Theory of Behavioral Power Functions

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Data in operant conditioning and psychophysics are often well fitted by functions of the form $y = qx^s$. A simple theory derives these power functions from the simultaneous equations $dx/x = a_1 f(z)dz$ and $dy/y = a_2 f(z)dz$, where z is a comparison variable that is equated for the effects of x and y, and a_1 and a_2 are sensitivity parameters. In operant conditioning, x and y are identified with response rates; in psychophysics, with measures of stimulus and response. The theory can explain converging sets of power functions, solves the dimensional problems with the standard power function, and can account for the relation between Type I and Type II psychophysical scales.

Many empirical relations take on a linear form when plotted in double-logarithmic coordinates. In experimental psychology such relations have frequently been reported in operant studies of choice behavior and rate-dependent drug effects and in experiments on psychophysical scaling. Power functions of the form $y = qx^s$, where q and s are constants, are the simplest mathematical approximation to these empirical functions. While there has been considerable theoretical discussion about the possible processes responsible for the psychophysical power law, choice of power function fits in other cases has been largely a matter of empirical convenience. The theoretical basis even for the psychophysical power function is still a matter for dispute.

In this article, I propose a very simple theoretical basis for power functions. This derivation can be interpreted in terms of behavioral competition in operant conditioning situations and in terms of an internal "sensation" variable in psychophysical scaling experiments. The theory predicts that sets of power functions will in many cases converge on a common point; it solves problems of dimensional balance encountered when x and y are measured in the

same units; and it is consistent with recent work on the relation between different types of psychophysical scales. Data from numerous experiments on operant behavior and psychophysical scaling are consistent with the convergence prediction.

The derivation is most easily introduced in connection with a specific example, and this is done in the first section. The second section describes its application to rate-dependent drug effects, and the third section deals with the psychophysical power law.

Response Competition

The most obvious effect of reward and punishment is on the level of the rewarded or punished response. However, for a complete understanding of reinforcement, it is necessary to study not just direct effects on a particular response but also indirect effects on the way that behaviors interact with one another. The most frequent type of interaction is competition, since the time available for responding in instrumental (operant) conditioning situations is generally less than the time needed for all tendencies to action to find full expression (Atkinson & Birch, 1970; McFarland, 1974; Staddon, 1977a; Staddon & Ayres, 1975). Recent theoretical analyses of competitive interactions have shown that even quite simple quantitative treatments can sometimes integrate a surprisingly wide range of data (McFarland & Sibly, 1975; Staddon, 1977b). However, the topic of response competition has received relatively little direct experi-

mental attention, and until recently, few results were available comparable in quantitative precision to those from studies of the direct effect of reinforcement on rates of operant responding.

The most direct way to study competition is to look at the level of one activity while the level of a second, competing one is varied by some means known not to affect the level of the first activity directly. In practice, however, this method is difficult to apply because a causal factor sufficient to affect one activity may also have direct effects on the other. However, a recent series of experiments by Nevin (1974a, 1974b) appears to meet this objection and has yielded data that are susceptible of a relatively simple interpretation. In his most extensive experiment, Nevin (1974b) studied the behavior of hungry pigeons responding for food reinforcement on two separate response keys. Pecks on one key yielded brief access to grain for the first peck after 50 sec (fixed-interval [FI] 50-sec schedule). On the other key, two colors alternated at 10-sec intervals. Food was available intermittently at different rates in the presence of each color for pecking on this second key (multiple variable-interval variable-interval [VI VI] schedule). The relative frequency of food delivery for pecking on each of the two multiple-key colors was varied in different conditions of the experiment. In each condition, the animals were exposed to the schedules daily until their rates of responding on each key and to each color had settled down to stable levels.

On FI schedules, responding increases progressively through the time between food deliveries, rising to a maximum as the next feeding becomes imminent. The pigeons in this study responded in this fashion on the FI key. Responding on the other (multiple) key followed a complementary course: The pigeons pecked rapidly early in the (FI) interfood interval and more slowly toward the end. Food delivery for pecks on the multiple key was aperiodic. Hence, it is likely that the observed temporal variation in multiple-key responding was due primarily to progressive suppression by FI-key responding and not to a separate internal clock for responding on the multiple key.

The major result from this study is a set of functions showing how responding on each of the multiple-key colors was suppressed by the progressively increasing FI-key responding throughout the 50-sec FI interfood interval. The interval was divided into 10-sec blocks, and plots were made of response rate during comparable blocks in the presence of each of the multiple-key colors, that is, of response rate to one color versus response rate to the other, at comparable times in the 50-sec fixed interval. These response–response plots were in all cases acceptably fitted by power functions of the form

$$y = qx^s, \qquad (1)$$

where x and y are response rates in the presence of the two multiple-key colors, and q and s are parameters that varied with the frequencies of reinforcement in the two colors.

Equation 1 can be derived in a number of ways. However, the basis to be presented here is one of the simplest and may be the most general. I suppose only that the effect on behaviors x or y of changes in the competing behavior z, the level of FI responding, depends solely on the initial levels of x or y. This assumption can be stated precisely as follows:

$$dx/x = a_1 f(z)dz \qquad (2a)$$

and

$$dy/y = a_2 f(z)dz. \qquad (2b)$$

In words, these equations simply affirm that a small change, dz, in the level of z produces a change, dx (or dy) in x or y that is proportional to the initial level of x, to the change in z, and to some unspecified function of z, $f(z)$. The constants of proportionality a_1 and a_2 represent the sensitivities of x and y, respectively, to these changes in z. The fact that $f(z)$ need not be specified simply indicates that the relation between x and y (which is about to be derived) does not depend on the form of the relation between x (or y) and z (except that it be the same for x and y). I return to this point in a moment. Since the interactions between x or y and z in Nevin's (1974b) experiment are assumed to be competitive, $f(z)$ is likely to be a negative function of some sort.
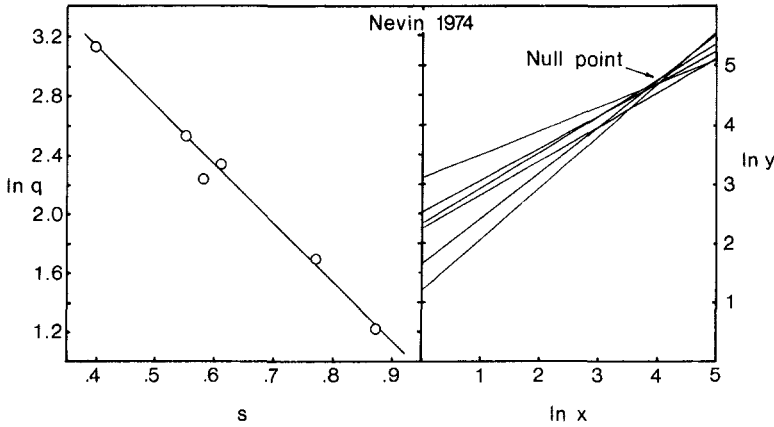
Figure 1. The right-hand panel shows lines fitted by Nevin (1974b) to the logarithms of response rates (ln y and ln x) in two multiple-schedule components at corresponding times in relation to periodic food delivered for an alternative response to a second response key. (Each line represents a different combination of reward frequencies for responding on the multiple-schedule key.) The left-hand panel shows the relation between the slopes (s) and intercepts (ln q) of the lines in the right-hand panel.

Integrating both sides of Equation 2 yields

$$\ln x + K_1 = a_1 \int f(z)dz \qquad (3a)$$

and

$$\ln y + K_2 = a_2 \int f(z)dz. \qquad (3b)$$

Dividing Equations 3a and 3b to eliminate expressions involving z, rearranging, and exponentiating yields

$$y = \exp(a_2 K_1/a_1 - K_2)\cdot x^{a_2/a_1}, \qquad (4)$$

where K_1 and K_2 are constants of integration.

Equation 4 is of the same form as Equation 1, but with

$$q = \exp(a_2 K_1/a_1 - K_2) \qquad (5)$$

and

$$s = a_2/a_1. \qquad (6)$$

Parameter s, the ratio of sensitivities, represents the *elasticity* of behavior y with respect to behavior x. It follows from Equations 5 and 6 that

$$\ln q = sK_1 - K_2. \qquad (7)$$

Nevin found that both s and q varied with the frequencies of food delivery for responding to the two multiple-key colors. The simplest assumption is that these changes are solely a reflection of changes in a_1 and a_2, the sensitivities of the two responses, so that K_1 and K_2 can be assumed constant across conditions. If this is so, then Equation 7 implies a linear relation between elasticity parameter s in Equation 1 and the logarithm of parameter q.

Nevin (1974b) obtained a number of functions that were fit by Equation 1, and he estimated s and q for each. A plot of ln q versus s from these data appears as the left-hand panel of Figure 1. The six points are very well fit by a straight line, with $K_1 = -3.96$ and $K_2 = -4.69$, in accordance with the prediction.

Geometrically, Equation 7 implies that the original power functions from which s and q were obtained should all pass through the point ln $x = -K_1$, ln $y = -K_2$. The right-hand panel of Figure 1 shows that the data are consistent with this prediction.[1] The point of

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[1] There is an important caveat to be noted here. Since the zero point of a logarithmic scale corresponds to the unity point of the corresponding linear scale, the location of zero depends on the units of measurement. The linear relation of Equation 7 simply represents the obvious fact that the slopes and intercepts of a set of straight lines that go through a point will be related—positively if the origin is to the right of the point of intersection, negatively otherwise. However, since on a log scale the origin can be moved as far as we please from the region of the empirical data simply by changing units, a linear relation between s and ln q can be forced by measuring rates in sufficiently extreme units. In this case the "point" through which the lines all go is nothing but the region of possible rates, which is limited at the top by physiological limitations and at the bottom by the time during which behavior is sampled. Since Nevin's (1974b) data cover almost the full range of the ln x axis on the right of Figure 1, the convergence shown there is not forced in this way.

convergence may be termed the *null point*, since it represents that pair of response rates that is unaffected by changes in *a*. If there were no preference for one color over the other and if the situation were symmetrical in every other way, one might expect K_1 and K_2 to be equal. This does not seem to be the case in this study: K_2 was greater than K_1 in absolute magnitude, and the birds tended to respond at higher rates on red (y) compared to green (x), a commonly observed preference in pigeons.

The derivation of Equation 1 just described does not depend on the form of the relation between x and z. However, Killeen (1975), in his discussion of Nevin's result, has assumed a particular relation between x and z (here considered not as a competing response but just as time, t, in the fixed interval) of the form

$$x = A_1 \exp(-t/C_1). \qquad (8)$$

This result can be obtained from Equation 2 by taking $f(z) = -1$; whereupon, on integration $x = \alpha \exp(-a_1 z)$, which is of the same form as Killeen's equation. Killeen suggests a particular identification for the constants A_1 and C_1 that goes beyond the present derivation. Since his model can be considered as a special case of the present one, it can make similar predictions about the relation between s and q in Equation 1. However, it also requires an exponential relation between x and t, and unfortunately, Nevin's data do not suggest a simple relation. Hence, there is virtue in a theoretical approach that is noncommittal about the function relating x (or y) and z. We shall see later that a similar uncertainty exists in the case of the psychophysical law.

Interpretation of Parameters

The parameters a_1 and a_2 in Equation 4 have already been interpreted as sensitivities. K_1 and K_2 are scale factors. This can be shown in two ways. First, unless x and y are considered to be dimensionless (e.g., as relative rates), Equation 1 cannot be made to balance dimensionally in its simple form (i.e., with q as an independent parameter) if x and y are measured in the same units.[2] For example, if x and y are considered as rates, having the dimensions T^{-1}, then q must have the dimensions T^{s-1} for dimensional balance, which implies dependence of q on s. However, if in Equation 4, the dimensions of K_1 and K_2 are taken as $\ln T$, the proper dimensional balance is achieved, as follows:

$$T^{-1} = \exp[s \ln (T) - \ln (T)] \cdot (T^{-1})^s;$$

whence, taking logarithms of both sides,

$$-\ln (T) = s \ln (T) - \ln (T) - s \ln (T)$$
$$= -\ln (T).$$

Hence K_1 and K_2 can be interpreted as the logarithms of the time bases with respect to which x and y are measured. The same conclusion can be arrived at by noting the effect of changing the units of x and y by multipliers, such that $x = \alpha x'$ and $y = \beta y'$. Then, a little algebra shows that $K'_1 = K_1 + \ln \alpha$ and $K'_2 = K_2 + \ln \beta$.

Rate-Dependent Drug Effects

Many drugs produce effects on responding in operant conditioning situations that are well described by Equation 1 (Dews, 1958, 1964; Gonzalez & Byrd, 1977). Since there is growing evidence that in such situations there is competition between the recorded operant response and other "schedule-induced" activities, the present analysis can be applied.

The usual procedure in these experiments is to compare response rates at equivalent points in a periodic schedule, such as fixed interval, between 2 days: a day on which no drug, or a control injection, is administered, and a second day (typically the next day) when a given drug dose is administered before the experimental session. (A few studies have compared response rates in the presence of different stimuli that sustain different rates under control conditions.) Traditionally, the results have been displayed as a plot of control data versus the ratio of drug and control rates for each segment of the fixed interval, collapsed over all the intervals in each experimental session. Such plots are linear in log–log coordinates, which implies a power relation of the same form as Equation 1 between drug

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[2] I am indebted to J. A. Nevin and M. C. Davison for pointing out the dimensional problem with Equation 1.
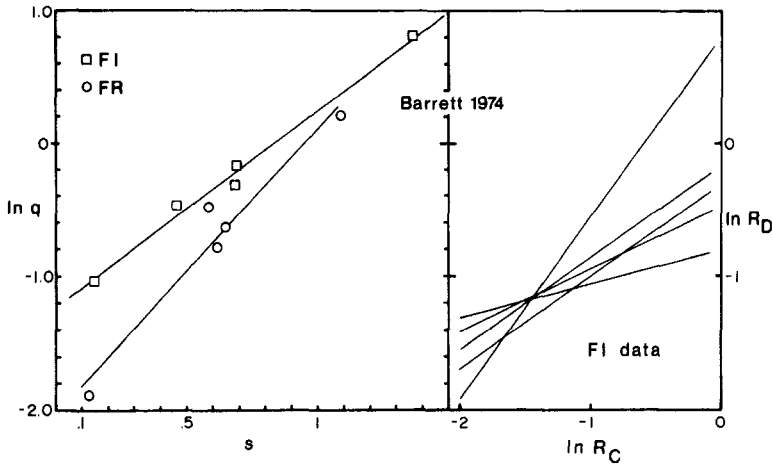
Figure 2. The right-hand panel shows lines fitted from Barrett (1974) to the logarithms of response rates of a single pigeon under drug (R_D; pentobarbital) and no-drug (R_C) conditions at comparable points in a periodic schedule. (Each line is the result of a different drug dose.) The lines in the left-hand panel show the relation between the slopes (*s*) and intercepts (ln *q*) of the fitted lines, both for responding on a key associated with a fixed-interval schedule (FI) and for responding on a key associated with a fixed-ratio schedule (FR).

and control rates plotted directly (Gonzalez & Byrd, 1977).

There is considerable evidence that on periodic schedules of food delivery there is competition between the instrumental response, which occurs with increasing frequency as the time when food will be available approaches, and other "interim" activities, which occur predominantly at the beginning of each interfood interval (Staddon, 1977b; Staddon & Ayres, 1975; Staddon & Frank, 1975). For example, if the interim activities are prevented in some way, instrumental responding begins earlier in the interval. These considerations set the stage for applying the theory to rate-dependent effects. It is necessary only to assume that the interim activities exert a suppressive effect on the instrumental response that is equivalent to the suppressive effect of FI-key responding on multiple-key responding in Nevin's (1974b) experiment. If the frequency of these interim activities is denoted by *z*, and the rates of the instrumental response on control and drug days are denoted by *x* and *y*, respectively, then Equation 2 can be applied directly. Proceeding as before, Equation 7 then describes the relation between the two parameters *s* and *q* of the power function relating drug and control response rates

on the assumption that the effect of the drug is solely on the sensitivity parameter: a_1 is then the sensitivity parameter under control conditions, and a_2 the sensitivity parameter under drug conditions. K_1 and K_2 are therefore assumed to be constant across conditions, that is, drug doses.

The slope and intercept of the function relating control and drug rates have been shown in many experiments with numerous drugs to vary with drug dose. Figure 2 shows data from an individual pigeon in an experiment by Barrett (1974). The animal was trained on a schedule in which pecks on one key (interval key) were reinforced with food once every 5 min, providing at least 10 pecks to a second key (ratio key) had occurred (conjunctive fixed-ratio [FR] 10 – FI 5). Various doses of pentobarbital were administered, and the response rates in successive 30-sec portions of the fixed interval on control and drug days were compared in the way just described. The left-hand panel of Figure 2 shows the relation between *s*, the slope, and ln *q*, the intercept of the power functions obtained by Barrett at different drug doses. Two sets of data are shown: one for responding on the ratio key and the other for responding on the interval key. In both cases, the points are

well fitted by a straight line. The right-hand panel of Figure 2 shows the actual power functions fitted to his data by Barrett,[3] and as implied by the linear relations in the left-hand panel, they converge on approximately the same point. If, as I have proposed, the drug affects only parameter a, then with no drug injection, $a_1 = a_2$, and y should equal x. Hence, from Equation 4, K_1 should equal K_2. This is approximately true for the ratio data in Figure 2, but lines fitted to the FI data converge on a point quite far removed from equality. This implies that control saline injections may have had an effect on K in this experiment.

I have examined a total of 14 studies involving 21 different drugs, in which rate-dependent relations were obtained (Barrett, 1974; Bond, Sanger, & Blackman, 1975; Branch & Gollub, 1974; Byrd, 1975; Dews, 1964; Leander, 1975; MacPhail & Gollub, 1975; Marr, 1970; McKearney, 1970; Mc-Millan, 1973a, 1973b; Stitzer, 1974; Wenger & Dews, 1976; Wüttke, 1970). These studies contain a total of 69 sets of power functions similar to those shown in Figure 2, obtained with six species of animals.[4] Fifty-eight of these sets comprise three or more functions, permitting a test of the linearity prediction of Equation 7. No study shows a systematic deviation from linearity. Twenty-five studies (43%) show linear fits with coefficients of determination (r^2) greater than .80; 9 were between .60 and .79, 14 between .20 and .59, and 10 less than .20.

Most studies therefore fit the theory quite well. Those that do not fit (i.e., show r^2 values less than .6) can be classified into three groups. (a) Some studies fail because the range of drug doses used yielded only a small range of slopes. It is clear from the geometry of the situation that if many slopes are close together, even small errors in determining slopes will produce large variations in the points of intersection of the lines. Hence, a good fit to the model is not to be expected if the range of slopes obtained is small. (b) The geometry also indicates that a poor fit will be obtained if the units are such that the point of intersection of the lines (the null point) is in the vicinity of a control rate of unity (zero on the log scale). In this case, there will be variation

in slope but little variation in intercept. (c) Finally, studies may yield an adequate range of slopes, with no common intersection point, that is, clear failure to agree with the model. Careful examination of the 25 or so sets of functions that yielded low r^2 values showed that only a handful, five or six, are clearly discrepant with the model. The clearly discrepant results are from studies by Leander (1975, three examples), McKearney (1970, Figure 5, S^p), and McMillan (1973a, Table 2, pentobarbital, unpunished; Table 4, imipramine, unpunished). As suggested by the studies cited previously, there are a number of intermediate cases. All the other cases fall into Categories a and b.

For most sets of functions, K_1 and K_2 are within 20% of each other, as implied by the assumption that different drug doses affect only parameter a. However, the null point is usually different for different drugs, implying a dose-independent effect of the drug on K. These characteristics are illustrated by data from an extensive study by Leander (1975), which are shown in Figure 3. The figure shows log (control rate) versus log (drug rate) functions fitted by Leander to data produced under different doses of 11 major tranquilizers. Three of the drugs (triflupromazine, trifluoperazine, and fluphenazine) fit the model poorly; the remaining 8 (and the three injection vehicles) show the predicted linear relations. The null points for the different drugs are clearly different. Other studies show that the null point depends on whether responding is punished or unpunished (McMillan, 1973a, 1973b), whether it is associated with a positive (reinforcement associated) or negative (associated with the absence of reinforcement)

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[3] Barrett's (1974) data were plotted in the usual form: log (drug rate/control rate) versus log (control rate). I estimated slopes and intercepts from the fitted lines in his Figure 3 and then transformed the functions to yield the log (drug rate) versus log (control rate) lines shown in Figure 2.

[4] In the Wüttke (1970) study, power functions were not given. I read off the relevant data from his Figure 5 and fitted power functions in the usual way. All the other studies presented linear fits of data in log–log plots, usually of control rate versus drug rate/control rate, occasionally of drug rate versus control rate. I estimated s and ln q (or log q) from these lines or computed them from tables, if available.
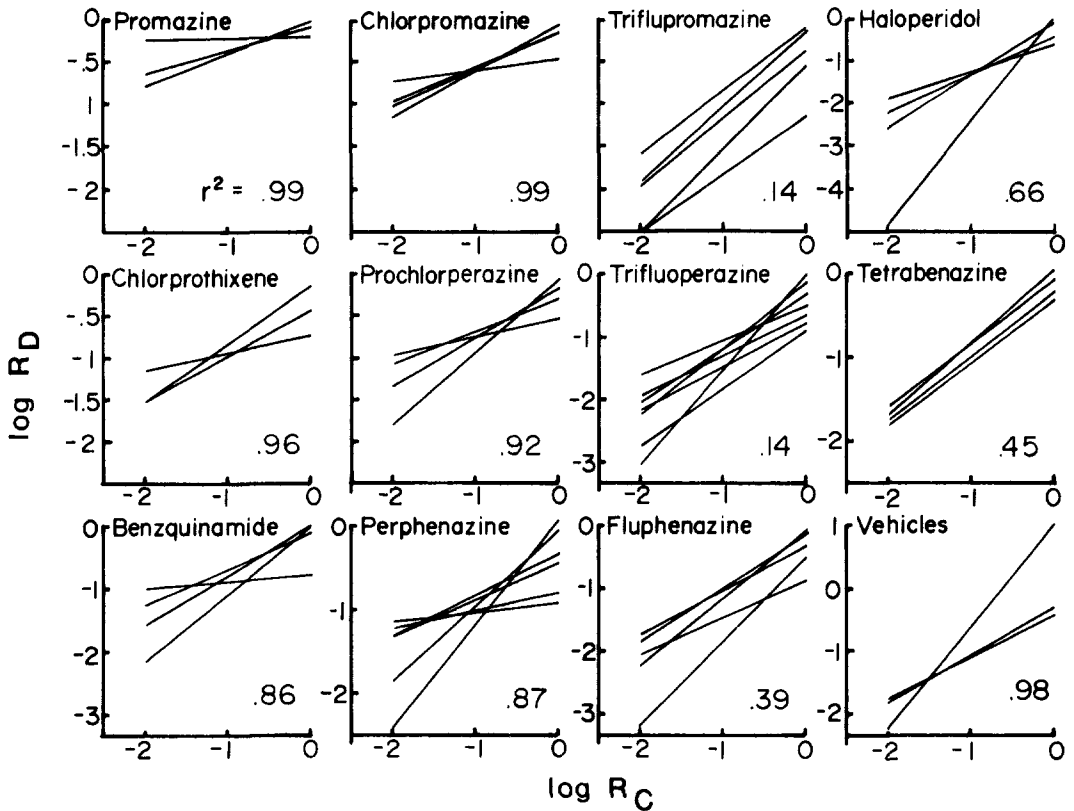
Figure 3. The relation between the logarithm of drug rate (log R_D) and control rate (log R_C) in a periodic schedule for a number of tranquillizers studied by Leander (1975). [The lines are derived from the slopes and intercepts of lines fitted to plots of 100 log (R_D/R_C) versus log R_C by Leander (his Table 1).]

stimulus (McKearney, 1970), and on the type of reinforcement schedule associated with the response (see Figure 2). Generally, the null point is higher on both axes for unpunished conditions, but this is reversed for some drugs, most notably, morphine. The null point is higher for responding in the positive stimulus (S^p). Individual animals also show different null points. Of course, no experiment has attempted explicitly to study the effects of these or any other variables on the null point, so that these correlations must be accepted with caution. For example, in most experiments, different doses of the same drug are administered in succession rather than being intermixed with doses of other drugs. Hence, effects of drug type are usually confounded with time of administration and order: If K values change slowly over time, then the sets of dose-response power functions obtained with differ-

ent drugs will tend to show different null points.

The identification of the a and K parameters with sensitivity and scale factors, respectively, provides an objective basis for the distinction between rate-dependent and rate-independent effects. If there is a fixed null point, and moreover, K_1 and K_2 are equal, then the effect of the drug is strictly on the a parameter, a pure rate-dependent effect. However, if K_1 and K_2 are not equal, then in addition to a dose-related rate-dependent effect, there is a dose-independent and rate-independent effect. Finally, if there is no fixed null point, the different drug doses have dose-related effects that are both rate dependent (on the a parameter) and rate independent (on the K parameter). K_1 and K_2 are approximately equal for most of the studies discussed here, so that the drug effects are indeed rate dependent.

The competition model appears to fit most of the available data on rate-dependent drug effects. Most of the apparently discrepant data can be attributed either to a limited range of slopes for the log (control rate) versus log (drug rate) functions or to an infelicitous choice of units. Most results show a dose-independent effect of drug type on the null point, and data that fail to fit may perhaps be attributable to a dose-dependent effect of the drug on the null point. The null point appears to depend on a number of situational factors, but controlled experiments are obviously required to map out these relations in detail.

Psychophysical Law

The sensation produced by a stimulus such as a sound or a light is not simply proportional to the stimulus intensity measured in physical units. For most stimulus dimensions, the response measure grows more slowly than physical intensity. On so-called prothetic (intensive) continua, the appropriate relation appears to be a power function of the same form as Equation 1, where y is the response measure, x the physical stimulus measure, and s an exponent characteristic of the stimulus dimension (Richardson & Ross, 1930; Stevens, 1957).[5] In a long series of papers, S. S. Stevens (e.g., 1957, 1975) presented evidence and arguments for the power relation and established that comparisons between different modalities (stimulus dimensions) generally behave as predicted by the law. For example, if the relation between perceived intensity and physical stimulus intensity for continuum A is $y_A = x_A{}^s$, and for continuum B, is $y_B = x_B{}^r$, then the relation between A and B when the level of A is adjusted so that A and B have the same perceived intensity should be $x_A = x_B{}^{r/s}$. In cross-modality matching experiments, the appropriate relations between exponents have been frequently demonstrated.

The relation of S. S. Stevens' law to two other types of psychophysical measurement, scales based on just-noticeable differences (JNDs) and so-called partition scales, is still a matter of debate. The present theory has straightforward implications for these questions. In some measure, it is a restatement in a convenient form of previous proposals. How-

ever, in this form, it can account for convergent power functions. It also makes clear the relations between Fechnerian and magnitude scales implied by different assumptions about the mapping of stimuli onto sensations.

The power law can be immediately derived from the present theory, given the following identifications: x is identified with the physical stimulus and y with the response measure, which may be a number (as in magnitude estimation) or some other physical measure (as in cross-modal matching). The comparison variable z must then be identified with sensation, that is, the level of some internal variable that is affected by both the stimulus x and the response y.

The suggestion that *number* can be considered a sensory dimension was first made by Attneave (1962) and has recently received considerable support from other sources (e.g., Rule, Curtis, & Markley, 1970; Teghtsoonian, 1974; Wagenaar, 1975). The idea that the central sensation variable bears the same relation to stimuli as to responses has also been frequently proposed. If that relation conforms to Weber's law (a possibility discussed more fully below), then, according to Teghtsoonian (1974), "if Weber's law holds, then it applies to *both* the target continuum plotted on the abscissa, *and* the matching continuum plotted on the ordinate" (p. 170). The assumption that stimuli and responses (including number) are related to sensation in a similar way is more symmetrical than the alternative view that responses are a direct measure of sensation, but stimuli are not. While symmetry is not proof, the fact that the present theory requires a symmetrical assumption should not be a cause for concern.

By the terms of the scaling experiment, the subject can be assumed to adjust his response y so that the level of sensation z associated with it is the same as that associated with the stimulus x. Thus, the expressions involving z

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[5] I exclude here any consideration of second-order or molecular effects, such as the biases associated with different scaling methods, hysteresis, and sequential effects. These require explanation at a different level than that attempted here. Refinements of method such as the widely used threshold correction are also taken for granted.

on the right-hand side of Equations 2a and 2b can be set equal, and the derivation proceeds as before. The relation between stimulus and response dimensions is then given by Equation 4. Parameters a_1 and a_2 are identified with the sensitivities of the individual to the stimulus and response dimensions, respectively. We may suspect that these parameters are related to discriminability, and I return to this in a moment. The K parameters are scale factors as before.

It is obvious that this derivation immediately predicts the results of cross-modality matching experiments. Every situation, including direct magnitude estimation, is regarded as involving both a stimulus and a response dimension, both of which bear a similar relation to the central sensation variable; and s, the exponent of the power function, is already expressed as the ratio of sensitivities of the stimulus and response systems.

The major empirical prediction of the present theory is that in many cases, variables that affect both the slope and the intercept of the power function act solely by affecting the sensitivity parameter. Hence, the family of power functions so produced should converge on a null point. J. C. Stevens (1974) has recently collected 13 sets of such converging functions. One example, from a study on glare inhibition by S. S. Stevens and Diamond (1965), is shown in Figure 4. In this experiment, three subjects adjusted the luminance of a matching field (seen only by the right eye) so that it matched the apparent brightness of a target field seen by the left eye. A small, intense glare source was also visible to the left eye. The experimenters adjusted the brightness of the target field and thus obtained a series of matches. The relation between target luminance and matching field luminance was a power function whose slope and intercept (in log–log coordinates) depended on the visual angle between glare source and target. S. S. Stevens and Diamond obtained a total of 14 power functions in this way for different glare angles. I fitted lines (by the method of least squares) to the corrected decibel data in their Table 1 and obtained 14 slope–intercept pairs. Figure 4 shows that these are very well fitted by a straight line ($r^2 = .995$), with slope and intercept $K_1 = -109$ dB, $K_2 = -108$ dB.
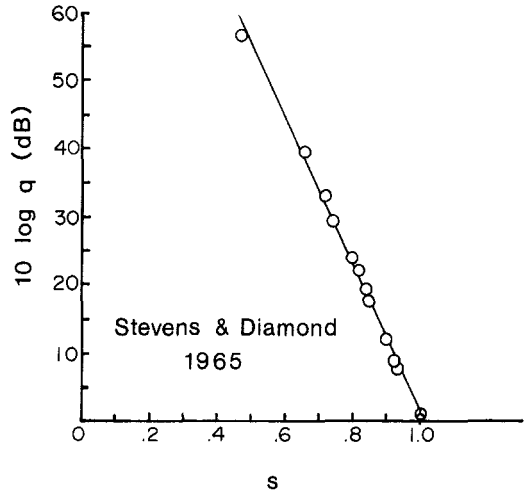

Figure 4. The relation between the slopes (s) and intercepts (10 log q) of power functions fitted by S. S. Stevens and Diamond (1965) to matching data from comparisons of the perceived brightness of a match field and a target field inhibited by an adjacent glare source.

K_1 and K_2 are approximately equal, as they should be for this symmetrical situation.

Data presented by Marks (1974b) and several studies reviewed by S. S. Stevens (1965) can also be treated in this way. However, some studies are available in which the treatments affected only the intercepts, leaving the slopes unchanged (e.g., Babkoff, 1976). In terms of the present theory, this corresponds to a change in one or both of the K parameters. In other studies (e.g., Pollack, 1949; replotted in Stevens, 1965), the convergence is less perfect than that shown in Figure 4, perhaps reflecting some effect on the K parameters. Nevertheless, when there is an effect on slope, the intercept usually changes in the way to be expected from Equation 7, and the convergence prediction is amply confirmed.

J. C. and S. S. Stevens have provided interpretations for the null point in various experiments. For example, in the S. S. Stevens and Diamond (1965) study, the coordinates of the null point are approximately equal to the luminance of the glare source, which was 118 dB. In other cases, the null point is identified with the stimulus level at which a qualitative change in sensation occurs, as from loudness to tickle or pain, or with the system's physiological limit, as in maximum heft. In some

cases, however, the interpretation is less plausible. For example, the family of power functions obtained by J. C. Stevens and S. S. Stevens (1963) in a study of adaptation converges on a null point whose luminance coordinate is between 150 and 160 dB. This approximately matches the luminance of the solar disc, but J. C. Stevens (1974) acknowledges that this identification is far from compelling. The present derivation does not preclude any particular interpretation for the null point. However, it also allows for the possibility that the null point may have no universal significance and is simply a consequence of constant scale factors and the generating process described by Equation 2.

Relation to Weber's and Fechner's Laws

Setting $f(z) = 1$ in Equation 2 yields the equations $dx/x = a_1 dz$ and $dy/y = a_2 dz$, which are simply the familiar Fechnerian expressions of Weber's law, with a_1 and a_2 proportional to the Weber fractions for the stimulus and response dimensions.[6] When integrated, these equations yield Fechner's logarithmic relation between stimulus intensity and sensation (see the Appendix). Thus, Equation 2 embraces the derivation of S. S. Stevens' law from logarithmic functions applied to both input and output (Ekman, 1964; MacKay, 1963; Treisman, 1964; see also Fechner, cited in Stevens, 1957). However, this derivation does not *require* that $f(z) = 1$; hence, it is consistent with Phillips' (1964) logical demonstration that no two-stage (i.e., stimulus–sensation–response) power law model can uniquely define the relation between stimulus (or response) and sensation.

Brentano (e.g., see Stevens, 1975, p. 234) suggested that JNDS yield a sensation that is proportional to the initial sensation level rather than being constant. This implies that in Equation 2, $f(z) = 1/z$. Integrating both sides of Equation 2a then yields $\ln z = (1/a_1) \times \ln x + K_1 - (1/a_1)K_2$, which is a power function but now relating stimulus and sensation (rather than response). It is noteworthy that the same slope–intercept relation holds for this one-stage derivation as for the two-stage derivation. Hence, if the response measure is identified directly with a sensation,

as S. S. Stevens (1957) proposed for magnitude estimation, the convergence property can be deduced directly. Unfortunately, the simple one-stage model cannot easily accommodate the relations between different types of psychophysical scales. However, Brentano's assumption reappears in the two-stage analysis of these relations, as we shall see.

The Fechner's law form for Equation 2 [i.e., $f(z) = 1$] suggests the hypothesis that the sensitivity parameter may be proportional to the Weber fraction (cf. Teghtsoonian, 1974), although this is not logically forced. It might be that JNDS on one continuum, electric shock, say, have a larger effect on sensation than JNDS on another, say, luminance. This proposition is readily tested, since if a_i bears a fixed proportion to the corresponding Weber fraction, then for any pair of stimulus and response dimensions, $s = a_2/a_1 = W_2/W_1$; hence,

$$sW_1 = W_2, \qquad (9)$$

where s is the exponent of the power function, and W_1 and W_2 are the Weber fractions for the stimulus and response dimensions, respectively. If *number* is the fixed response dimension (as in magnitude estimation), Equation 9 implies that the product of the exponent times the Weber fraction for the stimulus dimension should be constant. Teghtsoonian (1974) has provided evidence that it is. For example, in his Table 1, he presents values of W and s for

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[6] Auerbach (1971; see also Luce & Edwards, 1958) has shown by adding up JNDS that if Weber's law applies to both dimensions in the cross-modality matching experiment, then the power law exponent will be $s = \ln (1 + W_1)/\ln (1 + W_2)$, where W_1 and W_2 are the Weber fractions for the two dimensions. The present derivation yields $s = W_1/W_2 (= a_1/a_2)$. This apparent contradiction is due simply to the difference between adding up discrete JNDS and adding up JDNS that are assumed to be infinitesimal, as implied by Fechnerian integration. The contradiction is immediately resolved by noting that the series expansion of $\ln (1 + W) = W - W^2/2 + W^3/3 - \ldots, (-1 < W \leqslant 1)$, since when W is small, terms in W^2 and above can be neglected; whereupon, $\ln (1 + W) = W$. Since the actual value for W depends on the percentile criterion used, and this is simply a matter of convention, W can in practice be made as small as we please (cf. Eisler, 1963b). If the criterion is chosen to be within the linear range of the psychometric function, relative values of W for different continua will be independent of the criterion.

nine perceptual continua. The value of sW_1 computed from these data ranges from .026 to .034 (compared to a range from .026 to .033 for a comparable quantity computed using Auerbach's method [see Footnote 6]). Note, however, that in terms of the present theory, Teghtsoonian's finding implies only that $W_i = ka_i$, where k is approximately constant across perceptual dimensions; the finding does not require that Fechner's law hold, since $f(z)$ need not be defined for the relation between W_i and the exponent of the power function to be fixed.

Thus, neither the power law data nor Teghtsoonian's invariance can uniquely define the form of $f(z)$ in the present theory. However, a choice between the two simple possibilities can be made by looking at the differences between what Marks (1974a) has recently termed Type I (ratio) and Type II (partition) scales.

Partition Scales

The experiments from which power law results have been produced—magnitude estimation and production and cross-modal matching—have been analyzed here as instances of matching: The subject is presumed to match the sensation produced by a stimulus to that produced by another stimulus, either a number or a stimulus on another dimension. This approach allows for the possibility that other instructions may induce the subject to perform other operations with his sensations. The simplest such operation is "differencing," that is, the equation of perceptual differences along a single stimulus dimension.

A more complicated possibility is the computation by the subject of sensation ratios. S. S. Stevens, because of his theory of measurement, has tended to assume that magnitude estimation, because it yields a ratio scale, must therefore involve actual judgments of stimulus or sensation ratios. However, the present theory shows that the power law can be derived by assuming only matching by the subject. Wagenaar (1975) has also argued for the matching interpretation on purely empirical grounds. Moreover, Schneider, Parker, Farrell, and Kanow (1976), in an experiment explicitly designed to require judgments of

loudness ratios, showed that their subjects were actually judging sensation differences. Hence, the assumption that subjects can compute sensation ratios is neither required by theory nor well supported by experiment.

Partition scales are produced when subjects are required to place stimuli into a fixed number of categories (category scaling) or to divide a stimulus interval into equal-appearing parts (equisection). The scales that result from these operations are nonlinearly related to magnitude scales, as well as being especially subject to memory-related "hysteresis" and stimulus-spacing effects (Anderson, 1974; Marks, 1974a; Parducci, 1974; Stevens, 1975).

S. S. Stevens and others (e.g., Indow, 1974; Marks, 1974a; Torgerson, 1961) have suggested that partition scales are the result of a differencing operation by the subject. That is, instead of matching sensations (as implied by Equation 2 here) or matching response ratios to stimulus ratios, perhaps the subject is equating sensation differences. Equation 2 makes quite definite predictions about the form of the category scale to be expected, given the differencing assumption and an assumption about the form of $f(z)$. In the simplest case, where $f(z) = 1$, the predicted relation is logarithmic (cf. Torgerson, 1961). This can be seen as follows for the case of bisection:

Let z_1, z_2, and z_3 be the sensation values produced by three stimuli, where s_1 and s_3 are the anchor stimuli, and s_2 is the variable stimulus. Then, if the subject responds to the terms of the bisection experiment by differencing, s_2 is adjusted so that $z_1 - z_2 = z_2 - z_3$. Integrating both sides of Equation 2a and rearranging then yields

$$\ln x_1 - \ln x_2 = \ln x_2 - \ln x_3. \quad (10)$$

Hence, equal stimulus ratios yield equal sensation differences, a logarithmic scale.

If $f(z) = 1/z$, a similar set of manipulations yields

$$x_1^{1/a_1} - x_2^{1/a_1} = x_2^{1/a_1} - x_3^{1/a_1}, \quad (11)$$

that is, equal sensation differences are defined by equal differences between stimulus values raised to a power. Equation 10 is consistent with Fechner's version of Weber's law, and

Equation 11 with Brentano's version or with a Fechnerian version of the "near-miss" power function form of the law (see the Appendix).

These results, and others for different assumptions about $f(z)$, can be obtained more directly by noting that the differencing assumption implies that the category scale is linearly related to sensation. Hence, simply integrating Equation 2a for a given $f(z)$ at once yields the form of the category scale. For example, if $f(z) = 1/z$, then,

$$z = x^{1/a_1} \cdot \exp(K_1/a_1 - K_0), \qquad (12)$$

where K_1 and K_0 are constants of integration. The translation from z to a particular category scale requires a further arbitrary constant K added to z to allow for the fact that subjects will partition stimuli into categories labeled $1 - N$ in the same way as they will into categories labeled $M + 1 - M + N$.

These conclusions about the relations between category, JND, and magnitude scales are also compatible with a general model for intra-individual scale relations proposed by Eisler (1963a; see the Appendix).

Partly because of the sensitivity of partition scales to context and time effects[7] and partly because of their equivocal conceptual status (at least within S. S. Stevens' scheme), there has been disagreement on the proper form for partition scales. Earlier workers plumped for the logarithmic form (e.g., Luce & Galanter, 1963; Titchener, 1905). However, more recent work (e.g., Marks, 1968, 1974a; Schneider & Lane, 1963; Ward, 1975) favors a power function of the form

$$C + K = Ax^{s_c}, \qquad (13)$$

where C is the category value, s_c is the "virtual exponent" (generally smaller than the magnitude estimation exponent), and A and K are constants. Given that by the differencing assumption, C is linearly related to sensation level z, Equation 13 is then equivalent to Equation 12 and also to Equation A4 (see the Appendix), which is derived from the near-miss power function version of Weber's law.

Thus, the two-stage theory embodied in Equation 2 can be developed in two parallel directions, each internally consistent and dependent on the form of $f(z)$. On the one hand, if Weber's law holds, JNDs are subjectively equal $[f(z) = 1]$, and partition scales involve differencing, then magnitude estimation and cross-modal matching procedures will yield power functions, and partition scaling will yield a logarithmic relation between response and stimulus variables. On the other hand, if Brentano's version of Weber's law (sometimes called Ekman's law) holds $[f(z) = 1/z]$, then magnitude estimation and cross-modal matching procedures will yield power functions; and partition scaling, with an arbitrary constant added to the category value, will also yield power functions.

On the assumption that the power form is correct, it is possible to deduce a relation between the exponent of the category scale s_c and the exponent for the magnitude estimation (ratio) scale s_m. Comparison of Equations 4 and 12 shows that

$$s_m = a_2 s_c, \qquad (14)$$

where a_2 is the sensitivity parameter for number (the response dimension in magnitude estimation). a_2 can be estimated by the ratio s_m/s_c for a variety of continua or from the s_c value for number, given that $s_c = 1/a_1$. Marks (1968) in his Table 1 gives estimates of s_c and s_m for nine prothetic continua. The ratio s_m/s_c ($= a_2$) ranges from 1.42 to 2.54,[8] with four of the values lying between 2.26 and 2.54. The value of $1/s_c$ for the dimension of number is 1.85. The fact that s_c for number is less than unity is consistent with the fact that category scale exponents are reliably lower than their ratio scale counterparts. The values of s_c for loudness and brightness were estimated for a number of conditions differing in the number of categories used and other procedural details. These estimates vary quite a bit depending on the condition; for example, s_c estimates for brightness range from .055 to .22. Brightness and loudness are the two modalities for which most information is available, and the agreement here is quite good: The median values of s_c for these two dimensions from Marks' table are .13 and .25 compared with accepted

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[7] This characteristic is consistent with the greater memory load imposed by the differencing operation.

[8] I used the median s_c estimates for brightness and loudness and the estimate from the linear spacing condition for repetition rate.

s_m values of .33 and .6, yielding a_2 values of 2.54 in the first case and 2.40 in the second. Hence, the constancy of a_2 predicted by the present model is confirmed as well as can be expected.

The predictions of the present theory, on the assumption that $f(z) = 1/z$, are in accord with Marks' (1974a) conclusion that "the psychophysical functions that relate Type I and Type II scales to their corresponding physical scales are in both cases power functions, but the exponents that govern Type I functions are typically about twice as large" (p. 358). Thus, Equation 2 provides a natural representation for the relations among partition, magnitude, and JND scales on the assumption that Weber's law holds and that Brentano's conjecture is correct.

The formulation has some implications for the near-miss version of Weber's law that appears to apply to loudness under some conditions. If Fechner's assumption is correct, then the theory predicts that the sensation function will be a power function; if the relation between sensation and sound intensity is to be positive, then the exponent of the JND function should be negative, as it is (see the Appendix). Comparison of Equations A4 and 11 or 12 shows that the exponent of the near-miss power function should be predictable from s_c, since $s_c = 1/a_1 = -n$. Jesteadt, Wier, & Green (1977) present data on n for loudness for their study and five others. The values range from $-.035$ to $-.125$. The predicted values range from $-.13$ to $-.50$, using the s_c data in Marks' (1968) Table 1; all are too large. A possible reason is that Brentano's assumption about the relation between stimulus (x) and sensation (z) may be better than Fechner's; it yields an expression of the form $z = \alpha \exp(\beta x^{-n})$ for the near-miss form (see the Appendix). When $|n|$ is small, as is the case for loudness, and x ranges over two or three log units, this function is so close to a power function, with exponent somewhat larger than $|n|$, that the two may not be separable empirically.

The present theory shows the power law to be quite independent of the form of $f(z)$, and only two simple possibilities have been considered here. Brentano's version appears to be better than Fechner's, but it may be that still another alternative will be better than both and will resolve the remaining discrepancies. It may even be that $f(z)$ is not fixed, but depends on contextual factors, a distressing possibility for classical psychophysicists, but one that is not unlikely, given the strong sequential and other contextual effects that can be demonstrated in many scaling situations.

Conclusion

In operant conditioning, power function relations between response rates are often found in situations that involve successive comparison between each response and a third competing class of activities. Examples are certain choice studies and numerous experiments on rate-dependent drug effects. Equation 2 provides a natural account of this comparison process and explains the convergence property of many sets of power functions as well as dealing with the dimensional problems with the simple power function (Equation 1).

Equation 2 can also provide an account of the psychophysical power law, which is interpreted as the outcome of matching between the sensations produced by the stimulus and response. The approach immediately accounts for converging sets of power functions and, with the aid of a differencing assumption and an assumption about the form of the sensation function $f(z)$, can also account for the approximately fixed ratio between the exponents for magnitude (Type I) and partition (Type II) scales. The approach may also predict qualitative properties of the near-miss form to Weber's law, although there are still quantitative discrepancies that remain to be resolved.

Luce (1972) has pointed out the lack in psychophysics of a system of invariant relations among the measures that might justify the assimilation of the subject to classical physics. The simple theory embodied in Equation 2 begins to meet this criticism. However, the auxiliary assumptions (of matching or differencing) required to relate the theory to data tend to favor Luce's view of psychophysics as the study of a measuring instrument rather than Fechner's more grandiose vision of a field akin to physics.

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Appendix

Sensation Functions and Just-Noticeable Difference Functions

The relation between sensation, which is merely an intervening theoretical variable, and physical variables can be reduced to simple terms if our objective is just to explain the outcomes of various operations (i.e., scaling tasks) applied to the subject and, particularly, to understand the relations among them (cf. Luce & Edwards, 1958). Consider first the relation between the increment in a physical stimulus that is just noticeable, dx, and the value of the base stimulus x. Two simple possibilities have been reported. First, Weber's law states

$$dx/x = A, \qquad (A1)$$

where A is a constant. More recently, the following near-miss form of Weber's law has been reported for the intensity discrimination of pulsed sinusoids (Jesteadt et al., 1977):

$$dx/x = A(x/x_0)^n, \qquad (A2)$$

where n is negative and close to zero, and x_0 is the threshold stimulus value. What do these

two versions of Weber's law imply about the relation between the hypothetical sensation variable z and the value of the physical stimulus x?

This question can be answered only if some assumption is made about the relation between a just-noticeable increment in the physical stimulus and the increment in sensation associated with it. The simplest assumption is Fechner's, namely, that all JNDs yield the same constant increment in sensation level. Given this assumption, it remains simply to define the physically invariant unit that corresponds to a JND and integrate the result to obtain the expected relation between z and x. (Numerous papers have been written on the legitimacy or otherwise of Fechnerian integration [e.g., Eisler, 1963b; Falmagne, 1971; Krantz, 1971; Luce & Edwards, 1958]. The current consensus appears to be that the procedure is legitimate, subject to certain safeguards.) For Weber's law, Equation A1 gives this at once:

$$1 \text{ JND} = dz = (1/A)(dx/x);$$

hence, integration yields

$$z + C_1 = (1/A)(\ln x + C_2), \quad \text{(A3)}$$

which is Fechner's law. For the near-miss form, rearrangement yields

$$1 \text{ JND} = dz = (x_0{}^n/A)(dx/x^{n+1});$$

hence, integration yields

$$z + C_1 = (x_0{}^n/A)[(x^{-n}/- n) + C_2], \quad \text{(A4)}$$

which is a power function of the form $z + K_1 = K_2 x^m$, where K_1 and K_2 are constants, and m is the exponent (which is positive for loudness because n is negative).

If Brentano's conjecture is correct, then dz is replaced by dz/z, and Weber's law yields $z = x^{1/A} \cdot \exp(K_2/A - K_1)$, a power function. The near-miss form yields an expression of the form $z = \alpha \exp(\beta x^{-n})$, where α and β are constants.

Relation to the General Psychophysical Differential Equation

Eisler (1963a; Eisler & Montgomery, 1974) has proposed an equation to describe the intraindividual relation between different psychological scales (e.g., magnitude and category scales). If u and v are the scale values, and $\sigma_u(u)$ and $\sigma_v(v)$ are the corresponding Weber functions, estimated by the SDs of intraindividual judgments, the equation is

$$\frac{du}{dv} = \frac{\sigma_u(u)}{\sigma_v(v)}. \quad \text{(A5)}$$

Eisler's formulation can be reconciled with the present one as follows. First, Equation 2 is rewritten in its most general form, which is the following pair of separable differential equations:

$$\frac{dx}{g(x)} = \frac{dz}{f'(z)} \quad \text{(A6a)}$$

and

$$\frac{dy}{h(y)} = \frac{dz}{f'(z)}. \quad \text{(A6b)}$$

The functions $f'[= 1/f(z)]$, g, and h can be regarded as Weber functions in Eisler's sense. For example, if Weber's law holds for all three variables, then $f'(z) = z$, $g(x) = a_1 x$, and $h(y) = a_2 y$. (Note that Equations A6a and A6b yield the power law for magnitude estimation only if Weber's law holds for x and y.) We can therefore rewrite Equation A6b as follows:

$$\frac{dy}{\sigma_y(y)} = \frac{dz}{\sigma_z(z)}, \quad \text{(A7)}$$

where $\sigma_y(y)$ and $\sigma_z(z)$ are the Weber functions for y and z. From arguments in the text, it is clear that y and z are represented (but for additive constants) by the subject's magnitude and category judgments, respectively. Hence, granted that the σs can be estimated from judgmental SDs, Equation A7 is equivalent to Equation A5, the general psychophysical differential equation.