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The influence of Gribov copies on the gluon and ghost propagator [1]

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Abstract. The dependence of the gluon and ghost propagator in pure $SU(3)$ gauge theory on the choice of Gribov copies in Landau gauge is studied. Simulations were performed on several lattice sizes at $\beta = 5.8$, 6.0 and 6.2. In the infrared region the ghost propagator turns out to depend on the choice, while the impact on the gluon propagator is not resolvable. Also the eigenvalue distribution of the Faddeev-Popov operator is sensitive to Gribov copies.

Keywords: ghost and gluon propagator, Gribov problem, Faddeev-Popov operator eigenvalues
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Studying non-pertubative features of QCD such as confinement, there are two common approaches: lattice gauge theory and Dyson-Schwinger equations. From the latter approach there are promising results in recent years [1] about the infrared behavior of the gluon D and the ghost propagator G. Denoting by Z the dressing functions of the corresponding propagator, in Landau gauge they can be written as

$$D_{\mu\nu}(q^2) = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \frac{Z_{gl}(q^2)}{q^2} \quad \text{and} \quad G(q^2) = \frac{Z_{gh}(q^2)}{q^2} \, . \tag{1}$$

According to [1] in the low-momentum region the dressing functions are proposed to behave as $Z_{gl} \propto (q^2)^{2\kappa}$ and $Z_{gh} \propto (q^2)^{-\kappa}$ with a common value $\kappa \in (0.5, 1)$. The infrared suppression of the gluon propagator and the enhancement of the ghost propagator at low-momentum is in agreement with the Zwanziger-Gribov horizon condition [2–4] as well as with the Kugo-Ojima confinement criterion [5].

Zwanziger [2] has suggested that in the continuum the behavior of both propagators in Landau gauge results from restricting the gauge fields to the Gribov region Ω, where the Faddeev-Popov operator is non-negative. Generically, one gauge orbit has more than one intersection (Gribov copies) within the Gribov region Ω, but expectation values taken over this region are proposed to be equal to those over the fundamental modular region Λ. On a finite lattice, however, this is not expected [2]. In this contribution we assess the importance of the Gribov ambiguity on a finite lattice for the $SU(3)$ ghost and gluon propagators as well as for the lowest eigenvalues of the Faddeev-Popov operator.

To study these propagators in Landau gauge using lattice simulation, all thermalized gauge field configurations $\{U_{x,\mu}\}$ have to be fixed to this gauge. On the lattice the Landau gauge condition is implemented by searching for a gauge transformation

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[1] Talk presented by A. Sternbeck.

FIGURE 1. The upper parts show the dressing functions of the ghost Z_{gh} and gluon propagator Z_{gl} measured on best gauge copies as functions of the momentum q (scaled to physical units at $\beta = 5.8, 6.0$ and 6.2) using various lattice sizes. The lower parts show the ratio $\langle Z^{(\text{fc})} \rangle / \langle Z^{(\text{bc})} \rangle$ determined from the first (fc) and best (bc) gauge copies.

$^{g}U_{x,\mu} = g_x U_{x,\mu} g_{x+\hat{\mu}}^{\dagger}$, while keeping $U_{x,\mu}$ fixed, which maximizes the functional

$$F_U[g] \propto \sum_{x,\mu} \text{Re Tr} \, ^{g}U_{x,\mu} \, . \tag{2}$$

This functional has many different local maxima whose number increases as the lattice size increases or the inverse coupling β decreases. The different gauge copies corresponding to those maxima are called Gribov copies, due to its relation to the Gribov ambiguity in the continuum [4]. All Gribov copies $\{^{g}U\}$ belong to the gauge orbit created by U and satisfy the lattice Landau gauge condition $\partial_\mu{}^{g}A_{x,\mu} = 0$ with

$$^{g}A_{x+\hat{\mu}/2,\mu} = \frac{1}{2i} \left({}^{g}U_{x,\mu} - {}^{g}U_{x,\mu}^{\dagger} \right) \Big|_{\text{traceless}}. \tag{3}$$

In the literature it is widely accepted that the gluon propagator does not depend on the choice of Gribov copy, while an impact on the $SU(2)$ ghost propagator has been observed [6–8]. However, in a more recent investigation [9] an influence of Gribov copies on the $SU(3)$ gluon propagator has been demonstrated, too.

Here we report on a combined study of the $SU(3)$ gluon and ghost propagator in Landau gauge on the same gauge field configurations generated at $\beta = 5.8$, 6.0 and 6.2. For each configuration we have taken $N_{cp} = 30$, 40 and 10 random gauge copies for the lattice sizes 16^4, 24^4 and 32^4, respectively. A subsequent gauge-fixing was carried out using standard over-relaxation until $\max_x(\partial_\mu{}^{g}A_{x,\mu})^2 < 10^{-14}$ was reached.

On each first (fc) and each best (bc) gauge copy — that with largest functional value among N_{cp} copies — both the ghost and the gluon propagator have been measured. The results are shown in Fig. 1. The upper parts show the dressing functions of both propagators measured on the best gauge copies as a function of the momentum q scaled to energy units. In order to compare to other studies [9, 10] we have used $a^{-1} = 1.53$, 1.885 and 2.637 GeV for $\beta = 5.8$, 6.0 and 6.2, respectively. Looking at the lower parts
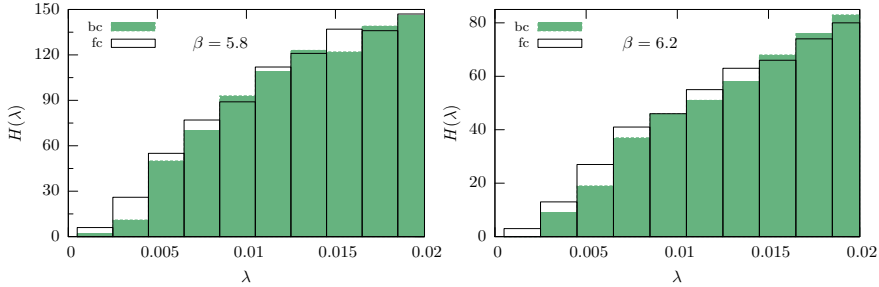
FIGURE 2. The frequency $H(\lambda)$ of the lowest eigenvalues λ of the Faddeev-Popov operator is shown. Full boxes represent the distribution obtained on the best gauge copies, while empty boxes represent those on the first gauge copies.

of this figure it becomes clear that the ghost propagator is affected by the choice of the Gribov copy the more the momentum is decreased. The impact on the gluon propagator stays inside the statistical error. For further details we refer to [11].

In trying to fit to the proposed power laws of the dressing function at lowest momenta, mentioned at the beginning, it turns out the lattice sizes used are to small to confirm such a behavior.

We also calculated the eigenvalue distribution of the Faddeev-Popov operator on the first and best gauge-fixed configurations as shown in Fig. 2. Looking at this figure it is obvious that the distribution $H(\lambda)$ of the lowest lying eigenvalues λ on the best gauge copies is slightly shifted towards larger eigenvalues compared to that determined on arbitrary first gauge copies. Thus better gauge-fixing seems to increase the gap between the lowest eigenvalues and the Gribov horizon.

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