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Assessment of Heart Disease using Fuzzy Classification Techniques

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In this paper we discuss the classification results of cardiac patients of ischemical cardiopathy, valvular heart disease, and arterial hypertension, based on 19 characteristics (descriptors) including ECHO data, effort testings, and age and weight. In this order we have used different fuzzy clustering algorithms, namely *hierarchical fuzzy clustering*, *hierarchical* and *horizontal fuzzy characteristics clustering,* and a new clustering technique, *fuzzy hierarchical cross-classification*. The characteristics clustering techniques produce fuzzy partitions of the characteristics involved and, thus, are useful tools for studying the similarities between different characteristics and for essential characteristics selection. The cross-classification algorithm produces not only a fuzzy partition of the cardiac patients analyzed, but also a fuzzy partition of their considered characteristics. In this way it is possible to identify which characteristics are responsible for the similarities or dissimilarities observed between different groups of patients.

KEY WORDS: fuzzy clustering, characteristics clustering, fuzzy cross-clustering, characteristics selection, cardiac disease, ischemical cardiopathy, valvular heart disease, arterial hypertension

DOMAINS: bioinformatics, medical informatics, biomathematics; cardiovascular biology; modeling; atherosclerosis

INTRODUCTION

The mathematics of fuzzy set theory was originated by L.A. Zadeh in 1965[15]. It deals with the uncertainty and fuzziness arising from interrelated humanistic types of phenomena such as subjectivity, thinking, reasoning, cognition and perception. This type of uncertainty is characterized by structures that lack sharp (well-defined) boundaries. This approach provides a way to translate a linguistic model of the human thinking process into a mathematical framework for developing the computer algorithms for computerized decision-making processes. The theory has grown very quickly[1,2,3,5].

There are two opposite approaches to hierarchical clustering, namely, *agglomerative* and *divisive* procedures. An agglomerative hierarchical classification places each object in its own cluster and gradually merges the clusters into larger and larger clusters until all objects are in a single cluster. The divisive hierarchical clustering reverses the process by starting with all the objects in a single cluster and subdividing it into smaller ones until, finally, each object is in a cluster of its own. The number of clusters to be generated may be either specified in advance or optimized by the algorithm itself according to certain criteria.

Among other interesting applications, the fuzzy clustering theory developed in References [3,4,6,8] has been used for the selection and the optimal combination of solvents[7,13], for the classification of Roman pottery[9], for the cross-classification of Greek muds[6], for the development of a fuzzy system of chemical elements[12,14], for producing a performant fuzzy regression algorithm[10], and for the cross-classification of thin layer chromatography data[11].

In this paper we analyze the possibility of identifying the correct diagnosis concerning the cardiac patients using different fuzzy clustering algorithms.

THEORETICAL CONSIDERATIONS

Fuzzy Substructure of a Fuzzy Set

In this section we will recall the so-called generalized fuzzy *n*-means algorithm[3,4,6]. This algorithm is a generalization of the well-known fuzzy *n*-means algorithm[1,4]. Let us consider a set of objects $X = \{ x^1, ..., x^p \} \subset \mathbf{R}^s$ and let C be a fuzzy set on X. We are searching for the fuzzy partition corresponding to the cluster substructure of the fuzzy set C. Let us suppose that this fuzzy partition is $\{A_1, ..., A_n\}$. We admit that each fuzzy class A_i may be represented by a prototype L^i from the representation space, $\mathbf{R}^s$. If L^i is from X, it is natural to suppose that L^i has the greatest membership degree to A_i, that is:

$$A_i(L^i) = \max_{x \in X} A_i(x) \tag{1}$$

Otherwise, if L^i is not from X, we cannot speak about its membership degree to the fuzzy set A_i, since the universe of the fuzzy set A_i is X.

Let us denote by d a distance in the space $\mathbf{R}^s$. For example, we may consider the distance induced by the norm of the space. The dissimilarity $D_i(x^j, L^i)$ between a point x^j and the prototype L^i is defined as the square local distance in the class A_i, $d_i(x^j, L^i)$, and is interpreted as a measure of the inadequacy of the representation of the point x^j by the prototype L^i.

If L^i is not a point from the data set X, then

$$D_i(x^j, L^i) = (A_i(x^j))^2 d^2(x^j, L^i). \tag{2}$$

If, on the contrary, L^i is a point from the data set X, then we have from (1) that $A_i(L^i) \geq A_i(x^j)$ for any x^j in X, and the relation (2) is still valid.

The inadequacy between the fuzzy partition P and its representation, $L = \{L^1, ..., L^n\}$ is given by the following function:

$$J(P,L)=\sum_{i=1}^{n}\sum_{j=1}^{p}(A_i(x^j))^2 d^2(x^j,L^i). \qquad (3)$$

$J(P, L)$ may also be interpreted as the representation error of P by L.

It is easy to observe that J is a criteria function of the type of square errors sum. The classification problem becomes the determination of the fuzzy partition P and its representation L for which the inadequacy $J(P, L)$ is minimal. We note that, intuitively, to minimize J means to give small membership degrees to A_i for those points in X for which the dissimilarity to the prototype L^i is large, and vice-versa. Another useful remark is that the fuzzy sets A_i, $i = 1,...,n$ are components of the fuzzy partition P of the fuzzy set C, and thus the obvious 'solution', $A_i = 0$ for all i is not acceptable since it does not form a fuzzy partition of C.

If we admit that d is a distance induced by the norm, we may write

$$J(P,L)=\sum_{i=1}^{n}\sum_{j=1}^{p}(A_i(x^j))^2\left\|x^j-L^i\right\|^2. \qquad (4)$$

If the norm is induced by the inner product, we have

$$\| x^j - L^i \|^2 = (x^j - L^i)^T M (x^j - L^i),$$

where M is a symmetrical and positively defined matrix. The transposing operation was denoted by ^{T}. The criteria function becomes:

$$J(P,L)=\sum_{i=1}^{n}\sum_{j=1}^{p}(A_i(x^j))^2\left(x^j-L^i\right)^T M\left(x^j-L^i\right). \qquad (5)$$

Because an algorithm to obtain an exact solution of the problem (5) is not known, we will use an approximate method in order to determine a local solution. The minimum problem will be solved using an iterative (relaxation) method, where J is successively minimized with respect to P and L.

Supposing that L is given, the minimum of the function $J(\bullet,L)$ is obtained[3, 4] for:

$$A_i(x^j)=\frac{C(x^j)}{\sum_{k=1}^{n}\frac{d^2(x^j,L^i)}{d^2(x^j,L^k)}}, i=1,...,n, \qquad (6)$$

for all j for which for every k, $d(x^j, L^k) \neq 0$. If, on the contrary, there exists a j so that for some values of k, $d(x^j, L^k) = 0$, then the membership degrees $A_i(x^j)$ fulfill the condition $A_i(x^j) = 0$ for all i so that $d(x^j, L^i) \neq 0$.

For a given P, the minimum of the function $J(P,\cdot)$ is obtained for:

$$L^i=\frac{\sum_{j=1}^{p}A_i(x^j)^2 x^j}{\sum_{j=1}^{p}A_i(x^j)^2}, i=1,...,n \qquad (7)$$

We observe[3,4] that L^i is the weighting center of the class A_i.

The iterative procedure for obtaining the cluster substructure of the fuzzy class C is called generalized fuzzy n-means (GFNM)[3]. Essentially, the GFNM algorithm works with Picard iterations using the relation (6) and (7). The iterative process begins with an arbitrary initialization of the partition P. The process ends when two successive partitions are close enough. To measure the distance between two partitions, we will associate to each partition P a matrix Q with the dimensions $n \times p$. Q is named the representation matrix of the fuzzy partition P and is defined as:

$$Q_{ij} = A_i(x^j),\ i = 1, 2, ..., n;\ j = 1, 2, ..., p. \tag{8}$$

Considering that Q_1 and Q_2 are the representation matrices of the partitions P_1 and P_2, we may define

$$d(P_1, P_2) = \|Q_1 - Q_2\| \tag{9}$$

where $\|Q\| = \max_{i,j}\left|A_i(x^j)\right|$.

The process ends at the r-th iteration if

$$d(P_r, P_{r+1}) < \varepsilon \tag{10}$$

where ε is an admissible error (usually, 10^{-5}).

For $C = X$ this procedure is the well-known algorithm fuzzy n-means (FNM)[1].

Fuzzy Divisive Hierarchical Clustering

Let us consider a fuzzy set C on X and a fuzzy binary partition $P = \{A_1, A_2\}$ of C, and the following function, called polarization degree:

$$R(P) = \frac{\sum_{x\in X}\left(A_{1,1/2}(x) + A_{2,1/2}(x)\right)}{\sum_{x\in X} C(x)}, \tag{11}$$

where $A_{i,1/2}$ is the 1/2-cut of A_i (i.e., $A_{i,1/2}(x)$ is equal to $A_i(x)$ if $A_i(x)$ is larger than $C(x)/2$ and is equal to zero otherwise). (For a complete study of the polarization degree and its properties, please see Reference [3].) We will say here only that *R(P)* is as larger as the partition P is more polarized, and *R(P)* is as small as the partition P is fuzzy. We say that the binary partition P describes 'real' clusters if the polarization degree *R(P)* is larger than a certain threshold $t \in (0,1)$ chosen *a priori*, and if for every class C_i there exists at least one point x in X so that $A_i(x) > C(x)/2$.

Using the FNM algorithm we may determine a binary fuzzy partition $P = \{A_1, A_2\}$ of the data set X. If the partition P does not describe 'real' clusters, the data set X does not have a substructure. If this partition describes 'real' clusters, we denote $P^1 = \{A_1, A_2\}$. Using the GFNM algorithm for two subclasses ($n = 2$) we may determine a binary fuzzy partition for each A_i of P^1. If this partition of A_i describes real clusters, these clusters will be attached to a new fuzzy partition, P^2. Otherwise, A_i will remain undivided. The class A_i will be marked and will be allocated to the partition P^2. The unmarked classes members of P^2 will follow the same procedure. The divisive procedure will stop when all the classes of the current partition P^l are marked, i.e., there are no more 'real' clusters.

The procedure described here is called the Fuzzy Divisive Hierarchical Clustering (FDHC) algorithm[3,9]. This procedure may be used to determine the optimal cluster substructure of the data set. The method is especially useful when the number of classes is unknown. We emphasize here that the ability of chose the value of the polarization threshold to be used allows us to stop the hierarchical analysis at that degree of refinement considered relevant for the application. If we decide to choose a high threshold we will obtain the fuzzy partition corresponding to the macroscopic structure of the data set, while by choosing a smaller threshold, we will have a more detailed image of the fuzzy substructure of the data. Moreover, we are not interested only in the final fuzzy partition; we are interested in the relationships between different fuzzy sets. These relationships may be observed very well from the binary classification tree[6,7,9,11,12,14].

Interpretation of the Final Fuzzy Partition

The fuzzy hierarchy obtained is richer in information (see Reference [5]) than a hierarchy based on classical sets, but sometimes is useful to have a classical partition also. For a complete discussion on the problem of passing from fuzzy partitions to classical partitions, see Reference [5]. We will only show the method used here for obtaining a classical partition.

Defuzzification of the final fuzzy partition will be realized using the maximum membership rule or a hierarchical assignment rule. This latter rule means that the classical sets corresponding to the fuzzy classes will be built in the same time with the respective fuzzy classes, based on the following rules (here, and in all that follows, $\widetilde{C}$ denotes the classical set obtained by defuzzification from the fuzzy set C): 1) initially, since X is a classical set, $\widetilde{X} = X$; 2) when we build the fuzzy partition $\{C_1, C_2\}$ of the fuzzy set C, we will say that:

$$x \in \widetilde{C}_1 \Leftrightarrow \left(x \in \widetilde{C} \text{ and } C_1(\mathrm{x}) \geq \mathrm{C}_2(\mathrm{x})\right)$$

and

$$x \in \widetilde{C}_2 \Leftrightarrow \left(x \in \widetilde{C} \text{ and } C_1(\mathrm{x}) < \mathrm{C}_2(\mathrm{x})\right)$$

Remark. It is obvious that $\{\widetilde{C}_1, \widetilde{C}_2\}$ is a hard partition of the classical set $\widetilde{C}$.

Finally, when obtaining the fuzzy hierarchy of the set X, we will also obtain the so-called classical hierarchy associated to that fuzzy hierarchy.

Associative Simultaneous Fuzzy *n*-Means Algorithm

Let $X = \{x^1, ..., x^p\} \subset \mathbf{R}^s$ be the set of objects to be classified. A characteristic may be specified by its values corresponded to the p objects. Thus, we may say that $Y = \{y^1, ..., y^s\} \subset \mathbf{R}^p$ is the set of characteristics. y_j^k is the value of the characteristic k with respect to the object j, so we may write $y_j^k = x_k^j$.

Let P be a fuzzy partition of the fuzzy set C of objects and Q a fuzzy partition of the fuzzy set D of characteristics. The problem of the cross-classification (or simultaneous classification) is to determine the pair (P, Q) which optimizes a certain criterion function.

By starting with an initial partition P^0 of C and an initial partition Q^0 of D, we will obtain a new partition P^1. The pair (P^1, Q^0) allows us to determine a new partition Q^1 of the characteristics. The algorithm consists in producing a sequence (P^k, Q^k) of pairs of partitions, starting from the initial pair (P^0, Q^0), in the following steps:

(i) $(P^k, Q^k) \to (P^{k+1}, Q^k)$;
(ii) $(P^{k+1}, Q^k) \to (P^{k+1}, Q^{k+1})$.

The rationale of the hierarchical cross-classification method[4,6] essentially supposes the splitting of the sets X and Y in two subclasses. The classes obtained are alternatively divided in two subclasses, and so on. The two hierarchies will be represented by the same tree, having in each node a pair (C, D), where C is an objects fuzzy set and D is a characteristics fuzzy set.

As a first step we wish to determine simultaneously the fuzzy partitions (as a particular case, the binary fuzzy partitions) of the classes C and D, so that the two partitions should be highly correlated. With the generalized fuzzy n-means algorithm, we will determine a fuzzy partition $P = \{A_1, ..., A_n\}$ of the class C, using the original characteristics.

In order to classify the characteristics, we will compute their values for the classes A_i, $i = 1, ..., n$. The value $\bar{y}_i^k$ of the characteristic k with respect to the class A_i is defined as:

$$\bar{y}_i^k = \sum_{j=1}^{p} A_i(x^j)x_k^j, i = 1,...,n; k = 1,...,s. \quad (12)$$

Thus, from the original s p-dimensional characteristics we computed s new n-dimensional characteristics which are conditioned by the classes A_i, $i = 1, ..., n$. We may admit that these new characteristics do not describe objects, but they characterize the classes A_i.

Let us consider now the set $\bar{Y} = \{\bar{y}^1,...,\bar{y}^s\}$ of the modified characteristics. We define the fuzzy set $\bar{D}$ on $\bar{Y}$ given by

$$\bar{D}(\bar{y}^k) = D(y^k), k = 1,..., s.$$

The way the set $\bar{Y}$ has been obtained lets us conclude that if we will obtain an optimal partition of the fuzzy set D, this partition will be highly correlated to the optimal partition of the class C. With the generalized fuzzy n-means algorithm we will determine a fuzzy partition $Q = \{B_1, ..., B_n\}$ of the class D, by using the characteristics given by the relation (12). We may now characterize the objects in X with respect to the classes of properties B_i, $i = 1, ..., n$. The value $\bar{x}_i^j$ of the object j with respect to the class B_i is defined as:

$$\bar{x}_i^j = \sum_{k=1}^{d} B_i(\bar{y}^k)x_k^j, i = 1,...,n; j = 1,..., p. \quad (13)$$

Thus, from the original p s-dimensional objects we have computed p new n-dimensional objects, which correspond to the classes of characteristics B_i, $i = 1, ..., n$.

Let us consider now the set $\bar{X} = \{\bar{x}^1,...,\bar{x}^p\}$ of the modified characteristics. We define the fuzzy set $\bar{C}$ on $\bar{X}$ given by

$$\bar{C}(\bar{x}^j) = C(x^j), j = 1,..., p.$$

With the generalized fuzzy n-means algorithm we will determine a fuzzy partition $P' = \{A'_1, ..., A'_n\}$, of the class C by using the objects given by the relation (13). The process continues until two successive partitions of objects (or characteristics) are closed enough to each other.

Considering $P = \{A_1, ..., A_n\}$ is the fuzzy n-partition of X and $Q = \{B_1, ..., B_n\}$ is the fuzzy n-partition of Y produced after this step of our algorithm. Let us remark that we made no explicit association of a fuzzy set A_i on X with a fuzzy set B_j on Y, i.e., what is the fuzzy set B_j that best describes the essential characteristics corresponding to the fuzzy set A_i. It only supposes that A_i is to be associated with B_i, and this is not always true.

Let us denote by S_n the set of all permutations on $\{1, ..., n\}$. We wish to build that permutation $\sigma \in S_n$ which best associates the fuzzy set A_i with the fuzzy set $B_{\sigma(i)}$, for every $i = 1, ..., n$. Our aim is to build some function $J : S_n \to \mathbf{R}$ so that the optimal permutation σ is that which maximizes this function. Let us consider the matrix $Z \in \mathbf{R}^{n,n}$ given by

$$z_{kl} = \sum_{j=1}^{p} \sum_{i=1}^{d} A_k(x^j) B_l(y^i) x_i^j, k, l = 1, ..., n. \tag{14}$$

Let us remark the similarity between the way we compute the matrix Z in (14) and the way we computed the new objects and characteristics in relation (12) and (13).

The experience enables us to consider the function J as given by

$$J = \prod_{i=1}^{n} z_{i,\sigma(i)}. \tag{15}$$

Thus, supposing that the permutation σ maximizes the function J defined above, we will be able to associate the fuzzy set A_i with the fuzzy set $B_{\sigma(i)}$, $i = 1, ..., n$. As we will see in the comparative study below, this association is more natural than the association of A_i with B_i, $i = 1, ..., n$. Based on these considerations we are able to introduce the following algorithm, the associative simultaneous fuzzy n-means algorithm (ASF):

S1. Set $l = 0$. With the generalized fuzzy n-means algorithm we determine a fuzzy n-partition $P^{(l)}$ of the class C by using the initial objects.

S2. With the generalized fuzzy n-means algorithm we determine a fuzzy n-partition $Q^{(l)}$ of the class D by using the characteristics defined in (12).

S3. With the generalized fuzzy n-means algorithm we determine a fuzzy n-partition $P^{(l+1)}$ of the class C by using the objects defined in (13).

S4. If the partitions $P^{(l)}$ and $P^{(l+1)}$ are close enough, that is, if $\|P^{(l+1)} - P^{(l)}\| < \varepsilon$, where ε is a preset value, then go to **S5**, otherwise increase l by 1 and go to **S2**.

S5. Compute the permutation σ that maximizes the function J given in relation (15).

S6. Re-label the fuzzy sets B_i, so that $B_{\sigma(i)}$ becomes B_i, $i = 1, ..., n$.

Let us remark now that, after steps **S5** and **S6**, we are able to associate the fuzzy set A_i with the fuzzy set B_i, $i = 1, ..., n$.

Let us also remark that the computation required in step **S5** is not an obvious one. But, as we will see, our purpose is to use this algorithm for developing a hierarchical technique. Thus, we will use the ASF algorithm in the particular case $n = 2$. In this case, the computation required in step **S5** becomes trivial.

Fuzzy Hierarchical Cross-Classification Algorithm

The method described below is the straightforward way of developing a hierarchical algorithm that should use at each node of the classification tree our ASF algorithm. We will first show the way to build the classification binary tree. The tree nodes are labeled with a pair (*C*, *D*), where *C* is a fuzzy set from a fuzzy partition of objects and *D* is a fuzzy set from a fuzzy partition of characteristics. The root node corresponds to the pair (*X*, *Y*). In the first step the two sub-nodes (A_1, B_1) and (A_2, B_2), respectively, will be computed by using the ASF algorithm. Of course, these two nodes will be effectively built only if the fuzzy partitions $\{A_1, A_2\}$ and $\{B_1, B_2\}$ describe real clusters. For each of the terminal nodes of the tree we try to determine partitions having the form $\{A_1, A_2\}$ and $\{B_1, B_2\}$, by using the ASF algorithm, modified as we have mentioned before. In this way the binary classification tree is extended with two new nodes, (A_1, B_1) and (A_2, B_2). The process continues until, for any terminal node, we are not able to determine a structure of real clusters, either for the set of objects or for the set of characteristics. The final fuzzy partitions will contain the fuzzy sets corresponding to the terminal nodes of the binary classification tree. This algorithm, termed the FHCCA algorithm, seems to be suitable for applications where the idea is to get most of the relationships between different classes of objects and different classes of characteristics. In Pop and Sârbu[11] we introduced two more variants of this algorithm, FHCCB and FHCCC.

Characteristics Clustering

In this section we address the problem of characteristics clustering. This may be useful in many situations. For example, the dimensionality reduction may be considered a characteristic classification process. The characteristics in the same class (which are, consequently, very similar to each other) will realize a reduced discrimination among the objects. On the contrary, the more distant the classes that contain two different characteristics, the greater their discrimination power. If the classes of characteristics are homogenous and well separated, a class may be replaced by the most representative characteristic. This characteristic represents an average of the properties of the class. The more compact the class, the smaller the loss of information produced by this replacement. In this way we realize a dimensionality reduction. By choosing a unique characteristic from each class, the number of selected characteristics is equal to the number of clusters in the set *Y*. Alternatively, we may not only select some of the existing characteristics, but we may replace them by new characteristics, by considering that each class of characteristics is replaced by the prototype characteristic. The technique obtained by using the fuzzy divisive hierarchical clustering algorithm on the set of characteristics will be called fuzzy hierarchical characteristics clustering (FHiCC). Similarly, the technique obtained by using the fuzzy *n*-means algorithm on the set of characteristics will be called fuzzy horizontal characteristics clustering (FHoCC).

CLASSIFICATION RESULTS AND DISCUSSIONS

Analysis of Human Experts

The data set analyzed in this paper consists of 72 cardiac patients described by 19 variables (descriptors), as follows: age (1), weight (2), ECHO data including left ventricle (3) and (4), right ventricle (5), left atrium (6) and contractility (7) and (8), respectively and effort testings for level of effort (9), duration (10), cardiac frequency (11), maximum cardiac frequency (12), systolic arterial tension (13), diastolic arterial tension (14), VTO_2 (MET) (15), index of tension-time (16), relative index of tension-time (17), body aerobic deficit (18), and myocardium aerobic deficit (19). Tables 1 and 2 show the values of these 19 descriptors for 72 patients.

TABLE 1
Original Values of the 19 Descriptors for 72 Cardiac Patients (Part I)

Patient	Characteristic (descriptor)									
	1	2	3	4	5	6	7	8	9	10
1	49	58	50	35	14	35	67	31	100	150
2	43	65	55	40	18	42	61	27.2	50	167
3	42	73	60	45	19	45	58	22	75	115
4	53	87	53	40	19	30	63	24	100	100
5	33	91	58	40	18	35	68	31	75	125
6	65	89	55	40	14	30	61	27.2	50	107
7	39	67	52	35	21	35	68	32	125	150
8	67	85	50	38	15	28	56.1	24	100	136
9	48	75	62	40	15	47	71	34	100	150
10	36	70	59	45	15	32	57	23.7	75	125
11	49	59	50	30	14	28	79	41	50	115
12	41	67	50	35	21	30	67	31	75	162
13	57	65	65	50	20	50	54	23	100	115
14	28	70	56	40	19	30	55	23	125	162
15	56	67	48	30	21	33	73	36	75	136
16	32	76	62	48	18	38	53.5	22.5	100	125
17	41	73	58	43	20	41	58	25	75	125
18	27	55	50	30	14	31	79	41	100	167
19	46	67	47	26	15	41	83.07	44.6	100	152
20	44	71	60	45	20	48	25	58	125	136
21	33	61	49	30	20	41	78	40	100	167
22	44	86	60	45	20	40	58	25	150	167
23	39	66	55	40	20	52	61	27.2	50	100
24	40	73	49	35	16	33	66	30	100	136
25	40	60	57	40	14	40	63	29.8	125	167
26	60	88	62	50	22	50	44	17	125	150
27	48	77	48	31	29	50	73	35.4	75	88
28	56	56	60	40	20	38	70	33.3	50	125
29	50	74	80	65	30	60	46.3	18.75	50	100
30	50	76	49	30	18	30	74	37	100	100
31	49	87	58	45	18	42	55	20	150	157
32	59	82	55	35	18	50	70	36	75	157
33	43	68	53	40	21	48	57	24.5	75	83
34	56	55	55	40	19	50	61	27.2	75	100
35	42	68	62	45	20	35	59	26	100	94
36	33	68	52	30	20	42	80	42	75	150
37	45	76	50	35	18	39	67	31	50	136
38	44	86	54	40	19	43	61	27	100	167
39	48	75	68	52	30	54	42	21	50	136
40	57	63	60	45	20	42	58	25	50	100
41	31	60	94	80	25	65	38.3	15.6	50	147
42	67	60	74	62	30	50	28	13.8	25	150
43	36	80	74	65	14	40	25.3	12.1	100	150
44	50	78	86	76	20	64	29	11.6	50	115
45	58	60	87	76	20	50	30	12	25	80
46	69	100	80	65	30	48	46	18.75	50	94
47	53	88	65	47	34	45	62.1	27.6	75	160
48	55	100	90	76	40	70	39.7	15.5	25	118
49	54	90	47	30	20	30	73.9	36	100	150

50	47	70	55	40	9	30	61	27.2	75	100
51	50	90	55	40	10	33	61	27.2	100	150
52	63	90	72	60	20	55	42	16.6	50	115
53	52	95	53	40	17	40	56	24	100	150
54	55	80	75	55	20	45	60	26.6	50	85
55	46	75	65	55	15	40	40	15.3	100	110
56	69	70	65	50	20	42	54	23	50	94
57	54	68	51	30	17	30	79.6	41	50	167
58	73	60	76	65	34	65	37	14.4	25	115
59	56	85	51	35	19	30	67.6	31.3	125	136
60	58	80	65	50	22	40	54	23	125	115
61	56	85	50	35	20	30	67	31	50	120
62	51	105	65	50	18	42	54	23	50	136
63	68	88	60	50	20	40	43	16	75	125
64	52	79	60	50	20	40	43	16	50	136
65	43	64	60	45	20	30	57.8	25	50	125
66	49	89	58	34	20	45	79.8	41.3	100	170
67	58	70	65	55	20	42	43	16	75	107
68	68	50	45	30	22	25	70.3	33.3	75	100
69	52	85	60	45	18	30	58	25	75	125
70	41	70	50	35	16	32	67	31	75	168
71	46	80	62	50	20	45	44	17	100	130
72	47	90	55	45	18	35	45.2	18.18	75	150

TABLE 2
Original Values of the 19 Descriptors for 72 Cardiac Patients (Part II)

	Characteristic (descriptor)								
Patient	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
1	161	190	100	25	7.14	31500	28500	22.81	16.46
2	167	135	85	14	4	32100	22545	60.99	33.50
3	178	155	90	17	4.85	32200	17825	52.86	44.51
4	167	155	85	16	4.57	31100	15500	53.20	50.16
5	177	150	90	13.3	3.8	33100	18750	66.90	45.63
6	145	175	90	10	2.85	29900	18725	68.89	42.64
7	171	230	110	26	7.42	32500	34500	17.96	1.06
8	153	180	95	20	5.71	29700	24480	46.55	24.70
9	162	165	90	20	5.71	31600	24750	42.92	26.37
10	174	115	80	17	4.85	32800	14375	63.45	68.20
11	161	150	90	15	4.28	31500	17250	56.98	48.59
12	169	175	90	18	5.14	32300	28350	53.07	16.67
13	163	190	100	23	6.57	30700	21850	31.36	28.82
14	182	155	90	25.5	7.28	33600	25885	34.96	25.56
15	154	165	90	17	4.85	30800	22440	46.55	32.31
16	178	165	95	20	5.71	33200	20625	50.44	40.29
17	169	135	85	17	4.85	32300	16875	54.45	50.39
18	183	170	90	27	7.71	33700	28390	32.26	18.49
19	164	165	95	21.5	6.14	31800	25905	36.80	23.20
20	166	195	105	25.5	7.28	32000	26520	34.75	21.64
21	177	160	80	25	7.14	33100	26720	33.49	22.51
22	176	180	100	38.3	10.94	32000	30060	34.75	11.19
23	171	145	85	14	4	32500	14500	64.25	57.37

24	170	135	75	21	6	32400	18360	46.1	43.3
25	170	180	85	30	8.57	32400	30060	23.07	11.79
26	160	215	95	21	6	30400	32250	37.87	2.03
27	162	180	110	15	4.28	31600	15840	57.19	52.87
28	154	185	90	16	4.57	30800	23125	52.49	30.24
29	170	140	85	12	3.42	31400	14000	71.49	58.20
30	160	170	90	12	3.42	31400	17000	65.41	49.25
31	171	235	95	23	6.57	31500	36895	31.17	9.94
32	161	155	95	15	4.28	30500	24335	56.28	26.20
33	167	135	85	17	4.85	32100	11205	55.83	65.09
34	154	130	70	22	6.28	30800	13000	34.67	60.78
35	168	155	95	21	6	32200	14570	32.74	57.10
36	177	150	90	17	4.85	33100	22500	54.77	34.45
37	165	145	90	12	3.42	31900	19720	66.24	41.63
38	166	110	85	16	5	32000	18370	51	46.72
39	172	160	90	12	3.42	31600	21760	65	31
40	153	197	95	14.5	4.14	30700	19500	56	36.5
41	189	170	80	15	4.28	33300	24990	60	25
42	153	135	70	10	2.85	29700	20250	68	32
43	174	165	85	19	6.24	32800	24750	48	24.5
44	170	115	75	11	3.14	31400	13225	68	57.88
45	162	116	70	10	2.85	30600	9280	70	69
46	151	150	90	9	2.6	29500	14100	70	52
47	167	190	100	13.3	3.8	31100	30880	60	7
48	165	143	70	6	1.71	30900	16874	80	45
49	156	215	90	16.7	4.77	31000	32250	50	0
50	163	160	85	17	4.85	31700	16000	52	50
51	160	180	95	16.7	4.77	31400	27000	50	14
52	157	155	90	10	2.85	30100	17825	70	41
53	158	165	75	16	4.57	31200	24750	53	20
54	165	200	115	11	3.14	30900	17000	67	45
55	174	190	95	20	5.71	31800	20900	43	34
56	151	180	85	13	3.71	29500	16920	58	42
57	156	195	95	13.5	3.85	31000	32565	60	0
58	147	145	90	10	2.85	29100	16675	67	43
59	164	160	90	21	6	30800	21760	37.6	30
60	162	160	90	22.5	6.42	30600	18400	32.53	39.86
61	154	175	90	10.5	3	30800	21000	70	36
62	159	220	110	9	2.57	31300	29920	74	4
63	152	185	95	13.3	3.8	29600	23125	58	22
64	158	135	85	11	3.14	31200	18360	68	41
65	167	165	90	14	4	32100	19625	61	39
66	171	160	85	16.7	4.77	31500	27200	52	14
67	162	165	85	17	4.85	30600	17655	50	42
68	142	180	95	24	6.8	29600	18000	24	39
69	168	145	90	14	4	31200	18125	59	42
70	169	190	95	17	4.85	32300	31920	53	0
71	174	135	70	19	5.4	31800	17500	50	48
72	163	160	90	13.3	3.8	31700	24000	60	24.29

The data set has been manually classified by human experts on the basis of paraclinical and clinical investigations into three principal groups: valvular heart disease (VHD) and fibrillation of aortic insufficiency (FAI) (1-38); dilatative cardiomiopathy (DCM), arterial hypertension (AH), and congestive cardiac insufficiency (CCI) (39-48); ischemic cardiopathy (IC), effort pectoralis angina (EPA), and myocardium infarct (MI) (49-72).

TABLE 3
Final Fuzzy Partition of the Cardiac Patients, without Data Normalization

Fuzzy class	Cardiac patients
A_{111}	8 9 28 32 43 53 63 72
A_{1121}	12 19 20 51 66
A_{1122}	14 18 21 41
A_{121}	1 22 25 47 62
A_{1221}	7 31
A_{12221}	26
A_{12222}	49 57 70
A_{2111}	2 13 15 36 39 42 59 61
A_{2112}	16 37 55 65
A_{2121}	3 5 11 17 24 30 38 64 69 71
A_{2122}	6 40 48 52 54 56 60 67 68
A_{221}	4 27 50 58
A_{2221}	10 23 29 34 35 44 46
A_{2222}	33 45

The medical specialists who realized this classification noted that many patients showed symptoms indicating more than one illness. This fact, together with the large number of descriptors used, represents good support for using a fuzzy clustering approach.

Fuzzy Hierarchical Classification of Cardiac Patients

The successive partition of the cardiac patients produced by using the 19 descriptors mentioned above is presented in Table 3. To ensure a more uniform participation of the various descriptors, we used a normalization (auto-scaling) of descriptors. The classification based on the same 19 descriptors but normalized—a procedure that avoids certain descriptors, expressed by larger numerical values, to prevail—gives finally just two classes. Table 4 shows the partition of the cardiac patient in this case.

TABLE 4
Final Fuzzy Partition of the Cardiac Patients, with 19 Normalized Characteristics

Fuzzy class	Cardiac patients
A_1	1 2 5 7 8 9 11 12 13 14 15 16 18 19 20 21 22 24 25 26 28 31 32 35 36 38 43 47 49 51 53 55 57 59 60 66 68 70
A_2	3 4 6 10 17 23 27 29 30 33 34 37 39 40 41 42 44 45 46 48 50 52 54 56 58 61 62 63 64 65 67 69 71 72

Comparing the results in Table 3 with the classes obtained by paraclinical and clinical investigations, we have observed certain differences. However, using data normalization the algorithm seems to produce better results providing only two classes: the first one including the majority of the valvular heart disease patients and the second one the majority of the dilatative cardiomiopathy and ischemic cardiopathy patients. In this case, a good agreement with the original diagnostics was established, as it may be seen in Table 5.

TABLE 5
Original Diagnostic and Memberships to the Classes of the Final Fuzzy Partition Produced with Data Normalization (the illnesses order has been established by clinical analysis)

Patient	Illness	NYHA	A_1	A_2
1	VHD	1	0.68002	0.31998
2	VHD	2/3	0.50467	0.49533
3	VHD	2/3	0.42350	0.57650
4	VHD	3	0.43222	0.56778
5	VHD	3	0.50049	0.49951
6	VHD	3	0.41299	0.58701
7	VHD	1	0.63278	0.36722
8	VHD, AH	2	0.56933	0.43067
9	VHD	2	0.76578	0.23422
10	VHD	3	0.44981	0.55019
11	VHD	2/3	0.53107	0.46893
12	VHD	2	0.71795	0.28205
13	VHD	2/3	0.55432	0.44568
14	VHD	1	0.62916	0.37084
15	VHD	3	0.62206	0.37794
16	VHD	2	0.57640	0.42360
17	VHD	3	0.40503	0.59497
18	VHD	1	0.63305	0.36695
19	VHD	2	0.69466	0.30534
20	VHD	1	0.58656	0.41344
21	VHD	1/2	0.65185	0.34815
22	VHD	1/2	0.60018	0.39982
23	VHD	3	0.38422	0.61578
24	VHD	2	0.61156	0.38844
25	VHD	1	0.64990	0.35010
26	VHD, FAI	2	0.56060	0.43940
27	VHD	2	0.47991	0.52009
28	VHD, FAI	2/3	0.51652	0.48348
29	VHD, FAI	3	0.33772	0.66228
30	VHD	2/3	0.49998	0.50002
31	VHD, AH	2/3	0.61521	0.38479
32	VHD, MI, FAI	3	0.56514	0.43486
33	VHD, FAI	2/3	0.38908	0.61092
34	VHD, MI, FAI	2/3	0.45803	0.54197
35	VHD, AH	2	0.51714	0.48286
36	VHD	2/3	0.62298	0.37702
37	VHD	3	0.45240	0.54760
38	VHD	3	0.52360	0.47640
39	DCM, AH, CCI	3	0.33589	0.66411
40	DCM, AH, CCI	3	0.37492	0.62508
41	DCM, CCI	3	0.45608	0.54392
42	DCM, CCI, FAI	3	0.38489	0.61511
43	DCM	2	0.50106	0.49894
44	DCM, CCI	3	0.37297	0.62703
45	DCM	3	0.37795	0.62205
46	DCM, CCI	3	0.36426	0.63574
47	DCM, AH, CCI	3	0.51120	0.48880
48	AH, DCM, FAI, CCI	3	0.40455	0.59545
49	IC, EPA, AH	2	0.63120	0.36880
50	IC, EPA	2/3	0.49107	0.50893
51	IC, EPA, AH	2	0.65957	0.34043

52	IC, EPA, CCI	3	0.31495	0.68505
53	IC, MI	3	0.55236	0.44764
54	IC, MI, AH, EPA	3	0.39859	0.60141
55	IC, MI, EPA	2	0.50264	0.49736
56	IC, EPA, MI	3	0.34103	0.65897
57	IC, EPA	3	0.59843	0.40157
58	IC, FAI	3	0.38734	0.61266
59	IC, MI	2	0.68343	0.31657
60	IC, EPA	2	0.51521	0.48479
61	MI	3	0.43440	0.56560
62	IC, EPA, AH	3	0.48213	0.51787
63	IC, AH, EPA	3	0.40022	0.59978
64	IC, EPA, AH	3	0.27272	0.72728
65	IC, EPA	3	0.40367	0.59633
66	MI	2	0.65184	0.34816
67	IC, EPA	2/3	0.26617	0.73383
68	IC, MI, EPA	2	0.53834	0.46166
69	MI, EPA	3	0.34337	0.65663
70	IC, EPA	2	0.68006	0.31994
71	IC, MI, EPA	2	0.41871	0.58129
72	IC, EPA	3	0.45662	0.54338

The table also presents the NYHA functional class as it was established by paraclinical and clinical investigations. Moreover, considering the relatively large number of variables, we attempt, in the next section, to reduce it by applying a fuzzy clustering algorithm.

Fuzzy Hierarchical and Horizontal Characteristics Clustering

The large number of available variables is always an issue, because of the extra computation required, and because not all the variables describe equally well the data. Following the fuzzy clustering of patients, our aim is to use fuzzy clustering in selecting the most relevant variables. We will next classify the data by using only these most relevant variables, and compare the results with those of the original fuzzy classification.

In order to develop the classifications presented in this section we applied the FHiCC procedure to the initial descriptors.

The characteristics clustering with the 19 descriptors for the 72 cardiac patients without data normalization produced the final partition presented in Table 6. The first descriptor separated from the others is 5, followed by 6 and then by 4. The cluster containing the descriptors from 7 to 19 is not subjected to any more splitting (their membership degrees, MD, to this cluster are all near 1). In the next step the descriptor 2 follows and, finally, 3 and 1.

The characteristics clustering with the same descriptors but with normalization (see Table 3) illustrates the same aspect, i.e., the high similarity of the last 13 descriptors based on effort testing and a large dissimilarity among the first descriptors including ECHO data and age and weight, respectively. This conclusion is supported also by the horizontal characteristics clustering, with the number of classes preset to seven, i.e., the number of classes produced by the hierarchic clustering procedure. The results of the horizontal characteristics clustering procedure are shown in Table 7. It is interesting to remark that all the divisions in Table 4 are clear-cut, the membership degrees to the different classes are all 1 or 0. We have to stress that the same treatment, but with a predefined number of eight classes, gives absolutely the same results, i.e., seven classes are clear-cut and one remains vacant, the MDs of all the descriptors to this class being zero.

TABLE 6
Membership Degrees of the 19 Descriptors to the Clusters of the Final Fuzzy Partition without and with Data Normalization, Respectively

Descriptor	Without normalization							With normalization	
	A_{111}	A_{11211}	A_{112121}	A_{112122}	A_{1122}	A_{12}	A_2	A_1	A_2
1	0.218	0.210	**0.526**	0.000	0.044	0.000	0.000	**0.537**	0.463
2	0.260	**0.643**	0.003	0.002	0.090	0.000	0.000	0.498	**0.502**
3	0.171	0.052	0.000	**0.681**	0.094	0.000	0.000	**0.561**	0.439
4	**0.991**	0.002	0.001	0.001	0.003	0.000	0.000	**0.528**	0.472
5	0.001	0.000	0.000	0.000	0.000	0.002	**0.996**	**0.589**	0.411
6	0.000	0.000	0.000	0.000	0.000	**0.759**	0.241	0.495	**0.504**
7	0.002	0.000	0.000	0.000	**0.997**	0.000	0.000	0.008	**0.992**
8	0.002	0.000	0.000	0.000	**0.997**	0.000	0.000	0.008	**0.992**
9	0.002	0.000	0.000	0.000	**0.997**	0.000	0.000	0.008	**0.992**
10	0.002	0.000	0.000	0.000	**0.997**	0.000	0.000	0.008	**0.992**
11	0.002	0.000	0.000	0.000	**0.997**	0.000	0.000	0.008	**0.992**
12	0.002	0.000	0.000	0.000	**0.997**	0.000	0.000	0.008	**0.992**
13	0.002	0.000	0.000	0.000	**0.997**	0.000	0.000	0.008	**0.992**
14	0.002	0.000	0.000	0.000	**0.997**	0.000	0.000	0.008	**0.992**
15	0.002	0.000	0.000	0.000	**0.997**	0.000	0.000	0.008	**0.992**
16	0.002	0.000	0.000	0.000	**0.997**	0.000	0.000	0.008	**0.992**
17	0.002	0.000	0.000	0.000	**0.997**	0.000	0.000	0.008	**0.992**
18	0.002	0.000	0.000	0.000	**0.997**	0.000	0.000	0.008	**0.992**
19	0.002	0.000	0.000	0.000	**0.997**	0.000	0.000	0.008	**0.992**

TABLE 7
Membership Degrees of the 19 Descriptors to the Seven Classes Obtained by Horizontal Classification

Descriptor	Fuzzy class						
	A_1	A_2	A_3	A_4	A_5	A_6	A_7
1	0.000	**1.000**	0.000	0.000	0.000	0.000	0.000
2	**1.000**	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	**1.000**	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	**1.000**	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	**1.000**	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	**1.000**	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000	**1.000**
8	0.000	0.000	0.000	0.000	0.000	0.000	**1.000**
9	0.000	0.000	0.000	0.000	0.000	0.000	**1.000**
10	0.000	0.000	0.000	0.000	0.000	0.000	**1.000**
11	0.000	0.000	0.000	0.000	0.000	0.000	**1.000**
12	0.000	0.000	0.000	0.000	0.000	0.000	**1.000**
13	0.000	0.000	0.000	0.000	0.000	0.000	**1.000**
14	0.000	0.000	0.000	0.000	0.000	0.000	**1.000**
15	0.000	0.000	0.000	0.000	0.000	0.000	**1.000**
16	0.000	0.000	0.000	0.000	0.000	0.000	**1.000**
17	0.000	0.000	0.000	0.000	0.000	0.000	**1.000**
18	0.000	0.000	0.000	0.000	0.000	0.000	**1.000**
19	0.000	0.000	0.000	0.000	0.000	0.000	**1.000**

We may conclude that the most significant descriptors, as shown by fuzzy clustering, are, in the order of their importance, 5 (ECHO data for right ventricle), 6 (ECHO data for left atrium), 4 (ECHO data for left ventricle), 2 (weight), 3 (ECHO data for left ventricle), 1 (age). This is because these descriptors have the highest discriminative power among the data set.

On the other side, the descriptors from 7 to 19 had the same membership degrees to all the produced classes, and this may indicate that they are representations of the same unique property. As such, we have added one of this descriptors to the set of six most relevant descriptors.

Fuzzy Hierarchical Clustering of Cardiac Patients Considering Only Seven Characteristics

Taking into account the results obtained above referring to the characteristics clustering, it appears more illuminating and intuitive to use for the classification of cardiac patients only the first seven descriptors, namely age (1), weight (2), and ECHO data (3-7). In order to validate our method, we will use the same fuzzy clustering procedures, but on the data set described only by these seven descriptors.

The results obtained in this case without and with normalization are presented in Table 8. By careful examination and comparison with the results in Table 9 concerning the membership degrees of the patients to the four final classes it is easy to observe a good agreement with the classification based on the paraclinical and clinical examinations (see Table 5).

This analysis confirms that the data set using only the seven selected descriptors conserve its discriminative power, and supports our decision to discard the less relevant descriptors.

TABLE 8
Final Fuzzy Partition of the Cardiac Patients Produced using Seven Characteristics without and with Data Normalization

Fuzzy class	Cardiac patients
	Without data normalization
A_{11}	3 4 5 6 8 9 16 17 22 31 32 35 38 40 49 51 53 59 61 66
A_{12}	1 2 7 10 11 12 14 15 18 19 21 23 24 25 27 28 30 33 34 36 37 0 57 65 68 70
A_{21}	13 20 26 39 47 54 55 56 60 62 63 64 67 71 72
A_{22}	29 41 42 43 44 45 46 48 52 58
	With data normalization
A_{11}	3 4 5 6 8 9 16 17 22 27 31 32 38 40 49 51 53 59 61 66 69 72
A_{12}	1 2 7 10 11 12 14 15 18 19 21 23 24 25 28 30 33 34 35 36 37 50 57 65 68 70
A_{21}	13 20 26 43 47 54 55 56 60 62 63 64 67 71
A_{22}	29 39 41 42 44 45 46 48 52 58

TABLE 9
Membership Degrees of the Cardiac Patients to the Classes of the Final Fuzzy Hierarchical Partition, without and with Data Normalization

Cardiac patient	Without normalization				With normalization			
	A_{11}	A_{12}	A_{21}	A_{22}	A_{11}	A_{12}	A_{21}	A_{22}
1	0.181	0.689	0.093	0.036	0.244	0.605	0.113	0.037
2	0.273	0.615	0.086	0.025	0.272	0.616	0.090	0.022
3	0.441	0.260	0.248	0.051	0.424	0.288	0.244	0.043
4	0.625	0.174	0.163	0.038	0.575	0.233	0.158	0.034
5	0.470	0.284	0.182	0.064	0.433	0.328	0.178	0.061
6	0.452	0.201	0.273	0.074	0.418	0.248	0.262	0.071
7	0.107	0.826	0.050	0.018	0.193	0.708	0.074	0.024
8	0.425	0.228	0.271	0.077	0.402	0.268	0.256	0.074
9	0.475	0.358	0.128	0.040	0.431	0.345	0.178	0.045
10	0.375	0.393	0.176	0.054	0.354	0.449	0.152	0.044
11	0.229	0.612	0.110	0.049	0.269	0.555	0.127	0.049
12	0.150	0.773	0.057	0.020	0.226	0.665	0.081	0.026
13	0.167	0.126	0.513	0.194	0.145	0.109	0.575	0.170
14	0.332	0.414	0.183	0.071	0.321	0.442	0.172	0.066
15	0.279	0.599	0.089	0.333	0.327	0.512	0.121	0.040
16	0.342	0.246	0.312	0.010	0.356	0.307	0.256	0.080
17	0.507	0.313	0.149	0.031	0.476	0.356	0.141	0.026
18	0.242	0.543	0.141	0.075	0.265	0.503	0.156	0.075
19	0.262	0.582	0.107	0.049	0.283	0.549	0.120	0.047
20	0.159	0.124	0.418	0.298	0.189	0.149	0.442	0.220
21	0.221	0.608	0.115	0.056	0.251	0.554	0.135	0.060
22	0.511	0.123	0.316	0.049	0.515	0.158	0.285	0.042
23	0.318	0.430	0.182	0.070	0.310	0.380	0.227	0.083
24	0.240	0.704	0.043	0.013	0.239	0.687	0.057	0.017
25	0.225	0.613	0.117	0.044	0.260	0.562	0.134	0.044
26	0.099	0.050	0.675	0.175	0.103	0.056	0.645	0.196
27	0.386	0.423	0.137	0.054	0.326	0.317	0.239	0.118
28	0.270	0.505	0.157	0.067	0.298	0.432	0.200	0.069
29	0.092	0.068	0.098	0.742	0.109	0.088	0.073	0.729
30	0.347	0.558	0.071	0.025	0.355	0.538	0.082	0.025
31	0.436	0.104	0.414	0.046	0.488	0.133	0.341	0.037
32	0.478	0.270	0.191	0.060	0.416	0.260	0.253	0.070
33	0.370	0.405	0.175	0.050	0.352	0.381	0.210	0.056
34	0.276	0.392	0.228	0.103	0.281	0.337	0.276	0.106
35	0.392	0.390	0.173	0.045	0.384	0.414	0.165	0.037
36	0.252	0.589	0.109	0.051	0.273	0.542	0.129	0.055
37	0.420	0.550	0.024	0.007	0.364	0.598	0.030	0.007
38	0.640	0.150	0.173	0.036	0.600	0.202	0.165	0.032
39	0.063	0.042	0.450	0.445	0.101	0.077	0.261	0.561
40	0.323	0.297	0.297	0.081	0.314	0.267	0.349	0.069
41	0.165	0.149	0.196	0.489	0.172	0.158	0.202	0.468
42	0.121	0.100	0.221	0.559	0.128	0.107	0.202	0.563
43	0.141	0.106	0.318	0.434	0.196	0.156	0.367	0.281
44	0.123	0.096	0.130	0.651	0.134	0.108	0.178	0.581
45	0.130	0.110	0.163	0.597	0.137	0.116	0.216	0.531
46	0.143	0.092	0.327	0.438	0.146	0.104	0.262	0.487
47	0.299	0.139	0.427	0.134	0.208	0.152	0.346	0.293
48	0.158	0.120	0.219	0.503	0.173	0.144	0.198	0.484

49	0.471	0.333	0.144	0.051	0.446	0.344	0.158	0.052
50	0.400	0.481	0.093	0.026	0.362	0.437	0.154	0.047
51	0.562	0.181	0.207	0.050	0.452	0.289	0.203	0.056
52	0.081	0.049	0.358	0.512	0.094	0.061	0.386	0.459
53	0.460	0.151	0.317	0.071	0.476	0.189	0.276	0.059
54	0.148	0.081	0.550	0.221	0.118	0.071	0.599	0.212
55	0.104	0.065	0.625	0.205	0.223	0.148	0.507	0.121
56	0.197	0.128	0.504	0.171	0.190	0.129	0.524	0.157
57	0.281	0.584	0.096	0.039	0.310	0.542	0.109	0.038
58	0.138	0.116	0.202	0.543	0.151	0.131	0.184	0.535
59	0.572	0.267	0.126	0.035	0.520	0.304	0.140	0.036
60	0.179	0.071	0.703	0.047	0.159	0.074	0.715	0.051
61	0.567	0.268	0.128	0.036	0.510	0.304	0.147	0.038
62	0.261	0.123	0.444	0.171	0.276	0.145	0.427	0.153
63	0.169	0.087	0.578	0.167	0.199	0.111	0.542	0.149
64	0.125	0.061	0.761	0.052	0.201	0.096	0.664	0.038
65	0.332	0.437	0.176	0.055	0.333	0.455	0.166	0.046
66	0.461	0.313	0.162	0.063	0.442	0.298	0.195	0.065
67	0.067	0.043	0.663	0.226	0.095	0.062	0.705	0.139
68	0.286	0.430	0.188	0.096	0.298	0.379	0.218	0.105
69	0.526	0.140	0.286	0.048	0.533	0.190	0.237	0.039
70	0.143	0.803	0.041	0.013	0.185	0.742	0.056	0.016
71	0.107	0.054	0.752	0.086	0.167	0.087	0.671	0.074
72	0.330	0.136	0.449	0.084	0.419	0.184	0.334	0.062

Fuzzy Horizontal Cardiac Patients Clustering Considering Only Seven Characteristics

We continue our analysis by clustering the set of patients with the seven descriptors without and with data normalization. Because the human experts indicated a classification of the patients in three classes, we will work here with the same number of classes. The fuzzy horizontal clustering distributes the cardiac patients according to the data presented in Table 10. It is interesting to remark in this case a better agreement with the classification obtained by paraclinical and clinical examinations. The class of arterial hypertension patients, A_3, is much better separated than the other ones. Concerning the class of valvular heart disease patients, A_1, and the class of the ischemic cardiac patients, A_3, each of them contains patients from the other one. However, we have to observe that in each of these classes we find the majority of patients indicated by paraclinical and clinical investigations (see Table 5) and this is a good validation of our technique.

The membership degrees of the cardiac patients to the classes of the final fuzzy partitions obtained by horizontal fuzzy clustering for seven descriptors without and with data normalization, presented in Table 11 illustrate also the efficiency of the fuzzy clustering approach. These fuzzy membership degrees are in good support with the medical practice that cardiac patients may present signs of more than one illness, since a clear-cut of the three groups of cardiac patients is practically impossible.

Fuzzy Hierarchical Cross-Clustering

In what follows our aim is to identify the descriptors responsible with the separation of each class of patients. We will achieve this by using our fuzzy hierarchical cross-clustering algorithm on the set of 72 cardiac patients characterized by the same seven descriptors.

TABLE 10
Results of Horizontal Fuzzy Clustering of the Cardiac Patients without and with Data Normalization, Considering only Seven Characteristics

Fuzzy class	Cardiac patients
	Without data normalization
A_1	1 2 7 9 10 11 12 14 15 18 19 21 23 24 25 27 28 30 33 34 35 36 37 49 50 57 65 66 68 70
A_2	3 4 5 6 8 13 16 17 22 26 31 32 38 40 47 51 53 54 56 59 60 61 62 63 64 69 71 72
A_3	20 29 39 41 42 43 44 45 46 48 52 55 58 67
	With data normalization
A_1	1 2 7 9 10 11 12 14 15 18 19 21 23 24 25 27 28 30 33 34 35 36 37 49 50 57 59 61 65 68 70
A_2	3 4 5 6 8 13 16 17 20 22 26 31 38 40 51 53 54 55 56 60 62 63 64 66 67 69 71 72
A_3	29 39 41 42 43 44 45 46 47 48 52 58

TABLE 11
Membership Degrees of the Cardiac Patients to the Classes of the Fuzzy Partition Obtained by Horizontal Fuzzy Clustering using only Seven Characteristics

Cardiac patient	Without normalization			With normalization		
	A_1	A_2	A_3	A_1	A_2	A_3
1	0.799	0.155	0.045	0.729	0.219	0.052
2	0.725	0.227	0.048	0.708	0.253	0.039
3	0.317	0.591	0.091	0.326	0.604	0.070
4	0.293	0.647	0.060	0.390	0.550	0.060
5	0.340	0.499	0.101	0.453	0.454	0.093
6	0.287	0.601	0.112	0.364	0.522	0.113
7	0.895	0.086	0.019	0.807	0.161	0.032
8	0.322	0.555	0.123	0.389	0.493	0.117
9	0.515	0.409	0.076	0.467	0.456	0.077
10	0.484	0.421	0.095	0.544	0.383	0.073
11	0.738	0.197	0.065	0.692	0.240	0.068
12	0.862	0.114	0.025	0.783	0.180	0.036
13	0.244	0.409	0.347	0.244	0.454	0.302
14	0.515	0.371	0.114	0.546	0.356	0.098
15	0.750	0.200	0.050	0.679	0.260	0.060
16	0.322	0.522	0.156	0.386	0.496	0.118
17	0.389	0.546	0.065	0.418	0.532	0.050
18	0.660	0.240	0.100	0.622	0.276	0.102
19	0.721	0.213	0.067	0.693	0.241	0.065
20	0.204	0.339	0.457	0.250	0.429	0.322
21	0.726	0.202	0.072	0.674	0.245	0.080
22	0.105	0.852	0.043	0.164	0.789	0.046
23	0.544	0.342	0.114	0.486	0.392	0.122
24	0.835	0.141	0.024	0.812	0.162	0.025
25	0.716	0.217	0.066	0.662	0.273	0.065
26	0.138	0.463	0.398	0.164	0.470	0.366

27	0.577	0.334	0.089	0.446	0.389	0.165
28	0.635	0.264	0.100	0.568	0.328	0.104
29	0.052	0.089	0.860	0.051	0.080	0.869
30	0.740	0.217	0.043	0.731	0.228	0.040
31	0.067	0.896	0.036	0.109	0.854	0.036
32	0.408	0.491	0.101	0.386	0.504	0.110
33	0.512	0.395	0.093	0.478	0.431	0.091
34	0.517	0.325	0.158	0.464	0.379	0.157
35	0.478	0.435	0.087	0.494	0.439	0.066
36	0.715	0.216	0.069	0.668	0.256	0.076
37	0.810	0.168	0.021	0.828	0.155	0.017
38	0.231	0.717	0.051	0.304	0.646	0.050
39	0.100	0.214	0.686	0.111	0.191	0.698
40	0.408	0.443	0.149	0.376	0.499	0.124
41	0.171	0.219	0.610	0.178	0.226	0.596
42	0.114	0.171	0.715	0.117	0.171	0.712
43	0.153	0.265	0.582	0.237	0.381	0.382
44	0.089	0.141	0.771	0.109	0.169	0.722
45	0.110	0.159	0.730	0.131	0.192	0.676
46	0.141	0.284	0.575	0.140	0.238	0.622
47	0.216	0.591	0.193	0.243	0.374	0.383
48	0.147	0.233	0.619	0.163	0.224	0.612
49	0.494	0.418	0.087	0.514	0.401	0.084
50	0.622	0.326	0.054	0.564	0.360	0.076
51	0.265	0.663	0.072	0.409	0.501	0.090
52	0.088	0.207	0.705	0.118	0.252	0.630
53	0.196	0.712	0.091	0.269	0.649	0.084
54	0.175	0.462	0.362	0.186	0.459	0.354
55	0.162	0.408	0.429	0.240	0.555	0.206
56	0.234	0.457	0.309	0.251	0.473	0.276
57	0.733	0.210	0.057	0.706	0.238	0.055
58	0.132	0.185	0.683	0.138	0.185	0.677
59	0.448	0.484	0.067	0.492	0.443	0.064
60	0.132	0.706	0.162	0.166	0.668	0.165
61	0.446	0.486	0.067	0.487	0.445	0.067
62	0.195	0.564	0.240	0.232	0.555	0.212
63	0.173	0.505	0.321	0.213	0.527	0.260
64	0.148	0.608	0.244	0.155	0.722	0.123
65	0.536	0.367	0.097	0.556	0.368	0.075
66	0.464	0.434	0.102	0.445	0.454	0.101
67	0.143	0.336	0.520	0.192	0.476	0.332
68	0.558	0.303	0.139	0.513	0.338	0.149
69	0.151	0.794	0.055	0.257	0.684	0.059
70	0.890	0.092	0.017	0.846	0.132	0.022
71	0.148	0.564	0.288	0.168	0.665	0.166
72	0.175	0.695	0.130	0.230	0.682	0.088

The classification hierarchies produced in this way using both non-normalized and normalized data are presented in Table 12. The partitioning of the cardiac patients in classes 1 and 2 is practically the same in both cases. What is different is the partitioning of the descriptors in the two cases. The descriptors associated to the class 1 (without normalization), comprising the majority of valvular heart disease patients, are age (1), left ventricle (4), right ventricle (5), and left atrium (6). The patients in class 2 (without normalization), majority of arterial hypertension and ischemic cardiopathy patients, have as main descriptors the weight (2), left ventricle (3), and contractility (7).

TABLE 12
Cross-Classification of the Cardiac Patients and Characteristics Produced with Seven Non-Normalized and Normalized Descriptors

Fuzzy class	Cardiac patients	Associated descriptors
	Without data normalization	
A_1	1 2 3 4 5 6 7 8 9 10 11 12 14 15 16 17 18 19 21 22 23 24 25 27 28 30 31 32 33 34 35 36 37 38 40 49 50 51 53 57 59 61 65 66 68 69 70	1 4 5 6
A_2	13 20 26 29 39 41 42 43 44 45 46 47 48 52 54 55 56 58 60 62 63 64 67 71 72	2 3 7
	With data normalization	
A_1	1 2 3 4 5 6 7 8 9 10 11 12 14 15 16 17 18 19 21 22 23 24 25 27 28 30 31 32 33 34 35 36 37 38 40 49 50 51 53 57 59 61 65 66 68 69 70 72	1 2 3 4 5 6
A_2	13 20 26 29 39 41 42 43 44 45 46 47 48 52 54 55 56 58 60 62 63 64 67 71	7

In the case with data normalization, the main descriptor associated to the class 2 is only the contractility (7), the rest, namely age (1), weight (2), left ventricle (3) and (4), right ventricle (5) and left atrium (6) are classified with the class 1, which includes the majority of the valvular heart disease patients and half from ischemic cardiopathy patients. We remark again a good agreement with the medical observations presented in Table 5.

CONCLUSIONS

Fuzzy classification algorithms applied to cardiac patients based on seven descriptors, namely ECHO data, and also age and weight, allow an objective interpretation of their similarities and dissimilarities. Moreover, the results obtained may be very useful in their reclassification. It is very interesting to study the classification of valvular heart disease and ischemic cardiopathy patients considering their membership degrees. Some of them belong practically with the same MD to the two classes, illustrating in this way the fuzziness of cardiac diseases. The new fuzzy approach, the fuzzy cross-classification algorithm, allows the qualitative and quantitative identification of the variables (descriptors) responsible for the observed similarities and dissimilarities between cardiac patients.

In addition, the fuzzy hierarchical characteristics clustering (FHiCC) and fuzzy horizontal characteristics clustering (FHoCC) procedures revealed a high similarity between the descriptors referring to the effort testing. This is one of the main conclusions and suggests their high redundant character concerning the diagnosis of cardiac diseases.

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