Research Article

Fractional Order Nonlinear Feedback Controller Design for PMSM Drives

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Fractional order integral is introduced into active disturbance rejection controller (ADRC) to establish the structure of fractional order proportional integral controller (FPI). Fractional order ADRC (FADRC) is designed by replacing the nonlinear state error feedback control law using nonlinear function combination in ADRC with FPI, which can combine the high performance of ADRC estimating disturbances with the characteristics of fractional order calculus more really describing the physical object and spreading the stable region of the system parameters. The proposed FADRC is applied to permanent magnet synchronous motor (PMSM) speed servo system in order to improve robustness of system against the disturbances. Compared with ADRC, simulation results verify that the proposed control method has given very good robust results and fast speed tracking performance.

1. Introduction

Permanent magnet synchronous motor (PMSM) is increasingly used in high precise AC servo driving control system due to its simple structures, high efficiency, low inertia, high power density and high torque-current ratio. PMSM is a nonlinear, strong coupling, and multivariable controlled object, which makes the system become nonlinear system with strong coupling [1–3]. To cope with parameter uncertainty, load disturbance, strong coupling, and measurement noise, and so forth, many control strategies, such as PI control [4, 5], adaptive control [6, 7], sliding mode control [8, 9], H_∞ control [10, 11], and intelligent control [12–15], have been developed and applied to the speed control of speed servo drives to obtain high driving performance and exploit advantage of PMSM. PI control can hardly meet the requirement of high performances although it was widely used. There exists complex adaptive control law in adaptive control. Chattering phenomenon can be caused in a conventional sliding mode control system. Due to these questions, the fields of use with those control techniques were restrained. Besides in recent years, active disturbance rejection control technique is applied in PMSM speed servo system [16–18], which can design the controller of controlled object by using

order of model, but not becoming dependent. It improves the response of system by using extended state observer to estimate and compensate disturbances. Reference [19] proposed fractional order proportional integral differential controller by using tracking differentiator to obtain tracking signal of output and its differential signal.

Fractional order calculus extends integer order ones to nonintegral order, which can model various real materials more adequately than integer order ones. This study introduces fractional order calculus into ADRC and establishes the structure of fractional order proportional integral controller. Fractional order ADRC is designed by replacing the nonlinear state error feedback control law using nonlinear function combination in ADRC with FPI, which can combine the high performance of ADRC estimating disturbances with the characteristics of fractional order calculus more really describing the physical object and spreading the stable region of the system parameters. The proposed FADRC is applied to permanent magnet synchronous motor speed servo system in order to improve robustness of system against the disturbances. Compared with ADRC, simulation results verify that the proposed control method has given very good robust results and fast speed tracking performance.

2. ADRC of PMSM Speed Servo System

On the basis of assumptions that the stator windings generate sinusoidal magnetic field, air gap is uniform and saturation is negligible. With reference to synchronous rotating reference frame, the dynamical rotate speed equation of PMSM may be expressed as follows:

$$\frac{d\omega}{dt} = -\frac{B}{J}\omega - \frac{T_L}{J} + p_n\frac{\psi_f}{J}i_q, \tag{1}$$

where i_q is stator q-axis currents; ψ_f is amplitude of the flux induced by the permanent magnets of the rotor in the stator phases; ω is angular speed of the motor; p_n is number of pole pairs; J is inertia of moment; T_L is the load torque; B is viscous friction coefficient.

To facilitate the design of ADRC, the rotate speed equation can be expressed as

$$\frac{d\omega}{dt} = f_0 + g_\omega + b_0 \cdot i_q, \tag{2}$$

where $f_0 = -(B/J)\omega$ is known information of model, function of which is to minish total observed disturbances and exploit known information of model to improve velocity of convergence and accuracy of observation in observation of ESO; $g_\omega = -T_L/J$ is disturbance term of model; $b_0 = ((3p_n\psi_f)/2J)$.

For the second-order system, type structure of ADRC includes tracking differentiator (TD), extended state observer (ESO), nonlinear state error feedback control law (NLSEF), and compensated control reference [20]. PMSM speed servo system using vector control is first-order model. In which the tracking differentiator plays a filter action role and can be omitted. So, ADRC of system is expressed as

ESO

$$e = \widehat{\omega} - \omega,$$

$$\dot{\omega} = g_\omega + z_0 - \beta_{01} \cdot \text{fal}\left(e, \frac{1}{2}, \delta\right) + b_0 \cdot i_q, \tag{3}$$

$$\dot{z}_0 = \beta_{02} \cdot \text{fal}\left(e, \frac{1}{4}, \delta\right),$$

where $\widehat{\omega}$ is observing signal of rotate speed; ω is rotate speed of motor; z_0 is disturbance estimation; β_{01} and β_{02} are gain factor, respectively.

NLSEF

$$e_0 = \int e_1 dt,$$

$$e_1 = \omega^* - \widehat{\omega}, \tag{4}$$

$$u_0 = \beta_1 \text{fal}\left(e_0, \frac{1}{2}, \delta\right) + \beta_2 \text{fal}\left(e_1, \frac{1}{4}, \delta\right),$$

where ω^* is input reference; e_0 is error integral; e_1 is error; β_1 and β_2 are gain factor, respectively. $\text{fal}(\cdot)$ is nonlinear function.

Disturbance compensation

$$i_q^* = u_0 - \frac{(f_0 + z_0)}{b_0} \tag{5}$$

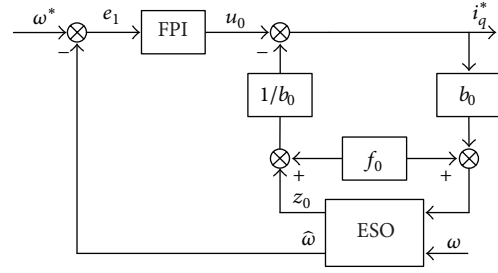i_q^* is input reference with disturbance compensation.

FIGURE 1: The block diagram of fractional order ADRC.

3. Design of FADRC

Fractional order calculus is a basic subject with mathematics to study differential coefficient and integral of random order, which can be expressed in calculus operator of fractional order and integral order [21]

$$D_t^\lambda = \begin{cases} \dfrac{d^\lambda}{dt^\lambda}, & R(\lambda) > 0 \\[2mm] 1, & R(\lambda) = 0 \\[2mm] \displaystyle\int_a^t (d\tau)^{-\lambda}, & R(\lambda) < 0, \end{cases} \tag{6}$$

where a is lower bound of calculus operator; t is upper bound of calculus operator; λ is order number of calculus operator; order number of calculus equals integral order when λ is integer; $R(\lambda)$ represents real number; D_t^λ indicates fractional differentiation when $R(\lambda)$ is greater than zero; D_t^λ indicates fractional differentiation when $R(\lambda)$ is less than zero.

Fractional order calculus extends the order number of calculus to fraction. Nonsmooth feedback of NLSEF in ADRC is replaced with the nonlinear fractional order PI controller, which combines the high performance of ADRC estimating disturbances with the characteristics of fractional order calculus more really describing the physical object and spreading the stable region of the system parameters. The structure of fractional order PI controller can be established by putting the $\lambda < 0$ into (4). So, the nonlinear error feedback in FADRC can be expressed as

$$e_1 = \omega^* - \widehat{\omega},$$
$$u_0 = k_1 e_1 + k_2 D_t^{-\lambda} e_1, \tag{7}$$

where k_1 is proportion coefficient; k_2 is integral coefficient; λ is order of integral; the block diagram of FADRC is shown in Figure 1.

By using Laplace transform, transfer function for u_0 and error is inferred as follows:

$$G(s) = \frac{U_0(s)}{E(s)} = k_1 + k_2 s^{-\lambda}. \tag{8}$$

In order to obtain discrete mathematical model of numerical control to be used, it is required that fractional order integral operator is approximately discretized. Using Euler
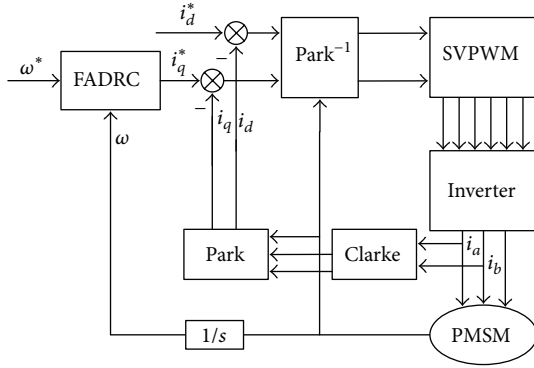
Figure 2: The block diagram of PMSM speed servo system.

Figure 3: Speed response to impulse load disturbance.

rule transform method, the fractional order system can be discretized into irrational equation of z, which is then transformed in the power series expansion (PSE) way to obtain discrete model of $s^{-\lambda}$, as follows:

$$Z\left(s^{-\lambda}\right) \approx \left(\frac{1}{T}\right)^{-\lambda} \text{PSE}\left[1 + \lambda z^{-1} + \frac{\lambda(1+\lambda)}{2!}z^{-2}\cdots\right], \quad (9)$$

where T is sampling period; when λ equals 0.3, T is 0.001 sec., and the degree of polynomial is 10, the approximate discretized model of fractional order integral is obtained as follows:

$$Z\left(s^{-0.3}\right) \approx \frac{1}{D}, \quad (10)$$

where D is expressed as follows:

$$\begin{aligned}
D = {} & 7.9428 - 2.3828z^{-1} - 0.834z^{-2} - 0.4726z^{-3} \\
& - 0.319z^{-4} - 0.2361z^{-5} - 0.1849z^{-6} \\
& - 0.1506z^{-7} - 0.1261z^{-8} - 0.1079z^{-9}.
\end{aligned} \quad (11)$$

The block diagram of PMSM speed servo system is shown in Figure 2.

4. Simulation Results and Analysis

The proposed method is investigated by means of simulations. The Matlab/Simulink environment is used for the simulations. The data of the PMSM are as follows: the nominal power is 1.8 kW; the nominal dc-link voltage is 72 V; the nominal torque is 5 N·m; the number of poles is 4; the moment of inertia is 0.012 kg·m²; the stator resistance is 0.09 Ω; the direct-axis inductance and quadrature-axis inductance are 0.4 mH and 0.5 mH, respectively; sampling period of current loop is set to 2 μs, in which the vector control of $i_d = 0$, two proportional-integral (PI) discrete-time current controllers for i_d and i_q, are used; speed loop is regulated by FADRC, $\delta = 0.01$; $\beta_{01} = 10000$ and $\beta_{02} = 500000$; fraction value of fractional order integral is selected by means of trial and error; the discrete mathematical model is given by (10).
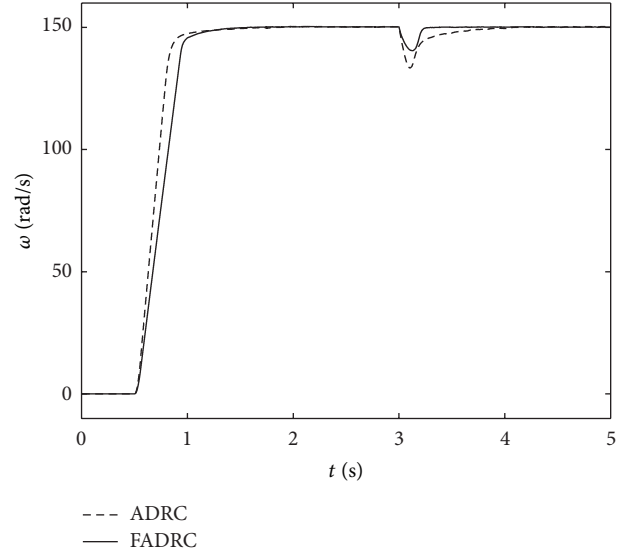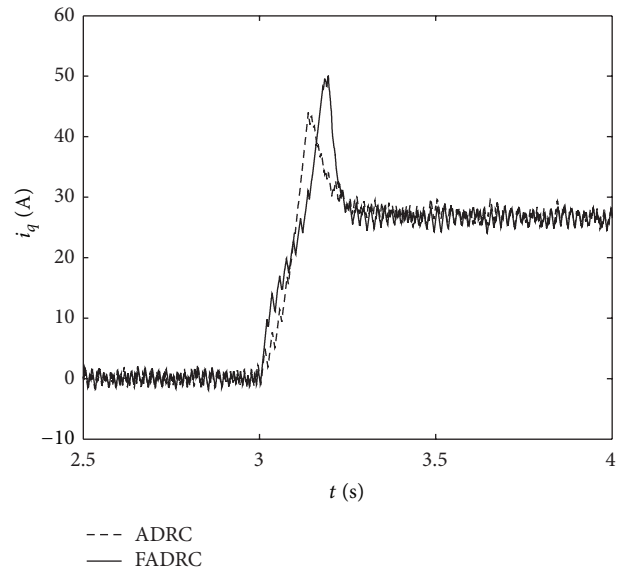

Figure 4: i_q under the control of ADRC and FADRC.

Figure 3 shows response of the PMSM as the load disturbances vary from 0 N·m to 5 N·m at 3 sec. when it is running at a steady speed of 150 rad/s. solid line represents the speed response of the PMSM under the control of FADRC. Dashed line represents the speed response of the PMSM under the control of ADRC. It can be observed that the PMSM returns to the regulated state very quickly. It indicates that FADRC can give faster compensation when load disturbances occur because fractional order calculus has the nice property of describing the physical object and spreading the stable region of the system parameters as well as the FADRC is simpler to reduce the computing time. The result can be inferred that the speed response of FADRC can still be faster than that of ADRC when load is variable.
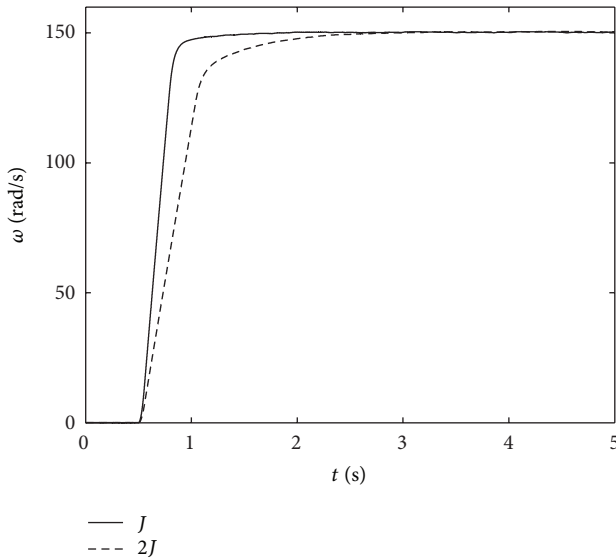
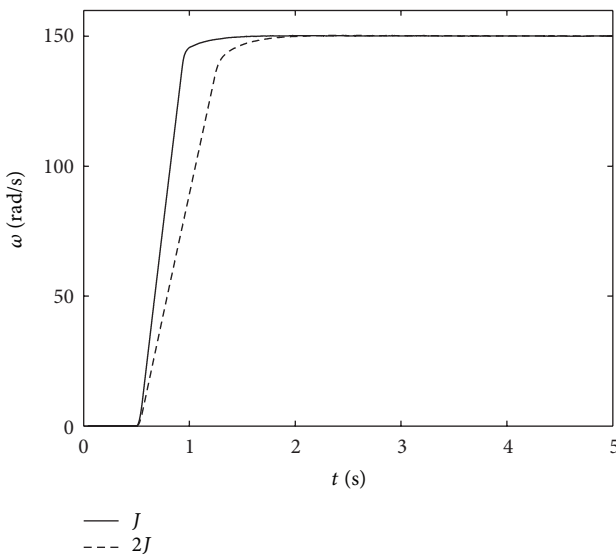Figure 5: Speed response by different inertia (J) values under the control of ADRC.

Figure 6: Speed response by different inertia (J) values under the control of FADRC.

Quadrature-axis currents of FADRC and ADRC are shown in Figure 4. The quadrature-axis current represents the electromagnetic torque. In Figure 4, the torque of the PMSM under the control of FADRC can be fast response to effectively restrain the load speed ripple caused by disturbances and improve the robustness of system.

Figures 5 and 6 are robustness of the ADRC and FADRC about PMSM inertia parameter varying twice. The results indicate that the regulation time of speed response of FADRC is shorter than that of ADRC. The proposed controller has better robustness.

5. Conclusions

A nonlinear feedback active rejection disturbance controlled permanent magnet synchronous motor drive is proposed. In this system fractional order integral is introduced to active rejection disturbance control to replace the nonlinear function in NLSEF, which can efficiently combine the high performance of active rejection disturbance technique observing disturbances with the characteristics of fractional calculus more really describing the physical object and spreading the stable region of the system parameters. It is applied to the permanent magnet synchronous drive. Simulation results show the feasibility and effectiveness of the proposed controller.

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