New correlations induced by nuclear supersymmetry

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Abstract. We show that the nuclear supersymmetry model (n-susy) in its extended version, predicts correlations in the nuclear structure matrix elements which characterize transfer reactions between nuclei that belong to the same supermultiplet. These correlations are related to the fermionic generators of the superalgebra and if verified experimentally can provide a direct test of the model.

The extended n-susy model [1] correlates spectroscopic properties of adjacent nuclei. It has been particularly sucessful in describing the quartet formed by the isotopes ^{194}Pt, ^{195}Pt, ^{195}Au and ^{196}Au using the $Spin(6)$ limit of the dynamical supersymmetry $U_\nu(6/12)\otimes U_\pi(6/4)$ [1, 2] in which the odd-proton is allowed to occupy the $\pi d_{3/2}$ orbit, the odd-neutron occupies the $\nu p_{1/2}$, $\nu p_{3/2}$ and $\nu f_{5/2}$ orbits, even-even nuclei are described by the $SO(6)$ limit of the interacting boson model.

In this framework, all states belong to the same irreducible representation (irrep) of the initial product of supergroups and can be labeled by different irreps of the subgroups present in the chain of groups. The states in the quartet are said to form a supermultiplet and the excitation spectra is obtained using the same hamiltonian. In this way the excitation energies are correlated. Transitions probabilities and moments are also correlated through the use of the same operators.

Both the hamiltonian and the electromagnetic transition operators are based on the bosonic generators of the superalgebra, which transform bosons into bosons and fermions into fermions, inducing transitions inside each nucleus. In this work we explore the fermionic generators, which transform bosons into fermions and viceversa. They are thus associated to transitions among different nuclei, which we can associate to transfer reactions.

The purpose of this contribution is to show that n-susy establishes correlations between different transfer reactions. We start with one-nucleon transfer reactions and then we present the first results, to our knowledge, of a susy analysis of the two-nucleon transfer reaction ^{198}Hg$(\vec{d},\alpha)^{196}$Au, in which the wave function correlations of proton-neutron clusters in the target can be tested.

CORRELATIONS IN ONE-NUCLEON TRANSFER REACTIONS

The nuclear structure information which can be extracted from one-nucleon transfer reactions is contained in the so called spectroscopic intensity, which is the modulus square of the reduced matrix element of the transfer operator $T^{(J)}$ between the ground state of the target nucleus in the reaction, and the final state of the residual nucleus

$$I = |\langle \alpha_f J_f || T^{(J)} || \alpha_i J_i \rangle|^2. \tag{1}$$

It is possible to use a transfer operator deduced from microscopic asumptions to calculate these quantities, but here we use an alternative method based on symmetry considerations. In this case, the transfer operator can be written as a tensor operator under the subgroups that appear in the group chain of the dynamical supersymmetry and has the advantage of giving rise to selection rules and closed expressions for the spectroscopic intensities [3, 4, 5].

For the one-proton transfer reactions, the transfer tensor operators read [5]

$$T_{1,\pi}^{\langle \frac{1}{2},\frac{1}{2},-\frac{1}{2}\rangle (\frac{1}{2},\frac{1}{2}) \frac{3}{2}} = -\sqrt{\frac{1}{6}} \left(\tilde{s}_\pi \times a^\dagger_{\pi,\frac{3}{2}} \right)^{(\frac{3}{2})} + \sqrt{\frac{5}{6}} \left(\tilde{d}_\pi \times a^\dagger_{\pi,\frac{3}{2}} \right)^{(\frac{3}{2})} \tag{2}$$

$$T_{2,\pi}^{\langle \frac{3}{2},\frac{1}{2},\frac{1}{2}\rangle (\frac{1}{2},\frac{1}{2}) \frac{3}{2}} = \sqrt{\frac{5}{6}} \left(\tilde{s}_\pi \times a^\dagger_{\pi,\frac{3}{2}} \right)^{(\frac{3}{2})} + \sqrt{\frac{1}{6}} \left(\tilde{d}_\pi \times a^\dagger_{\pi,\frac{3}{2}} \right)^{(\frac{3}{2})}, \tag{3}$$

where the upper indices specify the tensorial properties under $Spin(6)$, $Spin(5)$ and $Spin(3)$. The tensorial character under $Spin(6)$ of $T_{1,\pi}$ implies that it only excites the ground state of the odd-even nucleus from the ground state of the even-even nucleus. However, $T_{2,\pi}$ allows the transfer to an excited state in the odd-even nucleus. The ratio of the intensities for each transfer operator is given by [5]

$$R_1(\mathrm{ee} \to \mathrm{oe}) = \frac{I_{gs\to exc}}{I_{gs\to gs}} = 0, \tag{4}$$

$$R_2(\mathrm{ee} \to \mathrm{oe}) = \frac{I_{gs\to exc}}{I_{gs\to gs}} = \frac{9(N+1)(N+5)}{4(N+6)^2}, \tag{5}$$

where N is taken as the number of bosons in the odd-odd nucleus of the quartet and ee and oe refer to even-even and odd-even respectively. In the case of the one-proton transfer ${}^{194}\mathrm{Pt} \to {}^{195}\mathrm{Au}$, the second ratio is $R_2 = 1.12$ ($N = 5$), but the relatively small strength to excited $J = \frac{3}{2}$ states suggests that the operator $T_{1,\pi}$ can be used to describe the data.

Due to the F-spin symmetry structure of the wave functions it is possible to establish the following correlations with the reaction which involves the even-odd (eo) and odd-odd(oo) nuclei:

$$R_1(\mathrm{ee} \to \mathrm{oe}) = R_1(\mathrm{eo} \to \mathrm{oo}) = 0, \tag{6}$$

$$R_2(\mathrm{ee} \to \mathrm{oe}) = R_2(\mathrm{eo} \to \mathrm{oo}) = \frac{9(N+1)(N+5)}{4(N+6)^2}. \tag{7}$$

A different way to understand this result is through the use of a tensor operator which transforms as a scalar in the pseudo-l degree of freedom [6] (upper and lower indices specify the tensorial properties under $Spin(6)$ and $Spin(5)$, $Spin(3)$ and $SU(2)$ respectively)

$$P^{\langle 0,0,0\rangle}_{(0,0)0,\frac{1}{2}} = \left(\tilde{s}_\nu \times a^\dagger_{\nu,\frac{1}{2}}\right)^{\left(\frac{1}{2}\right)} - \sqrt{2}\left(\tilde{d}_\nu \times a^\dagger_{\nu,\frac{3}{2}}\right)^{\left(\frac{1}{2}\right)} + \sqrt{3}\left(\tilde{d}_\nu \times a^\dagger_{\nu,\frac{5}{2}}\right)^{\left(\frac{1}{2}\right)}. \quad (8)$$

This operator links the wave functions of the even-odd and odd-odd nuclei in the quartet to the wave functions of the even-even and odd-even nuclei, respectively. The use of this property and the fact that P conmutes with the one-proton transfer operators $T_{1,\pi}$ and $T_{2,\pi}$, allows to write the equations (6) and (7) [7, 8].

We thus find a direct correlation between two different one-proton reactions, which for the case of $^{195}\mathrm{Pt} \rightarrow ^{196}$ Au can be tested experimentally. Very recently this reaction has been measured [9] and the experimental data confirms that $T_{1,\pi}$ is capable of describing it, given that the strength to the ground state of ^{196}Au is relatively strong and the excited state which $T_{2,\pi}$ can excite is not seen populated in the measurements. This fact confirms also that the correlation seems to apply.

For the one neutron transfer reaction there is a similar correlation. Let us consider the following transfer tensor operator (labels like in (8)) [6]

$$T^{(2,0,0)}_{(1,0)2,j,\nu} = \sqrt{\frac{1}{2}}\left(\tilde{s}_\nu \times a^\dagger_{\nu,j}\right)^{(j)} - \sqrt{\frac{1}{2}}\left(\tilde{d}_\nu \times a^\dagger_{\nu,\frac{1}{2}}\right)^{(j)} \quad j = \frac{3}{2}, \frac{5}{2}. \quad (9)$$

The same argument about the link between the wave functions of the even-even and the odd-even nuclei permits us to correlate the one neutron transfer reaction even-even $\rightarrow$ odd-even with the inverse reaction odd-even $\rightarrow$ even-even [8]. Taking the following ratios

$$R(\mathrm{ee} \rightarrow \mathrm{oe}) = \frac{\left|\left\langle \mathrm{oe}; [N_1,N_2]\langle\sigma_1,\sigma_2,\sigma_3\rangle(1,0)2;J \left\| T^{(2,0,0)}_{(1,0)2,j,\nu} \right\| \mathrm{ee;\ g.s.}\right\rangle\right|^2}{\left|\left\langle \mathrm{oe}; [N+1,1]\langle N+1,1,0\rangle(1,0)2;J \left\| T^{(2,0,0)}_{(1,0)2,j,\nu} \right\| \mathrm{ee;\ g.s.}\right\rangle\right|^2}, \quad (10)$$

$$R(\mathrm{oe} \rightarrow \mathrm{ee}) = \frac{\left|\left\langle \mathrm{ee}; [N_1,N_2]\langle\sigma_1,\sigma_2,\sigma_3\rangle(1,0)2 \left\| \tilde{T}^{(2,0,0)}_{(1,0)2,j,\nu} \right\| \mathrm{oe;\ g.s.}\right\rangle\right|^2}{\left|\left\langle \mathrm{ee}; [N+1,1]\langle N+1,1,0\rangle(1,0)2 \left\| \tilde{T}^{(2,0,0)}_{(1,0)2,j,\nu} \right\| \mathrm{oe;\ g.s.}\right\rangle\right|^2}, \quad (11)$$

we find the following relations

$$R(\mathrm{oe} \rightarrow \mathrm{ee}) = R(\mathrm{ee} \rightarrow \mathrm{oe}) \text{ for } N_2 = 1,$$

$$R(\mathrm{oe} \rightarrow \mathrm{ee}) = R(\mathrm{ee} \rightarrow \mathrm{oe})\frac{(N+1)(N_\nu+1)}{(N_\pi+1)} \text{ for } N_2 = 0,$$

In table 1 we show these ratios and quote the experimental values [10] and the calculated ones for the isotopes ^{194}Pt and ^{195}Pt. We can observe a consistence between the experimental and calculated values, but clearly more experimental work is needed to confirm if these correlations hold.

Table 1. Intensity ratios for one-neutron transfer reactions.

$[N_1,N_2]\langle\sigma_1,\sigma_2,\sigma_3\rangle$	$R_{ee\to oe}$	$^{194}\mathrm{Pt}\to{}^{195}\mathrm{Pt}$ calc.	exp. $j=\frac{3}{2}$	exp. $j=\frac{5}{2}$	$^{195}\mathrm{Pt}\to{}^{194}\mathrm{Pt}$ calc.	exp.
$[N+2]\langle N+2,0,0\rangle$	$\frac{2(N+4)}{(N+1)(N+3)(N+6)}$	0.034	0.264	0.052	0.511	——
$[N+2]\langle N,0,0\rangle$	$\frac{N(N+4)(N+5)}{2(N+2)(N+3)(N+6)^2}$	0.033	——	——	0.498	——
$[N+1,1]\langle N,0,0\rangle$	$\frac{N^2(N+5)}{2(N+2)(N+6)^2}$	0.148	0.087	——	0.148	——

CORRELATIONS IN TWO-NUCLEON TRANSFER REACTIONS

In contrast to the one-nucleon transfer reactions in which the single particle structure of nuclear states is examinated, two-nucleon transfer reactions are very sensitive to the correlation between the transfered nucleons. As a consequence, this kind of transfer reactions supply a stringent test for the nuclear wave functions.

Two factors determine the strength of a two nucleon transfer reaction, which are related to the two-nucleon fractional parentage coefficients. On the one hand it depends on how similar the state of $A+2$ nucleons is to the state of A plus two additional nucleons, and on the other, if the correlation of these two nucleons in the $A+2$ state is similar to the correlation in the state of the light nucleus. The nuclear structure information that can be extracted from a model is related to both factors. We shall follow the formalism developed by Glendenning [11] and briefly sketch the main ingredients as applied to the reaction $^{198}\mathrm{Hg}(\vec{d},\alpha)^{196}\mathrm{Au}$, which has been measured recently to study the odd-odd nucleus $^{196}\mathrm{Au}$ [9].

The nuclear structure information is contained in the spectroscopic strengths G_{LJ}. This quantities are written as

$$G_{LJ} = \left| \sum_{N j_\nu j_\pi} \beta_{j_\nu j_\pi} G^{j_\nu j_\pi}_{NLJ} \right|^2 , \tag{12}$$

where

$$\beta_{j_\nu j_\pi} = \left\langle {}^{196}\mathrm{Au}; J \left\| T^{(J)}_{j_\nu j_\pi} \right\| {}^{198}\mathrm{Hg}; 0_{gs} \right\rangle \tag{13}$$

$$G^{j_\nu j_\pi}_{NLJ} = \sqrt{3}\hat{L}\hat{j}_\nu\hat{j}_\pi \left\{ \begin{array}{ccc} l_\nu & \frac{1}{2} & j_\nu \\ l_\pi & \frac{1}{2} & j_\pi \\ L & 1 & J \end{array} \right\} \langle n\,0\,N\,L\,|n_\nu\,l_\nu\,n_\pi\,l_\pi\,;L\rangle\,. \tag{14}$$

For the two nucleon transfer operator $T^{(J)}_{j_\nu j_\pi}$ we have chosen the simplest possible form:

$$T^{(J)}_{j_\nu j_\pi} = \alpha_{j_\nu} \left(a^\dagger_{j_\nu} \times a^\dagger_{j_\pi} \right)^{(J)} . \tag{15}$$

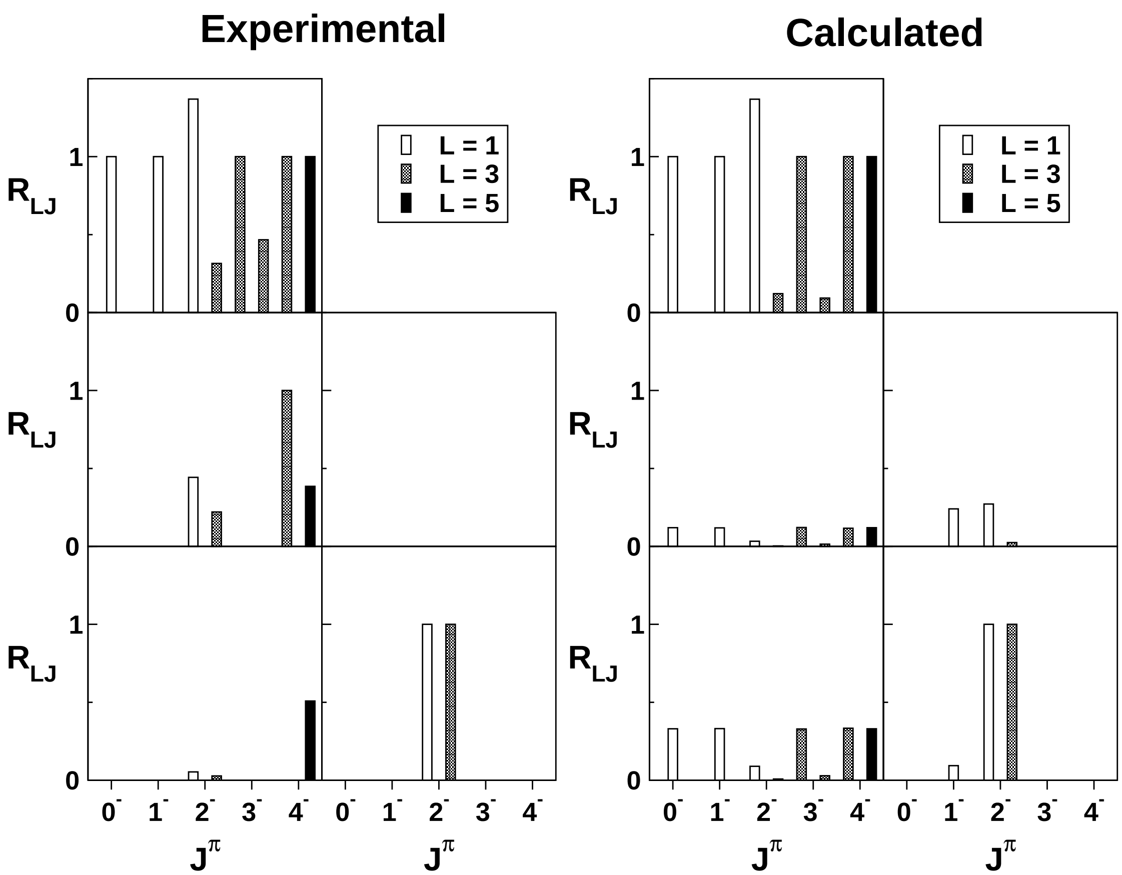

Figure 1. Ratios of spectroscopic strengths. The first column in each frame correspond to states with $\left(\frac{3}{2},\frac{1}{2}\right)$ $Spin(5)$ labels and the second with $\left(\frac{1}{2},\frac{1}{2}\right)$ labels, respectively. Each row from the bottom to the top corresponds to states with labels $[6,0]\langle 6,0\rangle(13/2,1/2,1/2)$, $[5,1]\langle 5,1\rangle(11/2,1/2,1/2)$ and $[5,1]\langle 5,1\rangle(11/2,3/2,1/2)$ for the groups $U^{BF}_{\nu\pi}(6)$, $SO^{BF}_{\nu\pi}(6)$ and $Spin(6)$.

The parameters α_{j_ν} are determined by using a least square fit of the spectroscopic strengths to the experimental values. To compare the experimental and calculated spectroscopic strengths we have choosen seven states of reference, each one corresponding to each of the seven possible LJ transfers that the n-susy model allows, and we have calculated for each LJ the transfer ratio

$$R_{LJ} = \frac{G_{LJ}}{G^{ref}_{LJ}}, \tag{16}$$

where G^{ref}_{LJ} is the spectroscopic strength for the reference state for a particular LJ tranfer. In figure 1 we show the experimental and calculated ratios R_{LJ}. We observe from this figure that the comparison is quite reasonable if we take into account the simple form we adopted for the two nucleon transfer operator. In concluding this section we can say that the n-susy model can reproduce remarkably well the main features of the neutron-proton correlations which are present in the nuclear states involved.

SUMMARY

We have found new correlations predicted by extended n-susy. These correlations relate spectroscopic strengths of different one-nucleon transfer reactions between the nuclei which are described by the model. We focused on the supermultiplet formed by ^{194}Pt, ^{195}Pt, ^{195}Au and ^{196}Au whose spectroscopic properties have been previously described in terms of this model. We have shown that these correlations are partially fulfilled with the availaible experimental data, but clearly more experimental work is necessary to test their full validity.

We have calculated the spectroscopic strengths associated to the two nucleon transfer reaction $^{198}\mathrm{Hg}(\vec{d},\alpha)^{196}$Au, which was recently measured. The comparison between the calculated and experimental data shows good agreement considering the simple form adopted for the two nucleon transfer operator.

We plan to search for other experimental examples to which extended n-susy and its correlations can be applied, eventually relaxing the constraints set by dynamical symmetry [12, 13]. We wish to emfasize that nuclear susy may be a model whose range of applicability is wider than was previously realized and which may lay the foundations of a new and unifying point of view in nuclear structure.

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