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Research Article

Generalization of Some Coupled Fixed Point Results on Partial Metric Spaces

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Using the setting of partial metric spaces, we prove some coupled fixed point results. Our results generalize several well-known comparable results of H. Aydi (2011). Also, we introduce an example to support our results.

1. Introduction and Preliminaries

The notion of coupled fixed point of a mapping $F : X \times X \to X$ was introduced by Gnana Bhaskar and Lakshmikantham in [1]. Later on, many authors investigated many coupled fixed point results in different spaces such as usual metric spaces, fuzzy metric spaces, generalized metric spaces, partial metric spaces, and partially ordered metric spaces (see [1–20]).

Definition 1.1 (see [1]). An element $(x, y) \in X \times X$ is called a coupled fixed point of a mapping $F : X \times X \to X$ if

$$F(x, y) = x, \qquad F(y, x) = y. \tag{1.1}$$

Matthews [21] in 1994 introduced the notion of partial metric spaces in such a way that each object does not necessarily have to have a zero distance from itself. Consistent with Matthews [21], the following definitions and results will be needed in the sequel.

Definition 1.2 (see [21]). A partial metric on a nonempty set X is a function $p : X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$:

(p_1) $x = y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$,

(p_2) $p(x,x) \leq p(x,y)$,

(p_3) $p(x,y) = p(y,x)$,

(p_4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.

A partial metric space is a pair (X,p) such that X is a nonempty set and p is a partial metric on X.

Each partial metric p on X generates a T_0 topology τ_p on X. The set $\{B_p(x,\varepsilon) : x \in X, \varepsilon > 0\}$, where $B_p(x,\varepsilon) = \{y \in X : p(x,y) < p(x,x)+\varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$, forms the base of τ_p.

If p is a partial metric on X, then the function $p^s : X \times X \to \mathbb{R}^+$ given by

$$p^s(x,y) = 2p(x,y) - p(x,x) - p(y,y) \tag{1.2}$$

is a metric on X.

Definition 1.3 (see [21]). Let (X,p) be a partial metric space. Then:

(1) a sequence (x_n) in a partial metric space (X,p) converges, with respect to τ_p, to a point $x \in X$ if and only if $p(x,x) = \lim_{n\to\infty} p(x,x_n)$,

(2) a sequence (x_n) in a partial metric space (X,p) is called a Cauchy sequence if there exists (and is finite) $\lim_{n,m\to\infty} p(x_n,x_m)$,

(3) A partial metric space (X,p) is said to be complete if every Cauchy sequence (x_n) in X converges, with respect to τ_p, to a point $x \in X$ such that $p(x,x) = \lim_{n,m\to\infty} p(x_n,x_m)$.

Lemma 1.4 (see [21]). *Let (X,p) be a partial metric space.*

(1) *(x_n) is a Cauchy sequence in (X,p) if and only if it is a Cauchy sequence in the metric space (X,p^s).*

(2) *A partial metric space (X,p) is complete if and only if the metric space (X,p^s) is complete. Furthermore, $\lim_{n\to\infty} p^s(x_n,x) = 0$ if and only if*

$$p(x,x) = \lim_{n\to\infty} p(x_n,x) = \lim_{n,m\to\infty} p(x_n,x_m). \tag{1.3}$$

Abdeljawad et al. [22–24], Altun et al. [25], Karapinar and Erhan [26–28], Oltra and Valero [29] and Romaguera [30] studied fixed point theorems in partial metric spaces. For more works in partial metric spaces, we refer the reader to [31–40].

Aydi [2] proved the following coupled fixed point theorems in partial metric spaces.

Theorem 1.5. *Let (X,p) be a complete partial metric space. Suppose that the mapping $F : X \times X \to X$ satisfies the following contractive condition for all $x, y, u, v \in X$:*

$$p(F(x,y), F(u,v)) \leq kp(x,u) + lp(y,v), \tag{1.4}$$

where k, l are nonnegative constants with $k + l < 1$. Then F has a unique coupled fixed point.

Theorem 1.6. *Let (X, p) be a complete partial metric space. Suppose that the mapping $F : X \times X \to X$ satisfies the following contractive condition for all $x, y, u, v \in X$:*

$$p(F(x,y), F(u,v)) \leq kp(F(x,y), x) + lp(F(u,v), u), \tag{1.5}$$

where k, l are nonnegative constants with $k + l < 1$. Then F has a unique coupled fixed point.

In this paper, we prove some coupled fixed point results. Our results generalize Theorems 1.5 and 1.6. Also, we introduce an example to support our results.

2. The Main Result

Theorem 2.1. *Let (X, p) be a complete partial metric space. Suppose that the mapping $F : X \times X \to X$ satisfies*

$$p(F(x,y), F(u,v)) \leq r \max\{p(x,u), p(y,v), p(F(x,y), x), p(F(u,v), u)\}, \tag{2.1}$$

for all $x, y, u, v \in X$. If $r \in [0,1)$, then F has a unique coupled fixed point.

Proof. Choose $x_0, y_0 \in X$. Let $x_1 = F(x_0, y_0)$ and $y_1 = F(y_0, x_0)$. Again let $x_2 = F(x_1, y_1)$ and $y_2 = F(y_1, x_1)$. By continuing in the same way, we construct two sequences (x_n) and (y_n) in X such that

$$\begin{aligned} x_{n+1} &= F(x_n, y_n), \quad n = 0, 1, 2, 3, \ldots, \\ y_{n+1} &= F(y_n, x_n), \quad n = 0, 1, 2, 3, \ldots. \end{aligned} \tag{2.2}$$

Then by (2.1), we have

$$\begin{aligned} p(x_{n+1}, x_{n+2}) &= p(F(x_n, y_n), F(x_{n+1}, y_{n+1})) \\ &\leq r \max\{p(x_n, x_{n+1}), p(y_n, y_{n+1}), p(F(x_n, y_n), x_n), \\ &\qquad p(F(x_{n+1}, y_{n+1}), x_{n+1})\} \\ &\leq r \max\{p(x_n, x_{n+1}), p(y_n, y_{n+1}), p(x_{n+1}, x_n), p(x_{n+2}, x_{n+1})\} \\ &\leq r \max\{p(x_n, x_{n+1}), p(y_n, y_{n+1})\} \\ p(y_{n+1}, y_{n+2}) &= p(F(y_n, x_n), F(y_{n+1}, x_{n+1})) \\ &\leq r \max\{p(y_n, y_{n+1}), p(x_n, x_{n+1}), \\ &\qquad p(F(y_n, x_n), y_n), p(F(y_{n+1}, x_{n+1}), y_{n+1})\} \\ &\leq r \max\{p(y_n, y_{n+1}), p(x_n, x_{n+1}), p(y_{n+1}, y_n), p(y_{n+2}, y_{n+1})\} \\ &\leq r \max\{p(y_n, y_{n+1}), p(x_n, x_{n+1})\}. \end{aligned} \tag{2.3}$$

Thus from (2.3), we have

$$\max\{p(x_n, x_{n+1}), p(y_n, y_{n+1})\} \leq r \max\{p(x_{n-1}, x_n), p(y_{n-1}, y_n)\}. \tag{2.4}$$

By repeating (2.4) n-times, we get that

$$\begin{aligned}\max\{p(x_n, x_{n+1}), p(y_n, y_{n+1})\} &\leq r \max\{p(x_{n-1}, x_n), p(y_{n-1}, y_n)\}\\ &\leq r^2 \max\{p(x_{n-2}, x_{n-1}), p(y_{n-2}, y_{n-1})\}\\ &\vdots\\ &\leq r^n \max\{p(x_0, x_1), p(y_0, y_1)\}.\end{aligned} \tag{2.5}$$

Letting $n \to +\infty$ in (2.5), we get that

$$\lim_{n \to +\infty} \max\{p(x_n, x_{n+1}), p(y_n, y_{n+1})\} = 0. \tag{2.6}$$

Therefore, we have

$$\begin{aligned}\lim_{n \to +\infty} p(x_n, x_{n+1}) &= 0,\\ \lim_{n \to +\infty} p(y_n, y_{n+1}) &= 0.\end{aligned} \tag{2.7}$$

For $n, m \in \mathbb{N}$ with $m > n$, we have

$$\begin{aligned}p(x_n, x_m) &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_m) - p(x_{n+1}, x_{n+1})\\ &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2}) + p(x_{n+2}, x_m)\\ &\quad - p(x_{n+1}, x_{n+1}) - p(x_{n+2}, x_{n+2})\\ &\vdots\\ &\leq \sum_{i=n}^{m-1} p(x_i, x_{i+1}) - \sum_{i=n}^{m-2} p(x_{i+1}, x_{i+1})\\ &\leq \sum_{i=n}^{m-1} p(x_i, x_{i+1}).\end{aligned} \tag{2.8}$$

By (2.5) and (2.8), we have

$$\begin{aligned} p(x_n, x_m) &\leq \sum_{i=n}^{m-1} r^i \max\{p(x_0, x_1), p(y_0, y_1)\} \\ &\leq \sum_{i=n}^{+\infty} r^i \max\{p(x_0, x_1), p(y_0, y_1)\} \\ &= \frac{r^n}{1-r} \max\{p(x_0, x_1), p(y_0, y_1)\}. \end{aligned} \tag{2.9}$$

Letting $n, m \to +\infty$ in (2.9), we get that

$$\lim_{n,m\to\infty} p(x_n, x_m) = 0. \tag{2.10}$$

Thus $\lim_{n,m\to\infty} p(x_n, x_m)$ exists and is finite. Hence (x_n) is a Cauchy sequence in (X, p). Similarly, we may show that

$$\lim_{n,m\to\infty} p(y_n, y_m) = 0, \tag{2.11}$$

and hence (y_n) is a Cauchy sequence in (X, p). By Lemma 1.4 there exist $x, y \in X$ such that $\lim_{n\to\infty} p^s(x_n, x) = 0$ (resp., $\lim_{n\to\infty} p^s(y_n, y) = 0$) if and only if

$$\begin{gathered} p(x, x) = \lim_{n\to\infty} p(x_n, x) = \lim_{n,m\to\infty} p(x_n, x_m) = 0, \\ \left(\text{resp.,}\ \ p(y, y) = \lim_{n\to\infty} p(y_n, y) = \lim_{n,m\to\infty} p(y_n, y_m) = 0\right). \end{gathered} \tag{2.12}$$

Now, we prove that $x = F(x, y)$. By (2.1), we have

$$\begin{aligned} p(F(x,y), x) &\leq p(F(x,y), x_{n+1}) + p(x_{n+1}, x) - p(x_{n+1}, x_{n+1}) \\ &\leq p(F(x,y), x_{n+1}) + p(x_{n+1}, x) \\ &\leq p(F(x,y), F(x_n, y_n)) + p(x_{n+1}, x) \\ &\leq r \max\{p(x, x_n), p(y, y_n), p(F(x,y), x), p(F(x_n, y_n), x_n)\} + p(x_{n+1}, x) \\ &= r \max\{p(x, x_n), p(y, y_n), p(F(x,y), x), p(x_{n+1}, x_n)\} + p(x_{n+1}, x). \end{aligned} \tag{2.13}$$

Letting $n \to \infty$ in the above inequality and using (2.12), we get that

$$p(F(x,y), x) \leq r p(F(x,y), x). \tag{2.14}$$

Since $r \in [0,1)$, we conclude that $p(F(x,y),x) = 0$. By (p_1) and (p_2), we have $F(x,y) = x$. Similarly, we may show that $F(y,x) = y$. Thus (x,y) is a coupled fixed point of F. To prove the uniqueness of the fixed point, we let (u,v) be a coupled fixed point of F. We will show that $x = u$ and $y = v$. By (2.1), we have

$$\begin{aligned} p(x,u) &= p(F(x,y),F(u,v)) \\ &\leq r\max\{p(x,u),p(F(x,y),x),p(y,v),p(F(u,v),u)\} \\ &= r\max\{p(x,u),p(y,v),p(x,x),p(u,u)\}. \end{aligned} \tag{2.15}$$

Since $p(x,x) \leq p(x,u)$ and $p(u,u) \leq p(x,u)$, we have

$$p(x,u) \leq r\max\{p(x,u),p(y,v)\}. \tag{2.16}$$

Also, from (2.1), we have

$$\begin{aligned} p(y,v) &= p(F(y,x),F(v,u)) \\ &\leq r\max\{p(y,v),p(F(y,x),y),p(x,u),p(F(v,u),v)\} \\ &= r\max\{p(x,u),p(y,v),p(y,y),p(v,v)\}. \end{aligned} \tag{2.17}$$

Since $p(y,y) \leq p(y,v)$ and $p(v,v) \leq p(y,v)$, we have

$$p(y,v) \leq r\max\{p(x,u),p(y,v)\}. \tag{2.18}$$

From (2.16) and (2.18), we have

$$\max\{p(x,u),p(y,v)\} \leq r\max\{p(x,u),p(y,v)\}. \tag{2.19}$$

Since $r < 1$, we have $\max\{p(x,u),p(y,v)\} = 0$. Hence $p(x,u) = 0$ and $p(y,v) = 0$. By (p_1) and (p_2), we have $x = u$ and $y = v$. □

Corollary 2.2. *Let (X,p) be a complete partial metric space. Suppose that there are $a,b,c,d \in [0,1)$ with $a+b+c+d < 1$ such that the mapping $F : X \times X \to X$ satisfies*

$$p(F(x,y),F(u,v)) \leq ap(x,u) + bp(y,v) + cp(F(x,y),x) + dp(F(u,v),u) \tag{2.20}$$

for all $x,y,u,v \in X$. Then F has a unique coupled fixed point.

Proof. The proof follows from Theorem 2.1 by noting that:

$$\begin{aligned} &ap(x,u) + bp(y,v) + cp(F(x,y),x) + dp(F(u,v),u) \\ &\quad \leq (a+b+c+d)\max\{p(x,u),p(y,v),p(F(x,y),x),p(F(u,v),u)\}. \end{aligned} \tag{2.21}$$

□

Remarks.

(1) Theorem 1.5 [2, Theorem 2.1] is a special case of Corollary 2.2.

(2) [2, Corollary 2.2] is a special case of Corollary 2.2.

(3) Theorem 1.6 [2, Theorem 2.4] is a special case of Corollary 2.2.

(4) [2, Corollary 2.6] is a special case of Corollary 2.2.

Now, we introduce an example satisfying the hypotheses of Theorem 2.1 but not the hypotheses of Theorems 2.1 and 2.4 of [2].

Example 2.3. Define $p : [0,1] \times [0,1] \to [0,1]$ by $p(x,y) = \max\{x,y\}$. Then $([0,1],p)$ is a complete partial metric space. Let $F : [0,1] \times [0,1] \to [0,1]$ be the mapping defined by

$$F(x,y) = \frac{|x-y|}{2}. \tag{2.22}$$

Then,

(a) $p(F(x,y),F(u,v)) \leq (1/2)\max\{p(x,u),p(y,v),p(F(x,y),x),p(F(u,v),u)\}$ for all $x,y,u,v \in [0,1]$,

(b) there are no $a,b \in [0,1)$ with $a+b<1$ such that $p(F(x,y),F(u,v)) \leq ap(x,u) + bp(y,v)$ for all $x,y,u,v \in [0,1]$.

(c) there are no $a,b \in [0,1)$ with $a+b<1$ such that $p(F(x,y),F(u,v)) \leq ap(F(x,y),x) + bp(F(u,v),u)$ for all $x,y,u,v \in [0,1]$.

Proof. To prove part (a), given $x,y,v,u \in [0,1]$. Then:

$$\begin{aligned} p(F(x,y),F(u,v)) &= \max\left\{\frac{|x-y|}{2},\frac{|u-v|}{2}\right\} \\ &= \frac{1}{2}\max\{|x-y|,|u-v|\} \\ &= \frac{1}{2}\max\{x-y,y-x,u-v,v-u\} \\ &\leq \frac{1}{2}\max\{x,y,u,v\} \\ &= \frac{1}{2}\max\{p(x,u),p(y,v)\} \\ &\leq \frac{1}{2}\max\{p(x,u),p(y,v),p(F(x,y),x),p(F(u,v),u)\}. \end{aligned} \tag{2.23}$$

To prove part (b), suppose that there are $a, b \in [0,1)$ with $a + b < 1$ such that $p(F(x,y), F(u,v)) \le ap(x,u) + bp(y,v)$ for all $x, y, u, v \in [0,1]$.

Since

$$\begin{aligned} p(F(1,0), F(0,0)) &= p\left(\frac{1}{2}, 0\right) = \frac{1}{2} \le ap(1,0) + bp(0,0) = a, \\ p(F(0,1), F(0,0)) &= p\left(\frac{1}{2}, 0\right) = \frac{1}{2} \le ap(0,0) + bp(1,0) = b, \end{aligned} \tag{2.24}$$

we have $a + b \ge 1$, which is a contradiction.

To prove part (c), suppose that there are $a, b \in [0,1)$ with $a + b < 1$ such that $p(F(x,y), F(u,v)) \le ap(F(x,y), x) + bp(F(u,v), u)$ for all $x, y, u, v \in [0,1]$.

Since

$$\begin{aligned} p(F(1,0), F(0,0)) &= p\left(\frac{1}{2}, 0\right) = \frac{1}{2} \\ &\le ap(F(1,0), 1) + bp(F(0,0), 0) \\ &= ap\left(\frac{1}{2}, 1\right) + bp(0,0) \\ &= a \\ p(F(0,0), F(1,0)) &= p\left(0, \frac{1}{2}\right) = \frac{1}{2} \\ &\le ap(F(0,0), 0) + bp(F(1,0), 1) \\ &= ap(0,0) + bp\left(\frac{1}{2}, 1\right) \\ &= b, \end{aligned} \tag{2.25}$$

we have $a + b \ge 1$, which is a contradiction. □

Thus by Theorem 2.1, F has a unique coupled fixed point. Here, $(0,0)$ is the unique fixed point of F.

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