TI 2013-142/III
Tinbergen Institute Discussion Paper

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Interactions between eurozone and US booms and busts: A Bayesian panel Markov-switching VAR model*

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September 11, 2013

Abstract

Interactions between the eurozone and US booms and busts and among major eurozone economies are analyzed by introducing a panel Markov-switching VAR model well suitable for a multi-country cyclical analysis. The model accommodates changes in low and high data frequencies and endogenous time-varying transition matrices of the country-specific Markov chains. The transition matrix of each Markov chain depends on its own past history and on the history of the other chains, thus allowing for modelling of the interactions between cycles. An endogenous common eurozone cycle is derived by aggregating country-specific cycles. The model is estimated using a simulation based Bayesian approach in which an efficient multi-move strategy algorithm is defined to draw common time-varying Markov-switching chains. Our results show that the US and eurozone cycles are not fully synchronized over the 1991-2013 sample period, with evidence of more recessions in the eurozone, in particular during the 90's when the monetary union was planned. Larger synchronization occurs at beginning of the Great Financial Crisis. Shocks affect the US 1-quarter in advance of the eurozone, but these spread very rapidly among economies. There exist reinforcement effects in the recession probabilities and in the probabilities of exiting recessions for both eurozone and US cycles, and substantial differences in the phase transitions within the eurozone. An increase in the number of eurozone countries in recession increases the probability of the US to stay within recession, while the US recession indicator has a negative impact on the probability to stay in recession for eurozone countries. Moreover, turning point analysis shows that the cycles of Germany, France and Italy are closer to the US cycle than other countries. Belgium, Spain, and Germany, provide more timely information on the aggregate recession than Netherlands and France.

*We thank conference and seminar participants at the: 5th CSDA International Conference on Computational and Financial Econometrics 2012, 5th Italian Congress of Econometrics and Empirical Economics, 2013, European Seminar on Bayesian Econometrics 2012, 2nd CAMP Workshop 2012 on "Business cycles, monetary policy and nonlinear relationships", University of Venice Workshop on "Modern Tools in Macro-Econometrics", and Norges Bank. The views expressed in this paper are our own and do not necessarily reflect those of Norges Bank.

JEL codes: C11, C15, C53, E37.
Keywords: Bayesian Model, Panel VAR, Markov-switching, International Business Cycles, Interaction mechanisms.

1 Introduction

We investigate interactions between booms and busts of the eurozone and the US economies, where the eurozone is represented by its six largest countries, with a particular focus on similarities and cyclical co-movements, leads and lags, transmission mechanisms and turning points. Our modeling approach is based on a Bayesian panel Markov-switching model that describes cyclical behavior of the eurozone economy at a country specific level and at an aggregate level by comparing results with those of the US economy. Our modeling approach allows for transmission of shocks among different sectors, for example from the financial sector, modeled with the term spread, to the real sector, modeled with the industrial production index. In our empirical application, the transmission of shocks occurs among countries using endogenous aggregate eurozone and US business cycle factors. By comparing such factors one can discover which economy leads the other ones and by allowing each country to load on these factors, we can investigate differences among the countries business cycles.

Our analysis is intended to provide useful information on the sources of business cycle co-movements, i.e. the channels through which business cycle fluctuations are transmitted across countries that are part of an international economic system. In the literature there is no consensus on the international transmission of shocks. For example, Canova and Marrinan (1998) address the question, whether international business cycles originate from common shocks or from a common propagation mechanism. Monfort et al. (2003) aim at disentangling common shocks from spill-over effects. To this end, they estimate a Bayesian dynamic factor model for the G7 real output growth, featuring a global common factor and two area specific (North-American and Continental European) common factors, which, being modelled as a VAR process, are interdependent. They find empirical support for the presence of spill-over effects running from North-America to Continental Europe, but not vice versa.

Our approach and empirical application aim to contribute to this debate by describing the country specific cycles and their interactions and we thus also contribute to the literature on the analysis of the business cycle of large panel of countries. A complete description of this literature is beyond the scope of this paper but we refer to it. A first attempt to model an international business cycle is Gregory et al. (1997), who consider output, consumption and investment for G7 countries and estimate a dynamic factor model featuring a common cycle, a country-specific component and a series-specific one. The specification extends the Stock and Watson (1991) single index model and allow the authors to conclude that both the common and the country-specific factors capture a significant amount of the fluctuations. Kose et al. (2003) reach similar conclusions, using a larger data set on 60 countries and using a Bayesian dynamic factor model. Kose et al. (2008) find, however, that the relative importance of the common factor has been declining over time and that the cycle of emerging economies has become decoupled from that of industrialized countries. Lumsdaine and Prasad (2003) assess the relative importance of country specific versus common shocks, using industrial production growth for a set of 17 countries. They estimate the common component of international fluctuations by aggregation with time-varying weights.

In the present paper we contribute and generalize the literature in this direction by focusing on the business cycle of the eurozone, represented by the cycles of its six largest economies, and the US. We measure the cycle by using multivariate series and extract features and turning points of the country-specific business cycles in order to investigate the similarities of booms and busts between the eurozone cycle at an aggregated level and the US, and among the cycles of the eurozone countries.

Apart from presenting an empirical analysis, this paper also contributes to the econometric literature on heterogeneity in cross-country panel data models. In the context of these models, the more recent approaches have focused on two issues: the estimation of international cycles focusing on the nature of the co-movements using relatively large dimensional data sets and the introduction of country and time heterogeneity in multi-country vector autoregressive models. The first issue has been considered by Hallin and Liska (2008), Pesaran et al. (2004), and Dees et al. (2007) and the second by Canova and Ciccarelli (2004) and Canova and Ciccarelli (2009). Hallin and Liska (2008) extend the generalized dynamic factor model by Forni et al. (2000, 2001) to a panel of time series with a block structure, where the blocks are represented by countries. They show that the extension provides the means for the analysis of the interblock relationships, allowing the identification of strongly common factors, which are common to all the blocks (e.g. the international common factors), the strongly idiosyncratic factors, which are idiosyncratic for all blocks, and the weakly common/weakly idiosyncratic factors, that are common to at least one block, but idiosyncratic to at least another.

Multi-country VAR models provide a tool for examining the propagation of shocks across countries. Canova and Ciccarelli (2009) consider Bayesian multi-country VAR models with time varying parameters, lagged interdependencies and country specific effects. They avoid the curse of dimensionality on the number of parameters by a factor parameterization of the time varying VAR coefficients in terms of a number of continuous random effects that are linear in the number of countries and series. The authors propose a Monte Carlo Markov Chain sampling scheme for posterior approximation. Empirically, the transmission of shocks in the G7 countries is analyzed with a focus on four macroeconomic variables: real growth, inflation, employment growth and rent inflation; oil prices are considered as exogenous.

In this paper, we build on Canova and Ciccarelli (2009) and extend their panel VAR model in order to model asymmetry and turning points in the business cycles of different countries. Our paper is also extends Kaufmann (2010), where a panel of univariate Markov-switching (MS) regression models is considered, by constructing a multivariate panel MSVAR structure for the country-specific time series. We take the models of Hamilton

(1989) and Krolzig (2000) as points of departure and consider Markov-switching dynamics for low and high frequency components, that is, the means of the series and the covariance matrices of the country-specific equations (see also Billio et al. (2012a), Basturk et al. (2013) and Billio et al. (2012b)). We further build on Kaufmann (2011) and use an endogenous time-varying transition mechanism to model the transition matrix of the country-specific Markov-chains. In our model the transition of a country-specific chain may depend not only on its past history but (endogenously) also on the past history of the other chains of the panel. We develop an efficient algorithm to draw the common latent MS chain which uses as candidate the standard forwarding-filtering backward sampling (e.g., see Frühwirth-Schnatter (2006)). Moreover, in order to solve potential overfitting problems due to large number of parameters in the model, we follow the hierarchical prior specification strategy proposed by Canova and Ciccarelli (2009). Our paper is also related to Amisano and Tristani (2013), who propose a panel Markov-switching model to investigate transmission mechanisms in European sovereign bond markets. Our modeling and inference differ from theirs in that since we follow a hierarchical specification of the VAR and Markov-switching parameters. We make use of an endogenous transition that is based on alternative weighting rules with time-varying weights that account for differences in size and importance of the countries and our regime transition also accounts for the Harding and Pagan (2002) constraints in order to obtain well defined business cycle phases.

Our main empirical results can be summarized as follows. The US cycle leads the eurozone cycle, with evidence of more recessions in the eurozone, in particular during the 90's when the monetary union was planned. The larger synchronization is at beginning of the Great Financial Crisis: the shock affects the US 1-quarter in advance of the eurozone, but it spread among economies very rapidly. We find evidence of reinforcement effects in the recession probabilities for both the eurozone and the US cycles, and an asymmetric relationship between the eurozone and the US economic phase transitions: an increase in the number of eurozone countries in recession increases the probability of the US to stay into recession, while the US recession indicator has a negative impact on the probability to stay in recession for the eurozone countries. Evidence is similar in the probabilities of exiting the recession phase. Finally, as regards the turning point analysis, the cycles of Germany and, somewhat less, France and Italy are closer to the one of the US than other countries, but Belgium, Spain, and somewhat less Germany, seem to provide more timely information on the aggregate eurozone cycle.

The remainder of this paper is organized as follows. Section 2 presents the Bayesian panel MS-VAR model that has been used for the analysis. Section 3 discusses the prior choice and the Bayesian inference framework. Section 4 presents empirical evidence on such cross-country features as indicated before within the eurozone and also between the eurozone and the US. Finally, Section 5 concludes.

2 A panel Markov-switching VAR model

In this section, we introduce a general Panel Markov-switching VAR (PMS-VAR) model with endogenous transition and interaction. Moreover, we discuss the VAR parameter restrictions needed to avoid overfitting and define the endogenous time-varying transition of the unit specific Markov-chains. We will assume that the transitions are dependent on their own past history and on the history of other chains in order to capture the cycle interactions. Alternative interaction mechanisms such as weighting schemes and duration of states are presented.

2.1 Panel VAR specification

Let $\mathbf{y}_{it} \in \mathbb{R}^K$, $i = 1, \ldots, N$ and $t = 1, \ldots, T$, be a sequence of observations on K-dimensional vectors of economic variables. N is the number of units (countries) and T the number of time observations. A general specification of the PMS-VAR model reads

$$\mathbf{y}_{it} = \mathbf{a}_i(s_{i\,t}) + \sum_{j=1}^{N} \sum_{l=1}^{p} A_{ijl}(s_{i\,t}) \mathbf{y}_{jt-l} + D_i(s_{i\,t}) \mathbf{z}_t + \boldsymbol{\varepsilon}_{it}, \tag{1}$$

$i = 1, \ldots, N$, with $\boldsymbol{\varepsilon}_{it} \sim \mathcal{N}_K(\mathbf{0}, \Sigma_i(s_{i\,t}))$ and $\mathbf{z}_t \in \mathbb{R}^G$ a vector of variables, common to all units.

The $\{s_{i\,t}\}_t$ are unit-specific and independent M-states Markov-chain processes with values in $\{1, \ldots, M\}$ and time-varying transition probability $\mathbb{P}(s_{i\,t} = k | s_{i\,t-1} = j, V_t, \boldsymbol{\alpha}_i^{kj}) = p_{it,kj}$, $j \in \{1, \ldots, M\}$, where V_t is a set of G_v common endogenous covariates and $\boldsymbol{\alpha}_i^{kj}$ is a unit-specific vector of parameters.

The generality of this statistical model comes from the fact that the coefficients may vary both across units and across time. Moreover the interdependencies between units are allowed whenever $A_{ijl}(s_{i\,t}) \neq 0$ for $i \neq j$.

In order to define the parameter shifts more clearly and to simplify the exposition of the inference procedure we introduce the indicator variable $\xi_{ikt} = \mathbb{I}(s_{i\,t} = k)$, where

$$\mathbb{I}(s_{i\,t} = k) = \begin{cases} 1 & \text{if } s_{i\,t} = k \\ 0 & \text{otherwise} \end{cases}$$

for $k = 1, \ldots, M$, $i = 1, \ldots, N$, and $t = 1, \ldots, T$ and the vector of indicators $\boldsymbol{\xi}_{it} = (\xi_{i1t}, \ldots, \xi_{iMt})'$, which collects the information about the realizations of the i-th unit-specific Markov chain over the sample period. The indicators allow us to write the parameter shifts as

$$\mathbf{a}_i(s_{i\,t}) = \sum_{k=1}^{M} \mathbf{a}_{i,k} \xi_{ikt}, \quad A_{ijl}(s_{i\,t}) = \sum_{k=1}^{M} A_{ijl,k} \xi_{ikt}$$

$$D_i(s_{i\,t}) = \sum_{k=1}^{M} D_{i,k} \xi_{ikt}, \quad \Sigma_i(s_{i\,t}) = \sum_{k=1}^{M} \Sigma_{i,k} \xi_{ikt}.$$

where $\mathbf{a}_{i,k} = (a_{i1,k}, \dots, a_{iK,k})' \in \mathbb{R}^K$ are K dimensional column vectors representing the country- and regime-specific VAR intercept, $A_{ijl,k} \in \mathbb{R}^K \times \mathbb{R}^K$ K-dimensional matrices of unit- and regime-specific autoregressive coefficients, $D_{i,k} \in \mathbb{R}^K \times \mathbb{R}^G$ $K \times G$-dimensional matrices of regime-specific regression coefficients and $\Sigma_{i,k} \in \mathbb{R}^K \times \mathbb{R}^K$ K-dimensional unit- and regime-specific covariance matrices.

The large number of parameters makes our PMS-VAR a flexible model. Nevertheless, the overparameterization may lead to an overfitting problem, especially in applications to macroeconomics, where time series are characterized by a low number of observations, slowly changing means and time-varying variances (see Basturk et al. (2013)). These issues call for the use of a Bayesian approach to modeling and estimation. Since the Bayesian approach allows for including parameter restrictions, with different degrees of prior beliefs, through the specification of the prior (see, e.g., Litterman (1986), Sims and Zha (1998) for Bayesian VAR, Chib and Greenberg (1995) for Bayesian Seemingly Unrelated Regression and Canova and Ciccarelli (2009) for panel Bayesian VAR), the overfitting problems can be strongly reduced. These restrictions should be motivated by the specific application. In our application using monthly macroeconomic data on growth of the industrial production index and on the term spread we assume Markov-switching in means and variances to model the low and high frequency dynamics and constant autoregressive parameters, constant common variables and block structure for panel in order to avoid overfitting. More specifically, we assume the following restrictions to hold: $\mathbb{E}(\boldsymbol{\varepsilon}_{it}\boldsymbol{\varepsilon}_{jt}') = O_{K\times K}$ with $O_{n\times m}$ the $(n \times m)$-dimensional null matrix, and there are no interdependencies among the same variable across units, that is $A_{ijl,k} = A_{il,k}\mathbb{I}(i = j) + O_{K\times K}(1 - \mathbb{I}(i = j))$, when conditioning on the parameters. The dependence across units can be modelled through the hierarchical prior specification discussed later on in this paper. Furthermore, Clements and Krolzig (1998) found in an empirical study that most forecast errors are due to the constant terms in the prediction models. Apart from this, they also suggest considering MS models with regime-dependent volatility. In this paper, we follow Krolzig (2000), Billio et al. (2012a) and Basturk et al. (2013) and assume that both unit-specific intercepts, $\mathbf{a}_i(s_{it})$, and volatilities, $\Sigma_i(s_{it})$, are driven by the regime-switching variables $\{s_{it}\}_t$ and assume constant autoregressive coefficients $A_{il,k} = A_{il}$, $\forall k$ (see also Anas et al. (2008)). In the same spirit we assume that the coefficients of the common variables do not change over time, that is $D_{i,k} = D_i$, $\forall k, i$.

Let $\bar{\mathbf{w}}_{it}' = (1, \dots, \mathbf{y}_{it-1}', \cdots, \mathbf{y}_{it-p}', \mathbf{z}_t')$, $t = 1, \dots, T$ be the sequence of $(1 + Kp + G)$-dimensional column vectors of regressors for the PMS-VAR model, that includes the constant term, p lagged dependent variables, and the set of common variables. Moreover define the regressors, $W_{it} = \bar{\mathbf{w}}_{it}' \otimes I_K$, and coefficients, $A_{i,k} = (\mathbf{a}_{i,k}, A_{i1,k}, \cdots, A_{ip,k}, D_i)$, matrices of dimension $(K(1 + Kp + G) \times K)$ and $(K \times K(1 + Kp + G))$ respectively. By using the allocation variables $\boldsymbol{\xi}_{it}$ and the unit independence assumptions, given above, the PMS-VAR model can be rewritten as

$$\mathbf{y}_{it} = A_{i,1}W_{it}\xi_{i1t} + \ldots + A_{i,M}W_{it}\xi_{iMt} + \boldsymbol{\varepsilon}_{it}, \quad \boldsymbol{\varepsilon}_{it} \sim \mathcal{N}_K(0, \Sigma_i(s_{it})) \tag{2}$$

or in a more compact form as $\mathbf{y}_{it} = (\boldsymbol{\xi}_{it} \otimes W_{it})\text{vec}(B_i) + \boldsymbol{\varepsilon}_{it}$ where $B_i = (\text{vec}(A_{i,1}), \text{vec}(A_{i,2}), \cdots, \text{vec}(A_{i,M}))$, $\Sigma_i(s_{it}) = \Sigma_i(\boldsymbol{\xi}_{it} \otimes I_K)$ and $\Sigma_i = (\Sigma_{i1}, \cdots, \Sigma_{iM})$. For reason of convenience related to the derivation of the inference procedure, we also consider an alternative re-parameterization (e.g., see Frühwirth-Schnatter (2006)) based on a partition of the set of regressors $\bar{\mathbf{w}}_{it}$ into $M+1$ subsets $\bar{\mathbf{x}}_{i0t}$ and $\bar{\mathbf{x}}_{imt}$, $m = 1, \ldots, M$, that are a K_0-dimensional vector of regressors with regime-invariant coefficients and M vectors of K_m regime-specific regressors with regime-dependent coefficients. Moreover, in this paper we apply a model without exogenous regressors common to all countries.

Under the previous assumptions, one obtains $K_0 = 1$, $K_m = Kp$, $\forall m$ and $G = 0$ and the PMS-VAR model writes as

$$\mathbf{y}_{it} = X_{i0t}\boldsymbol{\gamma}_{i0} + \xi_{i1t}X_{i1t}\boldsymbol{\gamma}_{i1} + \ldots + \xi_{iMt}X_{iMt}\boldsymbol{\gamma}_{iM} + \boldsymbol{\varepsilon}_{it} \tag{3}$$

where $X_{i0t} = (\bar{\mathbf{x}}_{i0t} \otimes I_K)$, $X_{imt} = (\bar{\mathbf{x}}_{imt} \otimes I_K)$, with $\bar{\mathbf{x}}_{i0t} = (\mathbf{y}'_{it-1}, \ldots, \mathbf{y}'_{it-p})'$ and $\bar{\mathbf{x}}_{imt} = 1$, are the regime-invariant and the regime-specific regressors respectively and $\boldsymbol{\gamma}_{im} = (a_{i1,m}, \ldots, a_{iL,m})' \in \mathbb{R}^L$, $m = 0, \ldots, M$, $i = 1, \ldots, N$, are L-dimensional vectors with $L = KK_m$. The relationship between the new parameterization and the previous one is: $\boldsymbol{\gamma}_{i0} = (\text{vec}(A_{i1}), \cdots, \text{vec}(A_{ip}))$.

2.2 Transition mechanisms

Following Kaufmann (2011) we assume a centered parameterization of the transition probabilities

$$\mathbb{P}(s_{i\,t} = k | s_{i\,t-1} = j, V_t, \boldsymbol{\alpha}_i) = H(V_t, \boldsymbol{\alpha}_i^{kj}), \quad k, j = 1, \ldots, M \tag{4}$$

with

$$H(V_t, \boldsymbol{\alpha}_i^{kj}) = \frac{\exp\left((V_t - c_i)'\boldsymbol{\alpha}_{1i}^{kj} + \alpha_{0i}^{kj}\right)}{\sum_{l=1}^{M} \exp\left((V_t - c_i)'\boldsymbol{\alpha}_{1i}^{lj} + \alpha_{0i}^{lj}\right)}, \tag{5}$$

where $\boldsymbol{\alpha}_i^{lj} = (\alpha_{0i}^{lj}, \boldsymbol{\alpha}_{1i}^{lj\prime})'$ and c_i is a vector of threshold parameters that can be chosen to be the average of V_t. For identification purposes, we let M be the reference state and assume $\boldsymbol{\alpha}_{1i}^{kM} = \mathbf{0}$ and $\alpha_{0i}^{kM} = 0$. In order to simplify the exposition we denote with $\boldsymbol{\alpha}_i = \text{vec}\left((\boldsymbol{\alpha}_i^{11}, \ldots, \boldsymbol{\alpha}_i^{MM})\right)$ the collection of parameters of the sequence of transition matrices for the i-th unit.

As regards to the choice of the number M of regimes, we notice that for more recent data one needs an adequate business cycle model with more than two regimes (see also Clements and Krolzig (1998)) and a time-varying error variance. For example, Kim and Murray (2002) and Kim and Piger (2000) propose a three-regime (recession, high-growth, and normal-growth) MS model while Krolzig (2000) suggests the use of a model with regime-dependent volatility for the US GDP. In our paper we consider data on EMU industrial production, for a period of time including the 2009 recession and find that three regimes (high-recession, contraction or normal-growth, and high-growth) are necessary to capture some important features of the US and eurozone cycle.

As evidenced in Harding and Pagan (2011) and Harding (2010) the use of simple logit or probit models for modelling the transition probability of the phases of a business cycle may be inappropriate when the goal is to describe the feature of the business cycle. More specifically, minimum phase duration leads to impose restrictions on the parameters of the transition model. Extending the idea of Harding and Pagan (2011) to our panel MS-VAR model and focusing on the minimum recession duration, we specify the following transition

$$\mathbb{P}(s_{i\,t} = k | s_{i\,t-1} = j, s_{i\,t-2}, V_t, \boldsymbol{\alpha}_i) = \begin{cases} H_1(V_t, \boldsymbol{\alpha}_i^{kj}) & \text{if}\, s_{i\,t-2} \neq 1 \\ H_2(V_t, \boldsymbol{\alpha}_i^{kj}) & \text{if}\, s_{i\,t-2} = 1 \end{cases} \tag{6}$$

with

$$H_1(V_t, \boldsymbol{\alpha}_i^{kj}) = \mathbb{I}(k=1)\mathbb{I}(j=1) + (1 - \mathbb{I}(j=1))H(V_t, \boldsymbol{\alpha}_i^{kj})$$
$$H_2(V_t, \boldsymbol{\alpha}_i^{kj}) = ((1 - \mathbb{I}(k=1)) + \mathbb{I}(j=1)\mathbb{I}(k=1))\, H(V_t, \boldsymbol{\alpha}_i^{kj})$$

2.3 Interaction mechanisms

In this paper we explore several alternative specifications of the endogenous transition mechanism, which account for the possible interaction between the unit-specific cycles. In our models, we introduce dependence through the covariates V_t, $i = 1, \ldots, N$ that summarize the information contents of the N unit-specific Markov-chains s_{it}, $i = 1, \ldots, N$, used in the PMS-VAR model. In order to have a properly defined transition we assume that covariates at time t, which drive the state transition between $t-1$ and t, use the past values of the observables and are of a Markov-switching nature up to time $t-1$.

We define a general aggregation scheme as a map $\boldsymbol{\phi} : \Delta_{[0,1]^M}^N \mapsto \Delta_{[0,1]^M}$ where $\Delta_{[0,1]^M}$ is the standard M-dimensional simplex and define

$$\boldsymbol{\eta}_t = \boldsymbol{\phi}(s_{1t}, \ldots, s_{Nt}) \tag{7}$$

Some alternative aggregation schemes based on the current value of the chains are presented in the following.

2.3.1 Equal weights

The elements of $\boldsymbol{\eta}_t = (\eta_{1t}, \ldots, \eta_{Mt})'$ are defined by the following aggregation rule

$$\eta_{kt} = \frac{1}{N} \sum_{i=1}^{N} \mathbb{I}(s_{it-1} = k) \tag{8}$$

$k = 1, \ldots, M$, where we assigned equal weights to the unit-specific regime probabilities. When $k = 1$ we get a measure of the proportion of countries which are in a "‘strong recession"’ regime.

2.3.2 Unit-specific weights

The elements of $\boldsymbol{\eta}_t = (\eta_{1t}, \ldots, \eta_{Mt})'$ are defined by the weighted average

$$\eta_{kt} = \sum_{i=1}^{N} \omega_{it} \mathbb{I}(s_{it-1} = k) \tag{9}$$

where, in order to have a properly defined vector of probability, we assume $(\omega_{1t}, \ldots, \omega_{Nt})' \in \Delta_{[0,1]^N}$, for all t. The unit-specific weight ω_{it}, can be driven, for example, by the relative IPI growth rate or size of the i-th unit at time $t-1$, with respect to the IPI growth rate or economic size of the other units. Distance measures based on other features of the units can be used to aggregate the hidden states. We shall notice that the aggregation weights can be included in the inference procedure leading to a more complex latent variable model both in terms of modelling and computation. One can use alternatively completely unobserved combination weights (e.g., see the modelling strategies in Billio et al. (2013)) or weights which are partially observed and driven by one or some of the variables mentioned above. Given the high number of latent variables in our model, the latter weight specification strategy should be preferred in order to avoid overfitting problems and to take advantage of all the information available. While this is a topic of substantial interest, it is beyond the scope of the present paper and we left it as a topic for future research.

2.3.3 Average duration

Also, we consider aggregation schemes which account for the duration of the states. For example:

$$\eta_{kt} = \frac{1}{N\tau} \sum_{i=1}^{N} \sum_{i=1}^{\tau} \mathbb{I}(s_{it-\tau} = k) \tag{10}$$

$k = 1, \ldots, M$, where we assigned equal weights to the unit-specific regime probabilities. When $k = 1$ we get a measure of the proportion of countries which are in a "‘strong recession"’ regime.

Average transitions

Schemes which account for the number of transitions between time $t-1$ and time t, from the other regimes to the specific regime k, are defined as:

$$\eta_{kt} = \frac{1}{N} \sum_{i=1}^{N} \sum_{l=1}^{M} \mathbb{I}(s_{it-1} = k)\mathbb{I}(s_{lt-1} = k) \tag{11}$$

$k = 1, \ldots, M$,

3 Bayesian Inference

The PMS-VAR model is estimated with a simulation based Bayesian procedure. In order to solve potential overfitting problems due to the large number of parameters, we use hierarchical prior distributions. Moreover, we develop an efficient algorithm, to draw the latent MS chain, which uses Metropolis candidate generated with the standard forwarding-filtering backward sampling (e.g., see Frühwirth-Schnatter (2006)).

3.1 Hierarchical prior

We follow a hierarchical prior specification strategy (see, e.g. Canova and Ciccarelli (2009)), which allows us to model dependence between the cross-sectional units through common latent variables and to avoid the potential overfitting problem. For the parameters of the VAR regression we assume

$$\boldsymbol{\gamma}_{i0} \sim \mathcal{N}_{K_0}(\boldsymbol{\lambda}_0, \underline{\Sigma}_{i0}) \tag{12}$$

$$\boldsymbol{\lambda}_0 \sim \mathcal{N}_{K_0}(\underline{\boldsymbol{\lambda}}_0, \underline{\Sigma}_0) \tag{13}$$

$$\boldsymbol{\gamma}_{im} \sim \mathcal{N}_{K_m}(\boldsymbol{\lambda}_m, \underline{\Sigma}_{im}), \quad m = 1, \ldots, M \tag{14}$$

$$\boldsymbol{\lambda}_m \sim \mathcal{N}_{K_m}(\underline{\boldsymbol{\lambda}}_m, \underline{\Sigma}_m), \quad m = 1, \ldots, M \tag{15}$$

$i = 1, \ldots, N$. We assume conditional independence across units, that is: $\mathbb{C}ov(\boldsymbol{\gamma}_{i0}, \boldsymbol{\gamma}_{j0} | \underline{\boldsymbol{\lambda}}_0) = O_{K_m \times K_m}$ and $\mathbb{C}ov(\boldsymbol{\gamma}_{im}, \boldsymbol{\gamma}_{jm} | \underline{\boldsymbol{\lambda}}_m) = 0$, for $i \neq j$. For the inverse covariance matrix Σ_{im}^{-1} we assume independent Wishart priors

$$\Sigma_{im}^{-1} \sim \mathcal{W}_K(\underline{\nu}_{im}/2, \Upsilon_m/2), \quad i = 1, \ldots, N \tag{16}$$

$$\Upsilon_m^{-1} \sim \mathcal{W}_K(\underline{\nu}_m/2, \underline{\Upsilon}_m/2), \tag{17}$$

$m = 1, \ldots, M$, that allow us to maintain the assumption of regime-specific degrees of freedom $\underline{\nu}_{im}$ and precision Υ_m parameters. We assume $\mathbb{C}ov(\Sigma_{im}^{-1}, \Sigma_{im}^{-1} | \underline{\Upsilon}_m^{-1}) = O_{K_m^2 \times K_m^2}$.

Note, that the hierarchical prior specification allow us to introduce dependence between units. Moreover, with the above given specification of the coefficients $\boldsymbol{\gamma}_{im}$ it is possible to have a regime-specific dependence structure.

When using Markov-switching processes, one should deal with the identification issue associated to the label switching problem. See for example Celeux (1998) and Frühwirth-Schnatter (2001) for a discussion on the effects that label switching and the lack of identification have on the results of a MCMC based Bayesian inference. In the literature, different routes have been proposed for dealing with the label switching (see Frühwirth-Schnatter (2006) for a review). One of the most efficient approach is the permutation sampler (see Frühwirth-Schnatter (2001)), which can be applied under the assumption of exchangeability of the posterior density. This assumption is satisfied when one assumes symmetric priors on the transition probabilities of the switching process. As an alternative one may impose identification constraints on the parameters. This practice is followed to a large extent in macroeconomics and is related to the natural interpretation of the different regimes as the different phases (e.g. recession and expansion) of the business cycle. We follow this latter approach and include the constraints

$$\gamma_{ij1} < \gamma_{ij2} < \ldots < \gamma_{ijM}$$

$j = 1, \ldots, K$ and $i = 1, \ldots, N$, that corresponds to a total ordering, across the different regimes, of the constant terms in the equations of the system.

Modeling dependence between the chains is another issues to deal with. We propose a flexible model, with regimes switching processes that are able to capture the different phases of the unit-specific business cycles. This flexibility has as a drawback the use of a large number of parameters that may lead to an overfitting problem. To avoid this, we suggest to use a hierarchical prior specification for the transition matrices. In particular, for the j-th row $\mathbf{p}_{it,j\cdot}$, $j = 1, \ldots, M$, of the i-th unit transition matrix, at time t, we assume

$$\boldsymbol{\alpha}_i^{jk} \sim \mathcal{N}_{G_v+1}(\boldsymbol{\psi}, \Upsilon_i) \quad i = 1, \ldots, N, \, k = 1, \ldots, M-1 \tag{18}$$

$$\boldsymbol{\psi} \sim \mathcal{N}_{G_v+1}(\underline{\boldsymbol{\psi}}, \underline{\Upsilon}) \tag{19}$$

3.2 Posterior simulation

We extend the Gibbs sampler of Krolzig (1997) and Frühwirth-Schnatter (2006) to our PMS-VAR model with the prior densities given in the previous sections. Under the hierarchical prior setting the full conditional posterior distributions of the equation-specific blocks of parameters are conditionally independent. Thus the Gibbs sampler can be iterated over different blocks of unit-specific parameters avoiding the computational difficulties associated with the inversions of large covariance matrices (see Canova and Ciccarelli (2009)). We derive the full conditional densities of the parameters in Eq. 2 and propose a further blocking step. Following the Markov-switching regression framework in Frühwirth-Schnatter (2006), we separate the unit-specific parameters into two different blocks: the regime-independent parameters and the regime-specific parameters.

We let $\mathbf{y}_i = \text{vec}\,((\mathbf{y}_{i1}, \ldots, \mathbf{y}_{iT}))$ be the set of observations collected over time, $\mathbf{y} = \text{vec}\,((\mathbf{y}_1, \ldots, \mathbf{y}_N)')$ the set of observations collected over time and panel units and $\boldsymbol{\xi} = \text{vec}\,((\Xi_1, \ldots, \Xi_N))$ the set of allocation variables, with $\Xi_i = (\boldsymbol{\xi}_{i1}, \ldots, \boldsymbol{\xi}_{iT})$. The complete data likelihood function associated to the PMS-VAR model writes as

$$p(\mathbf{y}|\boldsymbol{\xi}, \boldsymbol{\gamma}, \Sigma, \boldsymbol{\alpha}) = (2\pi)^{-\frac{TKN}{2}} \prod_{t=1}^{T} \prod_{i=1}^{N} |\Sigma_i(s_{it})|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\mathbf{u}_t'\Sigma_t^{-1}\mathbf{u}_t\right\} \prod_{i=1}^{N} \prod_{k,l=1}^{M} p_{it,kl}^{\xi_{ikt}\xi_{ilt-1}} \tag{20}$$

with $\mathbf{u}_t = \mathbf{y}_t - ((1, \boldsymbol{\xi}_{1t}', \ldots, \boldsymbol{\xi}_{Nt}') \otimes I_{NK}) X_t \boldsymbol{\gamma}$, $\boldsymbol{\gamma} = \text{vec}\,((\boldsymbol{\gamma}_1, \ldots, \boldsymbol{\gamma}_N))$ where $\boldsymbol{\gamma}_i = \text{vec}\,((\boldsymbol{\gamma}_{i0}, \boldsymbol{\gamma}_{i1}, \ldots, \boldsymbol{\gamma}_{iM}))$, $\Sigma = (\Sigma_1, \ldots, \Sigma_N)$ and $\boldsymbol{\alpha} = \text{vec}\,((\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_N))$. Under the conditional independence assumption, the likelihood factorises as

$$\prod_{i=1}^{N} p(\mathbf{y}_i|\Xi, \boldsymbol{\gamma}_i, \Sigma_i, \boldsymbol{\alpha}_i) \tag{21}$$

where

$$p(\mathbf{y}_i|\Xi, \boldsymbol{\gamma}_i, \Sigma_i, \boldsymbol{\alpha}_i) = (2\pi)^{-\frac{TK}{2}} \prod_{t=1}^{T} |\Sigma_{it}|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\mathbf{u}_{it}'\Sigma_{it}^{-1}\mathbf{u}_{it}\right\} \prod_{k,l=1}^{M} p_{it,kl}^{\xi_{ikt}\xi_{lkt-1}} \tag{22}$$

with $\mathbf{u}_{it} = \mathbf{y}_{it} - ((1, \boldsymbol{\xi}_{it}') \otimes I_K)X_{it}\boldsymbol{\gamma}_i$ and

$$X_{it} = \begin{pmatrix} X_{i0t} & X_{i1t} & & \mathbf{0} \\ \vdots & & \ddots & \\ X_{i0t} & \mathbf{0} & & X_{iMt} \end{pmatrix}$$

In order to describe the structure of the Gibbs sampler we define some more notation. Let us introduce the auxiliary variables $\mathbf{y}_{i0t} = \mathbf{y}_{it} - \xi_{i1t}X_{i1t}\boldsymbol{\gamma}_{i1} + \ldots + \xi_{iMt}X_{iMt}\boldsymbol{\gamma}_{iM}$ and the notation $\boldsymbol{\gamma}_{i(-m)} = (\boldsymbol{\gamma}_{i1}, \ldots, \boldsymbol{\gamma}_{im-1}, \boldsymbol{\gamma}_{im+1}, \ldots, \boldsymbol{\gamma}_{iM})$ and $\Sigma_{i(-m)} = (\Sigma_{i1}, \ldots, \Sigma_{im-1}, \Sigma_{im+1}, \ldots, \Sigma_{iM})$. The Gibbs sampler is in six blocks. In the blocks from one to three, the Gibbs iterates over the unit index, $i = 1, \ldots, N$, and simulates the unit-specific parameters

(i) $\boldsymbol{\gamma}_{i0}$ from $f(\boldsymbol{\gamma}_{i0}|\mathbf{y}_i, \Xi_i, \boldsymbol{\gamma}_i, \Sigma_i, \boldsymbol{\lambda}_0)$;

(ii) for $m = 1, \ldots, M$

- (ii.a) $\boldsymbol{\gamma}_{im}$ from $f(\boldsymbol{\gamma}_{im}|\mathbf{y}_i, \Xi_i, \boldsymbol{\gamma}_{i0}, \boldsymbol{\gamma}_{i(-m)}, \Sigma, \boldsymbol{\lambda}_m)$, for $m = 1, \ldots, M$;
- (ii.b) Σ_{im}^{-1} from $f(\Sigma_{im}^{-1}|\mathbf{y}_i, \Xi_i, \boldsymbol{\gamma}_{i0}, \boldsymbol{\gamma}_i, \Sigma_{i(-m)}, \Upsilon_m)$;

(iii) $\boldsymbol{\alpha}_i^{k1}, \ldots, \boldsymbol{\alpha}_i^{kM-1}$ from $f(\boldsymbol{\alpha}_i^{k1}, \ldots, \boldsymbol{\alpha}_i^{kM-1}|\mathbf{y}_i, \Xi, \boldsymbol{\gamma}_{i0}, \boldsymbol{\gamma}_i)$.

In the blocks from four to six, the Gibbs sampler simulates from the full conditionals of the common part of the hierarchical structure and jointly from the full conditional of all the Markov-switching processes, i.e.

(iv) For $m = 1, \ldots, M$:

- (iv.a) $\boldsymbol{\lambda}_m$ from $f(\boldsymbol{\lambda}_m|\boldsymbol{\gamma}, \Sigma)$;
- (iv.b) Υ_m^{-1} from $f(\Upsilon_m^{-1}|\boldsymbol{\gamma}, \Sigma)$;

(vi) Ξ from $p(\Xi|\mathbf{y}_{1:T}, \boldsymbol{\gamma}, \Sigma\boldsymbol{\alpha})$

All the full conditionals can be deduced from the joint density, that is proportional to the product of the prior densities, given in Section 3.1, and the completed likelihood given in Eq. 20. Further details on the MCMC algorithm proposed here are given in Appendix A.

We note that, for sampling the hidden states we propose a multi-move strategy. In Krolzig (1997) a multi-move Gibbs sampler (see Carter and Kohn (1994) and Shephard (1994)) is presented for Markov-switching vector autoregressive models as an alternative to the single-move Gibbs sampler given, for example, in Albert and Chib (1993). The multi-move procedure, also known as forward-filtering backward sampling (FFBS) algorithm, is particularly useful in highly parametrized model, because it can improve the mixing of the MCMC chain over a large parameter space, thus leading to a more efficient posterior approximation. Unfortunately, the FFBS does not apply easily to our model due to the presence of the chain interaction mechanism. In fact, the FFBS should be iterated jointly for all the Markov-switching processes of the panel implying large matrix operations and, therefore, a high computational cost. Alternatively, one could apply FFBS to a unit-specific chain, conditioning on the other chains. In our simulation experiments we found that this strategy may lead to a poor mixing of the MCMC chain. Thus, we propose a multi-move strategy, which makes use of the FFBS algorithm to generate proposals for each unit-specific chain within a global Metropolis-Hastings (M.-H.) step. The proposed procedure extends in two directions the Billio et al. (1999) global M.-H. for switching ARMA. First, we use a multi-move proposal instead of a single-move proposal within the global M.-H. step. Secondly, we extend to a multiple-chain multivariate model the global M.-H. given in Billio et al. (1999) for a single-chain univariate model. Our global M.-H. with multi-move proposal has two main advantages over the single-move proposal M.-H. First, the joint generation of the hidden state proposal improves the mixing of the MCMC chain. Secondly, the FFBS proposal leads to simple calculation of the M.-H. acceptance probability. Further details on the FFBS proposal are given in Appendix A.

4 Eurozone and US booms and busts

4.1 Data description

The main empirical focus of this paper is on whether and where the eurozone and US economies differ in periods of booms and busts. We consider the eurozone at the country level since the academic and economic debate is still open on whether European countries have synchronized and whether regional shocks still play a dominant role. Our analysis wants to contribute to the debate and provides evidence on this.

In our PMS-VAR we consider the US and the six largest economies in the eurozone, given as Belgium, France, Germany, Italy, Netherlands, and Spain. For each country, we consider two dependent variables: the Industrial Production Index (IPI), labelled as $y_{i1,t}$ and the term spread (TS), the short term (3 months) and long term (10 years) interest rate differentials, given as $y_{i2,t}$. The IPI is an economic indicator that measures changes in output for the manufacturing, mining, and utilities business sectors. Although these sectors contribute only to a small fraction of the GDP, and several countries have partially shifted from being production oriented to being service and consumer oriented, which reduces even further the contribution of these sectors, they are rather sensitive to variations in interest rates and consumer demand. This makes the IPI an important variable for forecasting the future economic performance of an economic system. The term spread has often been advocated as predictor of recession periods, see e.g. Harvey (1991). It can also be seen as a source of financial shocks, and therefore captures the transmission mechanism from the financial sector to the real one. Claessens et al. (2008) link shock transmissions from the financial sector to the real sector using a larger set of variables. Estrella and Hardouvelis (1991) use real GNP growth in US to examine the predictive ability of the term spread. The results show that term spread has significant predictive power on output growth, consumption, and investment. Plosser and Rouwenhorst (1994) find the term structure has significant predictive for economic growth in three industrial countries. However, there is no conclusive finding that the yield spread consistently contains information in explaining future economic activity. For example, Plosser and Rouwenhorst (1994) find the evidence that yield spreads contain useful information to forecast real economic activities in US, Canada and Germany, but not in France and UK. Harvey (1991) and Kim and Limpaphayom (1997) examine G7 economies and conclude that the yield spread does not consistently contain information about future economic activity. Hamilton and Kim (2002) address the theoretical model toward the nature of the term spread. They nicely present that the spread's forecasting contribution is attributed to two effects: an expectation effect that shows a sign of the public's expectation on the future economic activities and the term premium effect that represents the risk of investments in alternative assets. They find that both factors are relevant for predicting real GDP growth but respective contributions differ. The contributions are similar at short horizons but the effect of expected future short rates is much more important than the term premium for predicting GDP more than two years ahead.

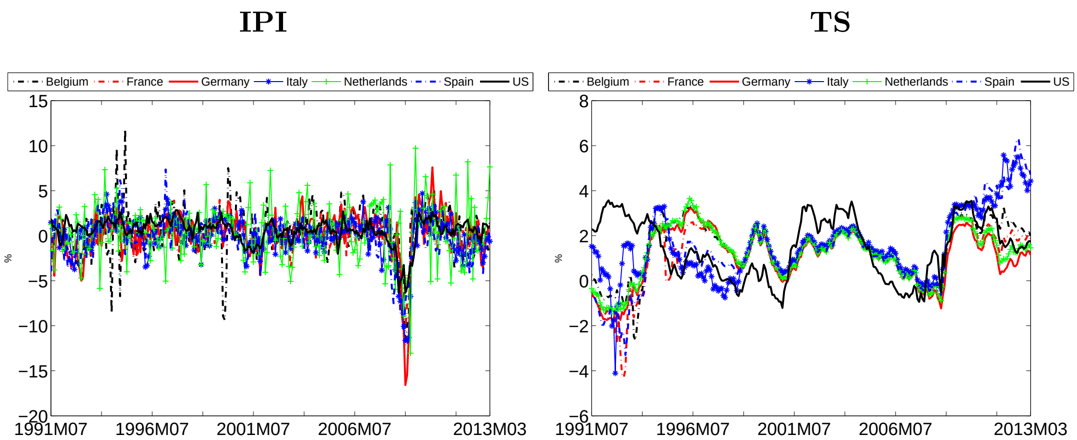

Figure 1: Country-specific endogenous variables: industrial production growth rate (IPI) and term structure (TS).

All data, from Eurostat and OECD databases, are sampled at a monthly frequency, from July 1991 to March 2013, and are seasonally and working day adjusted. Data is plotted in Figure 1

To avoid issues with possibly non-stationary series, we take the IPI in terms of log-changes. We set the number of regimes $M = 3$ for all countries in the panel, see e.g. Ferrara (2003), and impose the following restrictions on the intercept of the IPI growth rate $a_{i1,1} < 0$ and $a_{i1,1} < a_{i1,2} < a_{i1,3}$, $i = 1, ..., N$, in order to identify the regimes (see Section 3.1). We label regime 1 as recession; regime 2 as recovery or moderate expansion; and regime 3 as strong expansion.

One crucial aspect in studying interactions between the eurozone and the US and among Euro countries relates to the composition of the variable V_t. To investigate the interconnectedness between the eurozone and the US, we specify the set of common endogenous covariates V_t equal to the vector η_{1t} and $\mathbb{I}(s_{US,t-1} = 1)$. The indicator η_{1t} is a weighted average of the number of eurozone countries in the recession regime (regime 1) at time $t-1$; $\mathbb{I}(s_{US,t-1} = 1)$ takes value 1 when the US economy is in recession and 0 otherwise. Such assumptions allow us to have an endogenous interconnection mechanism between the two economies. Note that the information of the eurozone countries was discussed in Section 2.3. More precisely, we focus on the weighted interaction indicator given in equation (9) and use economic size unit-specific weights . We follow the Eurostat framework to eurozone variables aggregation and derive weights on relative value added, see Eurostat Regulation EC No 1165/98. Value added data are downloaded from the UNData database and Fig. 7 displays the weights.[1].

4.2 Country-specific features

We apply to our dataset the Gibbs sampler, given in Section 3 and obtain the posterior densities of the PMS-VAR model parameters. The posterior densities are then approximated through a kernel density estimator applied to a sample of 4,000 random draws from the posterior. In order to generate 4,000 i.i.d. samples from the posterior, we run the Gibbs sampler, for 50,000 iterations, discard the first 10,000 draws to avoid dependence from the initial condition, and finally apply a thinning procedure with a factor of 10 samples, to reduce the dependence between consecutive Markov-chain draws. See Appendix B for further details on choice of the number of iterations and of the burn in samples.

4.2.1 Unit- and variable-specific Markov-switching intercepts

Figures 2 and 3 show the approximated posterior densities of the parameters $\gamma_{im} = (a_{i1,m}, a_{i2,m})'$, $(\sigma_{i\,1,m})$ and $(\sigma_{i\,2,m})$, $m = 1, \ldots, M$ and $i = 1, \ldots, N$, that represent the value of the unit- and variable-specific time-varying intercepts and volatilities of the PMS-VAR model. A comparison of such posteriors provides useful information on whether and how individual countries differ over booms and busts. We recall that the regime identification follows from the parameter constraints $a_{i1,1} < 0$ and $a_{i1,1} < a_{i1,2} < a_{i1,3}$, on the intercept of the IPI growth rate equation.

The posterior densities for the IPI growth intercept, $a_{i1,m}$, $m = 1, 2, 3$ (see left column in Fig. 2), are not overlapping in most of the countries. This suggests that the regimes are well identified on the IPI growth data. Moreover, for all panel units the support of the

[1]We have also investigated different choices of weights, such as equal weights or IPI growth unit-specific weights. The sensitivity of the results is available upon request.

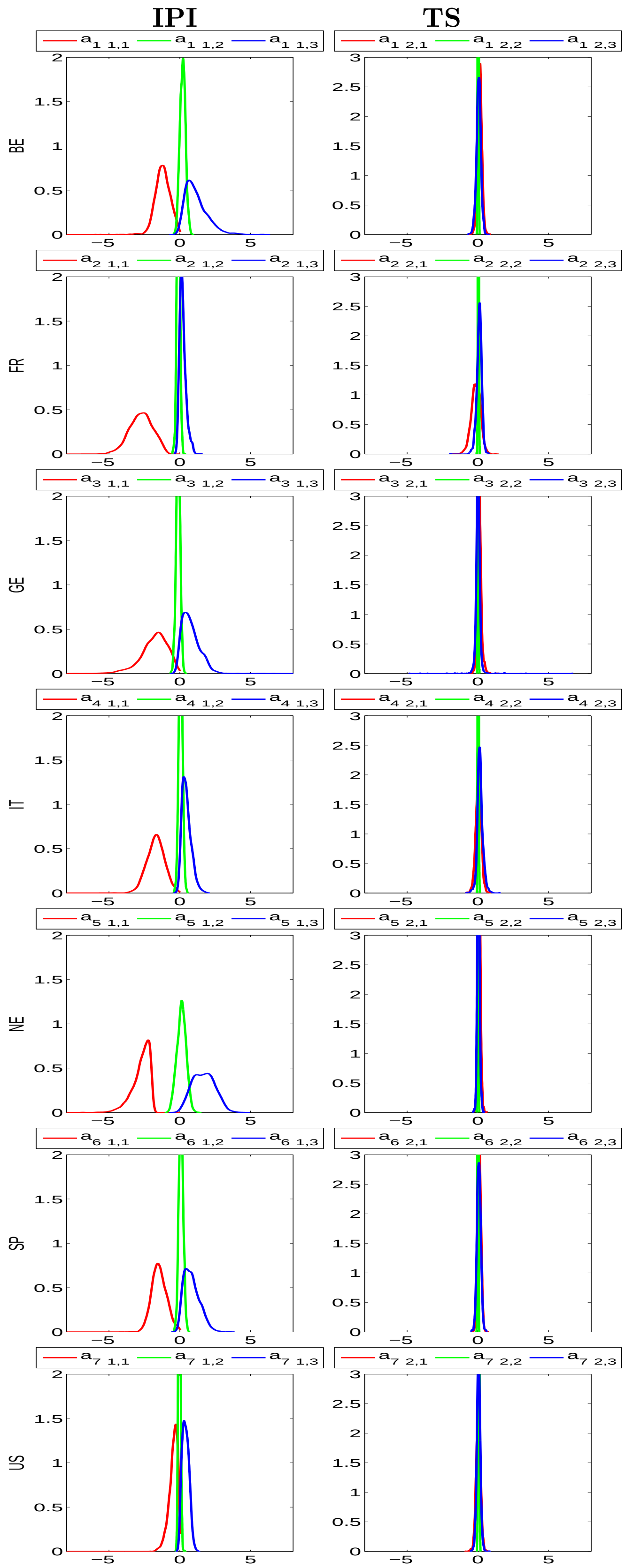

Figure 2: Posterior densities of the Markov-switching intercepts, $\gamma_{im} = (a_{i1,m}, a_{i2,m})'$, $i = 1, \dots, N$, $m = 1, \dots, M$ for IPI growth rate and TS.

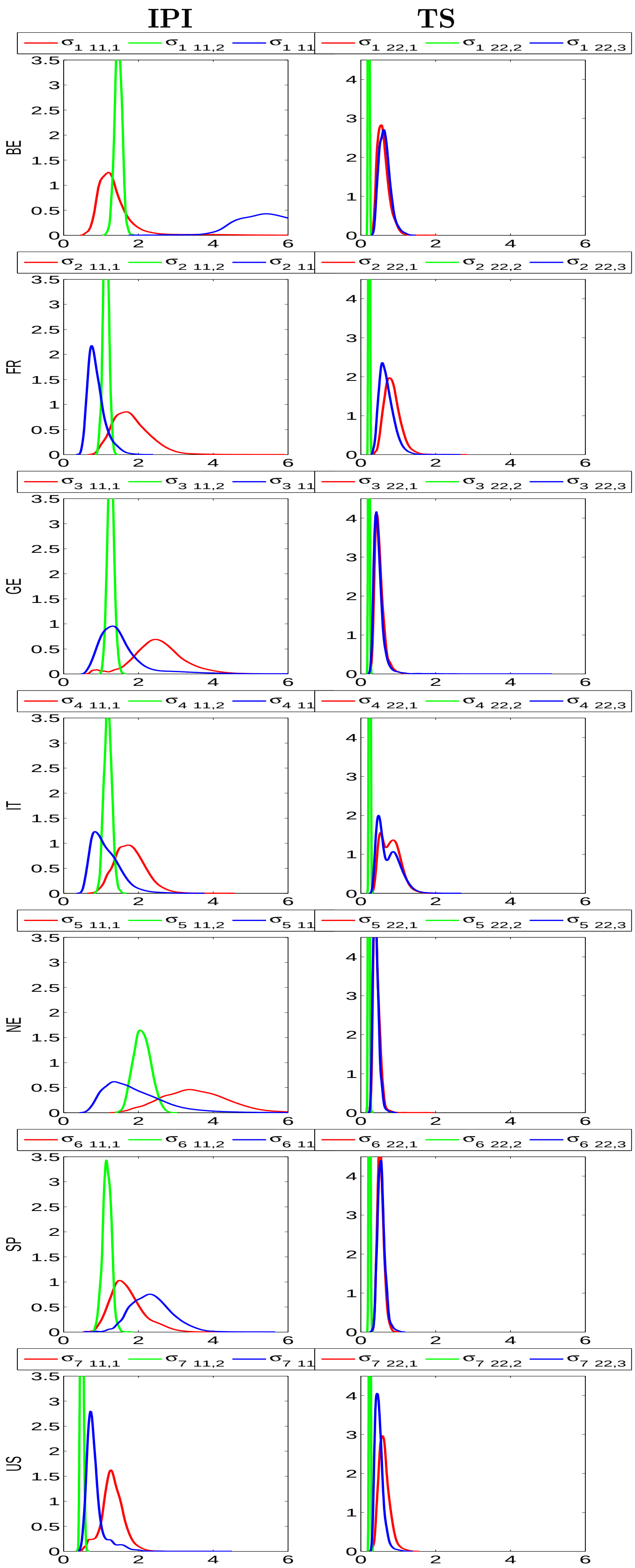

Figure 3: Posterior densities of the Markov-switching volatilities, $\sqrt{\sigma_{i\,jj,m}}$, for IPI and TS, with $\sigma_{i\,jj,m}$, $j = 1, 2$, diagonal elements of Σ_{im}, $i = 1, \ldots, N$ and $m = 1, \ldots, M$.

posterior density for $a_{i1,1}$, the intercept of the recession regime, is negative as we impose; whether $a_{i1,2}$, the moderate regime, is centered around zero; and $a_{i1,3}$, strong expansion, is positive. Nevertheless, there are substantial differences between European countries and US: the three posteriors are wider for European countries; and the posteriors of $a_{i1,1}$ are large and negative. Posteriors for US are more concentrated and closer to zero. For France and Italy, $a_{i1,2}$ and $a_{i1,3}$ overlap substantially, suggesting that the two countries have not experienced strong growth in our sample. The other four European countries have larger $a_{i1,3}$; in particular for Belgium, Germany and the Netherlands.

The posterior density of the term spread intercept (see right column in Fig. 2) is centered around zero for all countries, with larger dispersion for the recession and strong expansionary periods. The slope of the term structure is often flat during calm period and can be positive or negative both in recession and expansion periods. Our estimates display such uncertainty. Nevertheless, the overlapping supports of the posterior densities indicate a substantial equivalence of the mean TS value across regimes.

4.2.2 Markov-switching volatilities

The differences across regimes and across countries are larger for the posterior densities of the IPI and TS volatilities (see Fig. 3). As regards the IPI volatility, there is a large difference of the volatility behavior across regimes, between the US and the European countries. The general pattern is that volatility is higher during recessions and, for many countries, during expansion periods, and lower in recovery and moderate expansion periods, but with important differences among countries. The volatility posteriors for the three regimes do not overlap for Belgium and the US, whether this is not true for other countries. The US industrial production has larger switches during strong recession or expansion periods, which increase volatility estimates. Posterior mean estimates suggest such movements are transitory and do not imply large changes in the intercept. The eurozone estimates seem to be dominated by smoother transitions, resulting in lower volatilities but more evident differences in the intercept. Germany posteriors are the closest to US estimates. There is, however, an important difference in volatilities of the third regime for Belgium: estimates are higher, meaning that strong expansions have larger uncertainty in this country compared to the the rest of the eurozone and the US. There exist, on the contrary, not major differences for residual volatilities for the term spread. As regards the TS volatility, there is a strong evidence in favor of at least two regimes for the TS series. Again, the identification of the regimes for the IPI data is quite effective also for the TS series. For all countries, the posterior density for the TS volatility in the moderate regime is concentrated around 0.08 and its support set does not overlap with the ones of the recession and expansion regimes. The TS volatility in regimes 1 and 3 is larger and its posterior mean is between 0.3 and 0.5.

To sum up, we find some important differences in the parameter posterior densities of the eurozone and the US, both in the intercept and in the regime volatility. The heterogeneity is also important among eurozone economies, with mainly Germany more similar to the US than other countries.

4.3 Evidence on leading and lagging cycles

The PMS-VAR model allows us to study the business cycles fluctuations of each country in the panel, to analyse the transmission of shocks across cycles and predict the turning points of the country-specific cycles. We recall that the regime labeling is: recession, $s_{i,t} = 1$, recovery or moderate expansion, $s_{i,t} = 2$, and expansion, $s_{i,t} = 3$. The PMS-VAR model produces both country-specific smoothing probabilities for each regime (given in Fig.9-11) and eurozone and US aggregate smoothing probabilities. Specifically, the number of eurozone countries in recession and the similar measure for the US, used in the vector V_t, are reported in the first row of Fig. 4. The second row of the same figure reports the associated probabilities of the eurozone and US economies to be in recessions. These figures provide several interesting results and generally show that the eurozone and US economies are not fully aligned.

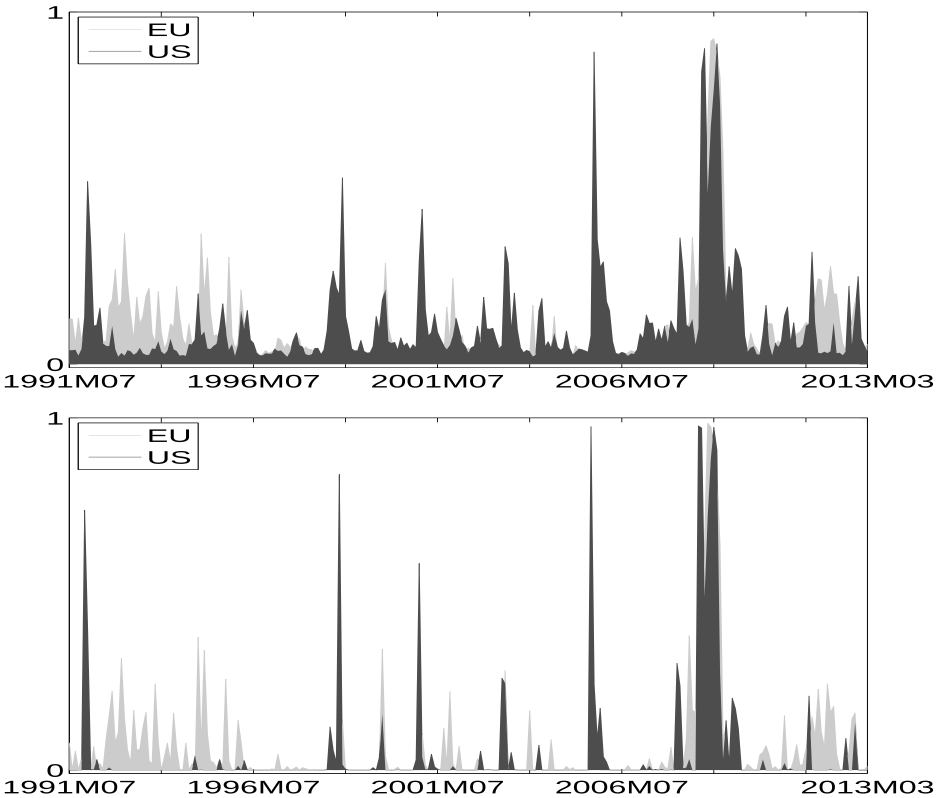

Figure 4: First row: fraction of eurozone countries in the recession regime, $\hat{\eta}_{1t}$, and US recession indicator $\mathbb{I}(\hat{s}_{7,t} = 1)$, $t = 1, \ldots, T$. Second row: smoothing probability of being in the recession regime (regime 1) using the indicator processes η_{1t} for the eurozone and $s_{7,t}$ for the US, $t = 1, \ldots, T$

.

In the first decade of our sample, the recession probability in the eurozone is more volatile than in the US, see also Fig. 9 in Appendix C, and this may be related to the construction of the European Monetary Union. In the second decade, the US apparently leads the eurozone cycle both in the short recessions in 2001-2002, and from 2007-2008. The internet bubble has generated small and short-lasting recessions in both economies, with instabilities up to 2003, and some calls for new recession in the US at the end of 2005 and in 2006. The largest recession probabilities are during the Great Financial Recession, with both economies having probabilities close to 1. The US seems to enter, and to reach the peak one-quarter in advance of the eurozone. Our model indicates precisely that the US economy enters in the recession phase in April 2008, with already increasing recession probabilities from August 2007 onwards, where the eurozone recession starts in August 2008. Both economies enter in 2009Q2 in a new regime generally defined recovery in our paper (see also the low probability levels in Fig. 9 and 11 and the high probability level in Fig. 10), but which is probably more accurate to interpret as stagnation. Furthermore, the eurozone has evidence of a new recession regime from June of 2011. We associate it to fiscal and debt problems in some European countries, in particular in Italy and Spain, see also next paragraph. In general, recessions in the US are shorter than in the eurozone.

Looking at the country specific smoothing probabilities we observe that the regimes are often highly persistent. Regime 2 is the most probable as we could anticipate since its definition can fit both stagnation, recovery and (moderate) expansion periods, which are appropriate definitions for most of our sample. The global financial crisis in 2008-2009 and its impact are evident, with most of the countries in recession. There is some evidence of a recession in 2001, see, for example, The Netherlands and Spain, but all short-lived. Larger differences exist during the European debt crisis, with Germany and the US the only two countries where the probability of regime 2 does not increase. The third regime has the lowest probabilities, in particular in the second decade. Finally, probabilities for Belgium seem the least related to US probabilities in the first decade of our sample, but converging in the second part of the sample. The large decline of mining in the 80's is a possible explanation for it.

The heterogeneity of the eurozone is present not only in the regime dynamics but also in the features of the regimes. The dynamic features of the cycle, in terms of conditional level and conditional variance, are given for each country in Appendix D. The mean conditional level (Fig. 12-14) and the mean conditional variance (Fig. 15-17) of the cycle and their high probability density regions account for both parameter and regime uncertainties. These figures allow, at each point in time, for a statistical analysis of the cycle fluctuations and for a comparison between the cycles of the different countries. A quick look at Fig. 12-14 reveals that during the Great Financial Recession the level is much more negative in Europe than the US. France, Germany, Italy and The Netherlands have all values below -1.5, compared to the 90% interval $[-0.5, -1]$ of the US. The Netherlands has the highest intercept in regime 2, whether France, Germany and Italy the lowest. The intercept of Germany increases after 2006 up to the crisis, probably due to the reforms they implemented. Fig. 15-17 tell us that the high volatilities in the recession are evident, with the US one of the smallest. Moreover, the US volatility in regime 2 is smaller than other countries.

The whole set of figures highlights the heterogeneity of the fluctuations in the eurozone and allows to study how recession and non-recession phases spread out across countries. In order to compare cycles and to better summarize the information about the diffusion of the recession (regime 1) and non-recession (regimes 2 and 3) phases across countries, we elaborate further the output of our PMS-VAR model by aggregating regimes 2 and 3 in the non-recession regime and by normalizing the mean conditional level of the eurozone countries by dividing it by the mean conditional standard deviation. The result is given in Fig. 5. The figure strengths the interpretation of previous results, and it is clear that the growth level during the Great Financial Recession is much more negative in Europe than US; Germany, France and Italy have the lowest expansion growth levels and closer to the US one, whether the Netherlands has the largest.

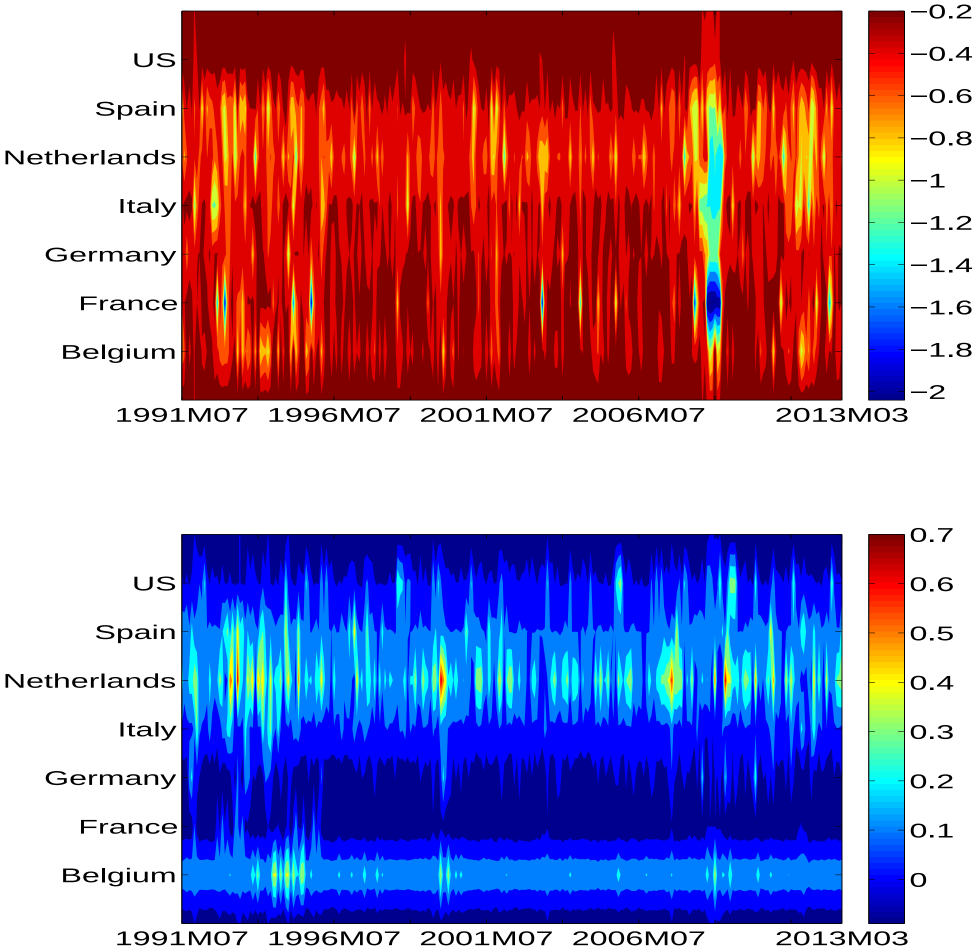

Figure 5: Heatmaps of the standardized conditional levels of the country-specific cylces during recession (up) and non-recession (bottom) phases. In each plot blue colors represent lower levels of the economic activity (note the different scales in the two panels). The levels are given by the ratio between the conditional interecepts (see Fig. 12-14, Appendix D) and the conditional standard deviations (see Fig. 15-17, Appendix D) of the country-specific cycles.

The evidence of strong heterogeneity of the cycles is one of the main results of our PMS-VAR model. Another relevant result regards the interaction between the cycles. The

Country		$p_{it,11}$		$p_{it,12}$	
i	Label	$\alpha_{1i}^{EU,11}$	$\alpha_{1i}^{US,11}$	$\alpha_{1i}^{EU,12}$	$\alpha_{1i}^{US,12}$
1	BE	1.70	-0.47	-0.20	-0.15
		(1.46, 1,99)	(-0.63,-0.26)	(-0.31, -0.02)	(-0.34,0.05)
2	FR	1.51	-0.32	0.07	0.17
		(1.39,1.74)	(-0.53,0.12)	(-012,0.21)	(0.01,0.34)
3	GE	1.69	-0.25	-0.16	-0.27
		(1.60,1.82)	(-0.50,-0.07)	(-0.51,0.00)	(-0.45,-0.09)
4	IT	1.68	0.10	0.22	0.41
		(1.45,1.87)	(-0.03,0.33)	(0.16,0.30)	(0.26,0.59)
5	NL	1.57	0.18	0.24	-0.23
		(1.47,1.71)	(0.10,0.27)	(0.08,0.47)	(-0.43,-0.08)
6	SP	1.76	0.08	0.09	-0.33
		(1.67,1.89)	(-0.01,0.19)	(-0.02,0.17)	(-0.48,-0.06)
7	US	1.58	1.00	0.00	0.22
		(1.53,1.65)	(0.94,1.04)	(-0.06,0.07)	(0.18,0.26)

Table 1: Posterior mean and 90% credible interval (in parenthesis) for the parameters, $\boldsymbol{\alpha}_{1i} = (\alpha_{1i}^{11}, \alpha_{1i}^{12})'$, with $\alpha_{1i}^{lj} = (\alpha_{1i}^{EU,lj}, \alpha_{1i}^{US,lj})'$, which are the coefficients of the interaction variables η_{1t} and $\mathbb{I}(s_{7,t} = 1)$ driving the Markov-switching transition probabilities.

posterior estimates of the loadings of V_t (see Table 1) provide further information on the interaction between the eurozone and the US cycles. Estimates of the coefficients $\alpha_{1i}^{EU,11}$, $i = 1, \ldots, 6$, associated with the eurozone recession indicator, η_{1t}, appearing in the country-specific probability to stay in recession (see Eq. 4-5), are all positive, large and significant. This means that there is a reinforcement effect, that is an increase in the probability to stay into the recession regime at time $t + 1$ due to the fact that the eurozone countries were in a recession phase at the previous time t. See the upper-left chart of Fig. 6 for a graphical illustration of the sensitivity of the recession probability $p_{it,11}$ to the values of η_{1t} when US is not in recession, i.e. $s_{7t} \neq 0$.

The posterior means of $\alpha_{1i}^{US,11}$ for the eurozone countries are negative, small and credible significant except for Italy and the Netherlands. This means that, when the US enters in the recession and the European countries are in recession, then the effect on the eurozone cycles is not homogeneous. The upper-right chart of Fig. 6 exhibits the effect of an increase of number of eurozone countries in recession, the η_{1t} indicator, on the probability of staying in regime 1 (recession), when the US is in recession, i.e. $s_{7,t} = 1$. A comparison of the upper-left and upper-right charts reveals the heterogeneous reactions of the eurozone countries to changes in the cycle of the US.

We find evidence of a reinforcement effect also for the US cycle. In fact, the posterior mean of $\alpha_{1i}^{US,11}$ for $i = 7$ (the US) is equal to one. Furthermore, there is strong evidence of a positive effect of the eurozone recession on the US probability to stay in recession, that corresponds to a posterior mean of $\alpha_{1i}^{US,11}$ for $i = 7$ equal to 1.58.

As regards the transition probability $p_{it,12}$, that is the probability to exit a recession phase, going from the recession regime (regime 1) to the moderate growth regime (regime

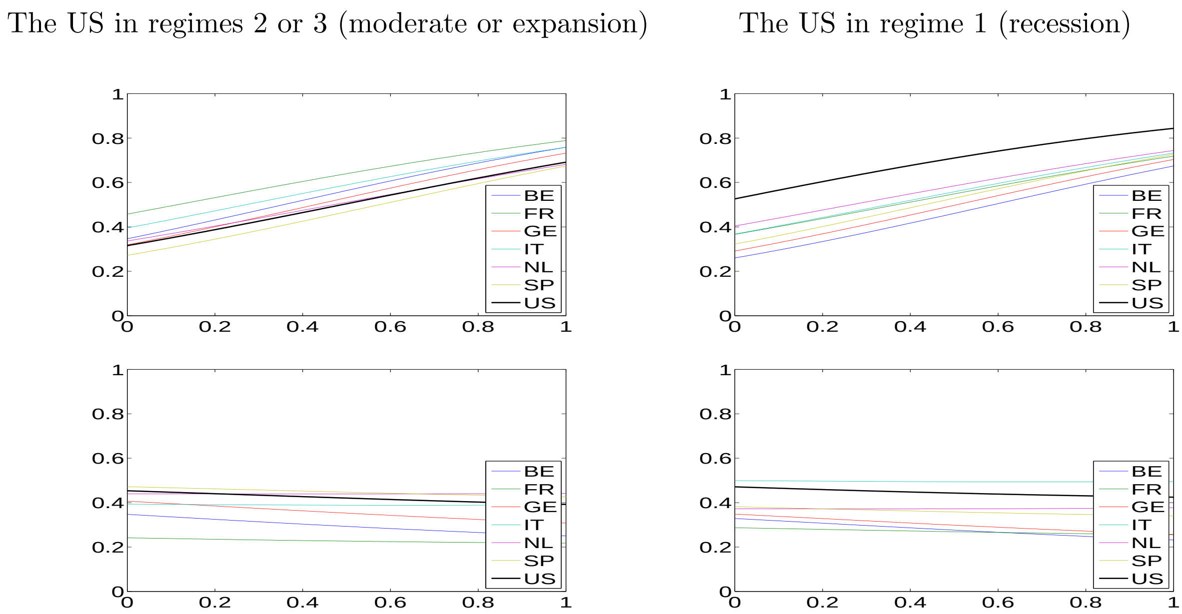

Figure 6: Reaction of the transition probabilities $p_{it,11}$ (first row) and $p_{it,12}$ (second row) to changes in η_{1t}, when conditioning on not recession for the US, i.e. $s_{7,t} \neq 1$ (left column) and recession for the US, i.e. $s_{7,t} = 1$, (right column).
.

2), we found evidence of differences between the eurozone countries (see Table 1). The estimate for $\alpha_{1i}^{US,12}$, $i = 7$, is positive and zero is outside the credible interval, where the sign and significance of $\alpha_i^{EU,12}$, with $i \neq US$, is more uncertain. The same evidence is true for $\alpha_i^{US,12}$, $i \neq US$. The discrepancies between European countries are clear from the charts in the second row of Fig. 6. When the US is not in a recession phase, an increase in η_{1t} (number of eurozone countries in recession) produces for some countries (i.e. Belgium and Germany) a decrease of the probability of exiting the recession.

As regard the relationship between the US and the eurozone cycles, the probability of exiting a recession phase for the US cycle is not affected by number of eurozone countries in recession. While the US recession has an effect on the exiting probability of the eurozone countries. This could be interpreted again as the fact that the US cycle is leading in eurozone when exiting a recession phase. As regards the effect of the US on the eurozone cycle when exiting a recession, we found evidence of heterogeneous behaviour of the eurozone countries (see bottom-right chart of Fig. 6).

4.4 Turning points detection

The previous sections describe how the US economy differs from the eurozone. It also highlights substantial differences among European countries. An important assumption for our model strategy refers to the weight scheme to aggregate country-specific cycles. We repeat that we use value added weights in equation (9). Fig. 7 shows that Germany has the largest weight, declining for most of the sample, but increasing at the end of it. France and Italy are the second and third largest economies, then Spain, the Netherlands; finally Belgium accounts for 5% of the sum. To further investigate how countries relate to the aggregate and possible synchronize with it, we study how each country cycle detects turning points of the business cycle. The contribution is not necessarily equal to the weights for several reasons. The link from individual countries to the aggregate depends on which measure is used. Some countries may lead the cycle, others may lag; value added weights may not contain such information.

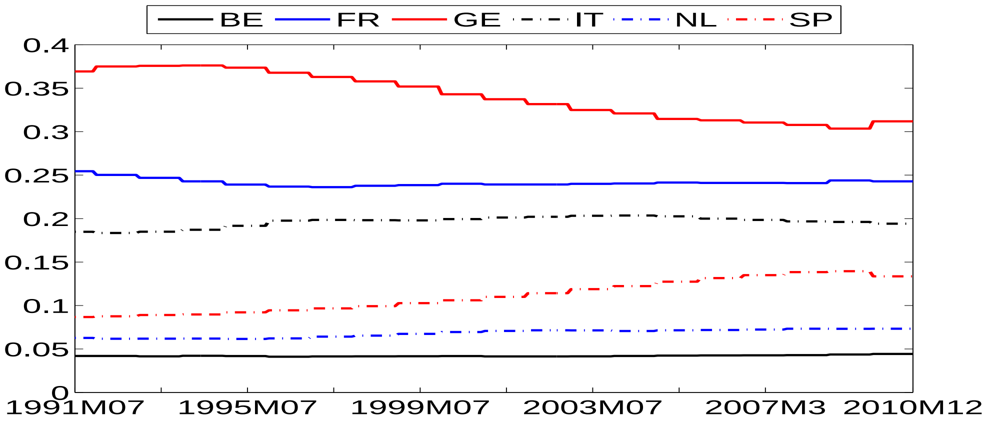

Figure 7: Value added eurozone weights.

We follow Billio et al. (2012a) and construct eurozone business cycle turning points by applying the Bry and Boschan (1971) (BB) rule, that identifies a downward turn (or peak) at time t for the variable of interest y_t if $y_{t-\kappa} < y_t, \ldots, y_{t-1} < y_t$ and $y_t > y_{t+1}, \ldots, y_t > y_{t+\kappa}$ and a upward turn (or trough) at time t if $y_{t-\kappa} > y_t, \ldots, y_{t-1} > y_t$ and $y_t < y_t, \ldots, y_t < y_{t+\kappa}$. Similarly, we define a non-downward turn at time t if $y_{t-\kappa} < y_t, \ldots, y_{t-1} < y_t$ and $y_t < y_{t+1}, \ldots, y_t < y_{t+\kappa}$ and a non-upward turn at time t if $y_{t-\kappa} > y_t, \ldots, y_{t-1} > y_t$ and $y_t > y_{t+1}, \ldots, y_t > y_{t+\kappa}$. The parameter κ reduces the number of false signals.[2] These definitions are standard in business cycle analysis (see for example Chauvet and Piger (2008)) and are also used (with some adjustments) by the NBER institute for building the reference cycle for the US.

In the following we apply an approximation of the BB rule and use only downward,

[2]Our analysis can be extended to include modifications of the BB rule (see for example Mönch and Uhlig (2005)), which account for asymmetries and time-varying duration across business cycle phases. Censoring rules preventing the algorithm from the detection of false signals could also be used.

$D_t(\kappa)$, and upward, $U_t(\kappa)$, turn signals, that are

$$D_t(\kappa) = \prod_{k=1}^{\kappa} \mathbb{I}_{[y_{t-k},+\infty)}(y_t)\mathbb{I}_{[y_{t+k},+\infty)}(y_t) \quad (23)$$

$$U_t(\kappa) = \prod_{k=1}^{\kappa} \mathbb{I}_{(-\infty,y_{t-k}]}(y_t)\mathbb{I}_{(-\infty,y_{t+k}))}(y_t) \quad (24)$$

respectively. Our analysis can be extended to include modifications of the BB rule (see for example Mönch and Uhlig (2005)), which account for asymmetries and time-varying duration across business cycle phases.

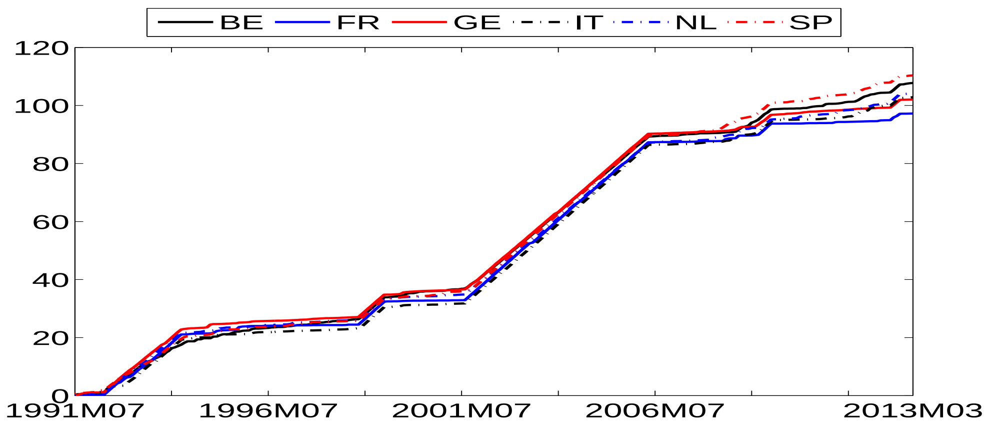

Figure 8: Cumulative concordance statistics of individual countries to predict eurozone cycle
.

Set y_t equal to the aggregate eurozone IPI growth used in Billio et al. (2012a) and download from the OECD database. The following indicator variable can be computed:

$$z_t = z_{t-1}(1 - D_t(\kappa)) + (1 - z_{t-1})U_t(\kappa)$$

that is equal to 1 in the expansion phases and 0 in the recession phases. We assume z_0 is given. We evaluate turning point detection ability of the different country chains by the concordance statistics (CS):

$$CS_i = \frac{1}{t+1+\kappa} \sum_{s=1}^{t+1-\kappa} \Big(\mathbb{I}(s_{i,t} = 1)z_s - (1 - \mathbb{I}(s_{i,t} = 1))(1 - z_s)\Big) \quad (25)$$

where we define a downward turn when switching to regime 1, i.e. $\mathbb{I}(s_{i,t} = 1)$, and upward turn otherwise, i.e. $(1 - \mathbb{I}(s_{i,t} = 1))$. This means that an upward turn can be a switch to regime 2 or 3 in our three-regime models. The CS statistics is a nonparametric measure of the proportion of time during which two series, in our case the country-specific cycle and the eurozone cycle, are in the same state. This measure ranges between 0 and 1, with 0 representing perfectly counter-cyclical switches, and 1 perfectly synchronous shifts. Fig. 8 shows the CS_i cumulated over time and it ranges between 0 and the sample size, that is 261 in our application. The countries with the highest CS have a business cycle which conserves over time a strong similarity to the cycle of the eurozone. At the end of the sample, the order of the countries following the CS values does not corresponds with the order obtained by applying the value added weights in Fig. 7. This constitutes evidence of a substantial difference between the size of the countries and the dynamic features of their economies in providing information about the aggregate cycle. Furthermore, we observe that there is change in the country ordering from 2008, which corresponds to the beginning of the recession phase dated in August 2008 following results in Fig. 4. In the first 15 years of our sample, Germany has the largest CS, but after the beginning of the Great Financial crisis and, above all, the European debt crisis, Spain and Belgium seem to provide more timely information on the aggregate recession; an increasing role for Italy; and France gives less accurate evidence.

5 Conclusion

We propose a new Bayesian panel VAR model with unit-specific time-varying Markov-switching latent factors and develop a suitable Gibbs sampling procedure for posterior inference. We apply our panel MS-VAR model to the analysis of the interconnections and the differences between the eurozone and the US business cycles and their turning points.

Our results show that the US cycle leads the eurozone cycle. The two cycles are not often fully synchronized over the 1991-2013 sample, with evidence of more recessions in the eurozone, in particular during the 90's when the monetary union was planned. The larger synchronization is at beginning of the Great Financial Crisis: the shock affects the US 1-quarter in advance of the eurozone, but it spreads among economies very rapidly. In general, recessions are shorter-lived in the US, and switches from recessions to expansions or from expansions to recessions are sharper, resulting in higher volatility in such regimes, compared to smoother transitions for the the eurozone countries.

We found evidence of reinforcement effects in the recession probabilities for both the eurozone and the US cycles, large differences in the phase transitions within the eurozone and an asymmetric relationship between the eurozone and the US economic phase transitions. More specifically an increase in the number of eurozone countries in recession increases the probability of the US to stay into recession, while the US recession indicator has a negative impact on the probability to stay in recession for the eurozone countries except for Italy and the Netherlands. We found a similar asymmetry and heterogeneity also in the probabilities of exiting the recession phase.

Finally, as regards the turning point analysis, the cycles of Germany and, somewhat less, France and Italy are closer to the one of the US than other countries. Affinity to the US cycle does not necessarily imply that the correspondent country cycle detects turning points for the eurozone accurately. Indeed, we find that Belgium, Spain, and somewhat less Germany, seem to provide more timely information on the aggregate recession; whether the Netherlands and France give less accurate evidence.

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A Computational details

A.1 Parameter full conditional densities

Updating $\boldsymbol{\gamma}_{i0}$. Then the full conditional distribution of the regime-independent parameter $\boldsymbol{\gamma}_{i0}$ is a normal with density function

$$
\begin{aligned}
& f(\boldsymbol{\gamma}_{i0}|\mathbf{y}_i, \Xi_i, \boldsymbol{\gamma}_i, \Sigma_i, \boldsymbol{\lambda}_0) \propto \qquad (26) \\
& \propto \exp\left\{-\frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T}(\mathbf{y}_{i0t}-\boldsymbol{\gamma}_{i0})'\Sigma_{it}^{-1}(\mathbf{y}_{i0t}-\boldsymbol{\gamma}_{i0}) - \frac{1}{2}(\boldsymbol{\gamma}_{i0}-\boldsymbol{\lambda}_0)'\underline{\Sigma}_{i0}^{-1}(\boldsymbol{\gamma}_{i0}-\boldsymbol{\lambda}_0)\right\} \\
& \propto \exp\left\{-\frac{1}{2}\boldsymbol{\gamma}_{i0}'\left(\sum_{t=1}^{T}X_{i0t}'\Sigma_{it}^{-1}X_{i0t}+\underline{\Sigma}_{i0}^{-1}\right)\boldsymbol{\gamma}_{i0}+\boldsymbol{\gamma}_{i0}\left(\sum_{t=1}^{T}X_{i0t}'\Sigma_{it}^{-1}\mathbf{y}_{i0t}+\underline{\Sigma}_{i0}^{-1}\boldsymbol{\lambda}_0\right)\right\} \\
& \propto \mathcal{N}_{K_0}(\bar{\boldsymbol{\gamma}}_{i0}, \bar{\Sigma}_{i0})
\end{aligned}
$$

where $\bar{\boldsymbol{\gamma}}_{i0} = \bar{\Sigma}_{i0}(\underline{\Sigma}_{i0}^{-1}\boldsymbol{\lambda}_0 + \sum_{t=1}^{T} X_{i0t}'\Sigma_{it}^{-1}X_{i0t})$ and $\bar{\Sigma}_{i0}^{-1} = (\underline{\Sigma}_{i0}^{-1} + \sum_{t=1}^{T} X_{i0t}'\Sigma_{it}^{-1}X_{i0t})$.

Updating $\boldsymbol{\gamma}_{im}$. The full conditional distributions of the regime-dependent parameters $\boldsymbol{\gamma}_{im}$, with $m = 1, \ldots, M$ are normal with density function

$$
\begin{aligned}
& f(\boldsymbol{\gamma}_{im}|\mathbf{y}_i, \Xi_i, \boldsymbol{\gamma}_{i0}, \boldsymbol{\gamma}_{i(-m)}, \Sigma, \boldsymbol{\lambda}_m) \propto \qquad (27) \\
& \propto \exp\left\{-\frac{1}{2}\sum_{t\in\mathcal{T}_{im}}\mathbf{u}_{it}'\Sigma_t^{-1}\mathbf{u}_{it} - \frac{1}{2}(\boldsymbol{\gamma}_{im}-\boldsymbol{\lambda}_m)'\underline{\Sigma}_{im}^{-1}(\boldsymbol{\gamma}_{im}-\boldsymbol{\lambda}_m)\right\} \\
& \propto \exp\left\{-\frac{1}{2}\boldsymbol{\gamma}_i'\left(\sum_{t\in\mathcal{T}_{im}}X_{imt}'\Sigma_{it}^{-1}X_{imt}+\underline{\Sigma}_{im}^{-1}\right)\boldsymbol{\gamma}_i+\boldsymbol{\gamma}_i'\left(\sum_{t\in\mathcal{T}_{im}}X_{imt}'\Sigma_{it}^{-1}\mathbf{y}_{imt}+\underline{\Sigma}_{im}^{-1}\boldsymbol{\lambda}_m\right)\right\} \\
& \propto \mathcal{N}_{K_m}(\bar{\boldsymbol{\gamma}}_{im}, \bar{\Sigma}_{im})
\end{aligned}
$$

with $\bar{\boldsymbol{\gamma}}_{im} = \bar{\Sigma}_{im}^{-1}(\underline{\Sigma}_{im}^{-1}\boldsymbol{\lambda}_m + \sum_{t\in\mathcal{T}_{im}} X_{imt}'\Sigma_{it}^{-1}X_{imt})$ and $\bar{\Sigma}_{im}^{-1} = (\underline{\Sigma}_{im}^{-1} + \sum_{t\in\mathcal{T}_{im}} X_{imt}'\Sigma_{it}^{-1}X_{imt})$, where we defined $\mathcal{T}_{im} = \{t = 1, \ldots, T|\xi_{imt} = 1\}$ and $\mathbf{y}_{imt} = \mathbf{y}_{it} - X_{i0t}\boldsymbol{\gamma}_{i0}$.

Updating Σ_{im}^{-1}. The full conditional distributions of the regime-dependent inverse variance-covariance matrix Σ_{im}, $m = 1, \ldots, M$, are Wishart distributions with density

$$
\begin{aligned}
& f(\Sigma_{im}^{-1}|\mathbf{y}_i, \Xi_i, \boldsymbol{\gamma}_{i0}, \boldsymbol{\gamma}_i, \Sigma_{i(-m)}, \Upsilon_m) \propto \qquad (28) \\
& \propto \prod_{t=1}^{T}|\Sigma_{it}|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}\sum_{t\in\mathcal{T}_{im}}\mathbf{u}_{it}'\Sigma_{it}^{-1}\mathbf{u}_{it}\right\}|\Sigma_{im}^{-1}|^{\frac{\nu_{im}-K-1}{2}}\exp\left\{-\frac{1}{2}\mathrm{tr}\left(\Upsilon_m^{-1}\Sigma_{im}^{-1}\right)\right\} \\
& \propto |\Sigma_{im}^{-1}|^{\frac{\nu_{im}+T_{im}-K-1}{2}}\exp\left\{-\frac{1}{2}\mathrm{tr}\left(\left(\Upsilon_m^{-1}+\sum_{t\in\mathcal{T}_{im}}\mathbf{u}_{imt}\mathbf{u}_{imt}'\right)\Sigma_{im}^{-1}\right)\right\} \\
& \propto \mathcal{W}_K(\bar{\nu}_{im}, \bar{\Upsilon}_{im})
\end{aligned}
$$

where $T_{im} = \sum_{t=1}^{T}\mathbb{I}(\xi_{imt} = 1)$, $\mathbf{u}_{imt} = \mathbf{y}_{it} - X_{i0t}\boldsymbol{\gamma}_{i0} - X_{imt}\boldsymbol{\gamma}_{im}$, $\bar{\nu}_{im} = \underline{\nu}_{im} + T$ and

$\bar{\Upsilon}_{im}^{-1} = \Upsilon_m^{-1} + \sum_{t\in\mathcal{T}_{im}} \mathbf{u}_{imt}\mathbf{u}_{imt}'$.

Updating $\boldsymbol{\alpha}_i$. The full conditional distribution of the parameters in the k-th row of the transition matrix is

$$f(\boldsymbol{\alpha}_i^{k1},\ldots,\boldsymbol{\alpha}_i^{kM-1}|\mathbf{y}_i,\Xi,\boldsymbol{\gamma}_{i0},\boldsymbol{\gamma}_i) \quad\propto\quad \prod_{t=1}^{T}\prod_{j=1}^{M-1}(H(V_t,\boldsymbol{\alpha}_i^{kj}))^{\xi_{ijt}\xi_{ik\,t-1}}$$

We apply a Metropolis-Hastings step.

Updating λ_m. The full conditional distributions of the parameters $\boldsymbol{\lambda}_m$, $m=0,1,\ldots,M$, of the third stage of the hierarchical structure, are normal distributions with density functions

$$\begin{aligned}
& f(\boldsymbol{\lambda}_m|\boldsymbol{\gamma},\Sigma)\propto \\
&\propto \exp\left\{-\frac{1}{2}\sum_{i=1}^{N}(\boldsymbol{\gamma}_{im}-\boldsymbol{\lambda}_m)'\underline{\Sigma}_{im}^{-1}(\boldsymbol{\gamma}_{im}-\boldsymbol{\lambda}_m)-\frac{1}{2}(\boldsymbol{\lambda}_m-\underline{\boldsymbol{\lambda}}_m)'\underline{\Sigma}_m^{-1}(\boldsymbol{\lambda}_m-\underline{\boldsymbol{\lambda}}_m)\right\} \\
&\propto \exp\left\{-\frac{1}{2}\left[\boldsymbol{\lambda}_m'\left(\sum_{i=1}^{N}\underline{\Sigma}_{im}^{-1}+\underline{\Sigma}_m^{-1}\right)\boldsymbol{\lambda}_m-2\boldsymbol{\lambda}_m'\left(\sum_{i=1}^{N}\underline{\Sigma}_{im}^{-1}\boldsymbol{\gamma}_{im}+\underline{\Sigma}_m^{-1}\underline{\boldsymbol{\lambda}}_m\right)\right]\right\} \\
&\propto \mathcal{N}_{K_m}(\bar{\boldsymbol{\lambda}}_m,\bar{\Sigma}_m)
\end{aligned} \tag{29}$$

where $\bar{\Sigma}_m^{-1}=\sum_{i=1}^{N}\underline{\Sigma}_{im}^{-1}+\underline{\Sigma}_m^{-1}$ and $\bar{\boldsymbol{\lambda}}_m=\bar{\Sigma}_m\left(\sum_{i=1}^{N}\underline{\Sigma}_{im}^{-1}\boldsymbol{\gamma}_{im}+\underline{\Sigma}_m^{-1}\underline{\boldsymbol{\lambda}}_m\right)$.

Updating Υ_m^{-1}. The full conditional distributions of the Υ_m, $m=1,\ldots,M$, are Wishart distributions with density

$$\begin{aligned}
& f(\Upsilon_m^{-1}|\boldsymbol{\gamma},\Sigma)\propto \\
&\propto |\Upsilon_m^{-1}|^{\frac{\underline{\nu}_m-K-1}{2}}\exp\left\{-\frac{1}{2}\mathrm{tr}\left(\underline{\Upsilon}_m^{-1}\Upsilon_m^{-1}\right)\right\}\prod_{i=1}^{N}|\Upsilon_m^{-1}|^{\underline{\nu}_{im}/2}\exp\left\{-\frac{1}{2}\mathrm{tr}\left(\sum_{i=1}^{N}\Upsilon_m^{-1}\Sigma_{im}^{-1}\right)\right\} \\
&\propto \mathcal{W}_K(\bar{\nu}_m,\bar{\Upsilon}_m)
\end{aligned} \tag{30}$$

where $\bar{\nu}_m=\sum_{i=1}^{N}\underline{\nu}_{im}+\underline{\nu}_m$ and $\bar{\Upsilon}_m^{-1}=\underline{\Upsilon}_m^{-1}+\sum_{i=1}^{N}\Sigma_{im}^{-1}$.

A.2 Allocation variable full conditional distributions

In the simulation from the full conditional of the hidden allocation variables, we exploit the following factorization of the full conditional distribution of the $\boldsymbol{\xi}_{i\,1:T}$

$$p(\boldsymbol{\xi}_{i\,1:T}|\mathbf{y}_{1:T},\Xi,\boldsymbol{\gamma},\Sigma\boldsymbol{\alpha}) \propto \prod_{i=1}^{N}\prod_{t=1}^{T} p(\mathbf{y}_t|\mathbf{y}_{t-p-1:t-1},\boldsymbol{\xi}_{it},\boldsymbol{\gamma}_i,\Sigma_i) \prod_{k,l=1}^{M} p_{it,kl}^{\xi_{ilt}\xi_{ikt-1}} \tag{31}$$

$$\propto \prod_{t=1}^{T} p(\mathbf{y}_t|\mathbf{y}_{t-p-1:t-1},\boldsymbol{\xi}_{it},\boldsymbol{\gamma}_i,\Sigma_i) \prod_{k,l=1}^{M} p_{it,kl}^{\xi_{ikt}\xi_{ilt-1}} \prod_{\substack{j=1\\ j\neq i}}^{N}\prod_{k,l=1}^{M} p_{it,kl}^{\xi_{jkt}\xi_{jlt-1}}$$

$$\propto \left(p(\boldsymbol{\xi}_{iT}|\mathbf{y}_{1:T},\boldsymbol{\gamma}_i,\Sigma_i)\prod_{t=1}^{T-1} p(\boldsymbol{\xi}_{it}|\boldsymbol{\xi}_{it+1:T},\mathbf{y}_{1:T},\boldsymbol{\gamma}_i,\Sigma_i)\right)\left(\prod_{t=1}^{T}\prod_{\substack{j=1\\ j\neq i}}^{N}\prod_{k,l=1}^{M} p_{it,kl}^{\xi_{jkt}\xi_{jlt-1}}\right)$$

The first factor in the full conditional can be written recursively using the sequence of filtering densities and can be easily simulated through the FFBS procedure. The second part, is a proportionality factor that depends on the values $\boldsymbol{\xi}_{it}$, $t = 1,\ldots,T$. This factorization suggests that the FFBS algorithm can be used as proposal for the hidden states of the i-th chain of the model and that a Metropolis-Hastings step can be then used to account for the proportionality factor.

These conditional probabilities do not account for the interaction mechanism between the chains. Thus, we adjust for this discrepancy with a Metropolis-Hastings step, which use the FFBS as proposal and the full conditional $p(\boldsymbol{\xi}_{i1:T}|\boldsymbol{\xi}_{1\,1:T}\;\;,\ldots,\boldsymbol{\xi}_{i-1\,1:T},\boldsymbol{\xi}_{i+1,1:T},$ $\ldots,\boldsymbol{\xi}_{N\,1:T},\mathbf{y}_{1:t},\boldsymbol{\gamma}_i,\Sigma_i,\boldsymbol{\alpha})$ as target.

The steps of the FFBS algorithms are described in the following.

First, the filtering probability for the i-th Markov chain at time t, $t = 1,\ldots,T$, is determined by iterating the prediction step

$$p(\boldsymbol{\xi}_{it}=\boldsymbol{\iota}_j|\mathbf{y}_{1:t-1},\boldsymbol{\gamma}_i,\Sigma_i,\boldsymbol{\alpha}) = \sum_{l=1}^{M} p_{it,jl}\,p(\boldsymbol{\xi}_{it-1}=\boldsymbol{\iota}_l|\mathbf{y}_{1:t-1},\boldsymbol{\gamma}_i,\Sigma_i,\boldsymbol{\alpha}) \tag{32}$$

where $p_{it,jl} = p(\boldsymbol{\xi}_{it}=\boldsymbol{\iota}_j|\boldsymbol{\xi}_{it-1}=\boldsymbol{\iota}_l,V_t,\boldsymbol{\alpha}_i)$, with $\boldsymbol{\iota}_m$ the m-th column of the identity matrix, and the updating step

$$p(\boldsymbol{\xi}_{it}|\mathbf{y}_{1:t},\boldsymbol{\gamma}_i,\Sigma_i,\boldsymbol{\alpha}) \propto p(\boldsymbol{\xi}_{it}|\mathbf{y}_{1:t-1},\boldsymbol{\gamma}_i,\Sigma_i,\boldsymbol{\alpha})p(\mathbf{y}_t|\mathbf{y}_{t-1-p:t-1},\boldsymbol{\xi}_{it},\boldsymbol{\gamma}_i,\Sigma_i,\boldsymbol{\alpha}) \tag{33}$$

where $p(\boldsymbol{\xi}_{it}=\boldsymbol{\iota}_j|\boldsymbol{\xi}_{it-1}=\boldsymbol{\iota}_l) = p_{it,jl}$ with $p(\mathbf{y}_t|\mathbf{y}_{t-p-1:t-1},\boldsymbol{\xi}_{it})$ the conditional distribution of the variable y_t from a MSIH(m)-VAR(p).

We shall notice that the prediction step can be used at time t to find the predictive density of $\boldsymbol{\xi}_{it+1}$

$$p(\boldsymbol{\xi}_{it+1}|\mathbf{y}_{1:t},\boldsymbol{\gamma}_i,\Sigma_i,\boldsymbol{\alpha}) \propto P'_{it+1}\,p(\boldsymbol{\xi}_{it}|\mathbf{y}_{1:t},\boldsymbol{\gamma}_i,\Sigma_i,\boldsymbol{\alpha}) \tag{34}$$

Secondly, the smoothing probabilities given by

$$p(\boldsymbol{\xi}_{it} = \boldsymbol{\iota}_j|\mathbf{y}_{1:T}, \boldsymbol{\gamma}_i, \Sigma_i, \boldsymbol{\alpha}) \propto \sum_{l=1}^{M} p(\boldsymbol{\xi}_{it} = \boldsymbol{\iota}_j|\boldsymbol{\xi}_{it+1} = \boldsymbol{\iota}_l, \mathbf{y}_{1:t}, \boldsymbol{\gamma}_i, \Sigma_i, \boldsymbol{\alpha})p(\boldsymbol{\xi}_{it+1} = \boldsymbol{\iota}_l|\mathbf{y}_{1:T}, \boldsymbol{\gamma}_i, \Sigma_i, \boldsymbol{\alpha}) \tag{35}$$

are used evaluated recursively and backward in time for $t = T, T-1, \ldots, 1$ with initial condition $p(\boldsymbol{\xi}_{iT} = \boldsymbol{\iota}_j|\mathbf{y}_{1:T}, \boldsymbol{\gamma}_i, \Sigma_i, \boldsymbol{\alpha})$ given by the last filtering step. The conditional distribution

$$p(\boldsymbol{\xi}_{it}|\boldsymbol{\xi}_{it+1}, \mathbf{y}_{1:t}, \boldsymbol{\gamma}_i, \Sigma_i, \boldsymbol{\alpha}) \propto p_{it+1,lj} p(\boldsymbol{\xi}_{it} = \boldsymbol{\iota}_j|\mathbf{y}_{1:t})$$

is the building block of the smoothing probability formula and is used in the FFBS algorithm to sample the allocation variables from their joint posterior distribution sequentially and backward in time for $t = T, T-1, \ldots, 1$ (see Frühwirth-Schnatter (2006), ch. 11-13).

As discussed in previous sections, when using data-dependent priors the generation of the allocation variables should omit draws that yield to impropriety of the posterior. In our prior settings, the set of non-troublesome grouping, for the i-th unit, is $\mathcal{S}_i = \mathcal{S}_{i,\nu} \cap \mathcal{S}_{i,\sigma} = \mathcal{S}_{i,\sigma}$. Thus, each time the set of allocation variables $\boldsymbol{\xi}_{i\,1:T}$, does not assign at least two observations to each component of the dynamic mixture, the entire set $\boldsymbol{\xi}_{i\,1:T}$, is rejected and a new set is drawn until a proper set is obtained.

B MCMC convergence issues

As regards to the number of iterations, we should say that the choice of the initial sample size and the convergence detection of the Gibbs sampler remain open issues (see Robert and Casella (1999)). In our application we choose the sample size on the basis of both a graphical inspection of the MCMC progressive averages and the application of the convergence diagnostic (CD) statistics proposed in Geweke (1992). We let $n = 40,000$ be the MCMC sample size and $n_1 = 10,000$, and $n_2 = 30,000$ the sizes of two non-overlapping sub-samples. For a parameter θ of interest, we let

$$\hat{\theta}_1 = \frac{1}{n_1}\sum_{j=1}^{n_1}\theta^{(j)}, \quad \hat{\theta}_2 = \frac{1}{n_2}\sum_{j=n+1-n_2}^{n}\theta^{(j)}$$

be the MCMC sample means and $\hat{\sigma}_i^2$ their variances estimated with the non-parametric estimator

$$\frac{\hat{\sigma}_i^2}{n_i} = \hat{\Gamma}(0) + \frac{2n_i}{n_i - 1}\sum_{j=1}^{h_i} K(j/h_i)\hat{\Gamma}(j),$$

$$\hat{\Gamma}(j) = \frac{1}{n_i}\sum_{k=j+1}^{n_i}(\theta^{(k)} - \hat{\theta}_i)(\theta^{(k-j)} - \hat{\theta}_i)'$$

where we choose $K(x)$ to be the Parzen kernel (see Kim and Nelson (1999)) and $h_1 = 100$ and $h_2 = 500$ the bandwidths. Then the following statistics

$$CD = \frac{\hat{\theta}_1 - \hat{\theta}_2}{\sqrt{\hat{\sigma}_1^2/n_1 + \hat{\sigma}_2^2/n_2}} \tag{36}$$

converges in distribution to a standard normal (see Geweke (1992)), under the null hypothesis that the MCMC chain has converged.

C Smoothing probabilities

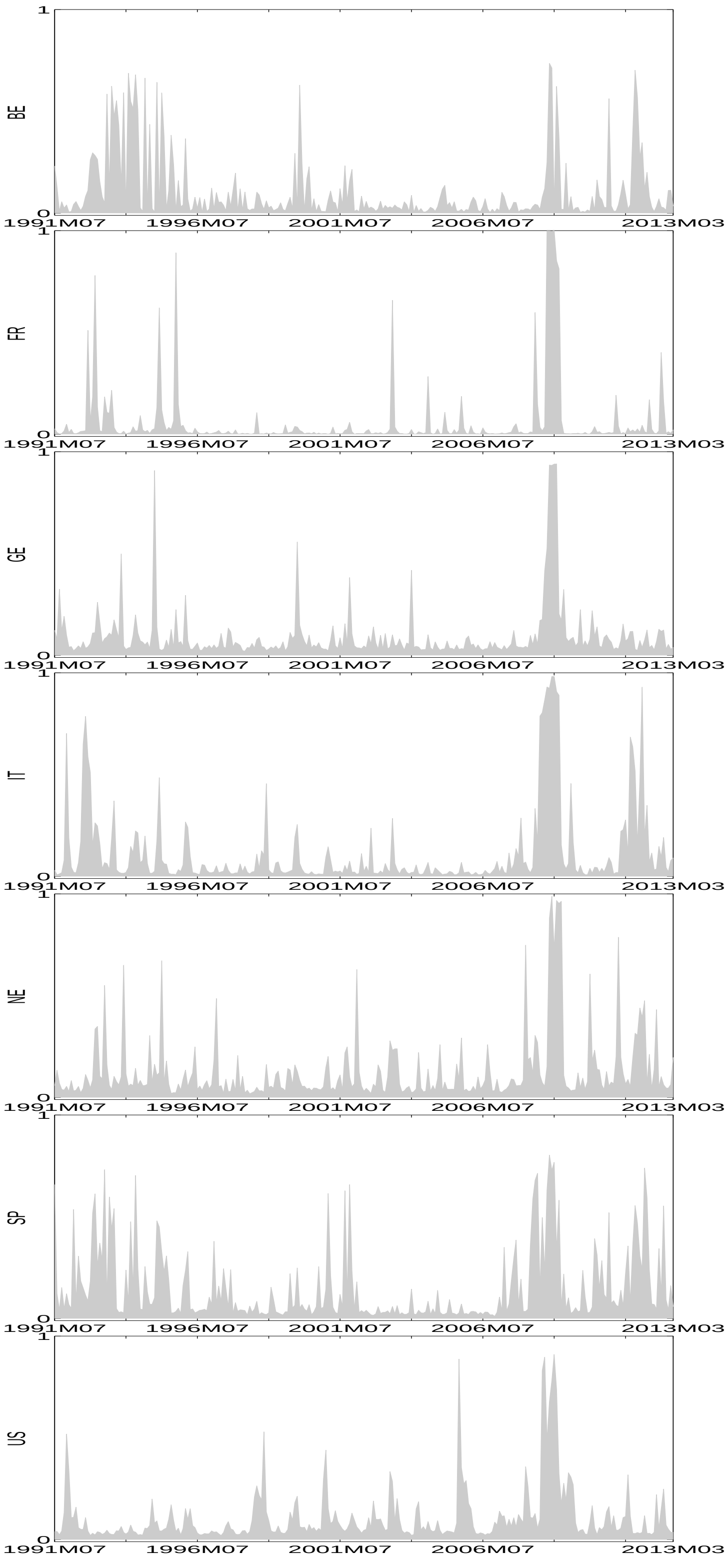

Figure 9: First regime (recession) smoothing probabilities for the Markov-switching processes $s_{i,t}$, $i = 1, \ldots, N$ and $t = 1, \ldots, T$.

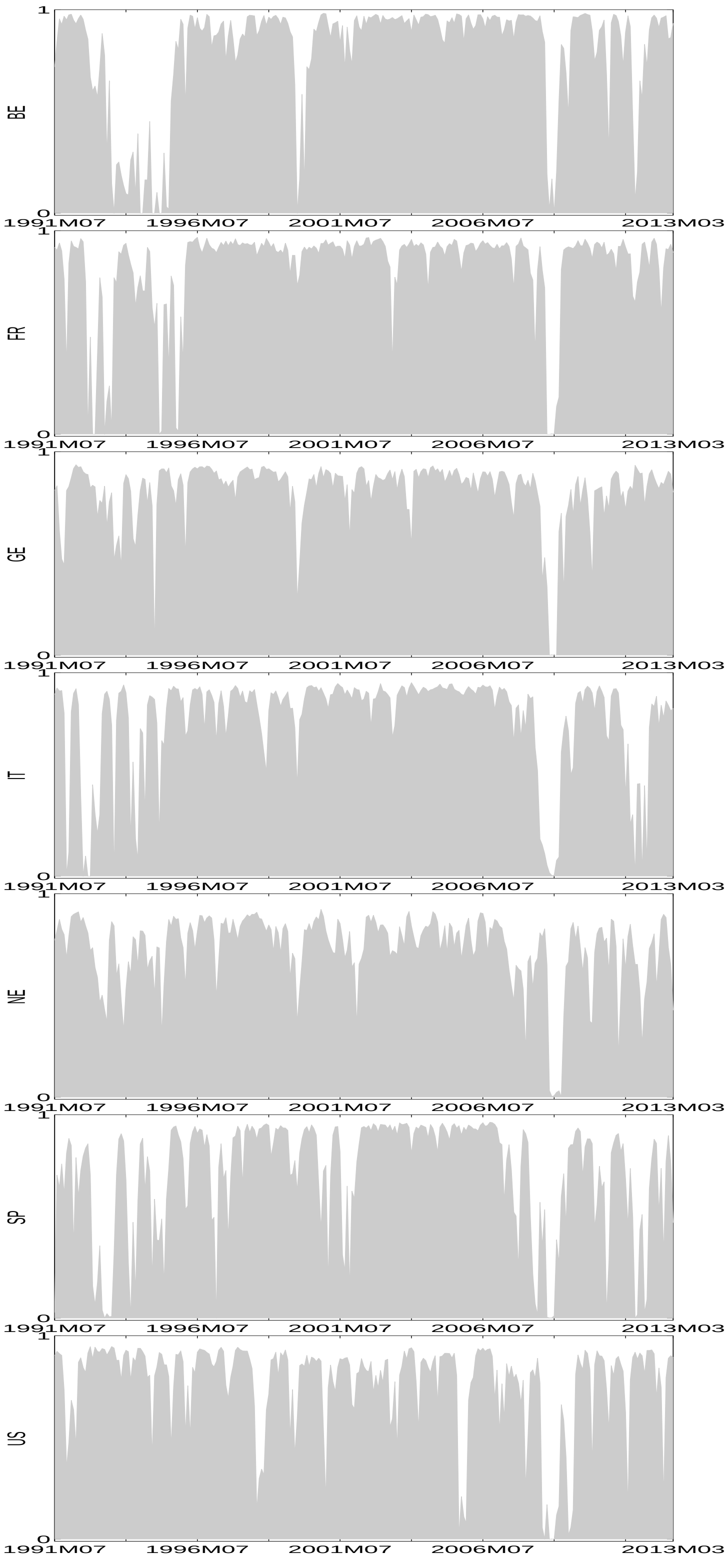

Figure 10: Second regime (moderate expansion) smoothing probabilities for the Markov-switching processes $s_{i,t}$, $i = 1, \ldots, N$ and $t = 1, \ldots, T$.

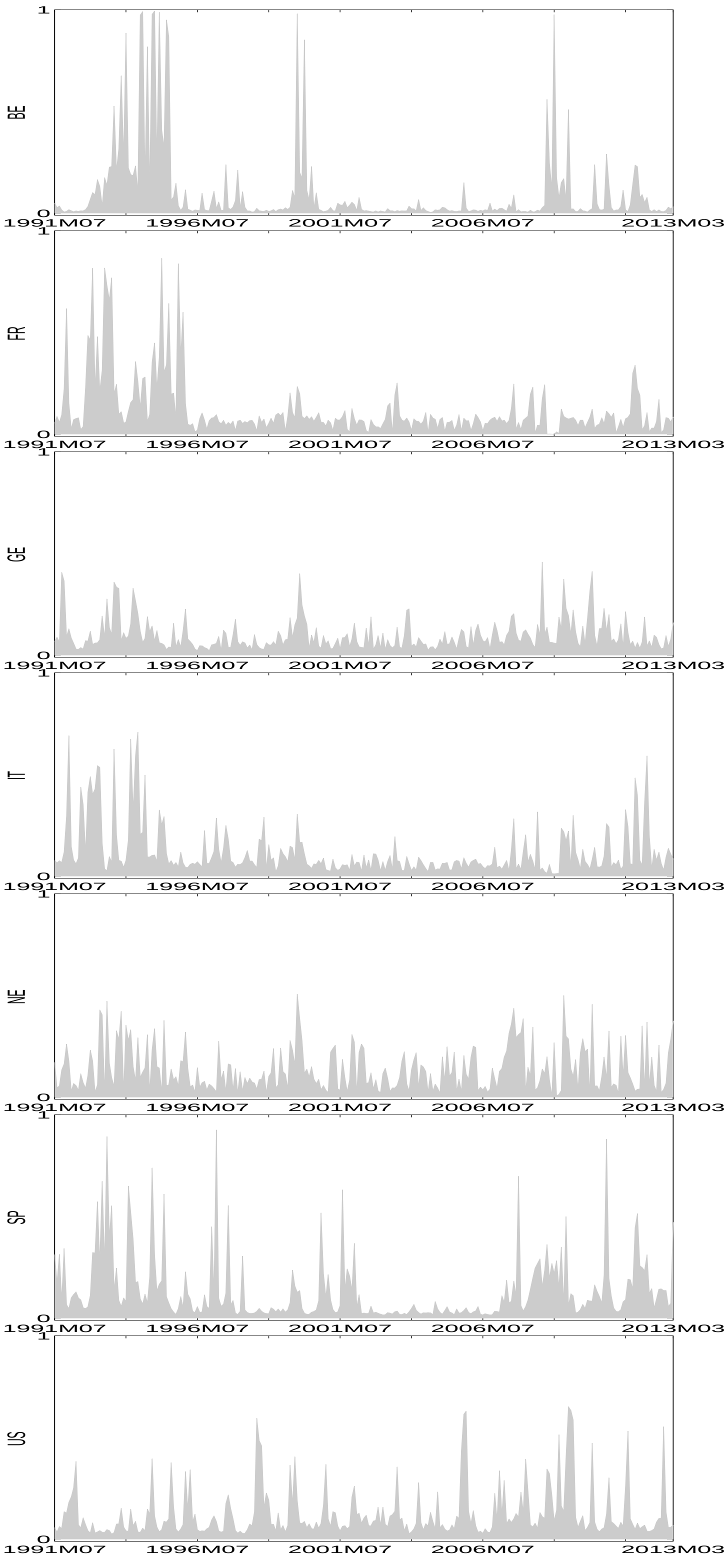

Figure 11: Third regime (strong expansion) smoothing probabilities for the Markov-switching processes $s_{i,t}$, $i = 1, \ldots, N$ and $t = 1, \ldots, T$.

D Cycle dynamic features

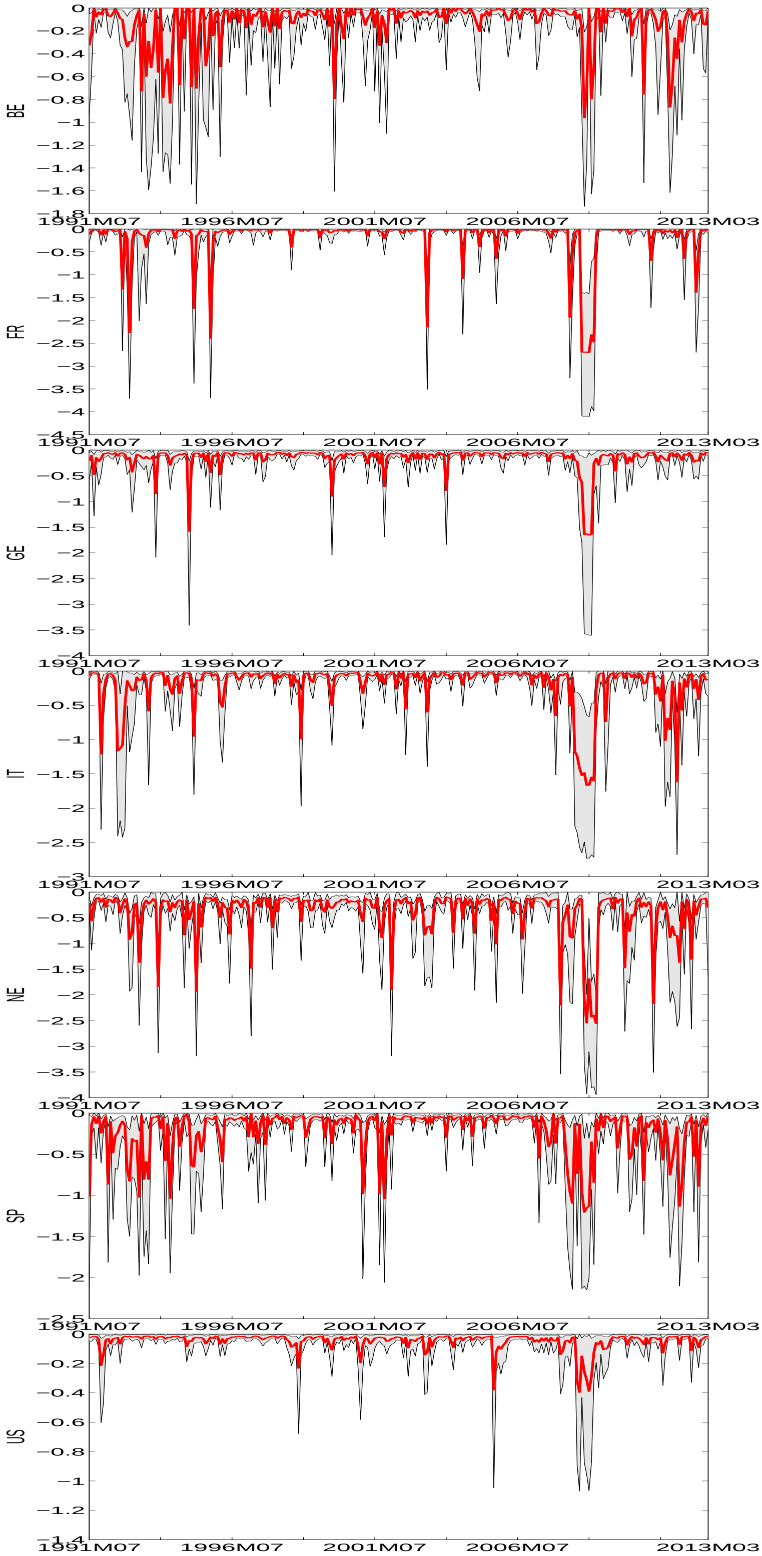

Figure 12: First regime (recession) unobserved conditional intercepts with 90% credible intervals.

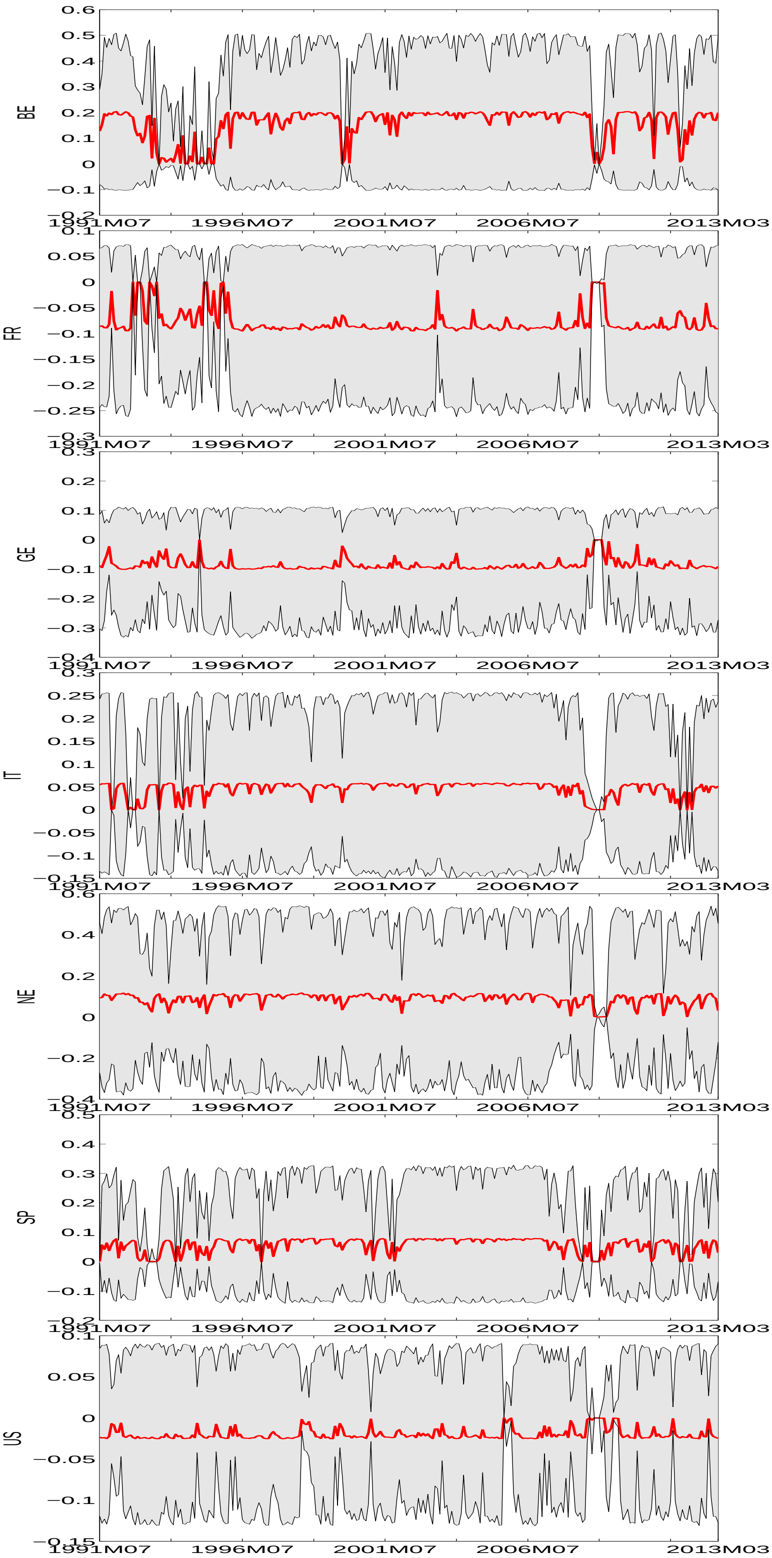

Figure 13: Second regime (moderate expansion) unobserved conditional intercepts with 90% credible intervals.

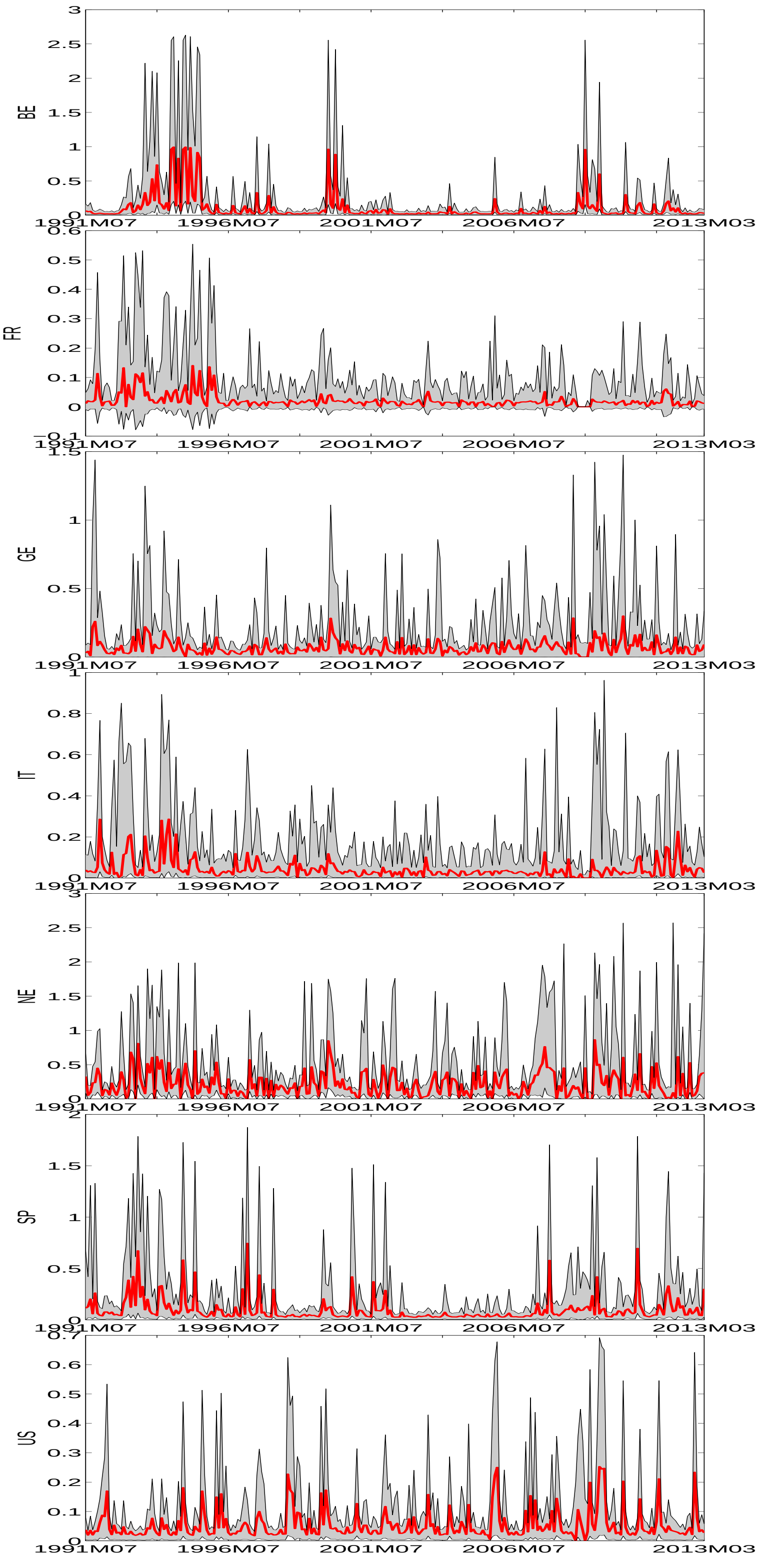

Figure 14: Third regime (strong expansion) unobserved conditional intercepts with 90% credible intervals.

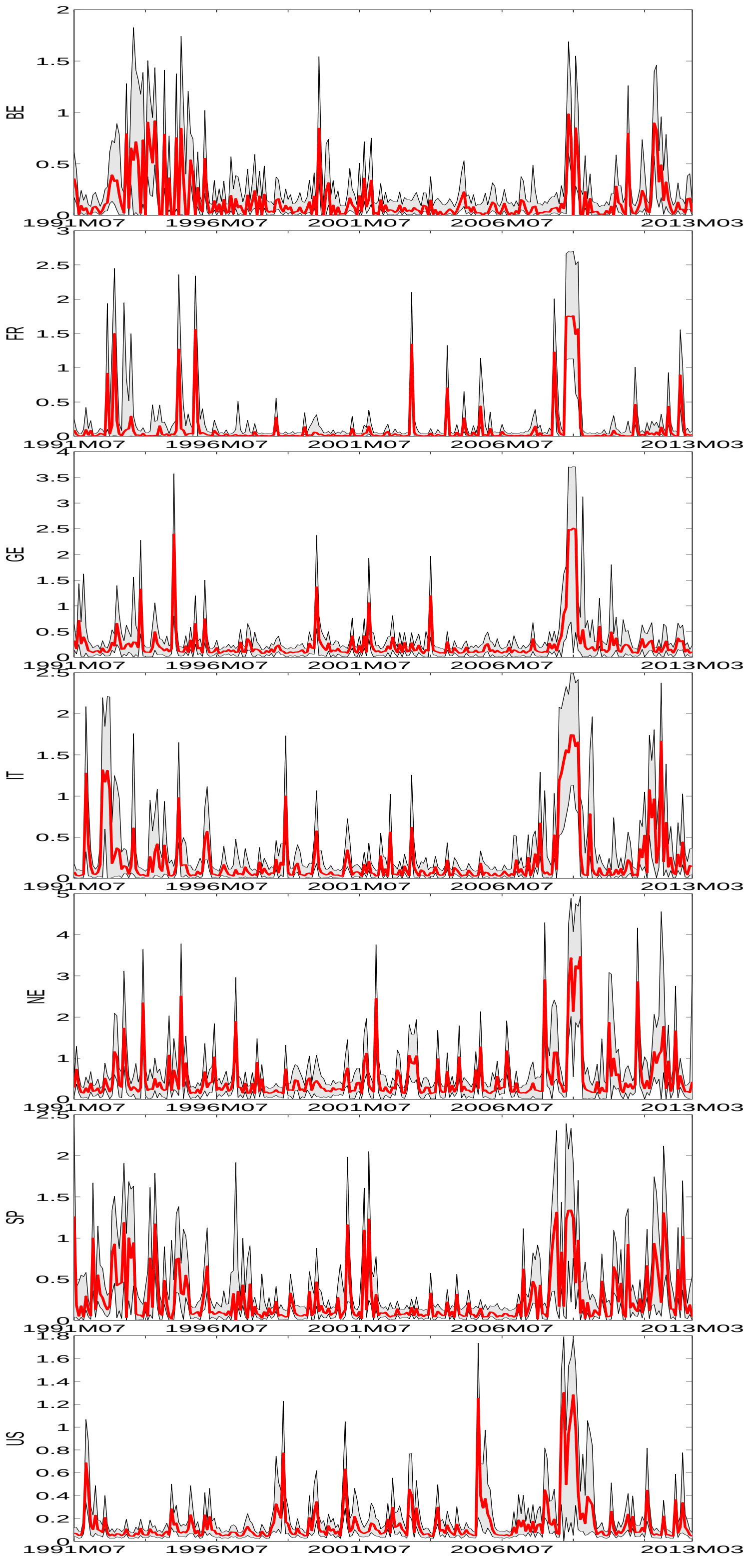

Figure 15: First regime (recession) unobserved conditional standard deviations with 90% credible intervals.

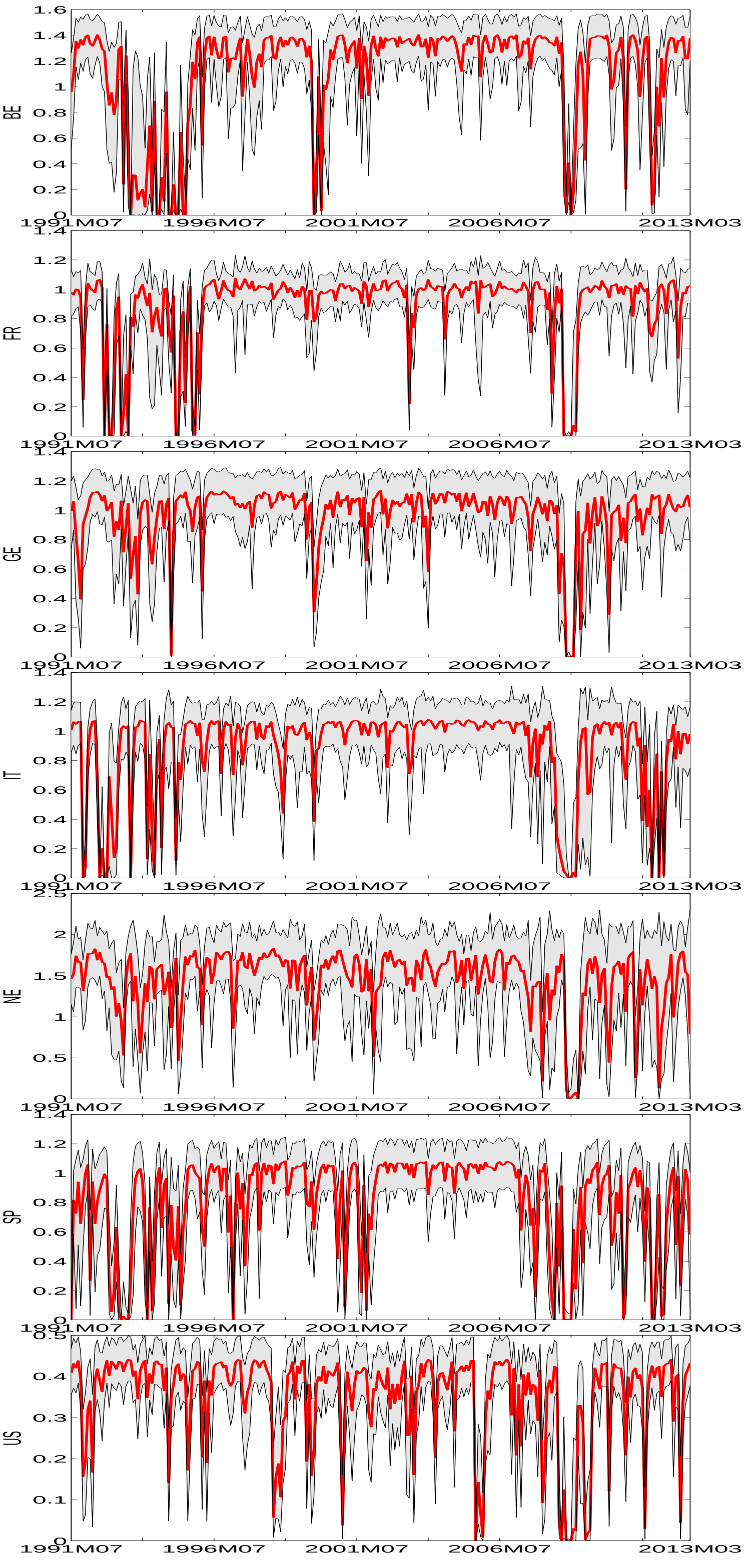

Figure 16: Second regime (moderate expansion) unobserved conditional standard deviations with 90% credible intervals.

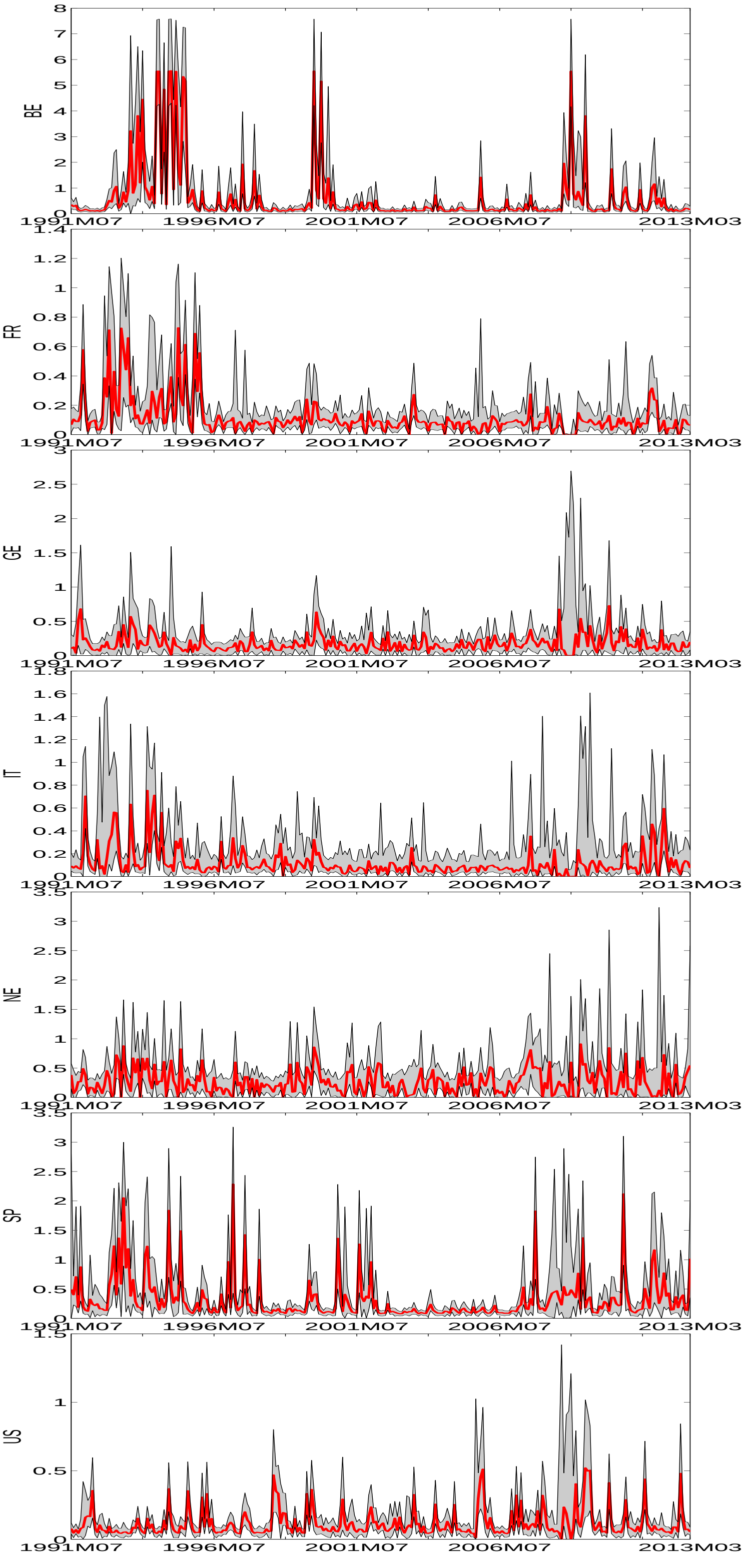

Figure 17: Third regime (strong expansion) unobserved conditional standard deviations with 90% credible intervals.