Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2016, Article ID 5185784, 11 pages
http://dx.doi.org/10.1155/2016/5185784

Research Article

Robust Stability, Stabilization, and H_∞ Control of a Class of Nonlinear Discrete Time Stochastic Systems

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Received 14 November 2015; Revised 20 March 2016; Accepted 31 March 2016

Academic Editor: Mingcong Deng

This paper studies robust stability, stabilization, and H_∞ control for a class of nonlinear discrete time stochastic systems. Firstly, the easily testing criteria for stochastic stability and stochastic stabilizability are obtained via linear matrix inequalities (LMIs). Then a robust H_∞ state feedback controller is designed such that the concerned system not only is internally stochastically stabilizable but also satisfies robust H_∞ performance. Moreover, the previous results of the nonlinearly perturbed discrete stochastic system are generalized to the system with state, control, and external disturbance dependent noise simultaneously. Two numerical examples are given to illustrate the effectiveness of the proposed results.

1. Introduction

Stochastic control has been one of the most important research topics in modern control theory. The study of stochastic stability can be traced back to the 1960s; see [1] and the recently well-known monographs [2, 3]. Stability is the first considered problem in system analysis and synthesis, while stabilization is to look for a controller to stabilize an unstable system. H_∞ control is one of the most important robust control approaches, which aims to design the controller to restrain the external disturbance below a given level. We refer the reader to [4–9] for stability and stabilization of Itô-type stochastic systems and [10–14] for stability and stabilization of discrete time stochastic systems. Stochastic H_∞ control of Itô-type systems starts from [15], which has been extensively studied in recent years; see [16–20] and the references therein. Discrete time H_∞ control with multiplicative noise can be found in [21–25].

Along with the development of computer technology, discrete time difference systems have attracted a great deal of attention, which have been studied extensively; see [26, 27]. The reason is twofold: Firstly, discrete time systems are ideal mathematical models in the study of satellite attitude control [28], mathematical finance [29], single degree of freedom inverted pendulums [21], and gene regulator networks [30]. Secondly, as said in [27], the study for discrete time systems has the advantage over continuous time differential systems from the perspective of computation; moreover, it presents a very good approach to study differential equations and functional differential equations.

From the existing works on stability, stabilization, and H_∞ control of discrete time stochastic systems with multiplicative noise, we can find that, except for linear stochastic systems where perfect results have been obtained [22–25], few works are on the stability of the general nonlinear discrete time stochastic system [12]

$$x(t+1) = f(x(t), w(t), t), \quad x(0) = x_0 \tag{1}$$

or the H_∞ control of affine nonlinear discrete time stochastic system [21]

$$\begin{aligned} x(t+1) &= f(x(t)) + g(x(t))u(t) + h(x(t))v(t) \\ &\quad + [f_1(x(t)) + g_1(x(t))u(t) + h_1(x(t))]w(t), \\ z(t) &= L(x(t)). \end{aligned} \tag{2}$$

Up to now, the results of the deterministic discrete time nonlinear H_∞ control [31] have not been perfectly generalized to the above nonlinear stochastic systems. For example, although some stability results in continuous time Itô systems [3] can be extended to nonlinear discrete stochastic systems [12], the corresponding criteria are not easily applied in practice; this is because the mathematical expectation of the trajectory is involved in the preconditions. In addition, [21] tried to discuss a general nonlinear H_∞ control of discrete time stochastic systems, but only the H_∞ control of a class of norm bounded systems was perfectly solved based on linear matrix inequality (LMI) approach. As said in [32], the general H_∞ control of nonlinear discrete time stochastic multiplicative noise systems remains unsolved. We have to admit such a fact that some research issues of discrete systems are more difficult to solve than those of continuous time systems. For instance, a stochastic maximum principle for Itô systems was obtained in 1990 [33], but a nonlinear discrete time maximum principle has just been presented in [34].

Recently, [7, 13] investigated the robust quadratic stability and feedback stabilization of a class of nonlinear continue time and discrete time systems, respectively, where the nonlinear terms are quadratically bounded. Such a nonlinear constraint possesses great practical importance and has been widely used in many types of systems, such as singularly perturbed systems with uncertainties [35, 36], neutral systems with nonlinear perturbations [37], impulsive Takagi-Sugeno fuzzy systems [38], and some time-delay systems [18]. It should be pointed out that the small gain theorem can also be used to examine the robustness as done in [39] for the study of the simple adaptive control system within the framework of the small gain theorem. In addition, the robustness of a class of nonlinear feedback systems with unknown perturbations was discussed based on the robust right coprime factorization and passivity property [40]. All these methods are expected to play important roles in stochastic uncertain H_∞ control.

This paper deals with a class of nonlinear uncertain discrete time stochastic systems, for which the system state, control input, and external disturbance depend on noise simultaneously, which was often called (x, u, v)-dependent noise for short [24] and which mean that not only the system state as in [21] but also the control input and external disturbance are subject to random noise. Hence, our concerned models have more wide applications. The considered nonlinear dynamic term is priorly unknown but belongs to a class of functions with a bounded energy level, which represent a kind of very important nonlinear functions, and has been studied by many researchers; see, for example, [41]. For such a class of nonlinear discrete time stochastic systems, the stochastic stability, stabilization, and H_∞ control have been discussed, respectively, and easily testing criteria have also been obtained. What we have obtained extends the previous works to more general models.

The paper is organized as follows: in Section 2, we give a description of the considered nonlinear stochastic systems and define robust stochastic stability and stabilization. Section 3 contains our main results. Section 3.1 presents a robust stability criterion which extends the result of [13] to more general stochastic systems. Section 3.2 gives a sufficient condition for robust stabilization criterion. Section 3.3 is about H_∞ control, where an LMI-based sufficient condition for the existence of a static state feedback H_∞ controller is established. All our main results are expressed in terms of LMIs. In Section 4, two examples are constructed to show the effectiveness of our obtained results.

For convenience, the notations adopted in this paper are as follows.

M' is the transpose of the matrix M or vector M, $M \geq 0$ ($M > 0$): M is a positive semidefinite (positive definite) symmetric matrix; I is the identity matrix; R^n is the n-dimensional Euclidean space; $R^{n\times m}$ is the space of all $n \times m$ matrices with entries in R; N is the natural number set; that is, N represents $\{0, 1, 2, \ldots\}$; N_t denotes the set of $\{0, 1, \ldots, t\}$; $l^2_w(N, R^n)$ denotes the set of all nonanticipative square summable stochastic processes

$$y = \{y_n : y_n \in L^2(\Omega, R^n),\ y_n \text{ is } \mathscr{F}_{n-1} \text{ measurable}\}_{n\in N}. \tag{3}$$

The l^2-norm of $y \in l^2_w(N, R^n)$ is defined by

$$\|y\|_{l^2_w(N,R^n)} = \left(\sum_{n=0}^{\infty} E\|y_n\|^2\right)^{1/2}. \tag{4}$$

Similarly, $l^2_w(N_T, R^n)$ and $\|y\|_{l^2_w(N,R^n)}$ can be defined.

2. System Descriptions and Definitions

Consider the discrete stochastic iterative system described by the following equation:

$$\begin{aligned} x(t+1) &= Ax(t) + h_1(t, x(t)) + Bu(t) \\ &\quad + (Cx(t) + h_2(t, x(t)) + Du(t))\,w(t), \\ &\qquad x(0) = x_0 \in R^n,\ t \in N, \end{aligned} \tag{5}$$

where $x(t) \in R^n$ is the n-dimensional state vector and $u(t) \in R^m$ is the m-dimensional control input. $\{w(t)\}_{t\geq 0}$ is a sequence of one-dimensional independent white noise processes defined on the complete filtered probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, \mathscr{P})$, where $\mathscr{F}_t = \sigma\{w(0), w(1), \ldots, w(t)\}$. Assume that $E[w(t)] = 0$, $E[w(t)w(j)] = \delta_{tj}$, where E stands for the expectation operation and δ_{tj} is a Kronecker function defined by $\delta_{tj} = 0$ for $t \neq j$ while $\delta_{tj} = 1$ for $t = j$. Without loss of generality, x_0 is assumed to be determined. The following is assumed to hold throughout this paper.

Assumption 1. The nonlinear functions $h_1(t, x(t))$ and $h_2(t, x(t))$ describe parameter uncertainty of the system and satisfy the following quadratic inequalities:

$$h_1'(t, x(t))\,h_1(t, x(t)) \leq \alpha_1^2 x'(t) H_1' H_1 x(t), \tag{6}$$

$$h_2'(t, x(t))\,h_2(t, x(t)) \leq \alpha_2^2 x'(t) H_2' H_2 x(t), \tag{7}$$

for all $t \in N$, where α_i is a constant related to the function h_i for $i = 1, 2$. H_i is a constant matrix reflecting structure of h_i.

We note that inequalities (6) and (7) can be written as a matrix form:

$$\begin{bmatrix} x \\ h_1 \\ h_2 \end{bmatrix}' \begin{bmatrix} -\alpha_1^2 H_1' H_1 - \alpha_2^2 H_2' H_2 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} x \\ h_1 \\ h_2 \end{bmatrix} \leq 0. \quad (8)$$

System (5) is regarded as the generalized version of the system in [13, 42]. We note that, in system (5), the system state, control input, and uncertain terms depend on noise simultaneously, which makes (1) more useful in describing many practical phenomena.

Definition 2. The unforced system (5) with $u = 0$ is said to be robustly stochastically stable with margins $\alpha_1 > 0$ and $\alpha_2 > 0$ if there exists a constant $\delta(x_0, \alpha_1, \alpha_2)$ such that

$$E\left[\sum_{t=0}^{\infty} x'(t)\, x(t)\right] \leq \delta(x_0, \alpha_1, \alpha_2). \quad (9)$$

Definition 2 implies $E\{\|x(t)\|^2\} \to 0$.

Definition 3. System (5) is said to be robustly stochastically stabilizable if there exists a state feedback control law $u(t) = Kx(t)$, such that the closed-loop system

$$\begin{aligned} x(t+1) &= (A + BK)\, x(t) + h_1(t, x(t)) \\ &\quad + ((C + DK)\, x(t) + h_2(t, x(t)))\, w(t), \quad (10) \\ &\quad x(0) = x_0 \in R^n, \ t \in N \end{aligned}$$

is robustly stochastically stable for all nonlinear functions $h_i(t, x(t))$ $(i = 1, 2)$ satisfying (6) and (7).

When there is the external disturbance $\nu(\cdot)$ in system (5), we consider the following nonlinear perturbed system:

$$\begin{aligned} x(t+1) &= Ax(t) + A_0 \nu(t) + Bu(t) + h_1(t, x(t)) \\ &\quad + (Cx(t) + C_0 \nu(t) + Du(t) + h_2(t, x(t)))\, w(t), \\ z(t) &= \begin{bmatrix} Lx(t) \\ M\nu(t) \end{bmatrix} \quad (11) \\ &\quad x(0) = x_0 \in R^n, \ t \in N, \end{aligned}$$

where $\nu(t) \in R^q$ and $z(t) \in R^p$ are, respectively, the disturbance signal and the controlled output. $\nu(t)$ is assumed to belong to $l_w^2(N, R^q)$, so $\nu(t)$ is independent of $w(t)$.

Definition 4 (H_∞ control). For a given disturbance attenuation level $\gamma > 0$, $u(t) = Kx(t)$ is an H_∞ control of system (11), if

(i) system (11) is internally stochastically stabilizable for $u(t) = Kx(t)$ in the absence of $\nu(t)$; that is,

$$\begin{aligned} x(t+1) &= (A + BK)\, x(t) + h_1(t, x(t)) \\ &\quad + [(C + DK)\, x(t) + h_2(t, x(t))]\, w(t) \end{aligned} \quad (12)$$

is robustly stochastically stable;

(ii) The H_∞ norm of system (11) is less than $\gamma > 0$; that is,

$$\begin{aligned} \|T\| &= \sup_{\nu \in l_w^2(N,R^q), u(t)=Kx(t), \nu \neq 0, x_0=0} \frac{\|z(t)\|_{l_w^2(N,R^p)}}{\|\nu(t)\|_{l_w^2(N,R^q)}} \\ &= \sup_{\nu \in l_w^2(N,R^q), u(t)=Kx(t), \nu \neq 0, x_0=0} \frac{\left(\sum_{t=0}^{\infty} E\|z(t)\|^2\right)^{1/2}}{\left(\sum_{t=0}^{\infty} E\|\nu(t)\|^2\right)^{1/2}} \quad (13) \\ &< \gamma. \end{aligned}$$

3. Main Results

In this section, we give our main results on stochastic stability, stochastic stabilization, and robust H_∞ control via LMI-based approach. Firstly, we introduce the following two lemmas which will be used in the proof of our main results.

Lemma 5 (Schur's lemma). *For a real symmetric matrix* $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$, *the following three conditions are equivalent:*

(i) $S < 0$.

(ii) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$.

(iii) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 6. *For any real matrices* U, $N' = N > 0$ *and* W *with appropriate dimensions, we have*

$$U'NW + W'NU \leq U'NU + W'NW. \quad (14)$$

Proof. Because $N' = N > 0$, $U'NW + W'NU = (U'N^{1/2})(N^{1/2}W) + (W'N^{1/2})(N^{1/2}W)$. Inequality (14) is an immediate corollary of the well-known inequality

$$X'Y + Y'X \leq X'X + Y'Y. \quad (15)$$

□

3.1. Robust Stability Criteria. Consider the following unforced stochastic discrete time system:

$$\begin{aligned} x(t+1) &= Ax(t) + h_1(t, x(t)) \\ &\quad + (Cx(t) + h_2(t, x(t)))\, w(t), \quad (16) \\ &\quad x(0) = x_0 \in R^n, \ t \in N, \end{aligned}$$

where $\{h_1(t, x(t))\}_{t \geq 0}$ and $\{h_2(t, x(t))\}_{t \geq 0}$ satisfy (8). The following theorem gives a sufficient condition of robust stochastic stability for system (16).

Theorem 7. *System (16) with margins* $\alpha_1 > 0$ *and* $\alpha_2 > 0$ *is said to be robustly stochastically stable, if there exists* *a symmetric positive definite matrix* $Q > 0$ *and a scalar* $\alpha > 0$ *such that*

$$\begin{bmatrix} -Q + 2\alpha_1^2\alpha H'H + 2\alpha_2^2\alpha H'H & A'Q & C'Q & 0 \\ * & -\frac{1}{2}Q & 0 & 0 \\ * & * & -\frac{1}{2}Q & 0 \\ * & * & * & Q-\alpha I \end{bmatrix} < 0. \tag{17}$$

Proof. If (17) holds, we set $V(x(t)) = x'(t)Qx(t)$ as a Lyapunov function candidate of system (16), where $0 < Q < \alpha I$ by (17). Note that $x(t)$ and $w(t)$ are independent, so the difference generator is

$$\begin{aligned} E\Delta V(x(t)) &= E[V(x(t+1)) - V(x(t))] \\ &= E\{x'(t)(A'QA + C'QC - Q)x(t) \\ &+ x'(t)A'Qh_1(t,x(t)) + h_1'(t,x(t))QAx(t) \\ &+ h_1'(t,x(t))Qh_1(t,x(t)) + x'(t)C'Qh_2(t,x(t)) \\ &+ h_2'(t,x(t))QCx(t) + h_2'(t,x(t))Qh_2(t,x(t))\}. \end{aligned} \tag{18}$$

Applying Lemma 6 and inequalities (6)-(7), by $0 < Q < \alpha I$, we have

$$\begin{aligned} & x'(t)A'Qh_1(t,x(t)) + h_1'(t,x(t))QAx(t) \\ &\quad \le x'(t)A'QAx(t) + h_1'(t,x(t))Qh_1(t,x(t)) \\ &\quad \le x'(t)(A'QA + \alpha_1^2\alpha H'H)x(t), \\ & x'(t)C'Qh_2(t,x(t)) + h_2'(t,x(t))QCx(t) \\ &\quad \le x'(t)(C'QC + \alpha_2^2\alpha H'H)x(t), \\ & h_1'(t,x(t))Qh_1(t,x(t)) + h_2'(t,x(t))Qh_2(t,x(t)) \\ &\quad \le x'(t)(\alpha_1^2\alpha H'H + \alpha_2^2\alpha H'H)x(t). \end{aligned} \tag{19}$$

Substituting (19) into (18), we achieve that

$$\begin{aligned} E\Delta V(x(t)) \le E\{x'(t)(2A'QA + 2C'QC - Q \\ + 2\alpha_1^2\alpha H'H + 2\alpha_2^2\alpha H'H)x(t)\}. \end{aligned} \tag{20}$$

By Schur's complement,

$$\begin{aligned} \Omega &:= 2A'QA + 2C'QC - Q + 2\alpha_1^2\alpha H'H + 2\alpha_2^2\alpha H'H \\ &< 0 \end{aligned} \tag{21}$$

is equivalent to

$$\begin{bmatrix} -Q + 2\alpha_1^2\alpha H'H + 2\alpha_2^2\alpha H'H & A'Q & C'Q \\ * & -\frac{1}{2}Q & 0 \\ * & * & -\frac{1}{2}Q \end{bmatrix} < 0, \tag{22}$$

which holds by (17). We denote $\lambda_{\max}(\Omega)$ and $\lambda_{\min}(\Omega)$ to be the largest and the minimum eigenvalues of the matrix Ω, respectively; then (20) yields

$$E\Delta V(x(t)) \le \lambda_{\max}(\Omega) E\|x(t)\|^2. \tag{23}$$

Taking summation on both sides of the above inequality from $t = 0$ to $t = T \ge 0$, we get

$$\begin{aligned} E[V(x(T))] - V(x_0) &= E\left[\sum_{t=0}^{T}\Delta V(x(t))\right] \\ &\le \lambda_{\max}(\Omega) E\left[\sum_{t=0}^{T} x'(t)x(t)\right]. \end{aligned} \tag{24}$$

Therefore,

$$\lambda_{\min}(-\Omega) E\left[\sum_{t=0}^{T} x'(t)x(t)\right] \le V(x_0), \tag{25}$$

which leads to

$$E\left[\sum_{t=0}^{T} x'(t)x(t)\right] \le \delta(x_0,\alpha_1,\alpha_2) := \frac{V(x_0)}{\lambda_{\min}(-\Omega)}. \tag{26}$$

Hence, the robust stochastic stability of system (16) is obtained by (26) via letting $T \to \infty$. □

Remark 8. From Theorem 7, if LMI (17) has feasible solutions, then, for any bounded parameters $\widehat{\alpha}_1$ and $\widehat{\alpha}_2$ on the uncertain perturbation satisfying $\widehat{\alpha}_1 \le \alpha_1$ and $\widehat{\alpha}_2 \le \alpha_2$, system (16) is robustly stochastically stable with margins $\widehat{\alpha}_1$ and $\widehat{\alpha}_2$.

3.2. Robust Stabilization Criteria. In this subsection, a sufficient condition about robust stochastic stabilization via LMI will be given.

Theorem 9. *System (5) with margins* α_1 *and* α_2 *is robustly stochastically stabilizable if there exist real matrices* Y *and* $X > 0$ *and a real scalar* $\beta > 0$ *such that*

$$\begin{bmatrix} -X & \alpha_1 XH' & \alpha_2 XH' & J_1' & J_2' & 0 \\ * & -\frac{1}{2}\beta I & 0 & 0 & 0 & 0 \\ * & * & -\frac{1}{2}\beta I & 0 & 0 & 0 \\ * & * & * & -\frac{1}{2}X & 0 & 0 \\ * & * & * & * & -\frac{1}{2}X & 0 \\ * & * & * & * & * & \beta I - X \end{bmatrix} < 0 \tag{27}$$

holds, where

$$\begin{aligned} J_1 &= AX + BY, \\ J_2 &= CX + DY. \end{aligned} \tag{28}$$

In this case, $u(t) = Kx(t) = YX^{-1}x(t)$ *is a robustly stochastically stabilizing control law.*

Proof. We consider synthesizing a state feedback controller $u(t) = Kx(t)$ to stabilize system (5). Substituting $u(t) = Kx(t)$ into system (5) yields the closed-loop system described by

$$\begin{aligned} x(t+1) &= \overline{A}x(t) + h_1(t, x(t)) \\ &\quad + \left(\overline{C}x(t) + h_2(t, x(t))\right) w(t), \\ & x(0) = x_0 \in R^n, \ t \in N, \end{aligned} \tag{29}$$

where $\overline{A} = A + BK$ and $\overline{C} = C + DK$. By Theorem 7, system (29) is robustly stochastically stable if there exists a matrix Q, $0 < Q < \alpha I$, such that the following LMI

$$\widetilde{\Omega} := \begin{bmatrix} -Q + 2\alpha_1^2 \alpha H'H + 2\alpha_2^2 \alpha H'H & \overline{A}'Q & \overline{C}'Q \\ * & -\frac{1}{2}Q & 0 \\ * & * & -\frac{1}{2}Q \end{bmatrix} < 0 \tag{30}$$

holds. Let $Q^{-1} = X$ and pre- and postmultiply

$$\operatorname{diag}[X, X, X] \tag{31}$$

on both sides of inequality (30), and it yields

$$\begin{bmatrix} -X + 2\alpha_1^2 \alpha XH'HX + 2\alpha_2^2 \alpha XH'HX & X\overline{A}' & X\overline{C}' \\ * & -\frac{1}{2}X & 0 \\ * & * & -\frac{1}{2}X \end{bmatrix} < 0. \tag{32}$$

In order to transform (32) into a suitable LMI form, we set $\beta = 1/\alpha$; then $0 < Q < \alpha I$ is equivalent to

$$\beta I - X < 0, \quad \beta > 0. \tag{33}$$

Combining (33) with (32) and setting the gain matrix $K = YX^{-1}$, LMI (27) is obtained. The proof is completed. □

3.3. H_∞ Control.

In this subsection, main result about robust H_∞ control will be given via LMI approach.

Theorem 10. *Consider system (11) with margins $\alpha_1 > 0$ and $\alpha_2 > 0$. For the given $\gamma > 0$, if there exist real matrices $X > 0$ and Y and scalar $\beta > 0$ satisfying the following LMI,*

$$\begin{bmatrix} -X & L' & \alpha_1^2 XH_1' & \alpha_2^2 XH_2' & J_1' & J_2' & 0 & 0 & 0 & 0 & J_1' & J_2' & 0 \\ * & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\frac{1}{3}\beta I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\frac{1}{3}\beta I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -X & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\gamma^2 I & A_0' & C_0' & M' & A_0 & C_0 & 0 \\ * & * & * & * & * & * & * & -X & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -X & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -X & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -X & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & \beta I - X \end{bmatrix} < 0, \tag{34}$$

where $J_1 = AX + BY$, $J_2 = CX + DY$, then system (11) is H_∞ controllable, and the robust H_∞ control law is $u(t) = Kx(t) = YX^{-1}x(t)$ for $t \in N$.

Proof. When $v(t) = 0$, by Theorem 9, system (11) is internally stabilizable via $u(t) = Kx(t) = YX^{-1}x(t)$, because LMI (34) implies LMI (27). Next, we only need to show $\|T\| < \gamma$.

Take $u(t) = Kx(t)$ and choose the Lyapunov function $V(x(t)) = x'(t)Qx(t)$, where

$$0 < Q < \alpha I, \quad \alpha > 0 \tag{35}$$

for some $\alpha > 0$ to be determined; then for the system

$$\begin{aligned} x(t+1) &= \overline{A}x(t) + A_0 v(t) + h_1(t, x(t)) \\ &\quad + \left(\overline{C}x(t) + C_0 v(t) + h_2(t, x(t))\right) w(t), \\ z(t) &= \begin{bmatrix} Lx(t) \\ Mv(t) \end{bmatrix}, \\ & x(0) = x_0 \in R^n, \ t \in N \end{aligned} \tag{36}$$

with $\overline{A} = A + BK$ and $\overline{C} = C + DK$, we have, with $x(t)$ and $\nu(t)$ independent of $w(t)$, in mind that

$$\begin{aligned}
E\Delta V(x(t)) &= E[V(x(t+1)) - V(x(t))] \\
&= E\left[x'(t+1)Qx(t+1) - x'(t)Qx(t)\right] \\
&= E\left\{x'(t)\left[\overline{A}'Q\overline{A} + \overline{C}'Q\overline{C} - Q\right]x(t)\right. \\
&+ x'(t)\overline{A}'Qh_1(t,x(t)) + x'(t)\overline{C}'Qh_2(t,x(t)) \\
&+ x'(t)\left[\overline{A}'QA_0 + \overline{C}'QC_0\right]\nu(t) \\
&+ h_1'(t,x(t))Q\overline{A}x(t) + h_1'(t,x(t))Qh_1(t,x(t)) \\
&+ h_1'(t,x(t))QA_0\nu(t) + h_2'(t,x(t))Q\overline{C}x(t) \\
&+ h_2'(t,x(t))Qh_2(t,x(t)) + h_2'(t,x(t))QC_0\nu(t) \\
&+ \nu'(t)\left(A_0'QA_0 + C_0'QC_0\right)\nu(t) \\
&+ \nu'(t)\left[A_0'Q\overline{A} + C_0'Q\overline{C}\right]x(t) \\
&\left.+ \nu'(t)A_0'Qh_1(t,x(t)) + \nu'(t)C_0'Qh_2(t,x(t))\right\}.
\end{aligned} \tag{37}$$

Set $x_0 = 0$, and then for any $\nu(t) \in l_w^2(N, R^p)$,

$$\begin{aligned}
&\|z(t)\|^2_{l_w^2(N_T,R^p)} - \gamma^2\|\nu(t)\|^2_{l_w^2(N_T,R^q)} \\
&= E\sum_{t=0}^{T}\left[x'(t)L'Lx(t) + \nu'(t)M'M\nu(t)\right. \\
&\left.- \gamma^2\nu'(t)\nu(t) + \Delta V(t)\right] - x'(T)Qx(T) \\
&\le E\sum_{t=0}^{T}\left\{x'(t)L'Lx(t) + \nu'(t)M'M\nu(t)\right. \\
&- \gamma^2\nu'(t)\nu(t) + \nu'(t)\left(A_0'QA_0 + C_0'QC_0\right)\nu(t) \\
&+ x'(t)\left[\overline{A}'Q\overline{A} + \overline{C}'Q\overline{C} - Q\right]x(t) \\
&+ x'(t)\overline{A}'Qh_1(t,x(t)) + x'(t)\overline{C}'Qh_2(t,x(t)) \\
&+ x'(t)\left[\overline{A}QA_0 + \overline{C}'QC_0\right]\nu(t) \\
&+ h_1'(t,x(t))Q\overline{A}x(t) + h_1'(t,x(t))Qh_1(t,x(t)) \\
&+ h_1'(t,x(t))QA_0\nu(t) + h_2'(t,x(t))Q\overline{C}x(t) \\
&+ h_2'(t,x(t))Qh_2(t,x(t)) + h_2'(t,x(t))QC_0\nu(t) \\
&+ \nu'(t)A_0'Qh_1(t,x(t)) + \nu'(t)C_0'Qh_2(t,x(t)) \\
&\left.+ \nu'(t)\left[A_0'Q\overline{A} + C_0'Q\overline{C}\right]x(t)\right\}.
\end{aligned} \tag{38}$$

Using Lemma 6 and setting $x'(t)\overline{A}' = U'$, $h_1(t,x(t)) = W$, and $N = Q > 0$, we have

$$\begin{aligned}
&x'(t)\overline{A}'Qh_1(t,x(t)) + h_1'(t,x(t))Q\overline{A}x(t) \\
&\quad\le x'(t)\overline{A}'Q\overline{A}x(t) + h_1'(t,x(t))Qh_1(t,x(t)).
\end{aligned} \tag{39}$$

Similarly, the following inequalities can also be obtained:

$$\begin{aligned}
&x'(t)\overline{C}'Qh_2(t,x(t)) + h_2'(t,x(t))Q\overline{C}x(t) \\
&\quad\le x'(t)\overline{C}'Q\overline{C}x(t) + h_2'(t,x(t))Qh_2(t,x(t)), \\
&h_1'(t,x(t))QA_0\nu(t) + \nu'(t)A_0'Qh_1(t,x(t)) \\
&\quad\le h_1'(t,x(t))Qh_1(t,x(t)) + \nu'(t)A_0'QA_0\nu(t), \\
&h_2'(t,x(t))QC_0\nu(t) + \nu'(t)C_0'Qh_2(t,x(t)) \\
&\quad\le h_2'(t,x(t))Qh_2(t,x(t)) + \nu'(t)C_0'QC_0\nu(t).
\end{aligned} \tag{40}$$

Substituting (39)-(40) into inequality (38) and considering (35), it yields

$$\begin{aligned}
&\|z(t)\|^2_{l_w^2(N_T,R^p)} - \gamma^2\|\nu(t)\|^2_{l_w^2(N_T,R^q)} \le E\sum_{t=0}^{T}\left\{x'(t)\left[-Q\right.\right. \\
&+ 2\overline{A}'Q\overline{A} + 2\overline{C}'Q\overline{C} + L'L + 3\alpha_1^2\alpha H_1'H_1 \\
&\left.+ 3\alpha_2^2\alpha H_2'H_2\right]x(t) + x'(t)\left[\overline{A}'QA_0 + \overline{C}'QC_0\right]\nu(t) \\
&+ \nu'(t)\left[A_0Q\overline{A} + C_0Q\overline{C}\right]x'(t) + \nu(t)\left(2A_0'QA_0\right. \\
&\left.\left.+ 2C_0'QC_0 - \gamma^2 I + M'M\right)\nu(t)\right\} = E\sum_{t=0}^{T}\begin{bmatrix}x(t)\\ \nu(t)\end{bmatrix}' \\
&\cdot \Xi\begin{bmatrix}x(t)\\ \nu(t)\end{bmatrix},
\end{aligned} \tag{41}$$

where

$$\Xi := \begin{bmatrix}\Xi_{11} & \Xi_{12}\\ * & \Xi_{22}\end{bmatrix} \tag{42}$$

with

$$\begin{aligned}
\Xi_{11} &= -Q + 2\overline{A}'Q\overline{A} + 2\overline{C}'Q\overline{C} + L'L + 3\alpha_1^2\alpha H_1'H_1 \\
&\quad + 3\alpha_2^2\alpha H_2'H_2, \\
\Xi_{12} &= \overline{A}'QA_0 + \overline{C}'QC_0, \\
\Xi_{22} &= 2A_0'QA_0 + 2C_0'QC_0 - \gamma^2 I + M'M.
\end{aligned} \tag{43}$$

Let $T \to \infty$ in (41); then we have

$$\begin{aligned}
&\|z(t)\|^2_{l_w^2(N,R^p)} - \gamma^2\|\nu(t)\|^2_{l_w^2(N,R^q)} \\
&\le E\sum_{t=0}^{\infty}\begin{bmatrix}x(t)\\ \nu(t)\end{bmatrix}'\Xi\begin{bmatrix}x(t)\\ \nu(t)\end{bmatrix}.
\end{aligned} \tag{44}$$

It is easy to see that if $\Xi < 0$, then $\|T\| < \gamma$ for system (11). Next, we give an LMI sufficient condition for $\Xi < 0$. Notice that

$$\Xi = \begin{bmatrix} \Xi_{11} - \overline{A}'Q\overline{A} & \overline{C}'QC_0 \\ * & \Xi_{22} - A_0'QA_0 \end{bmatrix} + \begin{bmatrix} \overline{A}' \\ A_0' \end{bmatrix} Q \begin{bmatrix} A+BK & A_0 \end{bmatrix} < 0$$

$$\Longleftrightarrow \begin{bmatrix} \Xi_{11} - \overline{A}'Q\overline{A} & \overline{C}'QC_0 & \overline{A}' \\ * & \Xi_{22} - A_0'QA_0 & A_0 \\ * & * & -Q^{-1} \end{bmatrix} < 0$$

$$\Longleftrightarrow \begin{bmatrix} \tilde{\Xi}_{11} & 0 & \overline{A}' & \overline{C}' \\ * & \tilde{\Xi}_{22} & A_0 & C_0 \\ * & * & -Q^{-1} & 0 \\ * & * & * & -Q^{-1} \end{bmatrix} < 0, \tag{45}$$

where

$$\tilde{\Xi}_{11} = \Xi_{11} - \overline{A}'Q\overline{A} - \overline{C}'Q\overline{C},$$
$$\tilde{\Xi}_{22} = \Xi_{22} - A_0'QA_0 - C_0'QC_0. \tag{46}$$

Using Lemma 5, $\Xi < 0$ is equivalent to

$$\begin{bmatrix} -Q & L' & \alpha_1^2 H_1' & \alpha_2^2 H_2' & \overline{A}'Q & \overline{C}'Q & 0 & 0 & 0 & 0 & \overline{A}' & \overline{C}' \\ * & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\frac{1}{3\alpha}I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\frac{1}{3\alpha}I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -Q & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -Q & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\gamma^2 I & A_0Q & C_0Q & M' & A_0 & C_0 \\ * & * & * & * & * & * & * & -Q & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -Q & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -Q^{-1} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -Q^{-1} \end{bmatrix} < 0. \tag{47}$$

It is obvious that seeking H_∞ gain matrix K needs to solve LMIs (47) and $Q - \alpha I < 0$. Setting $Q^{-1} = X$ and $\beta = 1/\alpha$, and pre- and postmultiplying

$$\operatorname{diag}[X, I, I, I, X, X, I, X, X, I, I, I] \tag{48}$$

on both sides of (47) and considering (35), (34) is obtained immediately. The proof is completed. □

4. Numerical Examples

This section presents two numerical examples to demonstrate the validity of our main results described above.

Example 1. Consider system (5) with parameters as

$$A = \begin{bmatrix} 0.8 & 0.3 \\ 0.4 & 0.9 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.3 & 0.2 \\ 0.4 & 0.2 \end{bmatrix},$$

$$H_1 = H_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B = D = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$x(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 50 \\ -50 \end{bmatrix}. \tag{49}$$

For the unforced system (16) with $\alpha_1 = \alpha_2 = 0.3$, its corresponding state locus diagram is made in Figure 1. From Figure 1, it is easy to see that the status values are of serious divergences through 50 iterations. Hence the unforced system is not stable.

To design a feedback controller such that the closed-loop system is stochastically stable, using Matlab LMI Toolbox, we

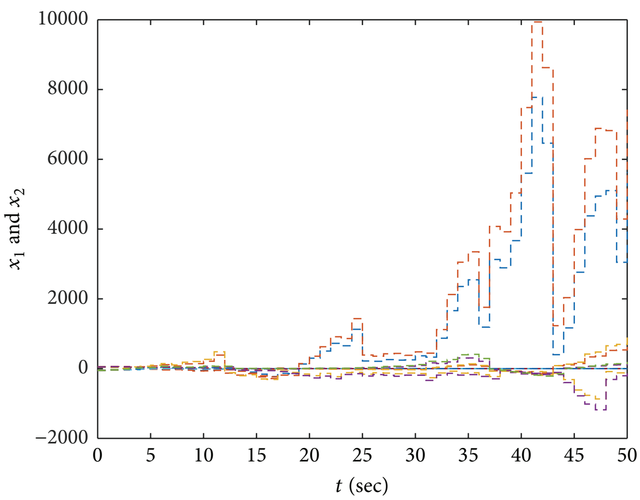


Figure 1: State trajectories of the autonomous system with $\alpha = 0.3$.

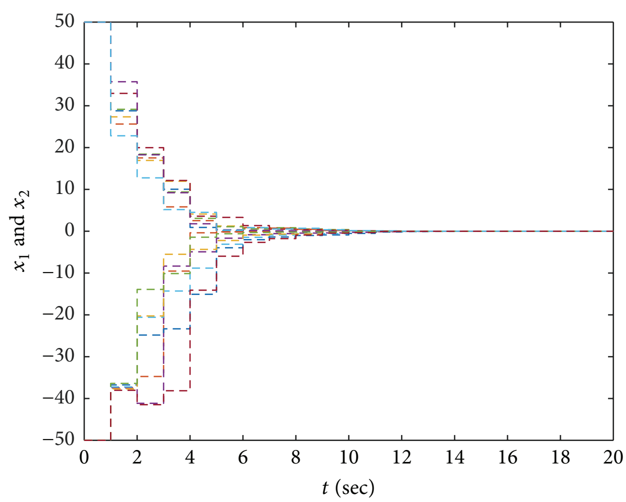


Figure 2: State trajectories of the closed-loop system with $\alpha = 0.3$.

find that a symmetric, positive definite matrix X, a real matrix Y, and a scalar β given by

$$X = \begin{bmatrix} 3.318 & 0.177 \\ 0.177 & 1.920 \end{bmatrix}, \quad Y = \begin{bmatrix} -1.587 & -0.930 \end{bmatrix}, \quad \beta = 1.868 \tag{50}$$

solve LMI (27). So we get the feedback gain $K = [-0.455 \ \ -0.443]$. Submitting

$$u(t) = \begin{bmatrix} -0.455 & -0.443 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{51}$$

into system (5), the state trajectories of the closed-loop system are shown as in Figure 2.

From Figure 2, one can find that the controlled system achieves stability using the proposed controller. Meanwhile,

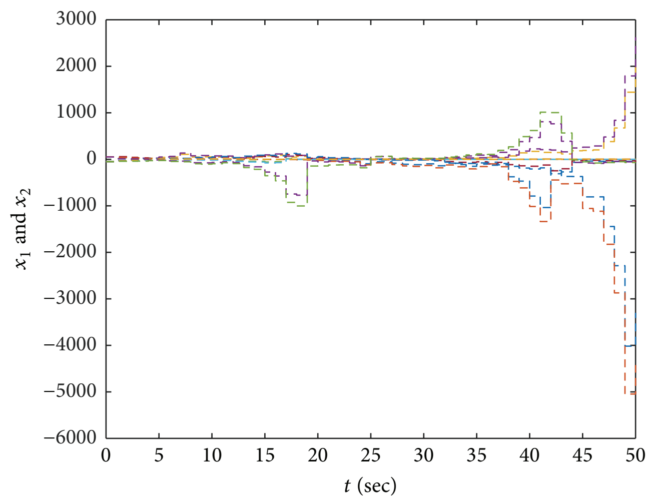


Figure 3: State trajectories of the autonomous system with $\alpha = 0.1$.

in the case $\alpha \leq 0.3$, the controlled system maintains stabilization.

In order to show the robustness, we use different values of α in Example 1 below. We reset $\alpha = 0.1$ and $\alpha = 1$ in system (5) with the corresponding trajectories shown in Figures 3 and 4, respectively. By comparing Figures 1, 3, and 4, intuitively speaking, the more value α takes, the more divergence the autonomous system (5) has. By using Theorem 9, we can get that, under the condition of $\alpha = 0.1$ and $\alpha = 1$, the corresponding controllers are $u_{\alpha=0.1}(t) = [-0.4750 \ \ -0.4000] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ and $u_{\alpha=1}(t) = [-0.4742 \ \ -0.4016] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, respectively. Substitute $u_{\alpha=0.1}(t)$ and $u_{\alpha=1}(t)$ into system (5) in turn, and the corresponding closed-loop system is shown in Figures 5 and 6, respectively.

From the simulation results, we can see that the larger the value α takes, the slower the system converges. This observation is reasonable, because the larger uncertainty the system has, the stronger the robustness of controller requires.

Example 2. Consider system (11) with parameters as

$$C_0 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad A_0 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad L = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad M = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad H_1 = H_2 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}. \tag{52}$$

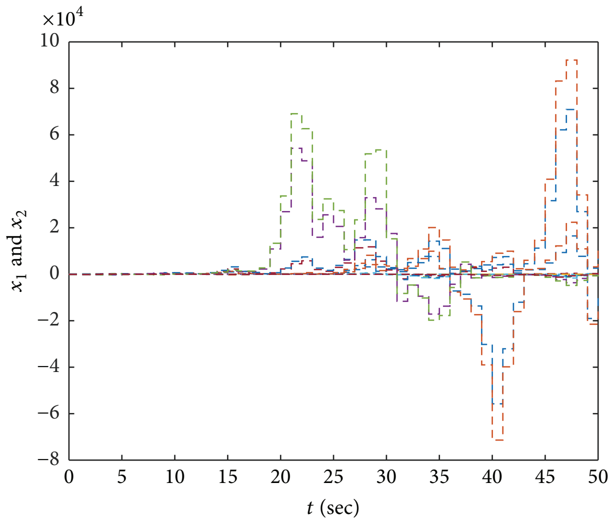


Figure 4: State trajectories of the autonomous system with $\alpha = 1$.

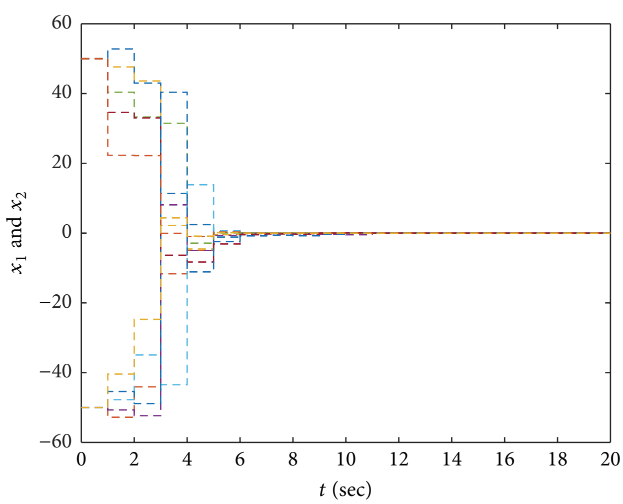


Figure 5: State trajectories of the closed-loop system with $\alpha = 0.1$.

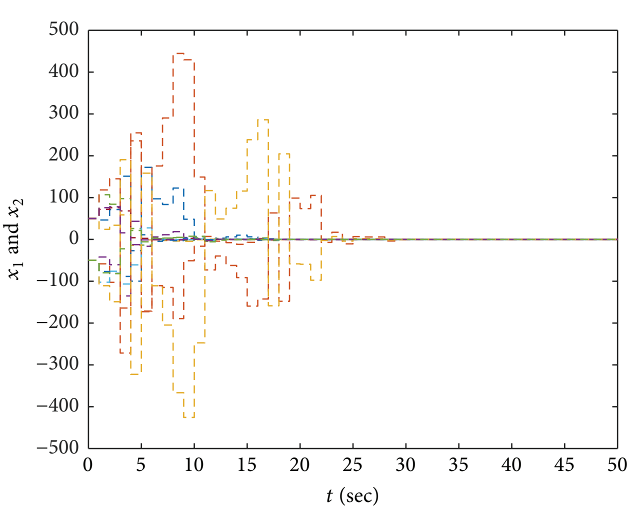


Figure 6: State trajectories of the closed-loop system with $\alpha = 1$.

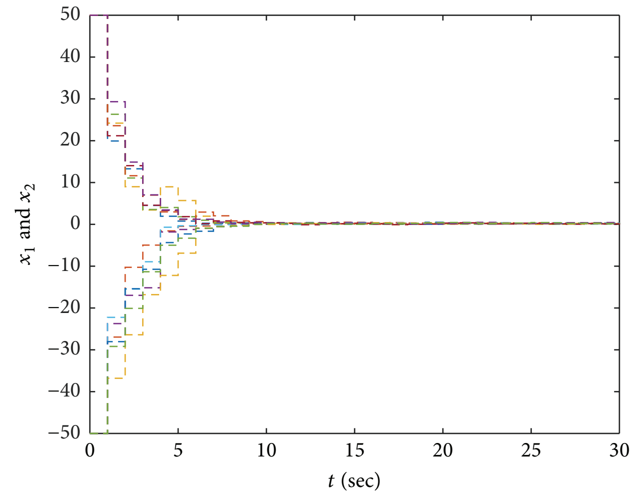


Figure 7: State trajectories of the closed-loop system with $\alpha = 0.3$.

A, B, C, D have the same values as in Example 1. Setting H_∞ norm bound $\gamma = 0.6$ and $\alpha_1 = \alpha_2 = 0.3$, a group of the solutions for LMI (34) are

$$X = \begin{bmatrix} 55.257 & 3.206 \\ 3.206 & 31.482 \end{bmatrix}, \quad Y = \begin{bmatrix} -26.412 & -15.475 \end{bmatrix}, \quad \beta = 30.596, \tag{53}$$

and the H_∞ controller is

$$u(t) = Kx(t) = YX^{-1}x(t) = \begin{bmatrix} -0.452 & -0.446 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}. \tag{54}$$

The simulation result about Theorem 10 is described in Figure 7. This further verifies the effectiveness of Theorem 10.

In order to give a comparison with $\alpha = 0.3$, we set $\alpha = 0.1$ and $\alpha = 1$. By using Theorem 10, the H_∞ controller for $\alpha = 0.1$ is $u(t) = \begin{bmatrix} -0.4737 & -0.4076 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ with the simulation result given in Figure 8. When $\alpha = 1$, LMI (34) is infeasible; that is, in this case, system (11) is not H_∞ controllable.

5. Conclusion

This paper has discussed robust stability, stabilization, and H_∞ control of a class of nonlinear discrete time stochastic systems with system state, control input, and external disturbance dependent noise. Sufficient conditions for stochastic stability, stabilization, and robust H_∞ control law have been, respectively, given in terms of LMIs. Two examples have also been supplied to show the effectiveness of our main results.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

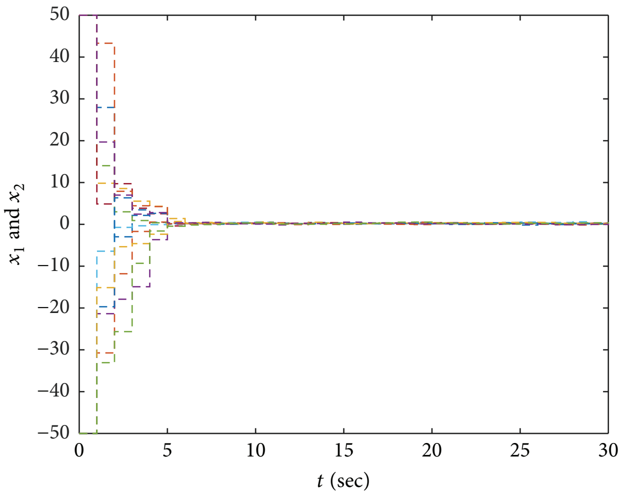


Figure 8: State trajectories of the closed-loop system with $\alpha = 0.1$.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (no. 61573227), the Research Fund for the Taishan Scholar Project of Shandong Province of China, the Natural Science Foundation of Shandong Province of China (no. 2013ZRE28089), SDUST Research Fund (no. 2015TDJH105), State Key Laboratory of Alternate Electrical Power System with Renewable Energy Sources (no. LAPS16011), and the Postgraduate Innovation Funds of China University of Petroleum (East China) (nos. YCX2015048 and YCX2015051).

References

[1] H. J. Kushner, *Stochastic Stability and Control*, Academic Press, New York, NY, USA, 1967.

[2] R. Z. Has'minskii, *Stochastic Stability of Differential Equations*, Sijtjoff and Noordhoff, Alphen, Netherlands, 1980.

[3] X. Mao, *Stochastic Differential Equations and Applications*, Ellis Horwood, Chichester, UK, 2nd edition, 2007.

[4] M. Ait Rami and X. Y. Zhou, "Linear matrix inequalities, Riccati equations, and indefinite stochastic linear quadratic controls," *IEEE Transactions on Automatic Control*, vol. 45, no. 6, pp. 1131–1143, 2000.

[5] Z. Lin, J. Liu, Y. Lin, and W. Zhang, "Nonlinear stochastic passivity, feedback equivalence and global stabilization," *International Journal of Robust and Nonlinear Control*, vol. 22, no. 9, pp. 999–1018, 2012.

[6] F. Li and Y. Liu, "Global stability and stabilization of more general stochastic nonlinear systems," *Journal of Mathematical Analysis and Applications*, vol. 413, no. 2, pp. 841–855, 2014.

[7] S. Sathananthan, M. J. Knap, and L. H. Keel, "Robust stability and stabilization of a class of nonlinear Ito-type stochastic systems via linear matrix inequalities," *Stochastic Analysis and Applications*, vol. 31, no. 2, pp. 235–249, 2013.

[8] Y. Xia, E.-K. Boukas, P. Shi, and J. Zhang, "Stability and stabilization of continuous-time singular hybrid systems," *Automatica*, vol. 45, no. 6, pp. 1504–1509, 2009.

[9] W. Zhang and B.-S. Chen, "On stabilizability and exact observability of stochastic systems with their applications," *Automatica*, vol. 40, no. 1, pp. 87–94, 2004.

[10] T. Hou, W. Zhang, and B.-S. Chen, "Study on general stability and stabilizability of linear discrete-time stochastic systems," *Asian Journal of Control*, vol. 13, no. 6, pp. 977–987, 2011.

[11] C. S. Kubrusly and O. L. Costa, "Mean square stability conditions for discrete stochastic bilinear systems," *IEEE Transactions on Automatic Control*, vol. 30, no. 11, pp. 1082–1087, 1985.

[12] Y. Li, W. Zhang, and X. Liu, "Stability of nonlinear stochastic discrete-time systems," *Journal of Applied Mathematics*, vol. 2013, Article ID 356746, 8 pages, 2013.

[13] S. Sathananthan, M. J. Knap, A. Strong, and L. H. Keel, "Robust stability and stabilization of a class of nonlinear discrete time stochastic systems: an LMI approach," *Applied Mathematics and Computation*, vol. 219, no. 4, pp. 1988–1997, 2012.

[14] W. Zhang, W. Zheng, and B. S. Chen, "Detectability, observability and Lyapunov-type theorems of linear discrete time-varying stochastic systems with multiplicative noise," http://arxiv.org/abs/1509.04379.

[15] D. Hinrichsen and A. J. Pritchard, "Stochastic H_∞," *SIAM Journal on Control and Optimization*, vol. 36, no. 5, pp. 1504–1538, 1998.

[16] B. S. Chen and W. Zhang, "Stochastic H_2/H_∞ control with state-dependent noise," *IEEE Transactions on Automatic Control*, vol. 49, no. 1, pp. 45–57, 2004.

[17] V. Dragan, T. Morozan, and A.-M. Stoica, *Mathematical Methods in Robust Control of Linear Stochastic Systems*, Springer, New York, NY, USA, 2006.

[18] H. Li and Y. Shi, "State-feedback H_∞ control for stochastic time-delay nonlinear systems with state and disturbance-dependent noise," *International Journal of Control*, vol. 85, no. 10, pp. 1515–1531, 2012.

[19] I. R. Petersen, V. A. Ugrinovskii, and A. V. Savkin, *Robust control design using H_∞-methods*, Springer, New York, NY, USA, 2000.

[20] W. Zhang and B.-S. Chen, "State feedback H_∞ control for a class of nonlinear stochastic systems," *SIAM Journal on Control and Optimization*, vol. 44, no. 6, pp. 1973–1991, 2006.

[21] N. Berman and U. Shaked, "H_∞ control for discrete-time nonlinear stochastic systems," *IEEE Transactions on Automatic Control*, vol. 51, no. 6, pp. 1041–1046, 2006.

[22] V. Dragan, T. Morozan, and A.-M. Stoica, *Mathematical Methods in Robust Control of Discrete-Time Linear Stochastic Systems*, Springer, New York, NY, USA, 2010.

[23] A. El Bouhtouri, D. Hinrichsen, and A. J. Pritchard, "H_∞-type control for discrete-time stochastic systems," *International Journal of Robust and Nonlinear Control*, vol. 9, no. 13, pp. 923–948, 1999.

[24] T. Hou, W. Zhang, and H. Ma, "Infinite horizon H_2/H_∞ optimal control for discrete-time Markov jump systems with $(x; u; v)$-dependent noise," *Journal of Global Optimization*, vol. 57, no. 4, pp. 1245–1262, 2013.

[25] W. Zhang, Y. Huang, and L. Xie, "Infinite horizon stochastic H_2/H_∞ control for discrete-time systems with state and disturbance dependent noise," *Automatica*, vol. 44, no. 9, pp. 2306–2316, 2008.

[26] S. Elaydi, *An Introduction to Difference Equation*, Springer, New York, NY, USA, 2005.

[27] J. P. LaSalle, *The Stability and Control of Discrete Processes*, Springer, New York, NY, USA, 1986.

[28] N. Athanasopoulos, M. Lazar, C. Böhm, and F. Allgöwer, "On stability and stabilization of periodic discrete-time systems with an application to satellite attitude control," *Automatica*, vol. 50, no. 12, pp. 3190–3196, 2014.

[29] O. L. V. Costa and A. de Oliveira, "Optimal mean–variance control for discrete-time linear systems with Markovian jumps and multiplicative noises," *Automatica*, vol. 48, no. 2, pp. 304–315, 2012.

[30] Q. Ma, S. Xu, Y. Zou, and J. Lu, "Robust stability for discrete-time stochastic genetic regulatory networks," *Nonlinear Analysis: Real World Applications*, vol. 12, no. 5, pp. 2586–2595, 2011.

[31] W. Lin and C. I. Byrnes, "H_∞ control of discrete-time nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 41, no. 4, pp. 494–510, 1996.

[32] T. Zhang, Y. H. Wang, X. Jiang, and W. Zhang, "A Nash game approach to stochastic H_2/H_∞ control: overview and further research topics," in *Proceedings of the 34th Chinese Control Conference*, pp. 2848–2853, Hangzhou, China, July 2015.

[33] S. G. Peng, "A general stochastic maximum principle for optimal control problems," *SIAM Journal on Control and Optimization*, vol. 28, no. 4, pp. 966–979, 1990.

[34] X. Lin and W. Zhang, "A maximum principle for optimal control of discrete-time stochastic systems with multiplicative noise," *IEEE Transactions on Automatic Control*, vol. 60, no. 4, pp. 1121–1126, 2015.

[35] Z. Du, Q. Zhang, and L. Liu, "New delay-dependent robust stability of discrete singular systems with time-varying delay," *Asian Journal of Control*, vol. 13, no. 1, pp. 136–147, 2011.

[36] K.-S. Park and J.-T. Lim, "Robust stability of non-standard nonlinear singularly perturbed discrete systems with uncertainties," *International Journal of Systems Science*, vol. 45, no. 3, pp. 616–624, 2014.

[37] S. Lakshmanan, T. Senthilkumar, and P. Balasubramaniam, "Improved results on robust stability of neutral systems with mixed time-varying delays and nonlinear perturbations," *Applied Mathematical Modelling*, vol. 35, no. 11, pp. 5355–5368, 2011.

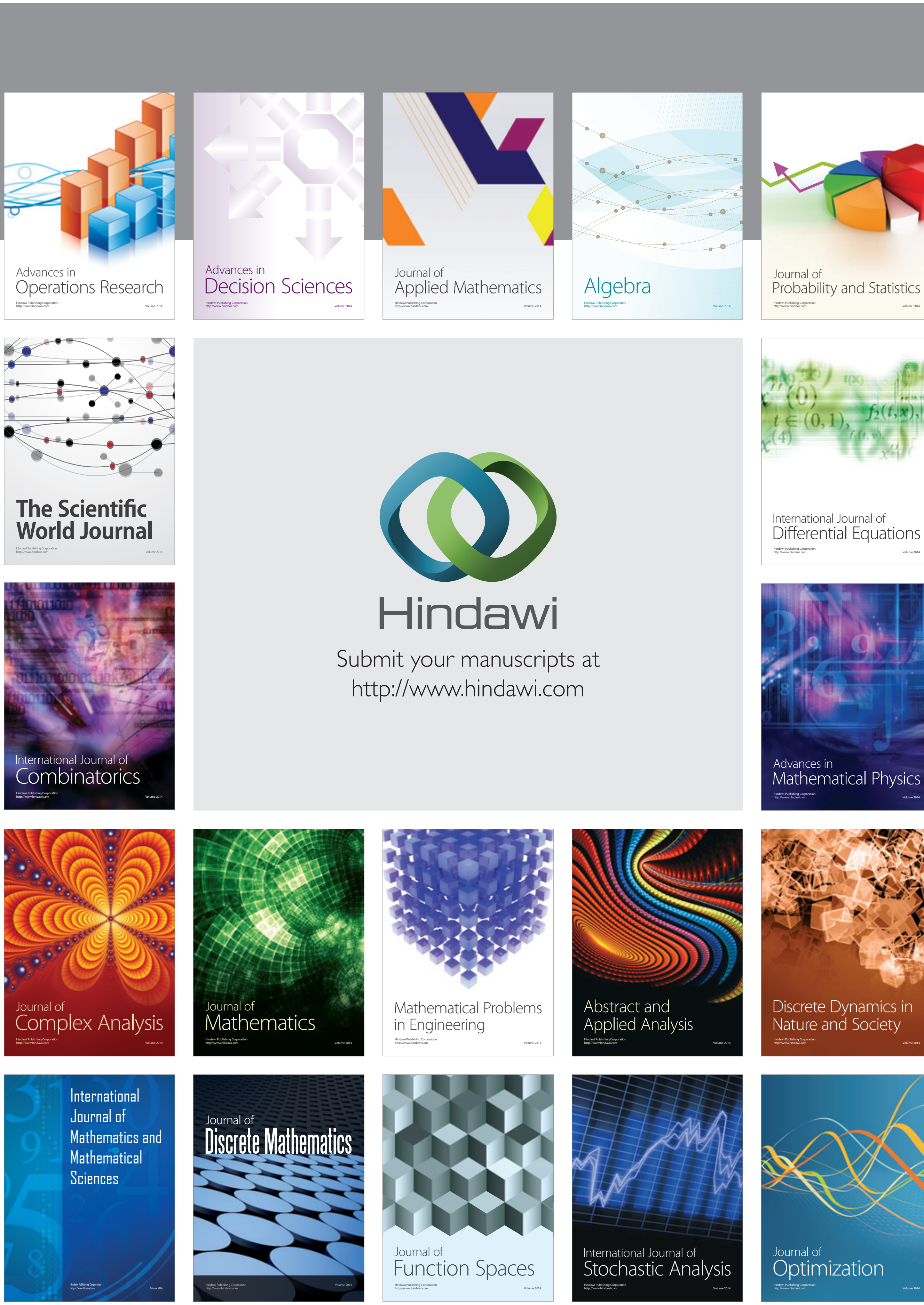
[38] X. Zhang, C. Wang, D. Li, X. Zhou, and D. Yang, "Robust stability of impulsive Takagi-Sugeno fuzzy systems with parametric uncertainties," *Information Sciences*, vol. 181, no. 23, pp. 5278–5290, 2011.

[39] S. Shah, Z. Iwai, I. Mizumoto, and M. Deng, "Simple adaptive control of processes with time-delay," *Journal of Process Control*, vol. 7, no. 6, pp. 439–449, 1997.

[40] M. Deng and N. Bu, "Robust control for nonlinear systems using passivity-based robust right coprime factorization," *IEEE Transactions on Automatic Control*, vol. 57, no. 10, pp. 2599–2604, 2012.

[41] Z. Y. Gao and N. U. Ahmed, "Feedback stabilizability of nonlinear stochastic systems with state-dependent noise," *International Journal of Control*, vol. 45, no. 2, pp. 729–737, 1987.

[42] D. D. Šiljak and D. M. Stipanovic, "Robust stabilization of nonlinear systems: the LMI approach," *Mathematical Problems in Engineering*, vol. 6, no. 5, pp. 461–493, 2000.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