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Axial irradiance and entropy of holographic optical elements under illumination with quasi-monochromatic light†

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Abstract. The chromatic sensitivity of holographic optical elements for use in optical interconnect systems is quantified using an entropy merit function. The influence of broad-band illumination on axial irradiance and intensity is studied showing that the best imaging point obtained with the minimum-entropy principle is stationary when a holographic optical element is reconstructed with a narrow special Gaussian shape. A new merit function that takes into account the axial irradiance and the entropy function is introduced. This new function is used to obtain the best imaging plane in the sense of minimal aberrations and maximum axial irradiance.

1. Introduction

Holographic lenses as the most commonly used diffractive optical elements are more and more widely employed in modern optical technology. One of the most important uses of holographic lenses is as an interconnect distribution network in optoelectronic computer systems. Holographic lenses have different advantages because they can combine several optical functions such as multiple focusing onto an image plane and beam splitting. Typical investigations of the imaging quality for classical optical systems or for a holographic optical element consists usually in analysing the image of a point object. This research usually includes both the estimation of the values of the coefficients characterizing the particular aberrations and an evaluation of the wave aberration or calculation of the aberration spot using the 'ray-tracing' method. These methods, however, do not allow one to calculate directly the light intensity distribution on the image plane. An evaluation of the lens imaging quality is possible by using the diffraction theory of aberrations. The diffraction image given by an optical system from a point is known as the point spread function (PSF) and this optical response function can be calculated from the design data. This is still of great use in studying the performance of an optical system with aberrations. Based on the light intensity distribution, the concept of entropy has been used to locate the position of the best image plane for holographic lenses [1, 2].

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When aberrations are small, it is possible to use the wave aberration variance to obtain the position of the best image plane in both conventional and holographic optical systems. However, when aberrations increase, this measurement is not sufficient and it does not provide the correct plane position for the best image. Standard deviation of the distribution of light and entropy are two alternative methods for evaluating imaging quality, but only entropy provides the best image plane when the value of these aberrations are increasing so much.

Aberrations are one of the main factors restricting the image quality of holographic optical elements (HOEs). This is due to the diffraction nature of holograms. In this communication we study the influence of wavelength on axial irradiance and entropy when a HOE is reconstructed with a narrow spectral Gaussian shape that takes into account the spectral distribution of the source. Let us consider a single holographic lens. Such a lens is used for imaging when a quasimonochromatic light source is used in the reconstruction step.

Typical research on the image quality of classical systems or of a hologram usually consists of the image of a point object. Imaging quality can be described by analysing the behaviour of the light intensity distribution in different image planes. The entropy merit function was introduced to analyse the intensity distribution on an image plane [1, 2] when optical elements work with monochromatic light. This present paper is devoted to researching one-point-object imaging under quasi-monochromatic illumination which enables us to determine the position of the best image plane and the influence of the broad-band $\Delta\lambda$ on this position, axial irradiance and entropy in holographic imaging.

2. Entropy calculation

In order to take into account the spectral distribution of the reconstruction source (which we suppose is a narrow spectral Gaussian), we introduce the weight factor

$$f(\lambda) = \frac{1}{\pi^{1/2}} \exp\left(\frac{-(\lambda - \lambda_r)^2}{\Delta\lambda^2}\right), \quad (1)$$

where λ_r is the recording wavelength. The intensity of the image in a plane normal to the chief ray at a distance z' from the centre of the HOE may be written as

$$I(x', y', z') = \frac{1}{B^2}\left|\int \frac{f(\lambda)}{\lambda}\,d\lambda \iint_S A(x, y) \exp\left[i\Delta(x, y, x', y'; z')\right] dx\,dy\right|^2, \quad (2)$$

where $A(x, y)$ takes into account any variation in the real amplitude over the wave-front (thus, for the case of uniform amplitude, we take $A(x, y) = 1$) and S represents the area of the exit pupil where the integration is done. In equation (2) the function $\Delta(x, y; x', y', z')$ is the wave-front phase corresponding to local coordinates (x', y') on the imaging plane (see appendix). B is the intensity at the Gaussian image point ($x' = y' = 0$) in the absence of aberrrations:

$$B = \int \frac{f(\lambda)}{\lambda}\,d\lambda \iint_S A(x, y)\,dx\,dy. \quad (3)$$

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The integral (2) does not generally have an analytical solution, and numerical methods have to be used to compute the diffraction patterns. In this study, the Hopkins–Yzuel [3] method is used to solve the spatial integral and the λ integral is obtained using the Simpson rule [4].

Let $I_k = I_{ij}(z'_k)$ be the matrix with dimensions $(P \times Q)$ whose components are the intensities at point (x'_i, y'_j) on the k'_k imaging plane. The intensity can be interpreted as a probability density function provided that it is adequately normalized [1]. Then, for each imaging plane, we can construct a probability matrix P_k whose components are

$$P_{ij} = I_{ij}(z'_k) \Big/ \sum_{p=1}^{P} \sum_{q=1}^{Q} I_{pq}(z'_k). \tag{4}$$

For an image plane situated at a z'_k position, we can construct the local entropy identifier for each P_k as

$$S(P_k) = -\sum_{p=1}^{P} \sum_{q=1}^{Q} P_{pq} \ln (P_{pq}). \tag{5}$$

$S(P_k)$ constitutes a criterion for obtaining the best imaging plane. For an optical system with a circular exit pupil, the aberration-free intensity distribution is the Airy pattern, and it is centred at the Gaussian image point. The quality of the image is limited then only by diffraction of the object radiation at the exit pupil of the system. Thus the value for the entropy obtained for this image plane, with a matrix probability P_g, will be the lowest value of the entropy for a realistic optical system.

In the presence of aberrations, the best imaging plane is different from the plane which contains the Gaussian image point. In all cases the entropy for a free aberration optical system verify that $S(P_g) < S(P_k)$, and so the plane with the lowest entropy is chosen as the best image plane because it is the most similar to the plane with matrix P_g. Therefore, we are interested in finding planes whose entropy $S(P_k)$, considered as a function of z'_k, is minimal.

Owing to normalization (4) the entropy function gives the same information for two different planes that have equal intensity distributions but different maximum intensity values. So as to take this fact into account we have introduced the axial irradiance averaged with the entropy function defined as

$$I_S(z'_k) = \frac{I_{00}(z'_k)}{S(P_k)}. \tag{6}$$

This function I_S gives a maximum value for the plane that has the best axial irradiance and entropy simultaneously.

When HOE is reconstructed with a narrow spectral Gaussian light, chromatic aberrations appear even though geometrical aberrations do not exist.

3. Numerical results

In order to illustrate the influence of 'Gaussian light' on quality image HOEs,

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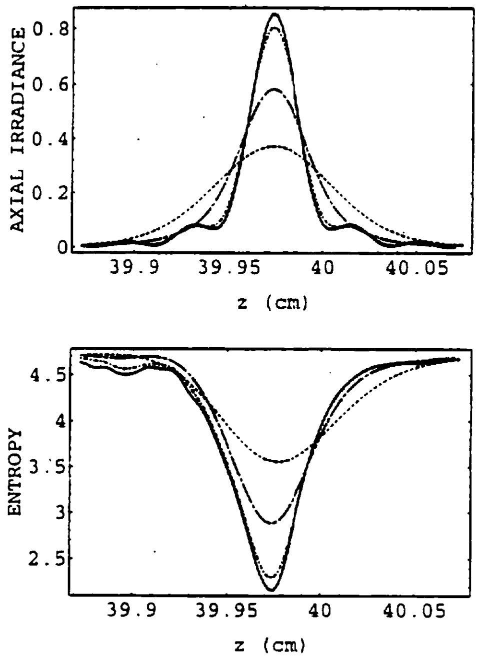


Figure 1. (*a*) Axial irradiance and (*b*) entropy for a holographic lens with a diameter of 5 cm for different values of $\Delta\lambda$: (———), $\Delta\lambda = 0$; (— · —), $\Delta\lambda = 0{\cdot}1$; (— - —), $\Delta\lambda = 0{\cdot}5$; (- - -), ■■.

we use the merit function entropy and I_S to obtain the best image plane for two in-line holographic lenses with spherical aberrations.

Using equation (2) we numerically evaluate entropy along the Z axis for two holographic lenses with diameters $D = 5$ cm and $D = 7$ cm recorded with a divergent spherical wave whose curvature radius is $R_r = -120$ cm and with a convergent spherical wave with the source point separated $R_0 = 60$ cm from the hologram centre and a recording wavelength $\lambda_r = 633$ nm. The two source points are situated on the axis perpendicular to the holographic plate. The hologram is reconstructed with a collimated beam ($R_c = \infty$) and the wavelength used in the recording step is given by the function of equation (1) with different parameters $\Delta\lambda$. The Gaussian point position (for $\lambda_c = \lambda_r$) is situated 40 cm from the HOE. Axial irradiance is also numerically evaluated from equation (2) with $x' = y' = 0$ and $A(x, y) = 1$. With this geometry, only spherical and chromatic aberrations are present. To obtain the entropy values for the lenses considered in the example, we chose the size of the imaging plane (where we make the integration) taking into account the values of the size of both the marginal and the paraxial planes using the larger of the two. Figure 1 shows the axial irradiance and entropy for different values of $\Delta\lambda$ for different image planes when the diameter is 5 cm. In this case the spherical aberration is negligible and there is only a maximum for axial irradiance for different values of $\Delta\lambda$, which coincides with the minimum-entropy position although as can be seen in figures 1 (*a*) and (*b*), when $\Delta\lambda$ increases, the value of maximum axial irradiance decreases as a result of the chromatic aberration.

Figure 2 shows the axial irradiance and entropy for different values of $\Delta\lambda$ for different image planes when the diameter is 7 cm. The spherical aberration in this

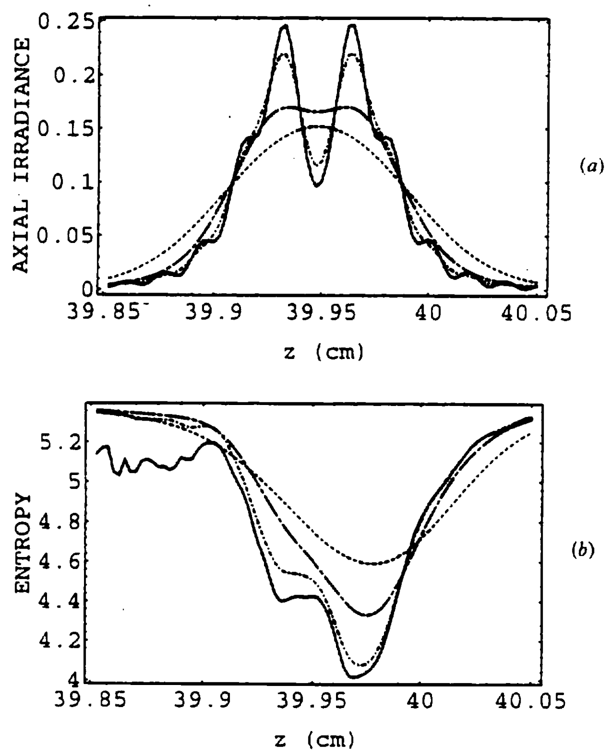


Figure 2. (*a*) Axial irradiance and (*b*) entropy for a holographic lens with a diameter of 7 cm for different values of $\Delta\lambda$: (———), $\Delta\lambda = 0$; (— · —), $\Delta\lambda = 0{\cdot}1$; (— - —), $\Delta\lambda = 0{\cdot}5$; (- - -), ▮▮.

example cannot be ignored, and there is more than one diffractional focus [4]. When $\Delta\lambda = 0$, the axial irradiance is symmetrical around the point where the spherical aberration compensates defocusing [5]. Then the entropy function shows a minimum for only one maximum. When $\Delta\lambda$ is not equal to zero, the symmetry of the axial irradiance is broken~~and, as can be seen~~ in figure 2 (*a*), the maximum with the lowest entropy is stationary for slight variations in wavelength. This maximum also shows the lowest entropy in this case. Figure 3 shows diffraction patterns for a lens diameter equal to 7 cm in the minimum-entropy plane for different values of $\Delta\lambda$.

Figure 4 shows the PSF I_s for different values of $\Delta\lambda$. As can be seen, this function shows the position of the best imaging plane in the sense of minimal aberrations and maximum axial irradiance.

The results presented above prove the usefulness of the method based on entropy in estimating holographic image quality, under not only monochromatic but also quasimonochromatic illumination.

Appendix

If the HOE is located in the (X, Y) plane, and the point source $Q(x_q, y_q, z_q)$ of a spherical wave is defined in terms of the parameters R_q, α_q and β_q, where ($q = r$, o, c, i) refers to the reference, object, reconstruction and trial image location

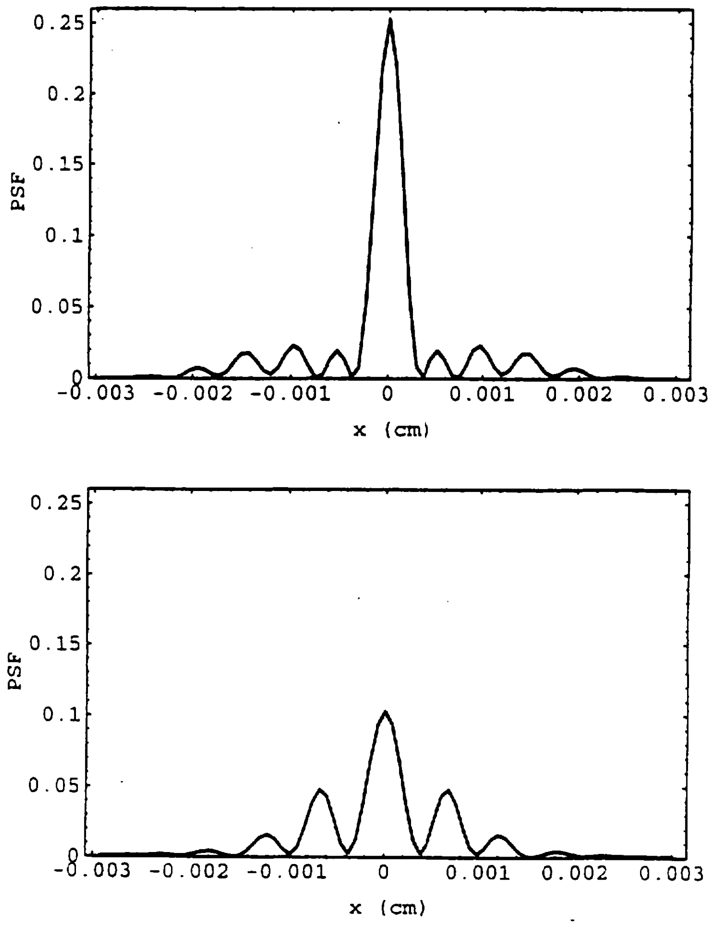


Figure 3. Diffraction patterns for a lens diameter of 7 cm for (*a*) $\Delta\lambda = 0$ (———), minimum-entropy plane; (– – –) first maximum irradiance), (*b*) $\Delta\lambda = 0{\cdot}1$ in the minimum-entropy plane and (*c*) $\Delta\lambda = 0{\cdot}5$ in the minimum-entropy plane.

respectively, the distance R_q (from the point source fo the HOE origin) can be calculated as follows:

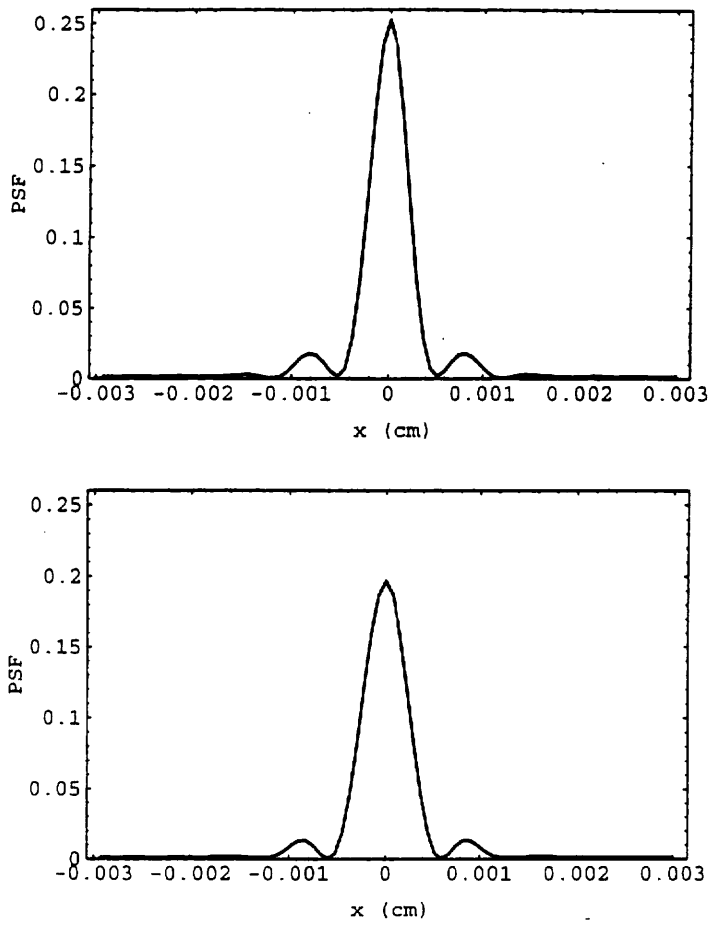
$$R_q = \sin(z_q)\,(x_q^2 + y_q^2 + z_q^2)^{1/2}. \tag{A 1}$$

α_q and β_q are the angles between R_q and the (Y, Z) and (X, Z) planes respectively:

$$\sin\alpha_q = \frac{x_q}{R_q}, \tag{A 2}$$

$$\sin\beta_q = \frac{y_q}{R_q}. \tag{A 3}$$

In equation (A 1) the sign of R_q is chosen so as to be equivalent to the sign of the



(c)

Figure 4. I_S for a lens diameter of 7 cm for (*a*) $\Delta\lambda = 0$, (*b*) $\Delta\lambda = 0{\cdot}1$ and (*c*) $\Delta\lambda = 0{\cdot}5$.

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corresponding coordinate of z_q. The algebraic distance r_q from the point source q to the point (x, y) on the HOE surface is given by

$$r_q = \sin(z_q)\,[(x - x_q)^2 + (y - y_q)^2 + z_q^2]^{1/2}. \tag{A 4}$$

We denote the recording and reconstruction wavelength as λ_r and λ_c respectively.

The aberration Δ of wave front is given by

$$\Delta = \phi_c - \phi_i \pm (\phi_o - \phi_r), \tag{A 5}$$

where the $\pm$ refers to the positive and negative first diffraction orders from the HOE, and ϕ_q, for q = r, o, c, i, is the phase of a spherical wave and its value in the plane of the hologram is

$$\phi_q(x, y) = \frac{2\pi}{\lambda_q}(r_q - R_q). \tag{A 6}$$

We introduce a function W such that

$$\Delta(x, y) = \frac{2\pi}{\lambda_c} W(x, y). \tag{A 7}$$

Using equations (A 5)–(A 7), $W(x, y)$ is given by

$$W = r_c - r_i \pm \mu(r_o - r_r) - [R_c - R_i \pm \mu(R_o - R_r)], \tag{A 8}$$

where μ denotes the wavelength shift λ_c/λ_r.

We characterize the Gaussian image point G by parameters R_g, α_g and β_g:

$$\frac{1}{R_g} = \frac{1}{r_c} \pm \mu\left(\frac{1}{R_o} - \frac{1}{R_r}\right), \tag{A 9}$$

$$\sin\alpha_g = \sin\alpha_c \pm \mu(\sin\alpha_o - \sin\alpha_r), \tag{A 10}$$

$$\sin\beta_g = \sin\beta_c \pm \mu(\sin\beta_o - \sin\beta_r). \tag{A 11}$$

Equation (9) defines the radius R_g of the 'reference' sphere centred on the Gaussian image point.

We introduce a local coordinate frame (X', Y', Z') fixed to the Gaussian image point G as the origin. The Z' axis is defined by the principal ray which runs from the centre of the hologram to the Gaussian image point G. For simplicity we choose the (X', Y') plane as the image plane, so that (x', y') are coordinates of a image point I in this plane while the coordinates of this image point in the (X, Y, Z) coordinate system are (x_i, y_i, z_i). The relationship between the coordinates of image point I in coordinate systems (X, Y, Z) and (X', Y', Z') can be expressed by means of the matricial relation

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$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{bmatrix} x_g \\ y_g \\ z_g \end{bmatrix} + \begin{bmatrix} n_g\Omega & -m_g l_g \Omega & l_g\Omega \\ 0 & \dfrac{1}{\Omega} & m_g \\ -l_g\Omega & -m_g n_g \Omega & n_g \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 0 \end{bmatrix} \tag{A 12}$$

where $W = (1 - m_g)^{-1/2}$ and (l_g, m_g, n_g) are the direction cosines of Gaussian image point G.

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