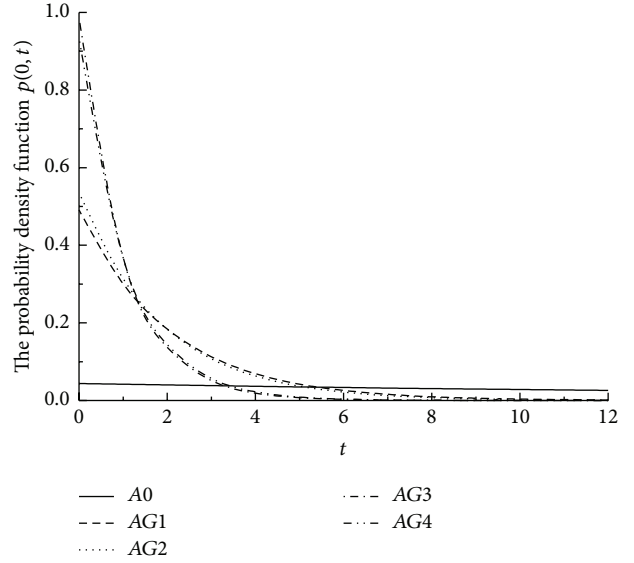
system as $t \in [0, +\infty)$ can be obtained. The probability density functions of the intrinsic data sequence $\mathbf{X}$ and the inspection data sequences $X_{A1} \sim X_{A4}$ are chosen to draw the graph as Figure 21, which can clearly reflect the running variation process of the manufacturing system. In Figure 21, the probability density function of the intrinsic data sequence $\mathbf{X}$ is described as the curve $A0$, and the probability density functions of the inspection data sequences $X_{A1} \sim X_{A4}$ are described as the curves $AG1 \sim AG4$, respectively.

In Figure 21, it is easy to see an intersection of the probability density function of the intrinsic data sequence and each inspection data sequence, namely, $X_{A1} \sim X_{A4}$, respectively, and there is only an intersection point of the curve $A0$ and the curves $AG1 \sim AG4$, respectively. And 4 intersection points are clearly observed as shown in Figure 21 and denoted as p_{G1}, p_{G2}, p_{G3}, and p_{G4}, corresponding to the curves $AG1 \sim AG4$, respectively.

Let the probability density function of the intrinsic data sequence be equal to the probability density function of the inspection data sequence, and the abscissa values $(T_{G1}, T_{G2}, T_{G3}, T_{G4})$ of the intersection points $(p_{G1}, p_{G2}, p_{G3}, p_{G4})$ of the intrinsic data sequence and the inspection data sequences are obtained according to (32) by solving equations, respectively; namely, $(T_{G1}, T_{G2}, T_{G3}, T_{G4}) =$ (5.4064, 5.0985, 3.3078, 3.4398). Via putting the abscissa values $(T_{G1}, T_{G2}, T_{G3}, T_{G4})$ into (35), the intersection areas $A(T)$ of the probability density functions of the inspection data sequences and the intrinsic data sequence are solved by subsection integral to (32), respectively; namely, $\{A(T_{G1}), A(T_{G2}), A(T_{G3}), A(T_{G4})\} = \{0.2805, 0.2649, 0.1728, 0.1797\}$, as shown in Figure 22. Using (36), the variation probability $P_B(T)$ of the inspection data sequence can be computed, respectively; namely, $\{P_B(T_{G1}), P_B(T_{G2}), P_B(T_{G3}), P_B(T_{G4})\} = \{0.7185, 0.7351, 0.8272, 0.8203\}$, as shown in Figure 23. The calculation results are shown in Table 4.

Figure 21: The variation of the reliability of the manufacturing system ($X_{A1} \sim X_{A4}$) (Case 5).

TABLE 4: The running state variation process of the manufacturing system (Case 5).

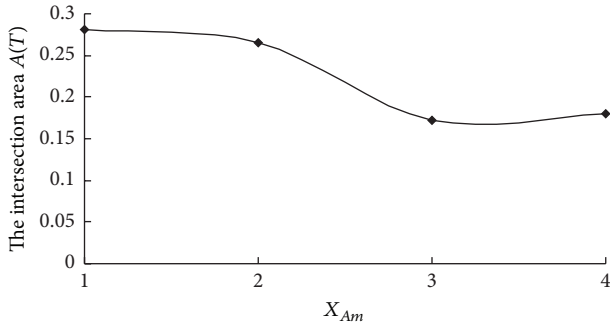| Number	Inspection data sequence	Abscissa value T	Intersection area $A(T)$	Variation probability $P_B(T)$
1	X_{A1}	5.4064	0.2805	0.7185
2	X_{A2}	5.0985	0.2649	0.7351
3	X_{A3}	3.3078	0.1728	0.8272
4	X_{A4}	3.4398	0.1797	0.8203

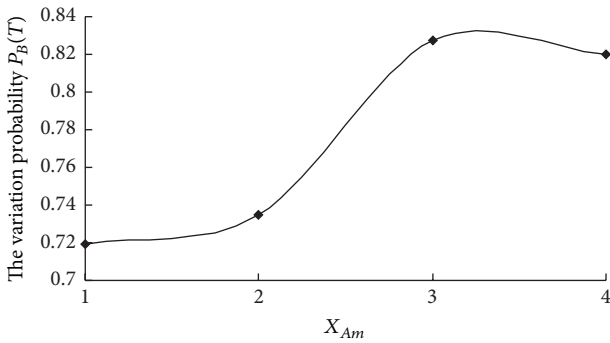FIGURE 22: The variation process of the intersection areas $A(T)$ (Case 5).

FIGURE 23: The variation process of the variation probability $P_B(T)$ (Case 5).

In Figure 22, based on the intrinsic data sequence **X**, it shows that the intersection area $A(T)$ appears as a downtrend as time accumulation from the point of view of the overall trend, except that the intersection area $A(T_{G4})$ of the probability density functions of X_{A4} and **X** is rather larger than $A(T_{G3})$ of the probability density functions of X_{A3} and **X**.

Case 5 is on the basis of Case 4 to evaluate the running state variation process of the manufacturing system with variation; hence 0.0057 is a significant value less than other data in X_{A1} of Case 5, which can affect the intersection area $A(T)$. However, the key factor of affecting the intersection area is the simulation trace linear component $y = y(s)$ in Case 5.

The simulated linear system can reflect the running state of the manufacturing system with variation by artificially adding the trace linear component $y = y(s)$ to the latter 24 raw datasets in Figure 13. The inspection data sequence **X**$_A$ of the running state variation process of the manufacturing system with variation is obtained and analyzed to simulate the running state variation process of the manufacturing system with variation. Figure 22 shows that the running state of the manufacturing system changes from good to moderate to bad and it can be reflected that the variation law of reliability of the manufacturing system changes from high to moderate to low. Obviously, the decreasing trend is consistent with the law that the manufacturing system performance gradually decays as the accumulation of processing time in the practical manufacturing process.

In Figure 23, on the basis of intrinsic data sequence **X**, it shows that the variation probability $P_B(T)$ appears as an increasing tendency as time accumulation as a whole, except that the variation probability $P_B(T_4)$ of the inspection data sequence X_{A4} is rather smaller than $P_B(T_3)$ of the inspection data sequence of X_{A3}.

Case 5 is on the basis of Case 4 to evaluate the running state variation process of the manufacturing system with variation; hence 0.0057 is a significant value less than other data in X_{A1} of Case 5, which can affect the variation probability $P_B(T)$. However, the key factor of affecting the intersection area is the simulation trace linear component $y = y(s)$ in Case 5.

The simulated linear system can reflect the running state of the manufacturing system with variation by artificially adding the trace linear component $y = y(s)$ to the latter 24 raw datasets in Figure 13. The inspection data sequence **X**$_A$ of the running state variation process of the manufacturing system with variation is obtained and analyzed to simulate the running state variation process of the manufacturing system with variation. Figure 23 shows that the running state of the manufacturing system changes from good to moderate to bad and it can be reflected that the variation law of reliability of the manufacturing system changes from high to moderate to low. Obviously, the decreasing trend is consistent with the law that the manufacturing system performance gradually decays as the accumulation of processing time in the practical manufacturing process.

4. The Sensitivity Analysis regarding the Size of the Samples

The sensitivity analysis is an approach to check the stability of the obtained results using the research method under a certain conditions. According to the research method presented in this paper, the variation intensity λ plays a decisive role to evaluate the running state of the manufacturing system in Sections 2.5 and 2.6; namely, the evaluation results of the running state of the manufacturing system depend on the variation intensity λ. Therefore, based on the raw
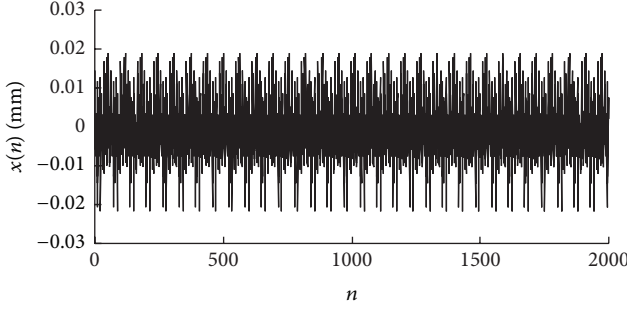
FIGURE 24: The 2000 simulation datasets obeying the normal distribution.

intrinsic data sequence **X**, the value of the variation intensity λ can be obtained according to the grey bootstrap method, the maximum entropy principle, and counting. Then, for the large research sample and the small research sample, the reliability degree r of achieving running quality, the intersection area $A(T)$ of the probability density functions, and the variation probability $P_B(T)$ can be computed with the aid of the variation intensity λ and Poisson process, so as to analyze the sensitivity degree of the research method proposed regarding the size of the samples, which can testify that the research method proposed in this paper is suitable for the research of small sample data.

Suppose that the mathematical expectation $E = 0$ mm and the standard deviation $s = 0.01$ mm are the known parameters, and 2000 simulation datasets obeying the normal distribution are generated by Monte Carlo simulation method to analyze the sensitivity of the research method proposed with regard to the size of the samples, as shown in Figure 24.

4.1. The Analysis of the Large Sample. The former 200 simulation datasets in Figure 24, namely, the large sample data, are selected as the elements of the raw intrinsic data sequence **X** ($N = 200$). Based on the raw intrinsic data sequence **X**, 20000 generated datasets are obtained using the grey bootstrap method and can constitute an intrinsic generated data sequence $\mathbf{X}_{GB}$ ($B = 20000$). Based on the intrinsic generated data sequence $\mathbf{X}_{GB}$, the probability density function $f(x)$ of the large sample data is obtained using the maximum entropy principle. Finally, let the confidence level $P = 95\%$, and the confidence interval of the large sample data is obtained that $[X_L, X_U] = [-0.01614 \text{ mm}, 0.0215 \text{ mm}]$. And the latter 1800 simulation datasets in Figure 24 are selected as the elements of the inspection data sequence $\mathbf{X}_A$ ($S = 1800$). Due to $S = 1800$, let $\mathbf{X}_{GA} = \mathbf{X}_A$ and $A = S$. According to (26), (28), and (29), it is obtained that there are 57 datasets within $\mathbf{X}_{GA}$ but outside the confidence interval $[-0.01614 \text{ mm}, 0.0215 \text{ mm}]$ by counting. Hence, the variation intensity λ_{large} of the large sample data is calculated that $\lambda_{\text{large}} = 57/1800 = 0.0316667$. The reliability function $R(t)$ of achieving running quality of the large research sample is obtained by Poisson process with zero-failure probability. Then let $t = 1$; the reliability degree r_{large} of achieving running quality of the large research sample

under the moment $t = 1$ is obtained according to (31); namely, $r_{\text{large}} = 0.968829471$.

4.2. The Analysis of the Small Sample. The former 8 simulation datasets in Figure 24, namely, the small sample data, are selected as the elements of the raw intrinsic data sequence **X** ($N = 8$). Based on the raw intrinsic data sequence **X**, 20000 generated datasets are obtained using the grey bootstrap method and can constitute an intrinsic generated data sequence $\mathbf{X}_{GB}$ ($B = 20000$). Based on the intrinsic generated data sequence $\mathbf{X}_{GB}$, the probability density function $f(x)$ of the small sample data is obtained using the maximum entropy principle. Finally, let the confidence level $P = 95\%$; the confidence interval of the small sample data is obtained that $[X_L, X_U] = [-0.01889 \text{ mm}, 0.02003 \text{ mm}]$. And the latter 1992 simulation datasets in Figure 24 are selected as the elements of the inspection data sequence $\mathbf{X}_A$ ($S = 1992$). Due to $S = 1992$, let $\mathbf{X}_{GA} = \mathbf{X}_A$ and $A = S$. According to (26), (28), and (29), it is obtained that there are 63 datasets within $\mathbf{X}_{GA}$ but outside the confidence interval $[-0.01889 \text{ mm}, 0.02003 \text{ mm}]$ by counting. Thus, the variation intensity λ_{small} of the small sample data is computed that $\lambda_{\text{small}} = 63/1992 = 0.0316265$. The reliability function $R(t)$ of achieving running quality of the small research sample is obtained by Poisson process with zero-failure probability. Then let $t = 1$; the reliability degree r_{small} of achieving running quality of the small research sample under the moment $t = 1$ is obtained according to (31); namely, $r_{\text{small}} = 0.968868381$.

4.3. The Sensitivity Analysis of Large and Small Samples. In order to visually judge the sensitivity degree of the research method proposed with respect to the size of the samples, it is necessary to solve the intersection area $A(T)$ of the probability density functions of the large sample data and the small sample data, as well as the variation probability $P_B(T)$ of the small sample data relative to the large sample data.

Let the probability density function of the large sample data be equal to the probability density function of the small sample data, by means of λ_{large} and λ_{small}, and the evaluation results are solved according to (32), (35), and (36). Specific results are as follows: the abscissa value T of the intersection point of the probability density functions of the large sample data and the small sample data is $T = 31.598988858$; the intersection area $A(T)$ of the probability density functions of the large sample data and the small sample data is $A(T) = 0.9995331437$; the variation probability $P_B(T)$ of the small sample data relative to the large sample data is $P_B(T) = 0.0004668563$.

Now on the basis of the research results of the large sample data, the comparative analysis of the research results of the large sample data and the small sample data can be put into effect to compute the relative error of the research results of them. In the light of the relative error of the research results of the large sample data and the small sample data, it is easy to judge that the sensitivity degree of the research method was proposed with respect to the large and small samples, which can realize the sensitivity analysis of the research method proposed regarding the size of the samples.

The related results of the comparative analysis are as follows: with the help of the analysis results of the large sample data, the variation intensity λ_{small} is only decreased by 0.1268242% relative to λ_{large}; the reliability degree r_{small} is increased by 0.0040162% relative to r_{large}. What is more, since the intersection area $A(T) = 0.9995331437$ and the variation probability $P_B(T) = 0.0004668563$, it can be inferred that the relative error of the intersection area of the raw intrinsic data sequence with large sample and the intersection area of the raw intrinsic data sequence with large sample is only 0.04668563%; and the relative error of the variation probability of the raw intrinsic data sequence with large sample and the variation probability of the raw intrinsic data sequence with large sample is only 0.04668563%. By the above comparative analysis results, the computed results obtained by the research method proposed in the paper have no essential change and have a good robustness.

The sensitivity analysis regarding the size of the samples shows that the size of the samples has no effect on the evaluation results of the research object. The sensitivity degree of the research method proposed in this paper with regard to the size of the samples is small, which is suffice to show that the research method proposed can solve the problem with small sample of the running state of the manufacturing system, and the evaluation results are trustworthy.

5. Discussions

5.1. Discussions on Evaluation for the Running State. According to the results of the former 3 cases, the possibility of achieving high quality running state for the manufacturing system is small in I region. The possibility of achieving low quality running state for the manufacturing system is larger in III region. In II region, the running quality of achieving the running state is in line with the possibility of achieving the quality of the running state, and both are moderate in II region. It shows that II region is a region of the good running state for the manufacturing system.

For the former 3 cases, the relationship between the confidence level P and the reliability degree r is drawn as shown in Figure 25. In Figure 25, it can be seen that three representative values of the confidence level P are chosen from Figure 25, namely, 0.9, 0.95, and 0.99, respectively, which can meet the reliability degree $r \in [0.7072, 0.9233]$ in II region. Then it is performed to determine the best value from three representative values with respect to the confidence level by hypothetical testing, so as to discover the best choice of the running state of the manufacturing system in II region. According to the statistics, the significant levels corresponding to the three confidence levels are 0.1, 0.05, and 0.01, respectively. According to the significant hypothesis testing principle, an assumption that II region is the region to keep the good running state of the manufacturing system is given and the significant level of the assumption can reach 0.1~0.01. It shows that the research results are significant, which are of theoretical significance and application value.

It can be found from the relationship of the reliability degree and the extended uncertainty that if the confidence
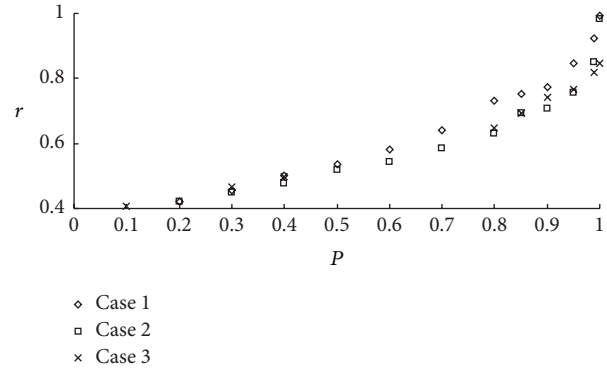

FIGURE 25: The relationship between the confidence level P and the reliability degree r.

level $P = 0.9$, the expanded uncertainty is small but the reliability degree r is about 0.75 and 0.75 is small and not desirable. If P increases from 0.9 to 0.95 and the reliability degree r is about 0.8 and its value increases by 6.7%, meanwhile the extended uncertainty increases by 3.4%~9.4%. If P increases from 0.9 to 0.99 and the reliability degree r is about 0.85 and its value increases by 13.3%, meanwhile the extended uncertainty increases by 20% ~60%. It is easy to see that the confidence level $P = 0.95$ is the best point in II region. The expanded uncertainty is moderate and the reliability degree r is about 0.8 under the confidence level $P = 0.95$. Therefore, the confidence level $P = 0.95$ is the best choice of the running state of the manufacturing system; namely, the consistency of the running quality of a manufacturing system and the possibility of achieving the running quality are the best.

The reliability evaluation for the running quality of the manufacturing system includes some elements, such as the confidence level, the confidence interval, 2 times the expanded uncertainty, and the reliability degree of achieving running quality with respect to the workpiece quality.

From the above discussion, the best running state of the manufacturing system in 3 cases can be evaluated, as follows:

(1) For Case 1, the running quality of the manufacturing system with the normal distribution is that the reliability degree r of achieving running quality is 0.8468 under the confidence level $P = 0.95$ and the confidence interval of the processing quality Q_1 is $[X_L, X_U] = [-15.75 \,\mu m, 12.78 \,\mu m]$ and 2 times the expanded uncertainty of the processing quality Q_1 is $2U = 28.53 \,\mu m$.

(2) For Case 2, the running quality of the manufacturing system with the Rayleigh distribution is that the reliability degree r of achieving running quality is 0.7552 under the confidence level $P = 0.95$ and the confidence interval of the processing quality Q_2 is $[X_L, X_U] = [12.23 \,\mu m, 28.63 \,\mu m]$ and 2 times the expanded uncertainty of the processing quality Q_2 is $2U = 16.4 \,\mu m$.

(3) For Case 3, the running quality of the rolling bearing inner raceway grinding machine with unknown

probability distribution is that the reliability degree r of achieving running quality is 0.7659 under the confidence level $P = 0.95$ and the confidence interval of the inner raceway roundness is $[X_L, X_U] = [0.65\,\mu m, 1.38\,\mu m]$ and 2 times the expanded uncertainty of the inner raceway roundness is $2U = 0.73\,\mu m$.

5.2. Discussions on Evaluation for the Running State Variation Process. Based on the discussion results of the above 3 cases, Cases 4 and 5 are under the best confidence level $P = 95\%$. The confidence interval is obtained that $[X_L, X_U] = [0.0099\,\mu m, 0.0121\,\mu m]$ and 2 times the expanded uncertainty $2U$ is obtained that $2U = 0.0031\,\mu m$ for the running state of the manufacturing system. Based on the above researches, there are two evaluations of the running state variation process of the manufacturing system with no variation and variation.

In order to evaluate the running state variation process in real time, the reliability evaluation for the running quality of the manufacturing system includes 3 elements, such as the reliability degree of achieving running quality, the intersection area $A(T)$, and the variation probability $P_B(T)$ of the probability density function of each inspection data sequence relative to the intrinsic data sequence.

In Case 4, the running quality of the running state variation process of the manufacturing system is that when $t = 1$, the value range allowed for the reliability degree r of achieving running quality is $r \in (0.6581, 0.7760)$. When $t \in [0, +\infty)$, the value range allowed for the intersection area is $A(T) \in (0.3117, 0.4259)$, and the value range allowed for the variation probability is $P_B(T) \in (0.5741, 0.6883)$.

In Case 5, the running quality of the running state variation process of the manufacturing system is that when $t = 1$, the value range allowed for the reliability degree r of achieving running quality is $r \in (0.3731, 0.6118)$. When $t \in [0, +\infty)$, the value range allowed for the intersection area is $A(T) \in (0.1728, 0.2805)$, and the value range allowed for the variation probability is $P_B(T) \in (0.7185, 0.8272)$.

By comparing the research results of Cases 4 and 5, based on Case 4, the reliable running quality of the running state variation process of the manufacturing system with variation can be defined as follows: when $t = 1$, the value range allowed for the reliability degree r of achieving running quality is $r > 0.65$, and when $t \in [0, +\infty)$, the value range allowed for the intersection area is $A(T) > 0.3$, and the value range allowed for the variation probability is $P_B(T) \le 0.7$. That is, only if the 3 elements of the running quality of the manufacturing system are in the above ranges, the running state variation process of the manufacturing system is reliable, which can meet the quality requirements of products.

In Case 5 relative to Case 4, when $t = 1$, the value range allowed for the reliability degree r of achieving running quality is $r < 0.65$, and when $t \in [0, +\infty)$, the value range allowed for the intersection area is $A(T) < 0.29$, and the value range allowed for the variation probability is $P_B(T) > 0.71$. At the moment, it means that the variation degree of the manufacturing system in Case 5 is too large to continue processing workpieces. It should timely stop manufacturing production and the manufacturing system should be conducted on inspection, adjustment or maintenance, and so forth, to ensure the good running state of the manufacturing system and guarantee the products up to standard.

6. Conclusions

The conclusions of this paper are as follows:

(1) A new evaluation method based on poor information is proposed to evaluate the reliability of the running state of manufacturing system under the condition of small sample size with known or unknown probability distributions in this paper. In the case of unknown and known probability distributions, small sample data obtained by detecting the workpiece quality are processed using the grey bootstrap theory and the maximum entropy principle to obtain the variation intensity of the running state of the manufacturing system by counting. With the help of Poisson process, the reliability model is established to realize reliability evaluation for the running state of the manufacturing system with no variation and variation. It is aimed to effectively determine the running quality of the manufacturing system, so as to ensure the product quality and reduce manufacturing costs.

(2) The evaluation results of the running state show that, based on the relationship between 2 times the expanded uncertainty and the reliability degree, II region is considered as the best choice of the good running state of the manufacturing system. Via hypothesis testing and contrastive analysis of the results, it is verified that the confidence level $P = 0.95$ is the best point of II region, namely, the best choice of the running state of the manufacturing system, which can provide theory basis for reasonable adjustment of the machine tool.

(3) The evaluation results of the running state variation process show that, under the best confidence level $P = 95\%$, the reliability graphs of the running state of the manufacturing system can predict the evolution of the running state of the manufacturing system in real time. Through the comparison and analysis, the reliable running quality of the running state variation process of the manufacturing system with variation is that the reliability degree r of achieving running quality is $r > 0.65$, the intersection area is $A(T) > 0.3$, and the variation probability is $P_B(T) \le 0.7$. The above researches are applied to discover the running state of the manufacturing system bad in time and avoid heavy economic losses.

(4) The sensitivity analysis regarding the size of the samples indicates that the size of the research sample does not affect the evaluation results of the running state of the manufacturing system by the research method proposed in the paper. The research method proposed is feasible to assess the reliability of the running state of the manufacturing system, which can acquire favorable evaluation effect.

Competing Interests

Acknowledgments

References

[1] G. Schuh, T. Potente, and A. Hauptvogel, "Methodology for the evaluation of forecast reliability of production planning systems," *Procedia CIRP*, vol. 17, pp. 469–474, 2014.

[2] Y. H. Zhou, "Analysis of the process reliability of machinery manufacturing," *Applied Mechanics and Materials*, vol. 329, pp. 215–218, 2013.

[3] J. Ma, Y. He, and C. Wu, "Research on reliability estimation for mechanical manufacturing process based on Weibull analysis technology," in *Proceedings of the 3rd Annual IEEE Prognostics and System Health Management Conference (PHM '12)*, Beijing, China, May 2012.

[4] W. J. Zhang, Y. P. Qian, W. Huang, and C. Yang, "Study on the conception of process reliability in mechanical manufacture," in *Proceedings of the 8th International Conference on Reliability, Maintainability and Safety (ICRMS '09)*, pp. 995–997, IEEE, Chengdu, China, July 2009.

[5] M. Samadani, S. Behbahani, and C. Nataraj, "A reliability-based manufacturing process planning method for the components of a complex mechatronic system," *Applied Mathematical Modelling*, vol. 37, no. 24, pp. 9829–9845, 2013.

[6] W. Dai, P. G. Maropoulos, and Y. Zhao, "Reliability modelling and verification of manufacturing processes based on process knowledge management," *International Journal of Computer Integrated Manufacturing*, vol. 28, no. 1, pp. 98–111, 2015.

[7] B. Li, D. R. Metzger, and T. J. Nye, "Reliability analysis of the tube hydroforming process based on forming limit diagram," *Journal of Pressure Vessel Technology, Transactions of the ASME*, vol. 128, no. 3, pp. 402–407, 2006.

[8] W. Li, D. Yuan, H. Liu, and T. Zhang, "Reliability research on the mechanism system wear simulation under the case of the small-scale sample," *Journal of Mechanical Engineering*, vol. 51, no. 13, pp. 235–244, 2015.

[9] Komal, "Fuzzy reliability analysis of plastic-pipe manufacturing system using non-linear programming approach," *International Journal of Industrial and Systems Engineering*, vol. 17, no. 1, pp. 98–114, 2014.

[10] Y.-K. Lin and P.-C. Chang, "Predecessor-set technique for reliability evaluation of a stochastic manufacturing system," *Journal of Systems Science and Systems Engineering*, vol. 24, no. 2, pp. 190–210, 2014.

[11] G. H. Lin, "Process reliability assessment with a Bayesian approach," *The International Journal of Advanced Manufacturing Technology*, vol. 25, no. 3-4, pp. 392–395, 2005.

[12] Q. Wang, Z.-Y. Wang, Z. P. Mourelatos, and J.-H. Fu, "Estimation of measurement results with poor information using the dynamic bootstrap grey method," *Measurement*, vol. 57, pp. 138–147, 2014.

[13] Q. S. He, S. F. Xiao, and X. E. Liu, "Research of fuzzy-norm method and its application in uncertainty metric with poor information," *Procedia Engineering*, vol. 15, pp. 1350–1354, 2011.

[14] T. Ferraro, "Data Rich but Information Poor: adopting procedures for efficient data sharing is a low-cost way to shorten development cycles," *Scientific Computing*, vol. 23, no. 5, pp. 18–19, 2006.

[15] X. T. Xia, "Reliability analysis of zero-failure data with poor information," *Quality and Reliability Engineering International*, vol. 28, no. 8, pp. 981–990, 2012.

[16] J. Zhang, J. Chen, and J. Du, "Reliability analysis of a mechanism with grey system parameters," *Advanced Materials Research*, vol. 538–541, pp. 3154–3159, 2012.

[17] Ş. Şenol, "Poisson process approach to determine the occurrence degree in failure mode and effect reliability analysis," *Quality Management Journal*, vol. 14, no. 2, pp. 29–40, 2007.

[18] X. X. Huang and J. Q. Chen, "Time-dependent reliability model of deteriorating structures based on stochastic processes and bayesian inference methods," *Journal of Engineering Mechanics*, vol. 141, no. 3, Article ID 04014123, pp. 1–11, 2015.

[19] S. Q. Wang, Y. M. Wu, M. Y. Lu, and H. F. Li, "Discrete nonhomogeneous poisson process software reliability growth models based on test coverage," *Quality and Reliability Engineering International*, vol. 29, no. 1, pp. 103–112, 2013.

[20] I. Iervolino, M. Giorgio, and B. Polidoro, "Reliability of structures to earthquake clusters," *Bulletin of Earthquake Engineering*, vol. 13, no. 4, pp. 983–1002, 2015.

[21] P. L. C. Saldanha, E. A. de Simone, and P. F. Frutuoso e Melo, "An application of non-homogeneous Poisson point processes to the reliability analysis of service water pumps," *Nuclear Engineering and Design*, vol. 210, no. 1–3, pp. 125–133, 2001.

[22] B. Efron, "Bootstrap methods: another look at the jackknife," *The Annals of Statistics*, vol. 7, no. 1, pp. 1–26, 1979.

[23] J. J. Reeves, "Bootstrap prediction intervals for ARCH models," *International Journal of Forecasting*, vol. 21, no. 2, pp. 237–248, 2005.

[24] J. L. Deng, "Introduction to grey system theory," *The Journal of Grey System*, vol. 1, no. 1, pp. 1–24, 1989.

[25] Y. Yatracos, "Assessing the quality of bootstrap samples and of the bootstrap estimates obtained with finite resampling," *Statistics & Probability Letters*, vol. 59, no. 3, pp. 281–292, 2002.

[26] E. T. Jaynes, "Information theory and statistical mechanics," *Physical Review*, vol. 106, no. 4, pp. 620–630, 1957.

[27] J. Trebicki and K. Sobczyk, "Maximum entropy principle and non-stationary distributions of stochastic systems," *Probabilistic Engineering Mechanics*, vol. 11, no. 3, pp. 169–178, 1996.

[28] S. Sankaran and N. Zabaras, "A maximum entropy approach for property prediction of random microstructures," *Acta Materialia*, vol. 54, no. 8, pp. 2265–2276, 2006.