

An AdS Crunch in Supergravity¹

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Abstract

We review some properties of N=8 gauged supergravity in four dimensions with modified, but AdS invariant boundary conditions on the $m^2 = -2$ scalars. There is a one-parameter class of asymptotic conditions on these fields and the metric components, for which the full AdS symmetry group is preserved. The generators of the asymptotic symmetries are finite, but acquire a contribution from the scalar fields. For a large class of such boundary conditions, we find there exist black holes with scalar hair that are specified by a single conserved charge. Since Schwarzschild-AdS is a solution too for all boundary conditions, this provides an example of black hole non-uniqueness. We also show there exist solutions where smooth initial data evolve to a big crunch singularity. This opens up the possibility of using the dual conformal field theory to obtain a fully quantum description of the cosmological singularity, and we report on a preliminary study of this.

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1 Introduction

One of the main goals of quantum gravity is to provide a better understanding of the big bang or big crunch singularities in cosmology. An issue that immediately comes to mind is whether cosmological singularities represent a true beginning or end of evolution. If this is the case it would raise the question what determines the boundary conditions at the singularity. A truly unified theory should then, besides specifying the dynamics, also include a principle that specifies the universe's quantum state. An appealing proposal in this context is the quantum state given by the no boundary wave function [1]. This describes the creation of an ensemble of universes with diverse properties. The no boundary proposal asserts that all information about a possible phase before the big bang that is in principle accessible to an observer in a given member of this ensemble is encoded in the no boundary instanton. But there is no real sense in which evolution continues through the singularity: the instanton describes the beginning of a new, disconnected universe that has a self-contained physical description. Because there is no boundary in the past, the no boundary condition naturally leads to a top down approach to cosmology [2]. In this approach, one first specifies a number of properties (as few as necessary, of course) of the universe at late times, which are then used to compute conditional probabilities predicting other features. The set of a posteriori conditions essentially select the histories that contribute to the path integral for a given member of the ensemble of universes.

Alternatively, it is possible that evolution continues through the singularity and that string theory itself determines the conditions at cosmological singularities. There may be some type of bounce, as envisioned by the pre-big bang [3] and cyclic universe models [4], or the transition could be chaotic, in which case one presumably needs to resort again to a top down approach to explain our observed universe. Even if evolution continues through the singularity the quantum state at the singularity may contain certain universal features. Perhaps the correct answer will turn out to be a combination of both scenarios: cosmological singularities could represent an endpoint of evolution only in certain situations, depending on the approach to the singularity. This would raise the interesting possibility that only certain 'special' cosmologies could be created from a pre-big bang phase.

Since our usual notions of space and time are likely to break down near cosmolog-

ical singularities, a particularly promising approach to study this issue might be to find a dual description in terms of more fundamental variables. In string theory we do not yet have a dual description of real cosmologies, but we do have the celebrated AdS/CFT correspondence [5] which provides a non-perturbative definition of string theory on asymptotically anti-de Sitter (AdS) spacetimes in terms of a conformal field theory (CFT). The dual CFT description has been used to study the singularity inside black holes [6], which is analogous to a cosmological singularity. Although some progress in this direction has been made, the fact that the singularity is hidden behind an event horizon clearly complicates the problem. This is because the CFT evolution is dual to bulk evolution in Schwarzschild time so the CFT never directly ‘sees’ the singularity.

It would be better to have examples of solutions in a low energy supergravity limit of string theory where smooth, asymptotically AdS initial data evolve to a big crunch singularity. Then AdS/CFT should provide a precise framework in which the quantum nature of cosmological singularities could be understood, at least with AdS boundary conditions. In this context, a big crunch singularity is simply any spacelike singularity which extends to infinity and reaches the boundary in finite time.

In this lecture we present examples of such solutions in the abelian truncation of gauged $\mathcal{N} = 8$, $D = 4$ supergravity in which one focuses on the $U(1)^4$ Cartan subgroup of $SO(8)$. Gauged $N = 8$ supergravity arises as the massless sector of eleven dimensional supergravity compactified on S^7 . The truncation to the $U(1)^4$ sector contains three scalar fields with $m^2 = -2$ in units of the AdS radius. We begin by reviewing the class of asymptotic conditions on these fields (and the metric components) that are invariant under the full AdS symmetry group. For each scalar we find (in addition to the standard ‘Dirichlet’ boundary conditions) there is a one parameter family of boundary conditions, labelled by f , that preserve the full set of AdS symmetries. When f vanishes, the dual CFT is the usual $2 + 1$ theory on a stack of M2-branes. Nonzero values of the parameter f correspond to modifying this theory by a triple trace operator. On the bulk side, the generators of the asymptotic symmetries are finite for all f , but acquire a contribution from the scalar fields.

For $f \neq 0$ we find there are static spherical black holes with scalar hair. These solutions are specified by a single conserved charge, namely their mass. Since Schwarzschild-AdS is a solution too for all boundary conditions, this provides an example of black

hole non-uniqueness. We then show there are also static solitons. We explain that the existence of solitons indicates AdS is nonlinearly unstable for these generalized AdS invariant boundary conditions. A particular manifestation of this is that for all nonzero f , there are bulk solutions where smooth, finite mass initial data evolve to a big crunch. We conclude this lecture with a preliminary discussion of the dual field theory description of the formation of a big crunch.

All this work was done in collaboration with G. Horowitz and K. Maeda, and the reader is referred to the original papers for more details [7, 8, 9].

2 AdS Invariant Boundary Conditions

We first consider gravity in $d + 1$ ($d \geq 2$) dimensions coupled to a single scalar field with a potential V that has a negative maximum at $\phi = 0$. This theory admits a pure AdS_{d+1} solution, with metric

$$ds_0^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = -(1 + \frac{r^2}{l^2}) dt^2 + \frac{dr^2}{1 + r^2/l^2} + r^2 d\Omega_{d-1} \quad (2.1)$$

where the AdS radius is given by

$$l^2 = -\frac{d(d-1)}{2V(0)} \quad (2.2)$$

Since we are assuming that the scalar mass m^2 is less than zero, solutions to the linearized wave equation $\nabla^2 \phi - m^2 \phi = 0$ with harmonic time dependence $e^{-i\omega t}$ all fall off asymptotically like

$$\phi = \frac{\alpha}{r^{\lambda_-}} + \frac{\beta}{r^{\lambda_+}} \quad (2.3)$$

with

$$\lambda_{\pm} = \frac{d \pm \sqrt{d^2 + 4l^2 m^2}}{2} \quad (2.4)$$

where we are assuming $m^2 \geq -\frac{d^2}{4l^2} = m_{BF}^2$. For fields that saturate the Breitenlohner-Freedman (BF) bound [10], $\lambda_+ = \lambda_- \equiv \lambda$ and the second solution asymptotically behaves like $\ln r/r^\lambda$.

We are interested in this lecture in nonlinear perturbations of (2.1) where the scalar asymptotically behaves as (2.3). Asymptotically anti-de Sitter spacetimes are defined by a set of boundary conditions at spacelike infinity which satisfy the requirements

set out in [11]. The standard set of boundary conditions on the metric components [11] that are left invariant under $SO(d-1, 2)$ are

$$\begin{aligned}
g_{rr} &= \frac{l^2}{r^2} - \frac{l^4}{r^4} + O(1/r^{d+1}) & g_{tt} &= -\frac{r^2}{l^2} - 1 + O(1/r^{d-3}) \\
g_{tr} &= O(1/r^d) & g_{ra} &= O(1/r^d) \\
g_{at} &= O(1/r^{d-3}) & g_{ab} &= \bar{g}_{ab} + O(1/r^{d-3})
\end{aligned} \tag{2.5}$$

These boundary conditions go together with (and indeed require) the standard ‘Dirichlet’ boundary conditions on the scalar field, which amount to taking $\alpha = 0$ in (2.3). It is well known that with these boundary conditions, a scalar field with negative mass squared does not cause an instability in anti de Sitter space. For boundary conditions of this form there is a positive energy theorem [12, 13, 14] which ensures that the total energy cannot be negative as long as the scalar does not violate the BF bound.

Recall that the energy, and more generally, conserved charges associated with asymptotic symmetries ξ^μ can be defined as follows [11]. One starts with the Hamiltonian (we have set $8\pi G = 1$)

$$H[\xi] = \int d^d x \xi^\mu H_\mu = \int d^d x (\xi^\perp \mathcal{H}_\perp(x) + \xi^i \mathcal{H}_i(x)) \tag{2.6}$$

where H_μ are the usual Hamiltonian and momentum constraints,

$$\begin{aligned}
\mathcal{H}_\perp &= \frac{2}{\sqrt{g}} (\pi^{ij} \pi_{ij} - \frac{\pi^2}{d-1} + \frac{p^2}{4}) + \sqrt{g} \left[-\frac{R}{2} + \frac{1}{2} (D\phi)^2 + V(\phi) \right], \\
\mathcal{H}_i &= -2\sqrt{g} D_j \left(\frac{\pi^j_i}{\sqrt{g}} \right) + p D_i \phi
\end{aligned} \tag{2.7}$$

and π^{ij} and p are the momenta conjugate to g_{ij} and ϕ . One then adds surface terms so that H has well defined functional derivatives, and one subtracts the analogous expression for the AdS_{d+1} background. For $\alpha = 0$ boundary conditions on the scalar field (together with (2.5)), this procedure yields the standard ‘gravitational’ surface term,

$$Q_G[\xi] = \frac{1}{2} \oint dS_i \bar{G}^{ijkl} (\xi^\perp \bar{D}_j h_{kl} - h_{kl} \bar{D}_j \xi^\perp) + 2 \oint dS_i \frac{\xi^j \pi^i_j}{\sqrt{g}} \tag{2.8}$$

where $G^{ijkl} = \frac{1}{2} g^{1/2} (g^{ik} g^{jl} + g^{il} g^{jk} - 2g^{ij} g^{kl})$, $h_{ij} = g_{ij} - \bar{g}_{ij}$ is the deviation from the spatial metric \bar{g}_{ij} of pure AdS, \bar{D}_i denotes covariant differentiation with respect to \bar{g}_{ij} and $\xi^\perp = \xi^\mu n_\mu$ with n_μ the unit normal to the surface.

However, for scalar fields with m^2 in the range $m_{BF}^2 + 1 > m^2 > m_{BF}^2$ we have recently found there exists an additional one-parameter family of AdS invariant boundary conditions on the scalar field and the metric components [8]. More precisely, we find that the asymptotic AdS symmetries are also preserved in solutions that belong to the following class,

$$\phi(r, t, x^a) = \frac{\alpha(t, x^a)}{r^{\lambda_-}} + \frac{f\alpha^{\lambda_+/\lambda_-}(t, x^a)}{r^{\lambda_+}} \quad (2.9)$$

$$\begin{aligned} g_{rr} &= \frac{l^2}{r^2} - \frac{l^4}{r^4} - \frac{\alpha^2 l^2 \lambda_-}{(d-1)r^{2+2\lambda_-}} + O(1/r^{d+2}) & g_{tt} &= -\frac{r^2}{l^2} - 1 + O(1/r^{d-2}) \\ g_{tr} &= O(1/r^{d-1}) & g_{ab} &= \bar{g}_{ab} + O(1/r^{d-2}) \\ g_{ra} &= O(1/r^{d-1}) & g_{ta} &= O(1/r^{d-2}) \end{aligned} \quad (2.10)$$

where x^a labels the coordinates on S^{d-1} and f is an arbitrary constant that labels the different boundary conditions. Notice that the boundary conditions on some of the metric components are relaxed compared to the standard set. For $f = 0$ we recover boundary conditions on the scalar corresponding to $\beta = 0$ in (2.3), which have been considered previously in the context of AdS/CFT [15]. Remarkably, however, the full AdS symmetry group is preserved for all values of f . In particular, it is easy to see that rescaling r leaves f unchanged. Since α depends on the particular solution and can vanish, each of these boundary conditions admits AdS_{d+1} as a solution.

For these more general boundary conditions, the usual energy (2.8) diverges as $r^{d-2\lambda_-}$. However, the purely gravitational surface term (2.8) no longer equals the conserved charge associated with the asymptotic symmetry $\xi = \partial_t$. Instead, by repeating the above procedure, one finds the conserved charges acquire an additional contribution from the scalar field. The conserved charges now read [8]

$$Q[\xi] = Q_G[\xi] + \frac{1}{2d} \oint \xi^\perp [(\nabla\phi)^2 - m^2\phi^2]. \quad (2.11)$$

For all finite f (including $f = 0$!) the scalar and gravitational terms separately diverge. The divergences, however, exactly cancel out yielding finite total charges $Q[\xi]$. By contrast, the scalar charges Q_ϕ vanish for the standard $\alpha = 0$ scalar boundary conditions. For the case $f = 0$ the scalar surface term is equivalent to the surface term $-\frac{1}{2} \oint \phi \nabla_i \phi dS^i$ introduced by Klebanov and Witten in $D = 4$ supergravity, to regularize the action of the α/r modes of the $m^2 = -2$ scalar [15].

For spherical solutions that are asymptotically of the form (2.9)-(2.10), it is easy to compute the total mass M . One obtains

$$M = Q[\partial_t] = \text{Vol}(S^{d-1}) \left(\frac{d-1}{2} M_0 - \frac{2fm^2\alpha^{d/\Delta_-}}{d} \right), \quad (2.12)$$

where M_0 is the coefficient of the $O(1/r^{d+2})$ correction to the g_{rr} component of the AdS metric. We emphasize again that in the theory defined by $f = 0$ boundary conditions, which is often used in AdS/CFT, the backreaction of the scalar relaxes the asymptotic falloff of some metric components, while preserving the asymptotic AdS symmetry group. Although there is no residual finite scalar contribution to the total mass M in this case, it is only the variation of the sum of both charges that is well defined.

Finally we briefly mention the case of a scalar saturating the BF bound, which generically behaves as $\ln r/r^\lambda$ near the boundary. One finds there is again a one-parameter family of boundary conditions, involving the logarithmic branch, that preserves the AdS symmetries [8, 16]. For all finite values of the parameter f that labels the different asymptotic conditions, the gravitational and scalar surface terms are logarithmically divergent. The divergences again cancel out, however, rendering the total charges (2.11) finite.

3 $D = 4$ Gauged Supergravity

We now consider the low energy limit of string theory with $AdS_4 \times S^7$ boundary conditions. The massless sector of the compactification of $D = 11$ supergravity on S^7 is $\mathcal{N} = 8$ gauged supergravity in four dimensions [17]. The bosonic part of this theory involves the graviton, 28 gauge bosons in the adjoint of $SO(8)$, 70 real scalars, and it admits AdS_4 as a vacuum solution. It is possible to consistently truncate this theory to its abelian $U(1)^4$ sector [18]. The resulting action is given by

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{2} \sum_{i=1}^3 [(\nabla\phi_i)^2 - 2 \cosh(\sqrt{2}\phi_i)] \right) + \dots \quad (3.1)$$

where the dots refer to gauge field terms that will be set to zero in this paper. Here we have chosen the gauge coupling so that the AdS radius is equal to one. Notice

that the potential is unbounded from below, and the scalars have mass

$$m_i^2 = -2. \quad (3.2)$$

The BF bound in four dimensions is $m_{BF}^2 = -9/4$. Therefore, in addition to the standard Dirichlet boundary conditions where asymptotically $\phi_i \sim \beta_i/r^2$, there is a class of asymptotically AdS solutions of the form

$$\phi_i(r, t, x^a) = \frac{\alpha_i(t, x^a)}{r} + \frac{f\alpha_i^2(t, x^a)}{r^2} \quad (3.3)$$

and

$$\begin{aligned} g_{rr} &= \frac{1}{r^2} - \sum_{i=1}^3 \frac{(1 + \alpha_i^2/2)}{r^4} + O(1/r^5) & g_{tt} &= -r^2 - 1 + O(1/r) \\ g_{tr} &= O(1/r^2) & g_{ab} &= \bar{g}_{ab} + O(1/r) \\ g_{ra} &= O(1/r^2) & g_{ta} &= O(1/r) \end{aligned} \quad (3.4)$$

where $x^a = \theta, \phi$ and f is an arbitrary constant labelling the different theories. The conserved charges for these boundary conditions acquire a scalar contribution and take the form

$$Q[\xi] = Q_G[\xi] + \frac{1}{6} \sum_{i=1}^3 \oint \xi^\perp [(\nabla\phi_i)^2 + 2\phi_i^2]. \quad (3.5)$$

We now turn to a more detailed analysis of this theory, with boundary conditions specified by (3.3)-(3.4). To simplify the analysis we concentrate on solutions with only one nontrivial scalar ϕ .

4 Black Holes with Scalar Hair

First we look for static, spherically symmetric AdS black hole solutions with scalar hair. The original no hair theorem of Bekenstein [19] proves there are no asymptotically flat black hole solutions with scalar hair for minimally coupled scalar fields with convex potentials. This result was generalized to the case of minimally coupled scalar fields with arbitrary positive potentials in [20]. Later it was shown [21] there are no hairy, asymptotically AdS black holes where the scalar field asymptotically tends to the true minimum of the potential. In [22], however, an example was given

of a hairy black hole where the scalar field asymptotically goes to a negative maximum of the potential. It is, however, not clear this solution can be regarded as being asymptotically AdS in a meaningful way, because its mass diverges.

More recently, however, a one-parameter family of AdS black holes with scalar hair was found in three dimensions [23]. Asymptotically the scalar field again tends to a negative maximum, but the potential satisfies the BF bound and the solutions belong to the class (2.9)-(2.10) in three dimensions. This raises the question if AdS black holes with scalar hair also exist in supergravity in four dimensions. In particular, it is possible that Bekenstein's no hair theorem applies to supergravity with some AdS invariant boundary conditions, but not with others.

Writing the metric as

$$ds_4^2 = -h(r)e^{-2\delta(r)}dt^2 + h^{-1}(r)dr^2 + r^2d\Omega_2^2 \quad (4.1)$$

the field equations read

$$h\phi_{,rr} + \left(\frac{2h}{r} + \frac{r}{2}\phi_{,r}^2 h + h_{,r}\right)\phi_{,r} = V_{,\phi} \quad (4.2)$$

$$1 - h - rh_{,r} - \frac{r^2}{2}\phi_{,r}^2 h = r^2V(\phi) \quad (4.3)$$

$$\delta_{,r} = -\frac{r\phi_{,r}^2}{2} \quad (4.4)$$

Regularity at the event horizon R_e requires

$$\phi'(R_e) = \frac{R_e V_{,\phi}}{1 - R_e^2 V} \quad (4.5)$$

Asymptotic AdS invariance requires ϕ asymptotically decays as

$$\phi(r) = \frac{\alpha}{r} + \frac{f\alpha^2}{r^2}, \quad (4.6)$$

where f is a given constant that is determined by the choice of boundary conditions. Hence asymptotically

$$h(r) = r^2 + 1 + \alpha^2/2 - \frac{M_0}{r}, \quad (4.7)$$

where M_0 is an integration constant.

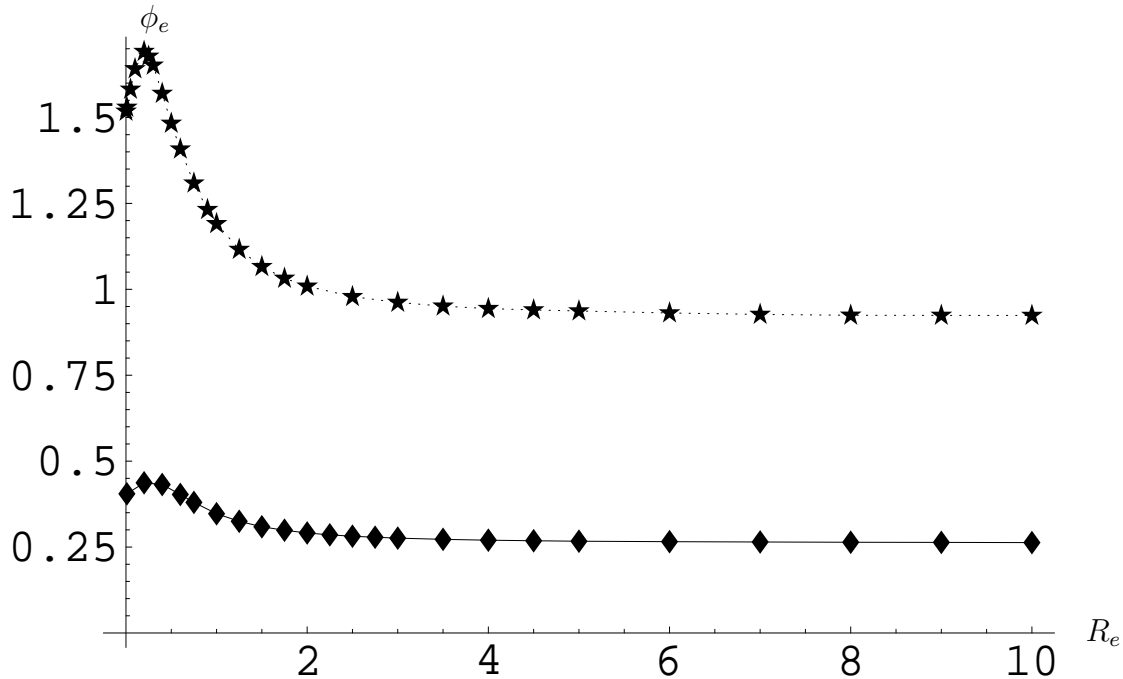


Figure 1: The value of the scalar field, ϕ_e , at the horizon of a hairy black hole as a function of horizon size R_e . The two curves show two one-parameter family of solutions of $D = 4$ $\mathcal{N} = 8$ supergravity, with two different AdS invariant boundary conditions, namely $f = -1$ (bottom) and $f = -1/4$ (top).

The Schwarzschild-AdS black hole with $\phi = 0$ everywhere outside the horizon is a solution for all AdS invariant boundary conditions. Its mass (3.5) is given by

$$M_s = Q[\partial_t] = 4\pi M_0 = 4\pi(R_e^3 + R_e), \quad (4.8)$$

which is the standard Schwarzschild-AdS mass. However, numerical integration of the field equations (4.2)-(4.3) shows that for a large class of boundary conditions there is in addition a one-parameter family of static spherically symmetric black hole solutions with scalar hair outside the horizon [8].

The value ϕ_e of the field at the horizon as a function of horizon size R_e is plotted in Figure 1. The two curves correspond to solutions with two different AdS invariant boundary conditions, namely $f = -1$ (bottom) and $f = -1/4$ (top). Generically, we obtain $\phi_e > 0$ if $f < 0$ and $\phi_e < 0$ for $f > 0$. Only for $f = 0$ and $f \rightarrow \infty$ there exist no regular hairy black hole solutions.

The integration constant M_0 in (4.7) is proportional to the finite gravitational

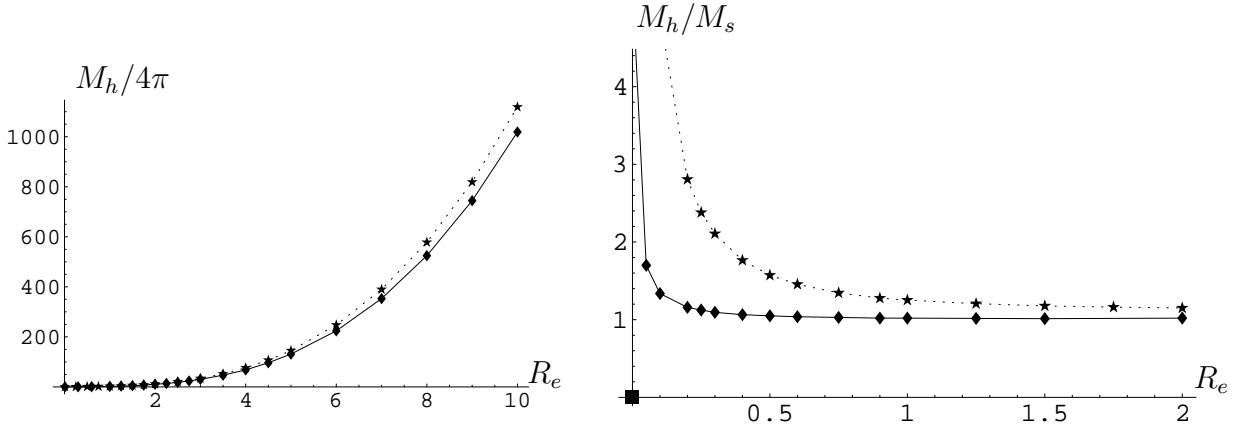


Figure 2: left: The total mass $M_h/4\pi$ of hairy black holes as a function of horizon size R_e , in $D = 4$ $\mathcal{N} = 8$ supergravity with two different AdS invariant boundary conditions $f = -1/4$ (top) and $f = -1$ (bottom). right: The ratio M_h/M_s as a function of horizon size R_e , where M_s is the mass of a Schwarzschild-AdS black hole of the same size R_e

contribution to the mass. It is, however, of little physical significance. Indeed the total gravitational mass diverges. The relevant quantity is the conserved charge $Q[\partial_t]$, which is given by

$$M_h = Q[\partial_t] = 4\pi \left(M_0 + \frac{4}{3} f \alpha^3 \right). \quad (4.9)$$

The total mass M_h is shown in Figure 2 as a function of horizon size R_e and for two different boundary conditions $f = -1/4$ (top) and $f = -1$ (bottom). We find $M_h > 0$ for all R_e and for all boundary conditions we have considered. For large R_e one has $M_h \sim R_e^3$. The mass is also compared with the mass M_s of a Schwarzschild-AdS black hole of the same size R_e . We find $M_h/M_s > 1$ for all R_e and $M_h/M_s \rightarrow 1$ for large R_e .

For given AdS invariant boundary conditions, there is at most one hairy black hole solution for a given total mass $Q[\partial_t]$, so the horizon size as well as the value of the scalar field at the horizon are uniquely determined by $Q[\partial_t]$. Thus we have found a one-parameter family of black holes with scalar hair, in a class of theories parameterized by f . Because Schwarzschild-AdS is a solution too for all boundary conditions we have two very different black hole solutions for a given total mass, one with $\phi = 0$ everywhere and one with nontrivial hair. The scalar no hair theorem, therefore, does not in general hold in $D = 4$ $\mathcal{N} = 8$ supergravity with asymptotically anti-de Sitter boundary conditions. Uniqueness is restored only in theories with $f = 0$

or for $f \rightarrow \infty$. The stability and thermodynamic properties of these hairy black hole solutions is currently under investigation [24].

5 Solitons

The existence of hairy black holes suggests there should also be regular static, spherically symmetric solitons that obey the same boundary conditions (3.3)-(3.4). Soliton solutions can similarly be found by numerically solving eqs (4.2-4.4). Regularity at the origin now requires $h = 1$, $h_{,r} = 0$ and $\phi_{,r} = 0$ at $r = 0$.

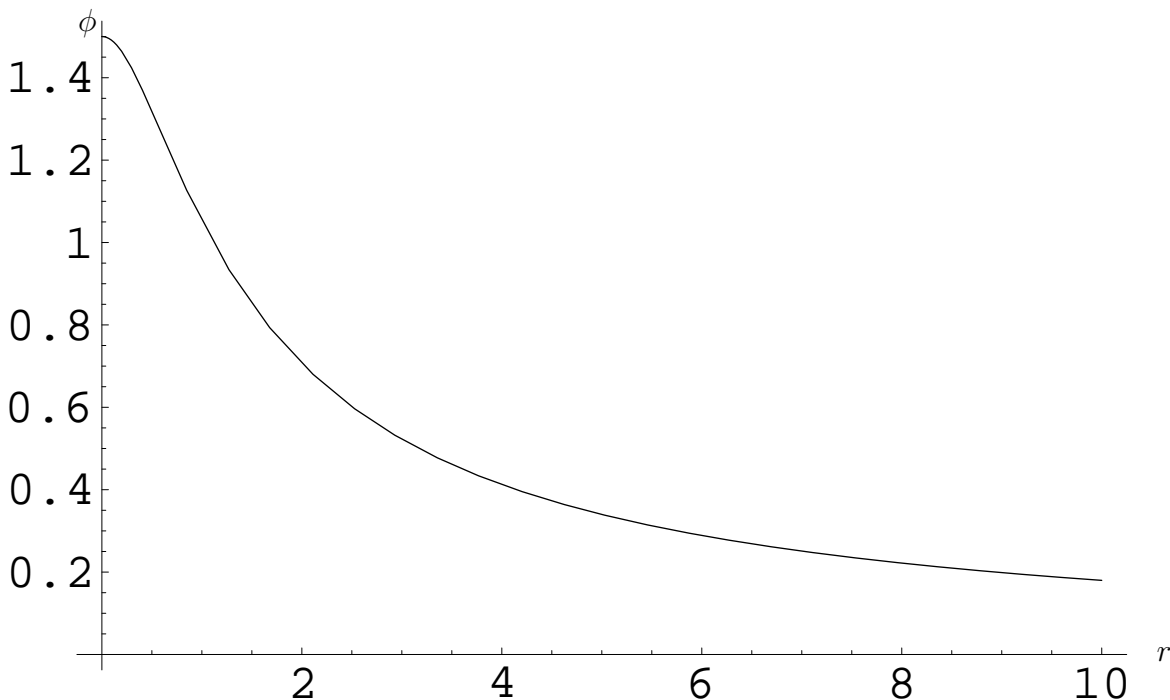


Figure 3: Soliton solution $\phi(r)$ in $D = 4$ supergravity with boundary conditions specified by $f = -1/4$.

For every nonzero ϕ_0 at the origin, the solution to (4.2) is asymptotically of the form (3.3) for some value of f . The staticity and spherical symmetry of the soliton mean $\alpha(t, x^a)$ is simply a constant. The scalar field value ϕ_0 at the origin uniquely determines f and vice versa: there is at most one static spherical soliton solution in each theory. We find [9] there is a regular soliton solution for all finite $f \neq 0$. When $|f| \rightarrow 0$ one finds $|\phi_0| \rightarrow \infty$ and for $|f| \rightarrow \infty$ one has $|\phi_0| \rightarrow 0$ so the nontrivial

soliton solution ceases to exist in this limit. As an example, in Figure 3 we show the soliton solution for $f = -1/4$ boundary conditions, which has $\phi_0 \approx 1.5$.

Most importantly, the existence of soliton solutions for a large class of AdS invariant boundary conditions implies supergravity with these boundary conditions does not admit a positive mass theorem [9]. This can be seen as follows. For the spherical solitons the constraint equation (4.3) can be integrated, which yields a formal expression for the gravitational surface term (2.8)

$$Q_G[\partial_t] = 2\pi \lim_{r \rightarrow \infty} \int_0^r e^{-\frac{1}{2} \int_{\tilde{r}}^r d\tilde{r} \hat{r}(\phi, r)^2} \left[2(V(\phi) - \Lambda) + \left(1 + \frac{\tilde{r}^2}{\ell^2} \right) \phi_{,\tilde{r}}^2 \right] \tilde{r}^2 d\tilde{r}. \quad (5.1)$$

One must add to this the scalar surface term to obtain the mass (3.5). Now consider a family of configurations $\phi_\lambda(r) = \phi_0(\lambda r)$ with mass M_λ where $\phi_0(r)$ is the static soliton profile. From (5.1) and the form of the scalar contribution one sees that the soliton mass M consists of the sum of a finite term M_1 (which includes the scalar contribution) that scales as the volume under rescalings $r \rightarrow \lambda r$ and a finite term M_2 that scales linearly in r . The latter comes from the gradient terms $\phi_{,r}^2$ in (5.1) and is manifestly positive. Therefore, one has

$$M_\lambda = \lambda^{-3} M_1 + \lambda^{-1} M_2 \quad (5.2)$$

Since the soliton extremizes the mass it follows that

$$\frac{dM_\lambda}{d\lambda} = -3M_1 - M_2 = 0 \quad (5.3)$$

Hence M_1 must be negative for the soliton. But this means rescaled configurations $\phi_\lambda(r) = \phi_0(\lambda r)$ with sufficiently small λ have negative mass. The AdS solution is unstable, therefore, with generalized boundary conditions (3.3) on the negative m^2 scalar.

Usually one discards unstable theories, saying they are not of physical interest. But here there should be a field theory dual to these bulk theories even if they are unstable. By studying the dual field theory description of various manifestations of the instability in the bulk, one can hope to gain insight into the quantum nature of such phenomena. Supergravity with generalized AdS invariant boundary conditions together with AdS/CFT thus provides a controlled setting to explore string theory away from the supersymmetric moduli space, where the theory is stable. In the

next sections we further explore this instability, concentrating on applications to cosmology. Finally in section 8 we turn to the dual field theory description of this theory.

6 Instantons

The existence of negative mass solutions means there must also be nontrivial zero mass solutions. The best known examples of such solutions are obtained from Euclidean instanton solutions which are usually interpreted as describing the decay of a false vacuum. An $O(4)$ -invariant instanton solution takes the form

$$ds^2 = \frac{d\rho^2}{b^2(\rho)} + \rho^2 d\Omega_3 \quad (6.1)$$

and $\phi = \phi(\rho)$. The field equations determine b in terms of ϕ

$$b^2(\rho) = \frac{2V\rho^2 - 6}{\rho^2\phi'^2 - 6} \quad (6.2)$$

and the scalar field ϕ itself obeys

$$b^2\phi'' + \left(\frac{3b^2}{\rho} + bb'\right)\phi' - V_{,\phi} = 0 \quad (6.3)$$

where prime denotes ∂_ρ . Regularity again requires $\phi'(0) = 0$.

From (6.3) it follows that asymptotically $\phi(\rho)$ has the same behavior as the Lorentzian scalar field solutions considered above,

$$\phi = \frac{\alpha}{\rho} + \frac{f\alpha^2}{\rho^2}. \quad (6.4)$$

We find that all boundary conditions that admit a spherical soliton solution also admit an $O(4)$ -invariant instanton solution. As for the solitons, f is determined by the field $\phi(0)$ at the origin. In Figure 4, the profile $\phi(\rho)$ is shown of the instanton with $f = -1/4$ boundary conditions.

The instanton also defines a Lorentzian solution which is obtained by analytical continuation across the equator of the three sphere. The fields on this slice of the instanton define time symmetric initial data for a Lorentzian solution. The Euclidean radial distance ρ simply becomes the radial distance r in the Lorentzian solution. The

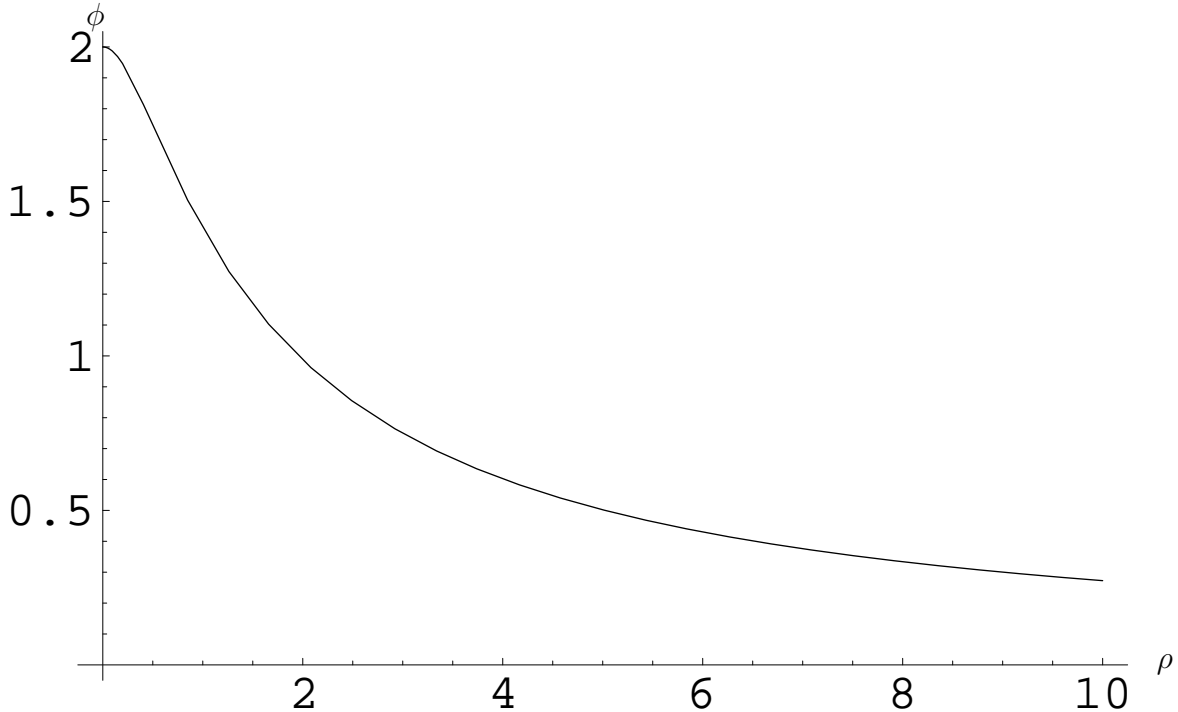


Figure 4: Instanton solution $\phi(\rho)$ in $D = 4$ supergravity with boundary conditions specified by $f = -1/4$.

total mass (3.5) of this initial data can be computed from the instanton geometry. Substituting (6.4) into (6.2) yields asymptotically

$$b^2(\rho) = \rho^2 + 1 + \frac{\alpha^2}{2} + \frac{4f\alpha^3}{3\rho} \quad (6.5)$$

This is of the form (3.4) required to have finite conserved charges. In fact, we see that $M_0 = -4f\alpha^3/3$ and hence (3.5) implies that the total mass is zero! This is consistent with the interpretation of the instanton as the solution AdS_4 decays into.

The quantum decay rate is determined in a semiclassical approximation by the Euclidean action of instanton. The action is given by

$$I = \int \left[-\frac{1}{2}R + \frac{1}{2}(\nabla\phi)^2 + V(\phi) \right] - \oint K + \frac{1}{6} \oint [(\nabla\phi)^2 - m^2\phi^2] \quad (6.6)$$

where the first surface term is the usual Gibbons-Hawking term, and the second is the surface term required so that the Hamiltonian constructed from this action (after subtracting the background) agrees with (3.5).

The relevant quantity for computing the rate of vacuum decay is the difference between the instanton action and the action for pure AdS: $\Delta I = I - I_{AdS}$. Subtracting I_{AdS} removes the leading divergences in I , but since ϕ goes to zero so slowly, there are two subleading divergences. If the coefficients of these terms were not exactly zero, ΔI would be infinite and there would be no probability for the vacuum to decay. We have shown [9] that both coefficients miraculously vanish. This involves nontrivial cancellations among the volume term and both surface terms in the action. Furthermore, the difference ΔI becomes small for large $|f|$ and goes to zero when $|f| \rightarrow \infty$.

7 Big Crunch Instability

We now turn to the evolution of the state AdS decays into. This is in light of the AdS/CFT correspondence potentially the most interesting manifestation of the supergravity instability. We will show that with generalized AdS invariant boundary conditions, there are supergravity solutions where regular initial data evolve to a big crunch singularity.

First let us return to the class of configurations $\phi_\lambda(r) = \phi_0(\lambda r)$, where ϕ_0 is the soliton profile discussed in section 5. The rescaled configurations $\phi_\lambda(r)$ specify initial data for time-dependent solutions in the same theory (i.e. with the same value of f). For large λ , the initial bubble is smaller than the soliton and probably collapses. On the other hand, by taking λ small one can arrange to have initially an arbitrarily large central region where ϕ is essentially constant and away from the maximum of the potential. It follows that the field must evolve to a spacelike singularity [25]. But the singularity that develops cannot be hidden behind an event horizon, because all spherically symmetric black holes have positive mass³ [8] while the total mass of the rescaled initial data is negative. Hence there is simply not enough mass to form a black hole, which encloses the singular region. Instead, one expects the singularity to continue to spread, cutting off all space⁴. Boundary conditions that admit a soliton solution, therefore, also admit solutions where finite mass configurations produce a

³We have demonstrated this for the $f = -1/4$ and $f = -1$ theories in section 4, but this is true in general.

⁴If V were bounded from below, it has been shown that the singularity cannot end or become timelike [26]. The same is likely to be true here.

big crunch.

A particular example of such a solution where the evolution is known explicitly is provided by the Euclidean instanton. The evolution of initial data defined by slicing the instanton across the three sphere is simply obtained by analytic continuation. This is discussed in detail in [27], but the basic idea is the following. The origin of the Euclidean instanton becomes the lightcone of the Lorentzian solution. Outside the lightcone, the solution is given by (6.1) with $d\Omega_3$ replaced by three dimensional de Sitter space. The scalar field ϕ remains bounded in this region. Inside the lightcone, the $SO(3, 1)$ symmetry ensures that the solution evolves like an open FRW universe,

$$ds^2 = -dt^2 + a^2(t)d\sigma_3 \quad (7.1)$$

where $d\sigma_3$ is the metric on the three dimensional unit hyperboloid. The field equations are

$$\frac{\ddot{a}}{a} = \frac{1}{3}[V(\phi) - \dot{\phi}^2] \quad (7.2)$$

$$\ddot{\phi} + \frac{3\dot{a}}{a}\dot{\phi} + V_{,\phi} = 0 \quad (7.3)$$

and the constraint equation is

$$\dot{a}^2 - \frac{a^2}{3} \left[\frac{1}{2}\dot{\phi}^2 + V(\phi) \right] = 1, \quad (7.4)$$

where $\dot{a} = \partial_t a$. On the light cone, $\phi = \phi(0)$ and $\dot{\phi} = 0$ (since $\phi_{,\rho} = 0$ at the origin in the instanton). Under evolution ϕ rolls down the negative potential, so the right hand side of (7.2) decreases. This ensures that $a(t)$ vanishes in finite time producing a big crunch singularity. For the purpose of understanding cosmological singularities in string theory, one can forget the origin of this solution as the analytic continuation of an instanton. We have simply found an explicit example of asymptotically AdS initial data which evolves to a big crunch.

We close this section with some comments on possible generalizations. In section 2 we have shown that one can generalize the boundary conditions while preserving the asymptotic AdS symmetries whenever one has a scalar field with $m_{BF}^2 \leq m^2 < m_{BF}^2 + 1$ which decouples from the rest of the matter. In particular, this includes $\mathcal{N} = 8$ supergravity in five dimensions, which involves a scalar field saturating the BF bound. In all cases one can construct similar solutions where a big crunch is produced

from smooth finite mass initial data [9]. The simplest solutions of this kind that we have presented here are constructed from time symmetric initial data, so they have a big bang singularity in the past as well. It would be interesting to construct solutions with only one singularity, in the future or the past.

8 Dual CFT description

Having shown that the bulk theory admits solutions which evolve to a big crunch, we now turn to the dual CFT description of this theory. The dual to string theory on $AdS_4 \times S^7$ can be obtained by starting with the field theory on a stack of N D2-branes. This is a $SU(N)$ gauge theory with seven adjoint scalars φ^i . One then takes the infrared (strongly coupled) limit to obtain the CFT. In the process, one obtains an $SO(8)$ symmetry. In the abelian case, $N = 1$, this can be understood by dualizing the three dimensional gauge field to obtain another scalar. But in general, it is not well understood.

This theory has dimension one operators $\mathcal{O}_T = Tr T_{ij} \varphi^i \varphi^j$ where T_{ij} is symmetric and traceless [28].⁵ One of these, \mathcal{O} , is dual to the bulk field we have been considering with the boundary conditions that $\phi = \alpha/r + O(r^{-3})$ for physical states. The field theory dual to the “standard” quantization, where physical states are described by modes with $\phi = \beta/r^2$ asymptotically, can be obtained by adding the double trace term $\frac{f}{2} \int \mathcal{O}^2$ to the action [29, 30]. This is a relevant perturbation and the infrared limit is another CFT in which \mathcal{O} has dimension two.

The AdS invariant boundary conditions we have considered here correspond instead to adding a triple trace term to the action

$$S = S_0 + \frac{f}{3} \int \mathcal{O}^3 \tag{8.1}$$

This follows from Witten’s treatment of multi-trace operators in AdS/CFT [29]. The extra term in (8.1) has dimension three, and hence is marginal and preserves conformal invariance, at least to leading order. One might wonder if this symmetry is exact, or whether the operator \mathcal{O}^3 has an anomalous dimension. The anomalous dimension can receive contributions proportional to $1/N$ or f . Since the large N limit corresponds

⁵Since there are only seven φ ’s and the theory has $SO(8)$ symmetry, there are other operators involving the gauge field which complete the $SO(8)$ representation.

to supergravity in the bulk with AdS invariant boundary conditions, and for every f there is a bulk solution corresponding to pure AdS, it seems likely that the theory remains conformally invariant for finite f (at least for large N).

More generally, Witten's procedure says that all AdS invariant boundary conditions discussed in Section 2 are dual to field theories that differ from each other by multi-trace deformations preserving conformal invariance. Thus one obtains a line of conformal fixed points in each case⁶.

We now turn to the dual field theory evolution of the big crunch solutions considered above. The Lorentzian solution obtained from the instanton takes the form (6.1) with $d\Omega_3$ replaced by three dimensional de Sitter space, dS_3 . So one might think that the natural dual would correspond to the CFT on dS_3 . This field theory certainly allows evolution for infinite time and is nonsingular. But this only corresponds to evolution for finite global time. We want to conformally rescale dS_3 to (part of) the cylinder $R \times S^2$. This is equivalent to a coordinate transformation in the bulk. The relation between the usual static coordinates (2.1) for AdS_4 and the $SO(3,1)$ invariant coordinates

$$ds^2 = \frac{d\rho^2}{1 + \rho^2} + \rho^2(-d\tau^2 + \cosh^2 \tau d\Omega_2) \quad (8.2)$$

is

$$\rho^2 = r^2 \cos^2 t - \sin^2 t \quad (8.3)$$

Since our bulk solution asymptotically has

$$\phi(\rho) = \frac{\alpha}{\rho} + \frac{f\alpha^2}{\rho^2} + O(\rho^{-3}) \quad (8.4)$$

This becomes

$$\phi(r) = \frac{\tilde{\alpha}}{r} + \frac{f\tilde{\alpha}^2}{r^2} + O(r^{-3}) \quad (8.5)$$

where $\tilde{\alpha} = \alpha/\cos t$. Notice that f is unchanged. Hence the evolution of the initial data defined by the instanton preserves the AdS invariant boundary conditions (3.3)-(3.4). The fact that $\tilde{\alpha}$ blows up as $t \rightarrow \pi/2$ is consistent with the fact that this is the time that the big crunch singularity hits the boundary. The coefficient of $1/r$

⁶In supergravity theories with more than one scalar with $m_{BF}^2 \leq m^2 < m_{BF}^2 + 1$ the different lines of conformal fixed points are parameterized by several dimensionless constants f_i .

is usually interpreted as the expectation value of \mathcal{O} in the CFT. Hence AdS/CFT predicts that in the large N approximation the latter diverges too.

A qualitative explanation for this is the following. The term we have added to the action is not positive definite. Since the energy associated with the asymptotic time translation in the bulk can be negative, the dual field should also admit negative energy states. This strongly suggests that the usual vacuum is unstable. It might decay via the (nongravitational) decay of the false vacuum. Perhaps a useful analogy is a scalar field theory with potential $V = m^2\varphi^2 - f\varphi^6$. The quadratic term is analogous to the coupling of φ to the curvature of S^2 , which is needed for conformal invariance. The second term is analogous to the second term in (8.1). Qualitatively this theory has the same behavior as the bulk. There are instantons which describe the semiclassical decay of the usual vacuum at $\varphi = 0$. For small f , the potential barrier is large, and the instanton action is large. So tunneling is suppressed. For large f , the barrier is small and tunneling is not suppressed. Classically one finds that after the tunneling the field rolls down the potential and becomes infinite in finite time. This means that in the semiclassical description of this analogous field theory, evolution ends in finite time. The fact that the field becomes infinite in this scalar field theory is analogous to the divergence of the expectation value of \mathcal{O} in the theory (8.1). Whether this means that evolution ends in the full quantum description of the CFT remains a fascinating open question, which we are currently investigating. If so, one could conclude that there is no bounce through the big crunch singularity in the bulk.

9 Conclusion

We have studied solutions of $\mathcal{N} = 8$, $D = 4$ supergravity where the $m^2 = -2$ scalar is the only excited matter field. Since its mass lies in the range $m_{BF}^2 \leq m^2 < m_{BF}^2 + 1$, there is a one-parameter family of boundary conditions on the scalar (and the metric components) that preserve the full AdS symmetry group. When the parameter vanishes, the dual CFT is the usual $2 + 1$ theory on a stack of M2-branes. Nonzero values of the parameter correspond to modifying this theory by a triple trace operator. We find that for all nonzero values, there exists a family of AdS black holes with scalar hair. Both the horizon size of the hairy black hole solutions and the value of the

scalar at the horizon are uniquely determined by a single conserved charge, namely the mass. Since Schwarzschild-AdS is a solution too for all boundary conditions, one has two very different black hole solutions for a given total mass. The uniqueness or no scalar hair theorem, therefore, does not hold in supergravity with generalized AdS invariant boundary conditions. It would be interesting to see how a microscopic string theory description distinguishes between both classes of black hole solutions.

Although the modified boundary conditions preserve the full set of asymptotic AdS symmetries and allow a finite conserved energy to be defined, we have shown this energy can be negative. Thus the AdS solution in supergravity with these boundary conditions is nonlinearly unstable. A particular manifestation of this is that there are asymptotically AdS solutions describing the evolution of regular finite mass initial data to a big crunch.

Our motivation to study supergravity in this regime is that there should be a dual CFT description of these bulk theories even if they are unstable. Most interestingly, the field theory should provide a complete quantum description of the big crunch singularity. If states in the CFT have a well defined evolution for all time, and one can reconstruct from it a semiclassical bulk metric at late time, then there must be a bounce through the singularity in the full string theory. However, if the CFT evolution ends after finite time, or a semiclassical metric cannot be constructed, then the bulk evolution would end at the big crunch.

As we mentioned, modifying the bulk boundary conditions corresponds to modifying the usual dual field theory on a stack of M2-branes by a triple trace operator. Since this term is not positive definite, it appears possible there will be certain CFT states which do not have well defined evolution for all time. We have seen this happening at the semiclassical level in the deformed $2 + 1$ theory for states that are dual to our big crunch supergravity solutions.

Moreover, we have good evidence that there are no bulk solutions that produce a big crunch in supergravity theories that are dual to stable CFT's. This is because solutions of this type would violate⁷ cosmic censorship [31], which is believed to hold in theories with a positive mass theorem, even in anti-de Sitter space [32, 33] (see also [34, 35]). Taken together, these results indicate that producing a big crunch

⁷There is no naked singularity, but one does not have well defined evolution for all time in the asymptotic region.

in AdS from smooth initial data *requires* boundary conditions that correspond to an unstable dual CFT. Therefore, the fact that the dual classical evolution ends in finite time and that the expectation value of the operator \mathcal{O} dual to the bulk scalar field diverges in the large N limit are, presumably, generic properties of a dual field theory description of a big crunch in the bulk, at least with AdS boundary conditions. Whether this means the big crunch is an endpoint of evolution in the full string theory remains a fascinating open question, which we are currently investigating. If it is an endpoint, that would raise the issue what determines the boundary conditions at cosmological singularities. Perhaps the AdS/CFT correspondence and the toy models of cosmologies we have constructed here could be useful to study this question further.

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