Dark Energy and The Dark Matter Relic Abundance

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Abstract. Two mechanisms by which the Quintessence scalar could enhance the relic abundance of dark matter particles are discussed. This effect can have an impact on supersymmetric candidates for dark matter.

INTRODUCTION

According to the standard paradigm, a particle species goes through two main regimes during the cosmological evolution. At early times it stays in thermal equilibrium, until the particle interaction rate Γ remains larger than the expansion rate H. Later on, the particles will be so diluted by the expansion of the universe that they will not interact anymore and H will overcome Γ . The epoch at which $\Gamma = H$ is called 'freeze-out', and after that time the number of particles per comoving volume for any given species will remain constant. This is how cold dark matter particle relics (neutralinos, for example) are generated.

As it can be easily understood, this scenario strongly depends on the evolution equation for H in the early universe, which is usually assumed to be radiation-dominated. However, as it was already noticed some time ago [1], there is little or no evidence that before Big Bang Nuclesynthesis (BBN) it was necessarily so. Non-standard scenarios are then worth exploring. In particular, if we imagine that for some time in the past the Hubble parameter was larger than usually thought (for example, due to the presence of some other component, in addition to radiation), then the decoupling of particle species would be anticipated, resulting in a net enhancement of their relic abundance.

A natural candidate for doing that is the Quintessence scalar, which is thought to consitute the Dark Energy fluid dominating the present universe. In most Quintessence models, the cosmological scalar is assumed to become the dominant component of the universe after a long period of sub-domination [2], playing little or no role in the earliest epochs. However, this has not always to be the case, as we will show in the following. We will focus on the possibility of modifying the past evolution of the Hubble parameter H, with the double aim of respecting all the post-BBN bounds for the expansion rate and of producing a mesurable enhancement of the dark matter particles relic abundance. In particular, we will report about two possible mechanism by which the past dynamics of the Quintessence scalar could significantly modify the standard evolution of the pre-BBN universe: an early "kination" phase and a "scalar-tensor" model.

KINATION ENHANCEMENT

If we imagine to add a significant fraction of scalar energy density to the background radiation at some time in the cosmological history, this would produce a variation in H^2 , depending on the scalar equation of state w_{ϕ} . If $w_{\phi} > w_r = 1/3$, the scalar energy density would decay more rapidly than radiation, but temporarily increase the global expansion rate. This possibility was explicitly considered in Ref. [3], where it was calculated that a huge enhancement of the relic abundance of neutralinos could be produced in this way.

In a flat universe, a scalar field with potential $V(\phi)$ obeys the equations

$$\ddot{\phi} + 3H\dot{\phi} + dV/d\phi = 0$$
; $H^2 \equiv (\dot{a}/a)^2 = 8\pi\rho/3M_p^2$. (1)

For any given time during the cosmological evolution, the relative importance of the scalar energy density w.r.t. to matter and radiation in the total energy density $\rho \equiv \rho_m + \rho_r + \rho_\phi$ depends on the initial conditions, and is constrained by the available cosmological data on the expansion rate and large scale structure. If the potential $V(\phi)$ is of the runaway type, the initial stage of the scalar evolution is typically characterized by a period of so-called 'kination' [2] during which the scalar energy density $\rho_{\phi} \equiv \dot{\phi}^2/2 + V(\phi)$ is dominated by the kinetic contribution $E_k = \dot{\phi}^2/2 \gg V(\phi)$, giving $w_{\phi} = 1$. After this initial phase, the field comes to a stop and remains nearly constant for some time ('freezing' phase), until it eventually reaches an attractor solution [2].

Then, if we modify the standard picture according to which only radiation plays a role in the post-inflationary era and suppose that at some time \hat{t} the scalar contribution was small but non negligible w.r.t. radiation, then at that time the expansion rate $H(\hat{t})$ should be correspondingly modified. During the kination phase the scalar to radiation energy density ratio evolves like $\rho_{\phi}/\rho_r \sim a^{-3(w_{\phi}-w_r)} = a^{-2}$, and so the scalar contribution would rapidly fall off and leave room to radiation. In this way, we can respect the BBN bounds and at the same time keep a significant scalar contribution to the total energy density just few red-shifts before. The increase in the expansion rate H due to the additional scalar contribution would anticipate the decoupling of particle species and result in a net increase of the corresponding relic densities. As shown in [3], a scalar to radiation energy density ratio $\rho_{\phi}/\rho_r \simeq 0.01$ at BBN would give an enhancement of the neutralino codensity of roughly three orders of magnitude.

The enhancement of the relic density of neutralinos requires that at some early time the scalar energy density was dominating the Universe. This fact raises a problem if we want to identify the scalar contribution responsible for this phenomenon with the Quintessence field [4]. Indeed, the initial conditions must be such that the scalar energy density is sub-dominant at the beginning, if we want the Quintessence field to reach the cosmological attractor in time to be responsible for the presently observed acceleration of the expansion [2]. For initial conditions $\rho_{\phi} \gtrsim \rho_r$ we obtain instead an 'overshooting' behavior: the scalar field rapidly rolls down the potential and after the kination stage remains frozen at an energy density much smaller than the critical one. However, as shown in [5], more complicated dynamics are possible if we relax the hypothesis of considering a single uncoupled scalar. The presence of several scalars and/or of a small coupling with the dark matter fields could modify the dynamics in such a way that the attractor is reached in time even if we started in the overshooting region. Consider a potential of the form $V(\phi_1, \phi_2) = M^{n+4} (\phi_1 \phi_2)^{-n/2}$, with M a constant of dimension mass. In this case, the two fields' dynamics enlarges the range of possible initial conditions for obtaining a quintessential behavior today. This is due to the fact that the presence of more scalars allows to play with the initial conditions in the fields' values, while maintaining the total initial scalar energy density fixed. Doing so, it is possible to obtain a situation in which for a fixed ρ_{ϕ}^{in} in the overshooting region, if we keep initially $\phi_1 = \phi_2$ we actually produce an overshooting behavior, while if we choose to start with $\phi_1 \neq \phi_2$ (and *the same* ρ_{ϕ}^{in}) it is possible to reach the attractor in time.

Suppose, instead, that the Quintessence scalar is not completely decoupled from the rest of the Universe. Among the possible interactions, two interesting cases are the following:

$$V_b = b H^2 \phi^2 \quad \text{or} \quad V_c = c \rho_m \phi \tag{2}$$

If we add V_b or V_c to $V = M^{n+4}\phi^{-n}$, the potential will acquire a (time-dependent) minimum and the scalar field will be prevented from running freely to infinity. In this way, the long freezing phase that characterizes the evolution of a scalar field with initial conditions in the overshooting region can be avoided. A more detailed discussion, together with numerical examples, can be found in Ref. [4].

SCALAR-TENSOR ENHANCEMENT

A different possibility arises if we consider Quintessence models in the framework of scalar-tensor (ST) theories of gravity (see [7] and references therein). These theories represent a natural framework in which massless scalars may appear in the gravitational sector of the theory without being phenomenologically dangerous, since they assume a metric coupling of matter with the scalar field, thus ensuring the equivalence principle and the constancy of all non-gravitational coupling constants [6]. Moreover a large class of these models exhibit an attractor mechanism towards GR [8], that is, the expansion of the Universe during the matter dominated era tends to drive the scalar fields toward a state where the theory becomes indistinguishable from GR.

ST theories of gravity are defined, in the so-called 'Jordan' frame, by the action

$$S_g = \frac{1}{16\pi} \int d^4x \sqrt{-\tilde{g}} \left[\Phi^2 \tilde{R} + 4 \,\omega(\Phi) \tilde{g}^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - 4 \tilde{V}(\Phi) \right] \,. \tag{3}$$

The matter fields Ψ_m are coupled only to the metric tensor $\tilde{g}_{\mu\nu}$ and not to Φ , *i.e.* $S_m = S_m[\Psi_m, \tilde{g}_{\mu\nu}]$. Each ST model is identified by the two functions $\omega(\Phi)$ and $\tilde{V}(\Phi)$. The matter energy-momentum tensor is conserved, masses and non-gravitational couplings are time independent, and in a locally inertial frame non gravitational physics laws take their usual form. Thus, the 'Jordan' frame variables $\tilde{g}_{\mu\nu}$ and Φ are also denoted as the 'physical' ones in the literature. By means of a conformal transformation,

$$\tilde{g}_{\mu\nu} \equiv A^2(\phi) g_{\mu\nu} , \quad \Phi^2 \equiv 8\pi M_*^2 A^{-2}(\phi)$$
 (4)

with

$$\alpha^{2}(\varphi) \equiv d\log A(\varphi)/d\varphi = 1/(4\omega(\Phi) + 6), \qquad (5)$$

it is possible to go the 'Einstein' frame in which the gravitational action takes the standard form, while matter couples to φ only through a purely metric coupling,

$$S_m = S_m[\Psi_m, A^2(\varphi)g_{\mu\nu}] .$$
(6)

In this frame masses and non-gravitational coupling constants are field-dependent, and the energy-momentum tensor of matter fields is not conserved separately, but only when summed with the scalar field one. On the other hand, the Einstein frame Planck mass M_* is time-independent and the field equations have the simple form

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T^{\phi}_{\mu\nu}/M_*^2 + T_{\mu\nu}/M_*^2 \quad , \qquad M_*^2 \partial^2 \phi + \partial V/\partial \phi = -\alpha(\phi)T/\sqrt{2} \,. \tag{7}$$

When $\alpha(\varphi) = 0$ the scalar field is decoupled from ordinary matter and the ST theory is indistinguishable from ordinary GR. The effect of the early presence of a scalar field on the physical processes will come through the Jordan-frame Hubble parameter $\tilde{H} \equiv d \log \tilde{a}/d\tilde{\tau}$:

$$\tilde{H} = H \left(1 + \alpha(\boldsymbol{\varphi}) \, \boldsymbol{\varphi}' \right) / A(\boldsymbol{\varphi}) \,, \tag{8}$$

where $H \equiv d \log a/d\tau$ is the Einstein frame Hubble parameter. A very attractive class of models is that in which the function $\alpha(\varphi)$ has a zero with a positive slope, since this point, corresponding to GR, is an attractive fixed point for the field equation of motion [8]. It was emphasized in Ref. [9] that the fixed point starts to be effective around matterradiation equivalence, and that it governs the field evolution until recent epochs, when the Quintessence potential becomes dominant. If the latter has a run-away behavior, the same should be true for $\alpha(\varphi)$, so that the late-time behavior converges to GR. For this reason, we will consider the following choice,

$$A(\varphi) = 1 + Be^{-\beta\varphi} \quad , \qquad \alpha(\varphi) = -\beta Be^{-\beta\varphi} / (1 + Be^{-\beta\varphi}) \,, \tag{9}$$

which has a run-away behavior with positive slope.

In Ref. [7] it was calculated the effect of ST on the Jordan-frame Hubble parameter \tilde{H} at the time of WIMP decoupling, imposing on the parameters B and β the constraints coming from GR test, CMB observations and BBN. Computing the ratio \tilde{H}/\tilde{H}_{GR} at the decoupling time of a typical WIMP of mass m = 200 GeV, it was found that it is possible to produce an enhancement of the expansion rate up to $O(10^5)$. As a further step, it was performed the calculation of the relic abundance of a DM WIMP with mass m and annihilation cross-section $\langle \sigma_{ann} v \rangle$. The effect of the modified ST gravity enters the computation of particle physics processes (like the WIMP relic abundance) through the "physical" expansion rate \tilde{H} defined in Eq. (8). We have therefore implemented the standard Boltzmann equation with the modified physical Hubble parameter \tilde{H} :

$$dY/dx = -s\langle \sigma_{\rm ann}v\rangle (Y^2 - Y_{\rm eq}^2)/\tilde{H} x$$
⁽¹⁰⁾

where x = m/T, $s = (2\pi^2/45) h_{\star}(T) T^3$ is the entropy density and Y = n/s is the WIMP density per comoving volume.

A numerical solution of the Boltzmann equation Eq. (10) is shown in Fig. 1 for a toy-model of a DM WIMP of mass m = 50 GeV and constant annihilation cross-section



FIGURE 1. Numerical solution of the Boltzmann equation Eq. (10) in a ST cosmology for a toy-model of a DM WIMP of mass m = 50 GeV and constant annihilation cross-section $\langle \sigma_{ann} v \rangle = 1 \times 10^{-7}$ GeV⁻². The temperature evolution of the WIMP abundance Y(x) clearly shows that freeze-out is anticipated, since the expansion rate of the Universe is largely enhanced by the presence of the scalar field φ . At a value $x = m/T_{\varphi}$ a re-annihilation phase occurs and Y(x) drops to the present day value.

 $\langle \sigma_{\rm ann} v \rangle = 1 \times 10^{-7} \text{ GeV}^{-2}$. The temperature evolution of the WIMP abundance Y(x)clearly shows that freeze-out is anticipated, since the expansion rate of the Universe is largely enhanced by the presence of the scalar field φ . This effect is expected. However, we note that a peculiar effect emerges: when the ST theory approached GR (a fact which is parametrized by $A(\varphi) \rightarrow 1$ at a temperature T_{φ} , which in our model is 0.1 GeV), \tilde{H} rapidly drops below the interaction rate Γ establishing a short period during which the already frozen WIMPs are still abundant enough to start a sizeable re-annihilation. This post-freeze-out "re-annihilation phase" has the effect of reducing the WIMP abundance, which nevertheless remains much larger than in the standard case (for further discussion on this aspect see [7]). The amount of increase in the relic abundance which is present in ST cosmology is shown in Fig. 2. The solid curve refers to an annihilation cross section constant in temperature, i.e. $\langle \sigma_{ann} v \rangle = a$, while the dashed line stands for an annihilation cross section which evolves with temperature as: $\langle \sigma_{ann} v \rangle = b/x$. In the case of *s*-wave annihilation the increase in relic abundance ranges from a factor of 10 up to a factor of 400. For a pure b/x dependence, the enhancement can be as large as 3 orders of magnitude. Needless to say, such potentially (very) large deviations entail new prospects on the WIMP characterization both for the choice of the CDM candidates and for their direct and indirect detection probes [7, 10].



FIGURE 2. Increase in the WIMP relic abundance in ST cosmology with respect to the GR case. The solid curve refers to an annihilation cross section constant in temperature, i.e. $\langle \sigma_{ann} v \rangle = a = 10^{-7}$ GeV⁻², while the dashed line stands for an annihilation cross section which evolves with temperature as $\langle \sigma_{ann} v \rangle = b/x = 10^{-7}$ GeV⁻²/x.

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