Radiative Corrections to Low-Energy Neutrino-Deuteron Reactions Revisited

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Abstract. The one-loop QED and electroweak radiative corrections to neutrino-deuteron scattering induced by the neutral current are reexamined, paying a particular attention to the constant term which has never been treated properly in literature. This problem is closely related to the definition of the axial-vector coupling constant g_A and requires thorough calculations of the constant terms in the charged current processes, too. We find that the radiative corrections to the neutral current induced reactions amount to 1.7 (1.5) per cent enhancement, if the Higgs boson mass is $m_H = 1.5\,m_Z (m_H = 5.0\,m_Z)$ This number happens to be close to that given by Kurylov et al., but we argue that this is accidental.

This talk is based on the collaboration with Masataka Fukugita [1] (see also [2, 3]), and is concerned with radiative corrections to the reactions occurring at the Sudbury Neutrino Observatory (SNO). The observation of the solar neutrinos at SNO has been playing important roles to resolve the solar neutrino problem [4]. The measurement of neutrino-deuteron scattering

$$\nu_e + d \longrightarrow e^- + p + p, \tag{1}$$

$$\nu_e + d \longrightarrow \nu_e + p + n \tag{2}$$

has now reached the level that radiative corrections should be included in the analyses [5]. The first step toward evaluation of the radiative corrections to (1) and (2) was taken by Towner [6]. Some subtle problems associated with soft photon emission were pointed out [7] and have been solved [8] by giving due consideration to the energy-dependence of the wave function overlap between initial and final states. There has remained, however, the problem as to the constant terms of the radiative correction, as remarked by the authors of [8]. They have evaluated the corrections to (1) by assuming implicitly that the inner correction to the Gamow-Teller part is the same as that to the Fermi transition.

The transition amplitudes squared of the charged current processes such as (1) are expressed in general on the $\mathscr{O}(\alpha)$ level as

$$A(\beta) = (1+\delta_{\rm out}(\beta))\left[f_V^2\left(1+\delta_{\rm in}^{\rm F}\right)\langle 1\rangle^2 + g_A^2\left(1+\delta_{\rm in}^{\rm GT}\right)\langle\sigma\rangle^2\right] \;, \tag{3}$$

where $f_V (\equiv 1)$ and g_A are the vector and axial-vector coupling constants. The Fermi and Gamow-Teller matrix elements are denoted by $\langle 1\rangle$ and $\langle\sigma\rangle$, respectively. The outer correction $\delta_{\rm out}(\beta)$ is a function of the electron velocity β and is process-dependent [9].

(See also ref. [10] for an approach based on effective field theory.) The inner corrections $\delta_{\rm in}^{\rm F}$ and $\delta_{\rm in}^{\rm GT}$ are in contrast independent of charged current processes considered and are universal constants [11, 2]. The Fermi part inner correction $\delta_{\rm in}^{\rm F}$ has been known for long time [12], while the Gamow-Teller part $\delta_{\rm in}^{\rm GT}$ was calculated only recently [2].

The axial-vector coupling constant $\tilde{g}_A$ usually quoted in literature is extracted from the neutron beta decay using the formula

$$A(\beta) = (1+\delta_{\rm out}(\beta))\left(1+\delta_{\rm in}^{\rm F}\right)\left[\langle 1\rangle^2 f_V^2 + \langle\sigma\rangle^2\tilde{g}_A^2\right] \ . \tag{4}$$

The polarized neutron beta decay is also used to extract $\tilde{g}_A$ instead of g_A [3]. The relation between g_A and $\tilde{g}_A$ is obviously given by

$$g_A^2 = \left(\frac{1+\delta_{\rm in}^{\rm F}}{1+\delta_{\rm in}^{\rm GT}}\right)\tilde{g}_A^2 \ , \tag{5}$$

and this indicates $g_A \neq \tilde{g}_A$, because of $\delta_{\rm in}^{\rm F} \neq \delta_{\rm in}^{\rm GT}$ as was shown by explicit computation in [2]. This, however, does not cause any practical problem, since the relation between the “bare" g_A and the “redefined " $\tilde{g}_A$ is universal, so far as we consider only charged current processes [11, 13]. Thus the analysis of (1) in [5] need not be corrected essentially.

If we consider neutral current processes such as (2), however, we have to be more careful about the finite radiative corrections to the axial-vector coupling constant. The reaction (2) is purely of the Gamow-Teller type and its amplitude squared is written as

$$B(\beta) = (1+\Delta_{\rm in}^{\rm GT})g_A^2\langle\sigma\rangle^2 \ , \tag{6}$$

where $\Delta_{\rm in}^{\rm GT}$ is the $\mathscr{O}(\alpha)$ radiative corrections. Using the relation (5), we see that (6) is expressed in terms of $\tilde{g}_A$ as

$$B(\beta) = (1+\Delta_{\rm in}^{\rm GT})\left(\frac{1+\delta_{\rm in}^{\rm F}}{1+\delta_{\rm in}^{\rm GT}}\right)\tilde{g}_A^2\langle\sigma\rangle^2 \ . \tag{7}$$

In [1], we have extracted $\Delta_{\rm in}^{\rm GT}$ on the bases of the work of Marciano and Sirlin [14] and have found

$$\Delta_{\rm in}^{\rm GT} = 0.0192 \qquad \text{for } m_H = 1.5\, m_Z, \tag{8}$$

$$\Delta_{\rm in}^{\rm GT} = 0.0173 \qquad \text{for } m_H = 5.0\, m_Z. \tag{9}$$

Combining these results with the previous calculation of the inner corrections [2]

$$\delta_{\rm in}^{\rm F} = 0.0237, \qquad \delta_{\rm in}^{\rm GT} = 0.0262, \tag{10}$$

we find that the cross section of (2) is enhanced by the factor

$$\begin{aligned}(1+\Delta_{\rm in}^{\rm GT})\left(\frac{1+\delta_{\rm in}^{\rm F}}{1+\delta_{\rm in}^{\rm GT}}\right) &= 1.017 \qquad \text{for } m_H = 1.5\, m_Z, \\ (1+\Delta_{\rm in}^{\rm GT})\left(\frac{1+\delta_{\rm in}^{\rm F}}{1+\delta_{\rm in}^{\rm GT}}\right) &= 1.015 \qquad \text{for } m_H = 5.0\, m_Z \ . \end{aligned} \tag{11}$$

Note that this is rather close to the number given in [8] and that the analysis of (2) in [5] using [8] need not be altered basically. We should, however, like to emphasize that the approximate agreement of our results (11) with that of [8] is simply due to an accidental cancellation of errors of the latter, between those caused by putative identification of constant terms for the Fermi and Gamow-Teller transitions for the charged current reactions and minor errors in their treatment of the constant terms for neutral current induced reactions.

Throughout the present work, the so-called one-body impulse approximation is used and the effect of the spectator nucleon is not included. See ref. [15] in this connection for an approach based on heavy-baryon chiral perturbation theory. We note as a final remark that the constant term for the radiative correction to the ratio of neutral to charged current reaction (after the usual outer correction [8, 9] for the charged current reaction) is -0.6%, which may be compared with the claimed error (0.5%) of nuclear calculations for the ratio of tree level cross sections [16].

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