The phase between the three gluon and one photon amplitudes in quarkonium decays

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Abstract

The phase between three-gluon and one-photon amplitudes in $\psi(2S)$ and $\psi(3770)$ decays is analyzed.

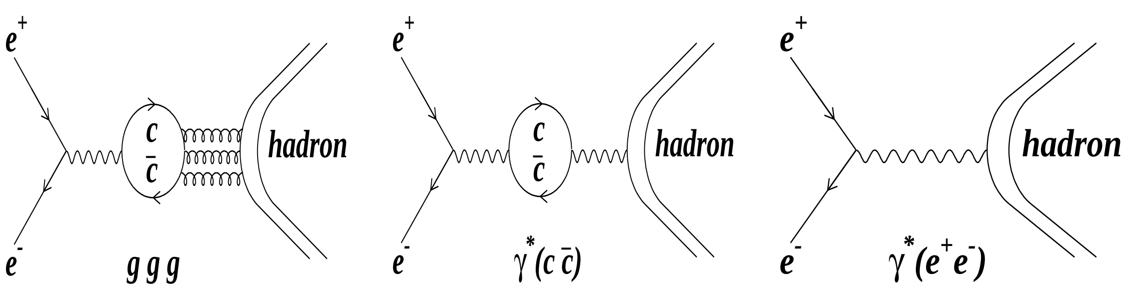
1 Motivations

It has been known that in J/ψ decays, the three gluon amplitude a_{3g} and one-photon amplitude a_γ are orthogonal for the decay modes 1^+0^- $(90°)$ [1], 1^-0^- $(106 \pm 10)°$ [2], 0^-0^- $(89.6 \pm 9.9)°$ [3], 1^-1^- $(138 \pm 37)°$ [4] and $N\overline{N}$ $(89 \pm 15)°$ [5].

J. M. Gérard and J. Weyers [6] augued that this large phase follows from the orthogonality of three-gluon and one-photon virtual processes. The question arises: is this phase universal for quarkonium decays? How about $\psi(2S)$, $\psi(3770)$ and $\Upsilon(nS)$ decays?

2 Quarkonium produced in electron-positron colliding experiments

Recently, more $\psi(2S)$ data has been available. Most of the branching ratios are measured in e^+e^- colliding experiments. For these experiments, there are three diagrams [7, 8], as shown in Fig. 1, which contribute to the processes. Although such formulas were written in the early years after J/ψ was discovered, but



(a) three-gluon annihilation (b) one-photon annihilation (c) one-photon continuum

Figure 1: The Feynman diagrams of $e^+e^- \rightarrow$ *light hadrons* at charmonium resonance.

the diagram in Fig. 1(c) is usually neglected. This reflects a big gap between theory and the actual experiments.

How important is this ampitude? For $\psi(2S)$, at first glance, $\sigma_{Born} = 7887$nb; while $\sigma_c \approx 14$nb. But for e^+e^- processes, initial state radiation modifies the Breit-Wigner cross section. With radiative correction, $\sigma_{r.c.} = 4046$nb; more important, the e^+e^- colliders have finite beam energy resolution, with Δ at the order of magnitude of MeV; while the width of $\psi(2S)$ is only 300KeV. Here Δ is the standard deviation of the guassian function which describes the C.M. energy distribution of the electron-positron. This reduces the observed cross section by an order of magnitude. For example, with $\Delta = 1.3$MeV

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(parameter of BES/BEPC at the energy of $\psi(2S)$ mass), $\sigma_{obs} = 640$nb. If $\Delta = 2.0$MeV (paramters of DM2/DCI experiment at the same energy), $\sigma_{obs} = 442$nb.

The contribution from direct one-photon annihilation is most important for pure electromagnetic process, like $\mu^+\mu^-$, where the continuum cross section is as large as the resonance itself and the interference is apparent. This is seen in the $\mu^+\mu^-$ cross section curve in the experimental scan of $\psi(2S)$ resonance, as shown in Fig. 2.

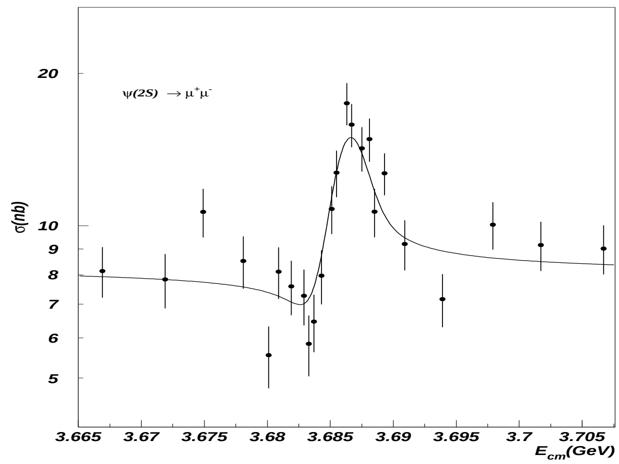


Figure 2: $\mu^+\mu^-$ curve at $\psi(2S)$ resonance scaned by BES

The observed cross section depends on experimental details: s_m, Δ, *etc.* [8]. The resonance cross section depends on the beam energy resolution of the e^+e^- collider; on the other hand, the continuum cross section depends on the invariant mass cut s_m in the selection criteria. This is seen from the treatment of the radiative correction [9]:

$$\sigma_{r.c.}(s) = \int_0^{1-\frac{s_m}{s}} dx F(x,s) \frac{\sigma_0(s(1-x))}{|1-\Pi(s(1-x))|^2} .$$

3 Pure electromagnetic decay

BES reports $\mathcal{B}(\psi(2S) \to \omega\pi^0) = (3.8\pm1.7\pm1.1)\times10^{-5}$. What it means is the cross section of $e^+e^- \to \omega\pi^0$ at $\psi(2S)$ mass is measured to be $(2.4 \pm 1.3) \times 10^{-2}$ nb. About 60% of this cross section is due to continuum [10]. This gives the form factor $\mathcal{F}_{\omega\pi^0}(M^2_{\psi(2S)})/\mathcal{F}_{\omega\pi^0}(0) = (1.6\pm0.4)\times10^{-2}$. It agrees well with the calculation by J.-M. Gérard and G.López Castro [11] which predicts it to be $(2\pi f_\pi)^2/3s = 1.66\times10^{-2}$ with f_π the pion decay constant. Similarly π form factor at $\psi(2S)$ is revised [10].

4 $\psi(2S) \to 1^-0^-$ and 0^-0^- decays

The $\psi(2S) \to 1^-0^-$ decays are due to three-gluon amplitude a_{3g} and one-photon amplitude a_γ. With these two amplitudes, a previous analysis [12] yielded $a_{3g} \approx -a_\gamma$, i.e. the phase ϕ between a_{3g} and a_γ is 180° and $\phi = 90°$ is ruled out. Here the SU(3) breaking amplitude ϵ is small compared with a_{3g}. But these branching ratios so far are all measured by e^+e^- experiments. So actually we have three diagrams and three amplitudes. The analysis should be based on Table 1:

modes	amplitude	B.R.(in 10^{-4})
$\rho^+\pi^-$ $(\rho^0\pi^0)$	$a_{3g}+a_\gamma+a_c$	< 0.09
$K^{*+}K^-$	$a_{3g}+\epsilon+a_\gamma+a_c$	< 0.15
$K^{*0}K^0$	$a_{3g}+\epsilon-2(a_\gamma+a_c)$	$0.41\pm0.12\pm0.08$
$\omega\pi^0$	$3(a_\gamma+a_c)$	$0.38\pm0.17\pm0.11$

Table 1: $e^+e^- \to \psi(2S) \to 1^-0^-$ process

In Table 1, a_{3g} interferes with $a_\gamma + a_c$, destructively for $\rho\pi$ and $K^{*+}K^-$, but constructively for $K^{*0}\overline{K^0}$ (ϵ is a fraction of a_{3g}). Fitting measured $K^{*+}K^-$ and $\rho\pi$ modes with different ϕ's are listed in Table 2.

It shows that a $-90°$ phase between a_{3g} and a_γ is still consistant with the data within one standard deviation of the experimental errors [13].

ϕ	$\mathcal{C} = \left|\frac{a_{3g}}{a_\gamma}\right|$	$\sigma_{pre}(K^{*+}K^-)$(pb)	$\mathcal{B}^0_{K^{*+}K^-}(\times 10^{-5})^1$	$\sigma_{pre}(\rho^0\pi^0)$(pb)	$\mathcal{B}^0_{\rho^0\pi^0}(\times 10^{-5})$
$+76.8°$	$7.0^{+3.1}_{-2.2}$	37^{+24}_{-23}	$5.0^{+3.2}_{-3.1}$	64^{+43}_{-41}	$9.0^{+6.1}_{-6.0}$
$-72.0°$	$5.3^{+3.1}_{-2.6}$	19^{+14}_{-14}	$3.1^{+2.3}_{-2.3}$	33^{+25}_{-24}	$5.5^{+4.1}_{-4.0}$
$-90°$	$4.5^{+3.1}_{-2.6}$	12^{+9}_{-9}	$2.0^{+1.5}_{-1.5}$	22^{+17}_{-17}	$3.7^{+2.9}_{-2.9}$
$180°$	$3.4^{+3.0}_{-2.2}$	$4.0^{+4.3}_{-3.2}$	$0.39^{+0.42}_{-0.31}$	$7.8^{+8.6}_{-6.7}$	$1.0^{+1.1}_{-0.8}$
BES observed		< 9.6		< 5.8	

Table 2: Calculated results for $\psi(2S) \to K^{*+}K^-$ and $\rho^0\pi^0$ with different ϕ.

The newly measured $\psi(2S) \to K_S K_L$ from BES-II [15], together with previous results on $\pi^+\pi^-$ and K^+K^-, is also consistant with a $-90°$ phase between a_{3g} and a_γ [14]. This is discussed in more detail by X.H. Mo in this conference.

5 $\psi(3770) \to \rho\pi$

J.L.Rosner [16] proposed that the $\rho\pi$ puzzle is due to the the mixing of $\psi(2S)$ and $\psi(1D)$ states, with the mixing angle $\theta = 12°$. In this scenario, the missing $\rho\pi$ decay mode of $\psi(2S)$ shows up instead as decay mode of $\psi(3770)$, enhanced by the factor $1/sin^2\theta$. He predicts $\mathcal{B}_{\psi(3770)\to\rho\pi} = (4.1 \pm 1.4) \times 10^{-4}$. With the total cross section of $\psi(3770)$ at Born order to be (11.6 ± 1.8) nb , $\sigma^{Born}_{e^+e^-\to\psi(3770)\to\rho\pi} = (4.8 \pm 1.9)$ pb .

But one should be reminded that for $\psi(3770)$, the resonance cross section, with radiative correction is only 8.17nb, while the continuum is 13nb. So to measure it in e^+e^- experiments, we must know the cross section $e^+e^- \to \gamma^* \to \rho\pi$. The cross section $\sigma_{e^+e^-\to\gamma^*\to\rho\pi}(s)$ can be estimated by the electromagnetic form factor of $\omega\pi^0$, since from SU(3) symmetry, the coupling of $\omega\pi^0$ to γ^* is three times of $\rho\pi$ [17]. The $\omega\pi^0$ form factor measured at $\psi(2S)$ is extrapolated to $\sqrt{s} = M_\psi(3770)$ by $|\mathcal{F}_{\omega\pi^0}(s)| = 0.531$ GeV$/s$. With this, the continuum cross section of $\rho\pi$ production at $\psi(3770)$ $\sigma^{Born}_{e^+e^-\to\gamma^*\to\rho\pi} = 4.4$ pb . Compare the two cross sections, the problem arises : how do these two interfere with each other?

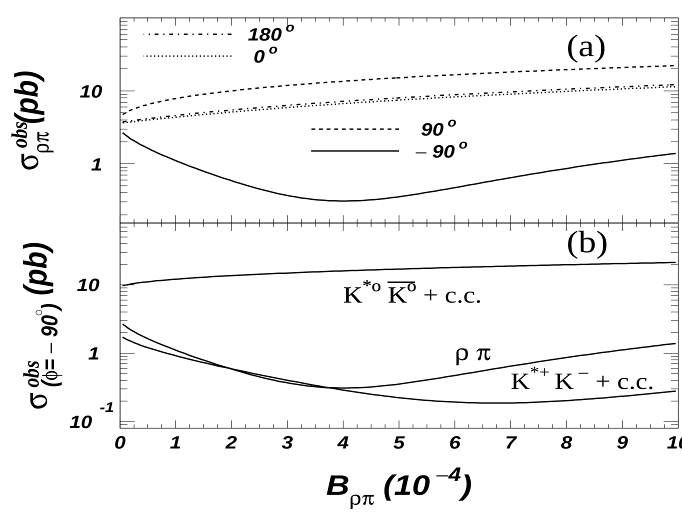
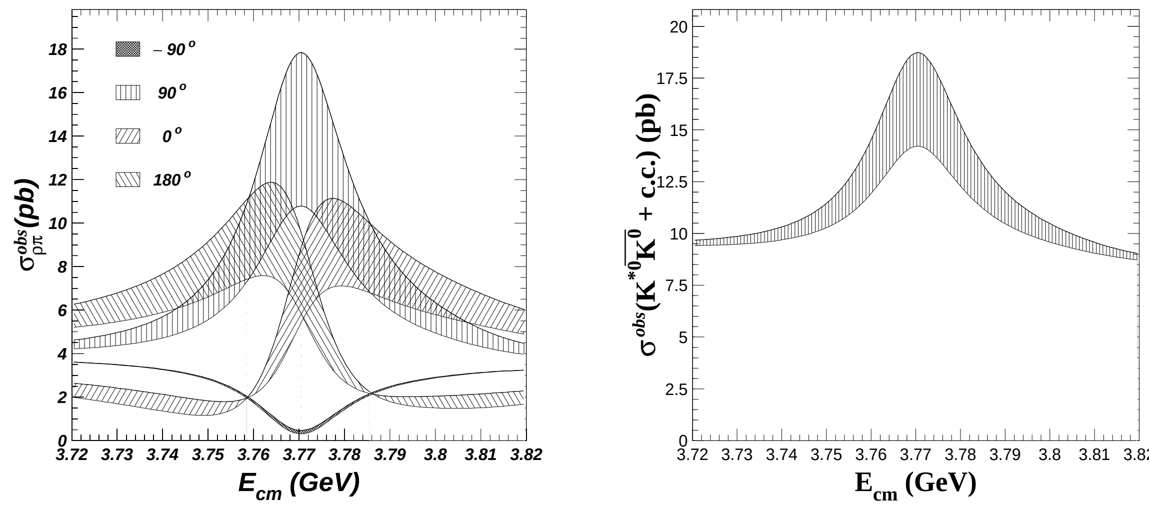


Figure 3: (a) $e^+e^- \to \rho\pi$ cross section as a function of $\mathcal{B}_{\psi(3770)\to\rho\pi}$ for different phases, and (b) $e^+e^- \to K^{*0}\overline{K^0}+c.c.$, $K^{*+}K^-+c.c.$, and $\rho\pi$ cross sections as functions of $\mathcal{B}_{\psi(3770)\to\rho\pi}$.

If the phase between a_{3g} and a_γ is $-90°$, then as in the case of $\psi(2S)$, the interference between a_{3g} and a_c is destructive in $\rho\pi$ and $K^{*+}K^-$ modes, but constructive in $K^{*0}\overline{K^0}$ mode [18]. The $e^+e^- \to \rho\pi$ cross section at $\psi(3770)$, as a function of $\mathcal{B}_{\psi(3770)\to\rho\pi}$ for different ϕ's are shown in Fig. 3(a); while the $e^+e^- \to \rho\pi, K^{*+}K^-, K^{*0}\overline{K^0}$ cross sections as functions of $\mathcal{B}_{\psi(3770)\to\rho\pi}$ for $\phi = -90°$ are shown in Fig. 3(b). To measure $\psi(3770) \to \rho\pi$ in e^+e^- collision, we must scan the $\psi(3770)$ peak (as we measure Γ_{ee}, Γ_{total} and $M_{\psi(3770)}$).

Fig. 4(a) shows the $e^+e^- \to \rho\pi$ cross section vs C.M. energy for different ϕ's. Fig. 4(b) shows the $e^+e^- \to K^{*0}\overline{K^0}$ cross section with $\phi = -90°$.

MARK-III gives $\sigma_{e^+e^-\to\rho\pi}(\sqrt{s} = M_{\psi(3770)}) < 6.3$ pb, at 90% C.L. [19]. It favors $-90°$.



(a) $e^+e^- \to \rho\pi$ (b)$e^+e^- \to K^{*0}\overline{K^0}$

Figure 4: (a) $e^+e^- \to \rho\pi$ cross section vs C.M. energy for different phases: $\phi = -90°$, $+90°$, $0°$, and $180°$ respectively. (b) $e^+e^- \to K^{*0}\overline{K^0}$ cross section vs C.M. energy with $\phi = -90°$.

$\psi(3770) \to 1^-0^-$ modes test the universal orthogonal phase between a_{3g} and a_γ in quarkonium decays as well as Rosner's scenario. A small cross section of $e^+e^- \to \rho\pi$ at $\psi(3770)$ peak means $\mathcal{B}(\psi(3770) \to \rho\pi) \approx 4 \times 10^{-4}$. (With radiative correction, the cancellation between a_{3g} and a_c cannot be complete. With a practical cut on the $\rho\pi$ invariant mass, the cross section is a fraction of 1pb.) It also implies the phase of the three gluon amplitude relative to one-photon decay amplitude is around $-90°$. These will be tested by the $20pb^{-1}$ of $\psi(3770)$ data by BES-II, or $5pb^{-1}$ of $\psi(3770)$ data by CLEO-c.

6 Summary

- The universal orthogonality between a_{3g} and a_γ found in various decay modes of J/ψ can be generalized to $\psi(2S)$ and $\psi(3770)$ decays. A $-90°$ phase between a_{3g} and a_γ is consistant with the data on $\psi(2S) \to 1^-0^-$ and 0^-0^- modes.
- The $\psi(3770) \to \rho\pi, K^{*+}K^-, K^{*0}\overline{K^0}$ test the universal $-90°$ phase, as well as Rosner's scenario on $\rho\pi$ puzzle. This should be pursued by BES-II and CLEO-c.
- The exisiting $\Upsilon(nS)$ data should be used to test the phase in bottomonium states.

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