Psychology and Aging
1991, Vol. 6, No. 3, 474–483

Copyright 1991 by the American Psychological Association, Inc.
0882-7974/91/$3.00

Cognitive Addition: Strategy Choice and Speed-of-Processing Differences in Young and Elderly Adults

David C. Geary and Judith G. Wiley
University of Missouri—Columbia

Sixty young and 60 elderly adults completed a pencil-and-paper addition test and solved 40 computer-presented simple addition problems. Strategies and problem solution times were recorded on a trial-by-trial basis and were classified in accordance with the distributions of associations model of strategy choices. The elderly group showed a performance advantage on the ability measure and for the developmental maturity of the mix of problem-solving strategies, but the young group showed an advantage for overall solution times. A componential analysis of the overall solution times for memory retrieval trials, however, showed no reliable age difference for rate of retrieving addition facts from long-term memory but did suggest that the elderly adults might have been slower than the younger adults for rate of encoding digits and verbally producing an answer. Overall results are interpreted within the context of the strategy choice model.

The first purpose of this study was to extend into adulthood normative information on the development of problem-solving strategies in addition. The second purpose was to compare the performance of young and elderly adults on a relatively well-understood cognitive task: the mental solution of simple addition problems (Ashcraft, 1982; Widaman, Geary, Cormier, & Little, 1989). The cognitive addition task provides a useful vehicle for examining age-related performance differences for several reasons. First, the array of problem-solving strategies used for this task is well documented. In fact, at least for children, problem-solving strategies can be reliably classified on a trial-by-trial basis (Geary, 1990; Siegler & Shrager, 1984). Second, for each of the strategies, the overall solution times can be decomposed into more elementary information processes that might index, for example, the rate of retrieving facts from long-term memory (Geary & Burlingham-Dubree, 1989). Finally, the performance characteristics (e.g., error rates) associated with this task can be interpreted within the context of a more general model of cognitive development: the distributions of associations model of strategy choices (Siegler, 1986). Thus, performance on the addition task should enable a theory-driven assessment of age-related differences in the mix of problem-solving strategies and in the rate of executing the underlying processes. An overview of the strategy choice model precedes the review of related information in the cognitive arithmetic and cognitive aging domains.

Strategy Choice Model

Skill development can be characterized, in part, by improvements in the adaptive use of alternative problem-solving strategies (Charness, 1981; Geary & Burlingham-Dubree, 1989; Kaye, 1986). For a given cognitive task an array of strategies is typically available for problem solving. These strategies will differ, for instance, in the probability of producing the correct answer, the duration of the problem-solving processes, and the demands the strategy places on working memory resources (Brainerd, 1983; Siegler, 1986). An adaptive strategy choice is usually based on a weighted combination of these factors. Children make adaptive strategy choices in many cognitive domains, including arithmetic, reading, spelling, time telling, and reasoning (Siegler, 1989a). For adults, the relationship between task demands and problem-solving strategies is not as well documented. Nevertheless, for some tasks, the slowing in the rate of information processing often associated with aging can be compensated for by a shift to more efficient problem-solving strategies (e.g., Charness, 1981), suggesting adaptive strategy choices might also be an important component of skill development in adulthood. The strategy choice model (Siegler, 1986) delineates the mechanisms governing the adaptive use of alternative problem-solving strategies in children and might therefore provide a useful theoretical context for the comparison of strategy choices in young and elderly adults.

Within the model, the two primary mechanisms governing strategy choices are *associative strength* and a *confidence criterion.* More precisely, the strategy chosen for problem solving is influenced by the peakedness of the distribution of associations between a problem and all potential answers to that problem. With a peaked distribution, "the preponderance of associative strength is concentrated on a single answer (the peak of the distribution)" (Siegler, 1988, p. 834), whereas for a flat distribution the associative strength is distributed among several potential answers. The more peaked the distribution of associations, the more readily an answer can be retrieved from long-term memory and the more likely a memory retrieval process

---

This research was partially supported by a summer research fellowship from the graduate division of the University of Missouri at Columbia and by a grant from the Weldon Spring Foundation to David C. Geary. Judith G. Wiley was supported by Grant AG08214 from the National Institute on Aging, awarded to Donald Kausler.

We thank Michelle Owens and Wendy Warren for their assistance with data collation and Mark Ashcraft, Peter Frensch, Donald Kausler, and an anonymous reviewer for insightful comments on earlier drafts.

Correspondence concerning this article should be addressed to David C. Geary, Department of Psychology, 210 McAlester Hall, University of Missouri, Columbia, Missouri 65211.

will be used to solve the presented problem. The confidence criterion represents an internal standard against which confidence in the correctness of the retrieved answer is gauged (see Siegler, 1986).

The problem-solving process begins with setting the value of the confidence criterion followed by an attempt to retrieve the correct answer. If an answer cannot be retrieved or if the retrieved answer does not exceed the value of the confidence criterion, then retrieval might be attempted a second time or a backup strategy might be invoked to complete problem solving (Siegler, 1983). Backup strategies consume more time than retrieval-based processes, but are of greater accuracy *if* correctly executed. For simple addition, two backup strategies are likely to be used by adults if the correct answer is not readily retrievable from long-term memory. The first of these involves the *decomposition* of the problem into more simple problems (Siegler, 1987). For example, the problem "6 + 9" might be solved by subtracting 1 from the 6, then adding the 1 to the 9, and finally adding 5 and 10. The second backup strategy involves *verbal counting.* Here, the process, typically termed *min* or *counting on* (Groen & Parkman, 1972), starts with the cardinal value of the larger digit (e.g., "9" in the problem "6 + 9") and then counting in a unit-by-unit fashion a number of times equal to the value of the smaller or minimum (min) integer (i.e., "6") until a sum is obtained.[1]

In a recent revision of the strategy choice model it was argued that strategies compete for expression (Siegler & Jenkins, 1989). Thus, the presentation of a problem such as "6 + 9" activates not only candidate answers, such as "15," in long-term memory but also simultaneously activates long-term memory representations of procedural knowledge, such as a schema for executing the verbal counting strategy (Greeno, Riley, & Gelman, 1984). If the level of activation of the counting schema exceeds the level of activation of a candidate answer, then the counting algorithm will be executed. If the counting algorithm is correctly executed, then the stated answer, "15," becomes associated with the problem. Thus, when the problem "6 + 9" is again presented for solution, the level of activation of the candidate answer (i.e., "15") is higher than it was before the previous successful count.

Gradually, after many successful counts, re-presentation of the problem will lead to a stronger activation of the candidate answer, as compared with the level of activation of the schema for the counting algorithm. At this point, memory retrieval will be used to solve the problem. The use of counting and in fact other backup procedures, such as decomposition, therefore leads to their extinction by ensuring that memory retrieval becomes the dominant problem-solving process (Siegler & Jenkins, 1989). Nevertheless, some form of confidence criterion probably also influences whether a retrieved answer is stated or whether a backup strategy is used to verify the accuracy of the retrieved answer (Geary & Brown, 1991; Siegler, 1988). In all, within the context of the strategy choice model, for the simple addition task, *expert* or *developmentally mature* performance would be represented by the error-free retrieval of all basic facts.

Development of Skill in Cognitive Addition

Following the acquisition of basic concepts (Gelman & Meck, 1983), the development of skill in cognitive arithmetic proceeds on two dimensions: strategy choices and speed of information processing. The use of increasingly mature problem-solving strategies cannot be characterized by the substitution of one strategy, such as memory retrieval, for another less mature strategy, such as verbal counting (Ashcraft, 1982). Rather, "development involves changes in the mix of existing strategies as well as construction of new ones and abandonment of old ones" (Siegler & Jenkins, 1989, p. 27). For simple addition, accurate retrieval of the answer from long-term memory represents the developmentally most mature strategy choice. Children as young as 4 years of age can accurately use the retrieval strategy to solve some simple problems (e.g., "1 + 2"), but will count or use some other backup strategy to solve more difficult problems (e.g., "5 + 4"). Gradually, with practice, the frequency with which memory retrieval is used for problem solving increases and the use of backup strategies decreases (Siegler, 1986).

In fact, it is assumed that memory retrieval is the exclusive problem-solving strategy for simple arithmetic problems by the end of the elementary school years and certainly in adulthood (Ashcraft & Battaglia, 1978; Ashcraft & Fierman, 1982; Campbell, 1987a; Campbell & Graham, 1985; Geary, Widaman, & Little, 1986; Geary, Widaman, Little, & Cormier, 1987; Miller, Perlmutter, & Keating, 1984; Widaman et al., 1989). Despite this strong assumption, the use of backup strategies by adults for solving simple arithmetic problems has not been empirically tested in terms of developmental models (but see Svenson, 1985).

The second dimension that changes with skill development in arithmetic is the rate with which the substantive elementary operations can be executed (Geary, Brown, & Samaranayake, in press; Kail, 1988; Widaman, Little, Geary, & Cormier, in press). The decrease in the time for fact retrieval across the ages of 8 to 21 years, for example, can be represented by an exponential decay function, with the asymptotic value being approached during the high school years (Kail, 1988; Widaman et al., in press). It is not clear, however, if the rate of fact retrieval remains constant through middle and old age. The few studies that have compared young and elderly adults on rate of solving simple arithmetic problems have yielded mixed results (Birren & Botwinick, 1951; Charness, 1987; Charness & Campbell, 1988). Birren and Botwinick found a faster rate of solving simple addition problems for young as compared with elderly adults but no age difference in error rate, whereas Charness and Campbell found no age difference in overall rate of solving simple multiplication problems but a lower error rate for elderly individuals. Finally, Schaie (1989) reported improvements in performance, after controlling for age-related changes in perceptual speed, on pencil-and-paper measures of addition throughout most of the life span.

Cognitive Aging

Two issues in cognitive aging research are germane to our study. The first is age-related differences in problem-solving

---

[1] We monitored for other types of strategies, such as counting on fingers (Siegler, 1986), but our subjects never reported using these alternative strategies and we never saw any indication that these strategies were in fact used for problem solving. Because these alternative backup strategies were apparently never used, for the sake of brevity, they are not described.

strategies and the second involves differences in the rate of information processing comparing young and elderly adults. The evidence is mixed, with regard to the former issue. Charness (1981) found that elderly chess players when selecting a chess move showed a more efficient search of the problem space relative to younger peers, whereas Hartley and Anderson (1983) found that young adults used more efficient problem-solving strategies than elderly adults for the solution of the Twenty Questions task. Finally, Salthouse and Somberg (1982) argued that the problem-solving strategies used by young and elderly adults did not differ across an array of simple information-processing tasks (also see Charness & Campbell, 1988; Myerson, Hale, Wagstaff, Poon, & Smith, 1990). The key to resolving these apparently contradictory claims probably resides in both the complexity (i.e., working memory demands) of the task (Salthouse, Mitchell, Skovronek, & Babcock, 1989) and the level of task familiarity of the young and elderly subjects (Charness & Bieman-Copland, in press).

With regard to the rate of information processing, the evidence is less mixed but certainly not definitive. Many scientists in the cognitive aging area have argued that the rate of information processing slows with age but disagree as to the exact nature of the function relating age to the slowing of central (e.g., computational) and peripheral (i.e., sensory-motor) mechanisms (Birren, 1965; Cerella, 1985; Charness, 1981; Hertzog, 1989; Hertzog, Raskind, & Cannon, 1986; Madden, 1985; Mueller, Kausler, & Faherty, 1980; Salthouse & Somberg, 1982; Schaie, 1989). More recently, Cerella (1990) and Myerson et al. (1990) argued that aging is associated with a general slowing of the rate of information processing across all domains. The locus of this generalized slowing is conceptualized at the level of the neural network. Alternative explanations for age-related changes in information-processing rate include disuse, speed–accuracy trade-offs, and strategy shifts (cf. Cerella, 1990).

The disuse hypothesis posits that the slower rate of information processing of elderly, relative to young, adults is related to a lack of practice. Elderly adults are sometimes described as being more cautious than younger adults and therefore might differ from younger adults on the speed–accuracy trade-off continuum associated with many information-processing tasks. For the simple addition task, this could translate into a more rigorous confidence criterion for elderly subjects and therefore result in either (a) a relatively high usage of backup strategies or (b) a high frequency of retrieval trials but with rather long latencies. The former could occur if elderly individuals retrieved an answer from memory and then resorted to a backup strategy to verify the accuracy of the retrieved answer. The latter could occur if elderly individuals retrieved a second fact from memory to verify the veracity of the first retrieved answer. Either way, the hypothesis of a more rigorous confidence criterion for elderly individuals would only be supported if elderly adults showed a lower proportion of retrieval errors than the younger adults (Siegler, 1988). The final hypothesis, strategy shift, argues that elderly adults can process basic information at the same rate as younger adults but show longer overall solution times because they use more time-consuming problem-solving strategies. This alternative explanation should also be testable with the addition task.

Regardless, one exception to the general finding of slower information processing in elderly as compared with young adults is in the rate of access to the meaning of words (Cerella & Fozard, 1984). Cerella and Fozard found no age difference in the rate of retrieving the meaning of single words when they compared young and elderly adults; however, age differences, favoring the young, do appear to emerge on such lexical access tasks when the procedure requires the retrieval and comparison of two words (e.g., Mueller et al., 1980).

This Study

This study provides normative information on strategy choices for addition in adults and should provide potentially useful information to cognitive aging researchers in both of the previously described areas: age differences in strategy choices and rate of information processing. As noted earlier, previous studies (e.g., Siegler, 1987) have shown that problem-solving strategies in simple addition can be reliably classified on a trial-by-trial basis. This task should therefore allow for an assessment of potential age differences in the mix of problem-solving strategies for solving addition problems. Moreover, because the problem-solving strategy is recorded for each trial, age differences in the rate of executing the processes defining different strategies can be accurately assessed. In previous cognitive aging research, *trial-by-trial* strategy classifications have not typically been done. Thus, if young and elderly adults differ in their mix of problem-solving strategies, then mean solution times for various information-processing tasks have been averaged across different strategies. These strategies will probably vary in rate of execution. In this circumstance, estimates of the age difference in rate of information processing would almost certainly be biased, if not inaccurate (see Siegler, 1987, 1989b). Finally, and perhaps most important, the performance characteristics of young and elderly adults on the addition task can be interpreted within the context of a general theoretical model of cognitive development, the strategy choice model (Siegler, 1986).

Method

Subjects

The subjects included 60 young (25 male, 35 female) and 60 elderly (19 male, 41 female) adults. The young adults were undergraduate students who received course credit for participating in the experiment. The elderly adults were recruited from the Columbia, Missouri, area from a list of university faculty and staff retirees and their spouses. Prior to participation in the study, all subjects had initially participated in another experiment on activity memory. The activity memory study required about 50 min to complete. The tasks associated with the activity memory experiment were nontaxing and unrelated to this study, except that scores from one of the ability measures were used as an index of addition skill in this study (i.e., the Addition Test; Ekstrom, French, & Harman, 1976). However, because the purpose of the activities study was to assess age-related differences in memory for activities (the Addition Test was one of these activities), only one form of the Addition Test was administered for 2 min instead of the 3 min indicated in the instruction manual.

Before participation in the experimental task, all subjects responded to several survey questions. The questions concerned the subject's age, self-reported health status (this was recorded using a 1 [*excellent*] to 5 [*bad*] Likert-type scale), years of education, and years since retirement; the associated descriptive information is displayed in Table 1. The group difference in age was, of course, highly reliable ($p < .0001$), with the age range for the young group being 18 to 31 years as compared with 60 to 82 years for the elderly group. A reliable group difference for health status ($p < .0001$) was also found. Relative to the young adults, the elderly subjects reported poorer health, although the group mean indicated that the overall health of the elderly adults was reported as good, as compared with good to excellent for the young subjects. Finally, the advantage of the elderly adults in mean years of education was reliable ($p < .005$).

In all, 5 of the elderly subjects had not completed high school; 9 were high school graduates; 15 reported some college but no degree; 12 were college graduates but reported no graduate education; 19 reported some graduate education; and 7 of these 19 had received doctoral-level degrees. The younger sample was much more homogeneous with regard to educational level; 46 were in their 1st or 2nd year of college, and the remaining 14 were in their 3rd or 4th year of college. Across groups, years of education was not correlated with performance on the Addition Test, $r = .002$, $p > .50$. Within groups, years of education was not correlated with performance on the Addition Test for either the elderly, $r = -.09$, $p > .25$, or young, $r = 0.06$, $p > .50$, samples. Thus, years of education beyond elementary school does not appear to have a substantive impact on basic addition skills.

Experimental Task

Addition stimuli. The experimental stimuli consisted of 40 pairs of vertically placed single-digit integers. Stimuli were constructed from the 56 possible nontie pairwise combinations of the integers 2 to 9 (a tie problem is, e.g., 4 + 4).[2] The frequency and placement of all integers were counterbalanced. That is, each integer appeared five times as the augend and five times as the addend, and the smaller value integer appeared 20 times as the augend and 20 times as the addend. No repetition of either the augend or the addend was allowed across consecutive problems, to prevent priming effects. Although priming effects can also occur for the problem's correct sum, these effects are most pronounced when the task involves verifying a stated sum (e.g., "3 + 4 = 7"; Campbell, 1987b) rather than producing an answer.

Apparatus. The addition problems were presented at the center of a 30-cm × 30-cm video screen controlled by an IBM PC-XT microcomputer. A Cognitive Testing Station clocking mechanism ensured the collection of reaction times (RTs) with ±1 ms accuracy. The timing mechanism was initiated with the presentation of the problem on the video screen and was terminated through a Gerbrands G1341T voice-operated relay. The voice-operated relay was triggered when the subject spoke the answer into a microphone connected to the relay.

For each problem, a READY prompt appeared at the center of the video screen for 1,000 ms, followed by a blank screen for 1,000 ms. Then, an addition problem appeared on the screen and remained until the subject responded. The experimenter initiated each problem presentation sequence by pushing a control key.

Table 1
Descriptive Information for Subject Characteristics

Variable	Young *M*	Young *SD*	Elderly *M*	Elderly *SD*	*F*	*p*
Age	19.8	2.2	71.3	5.3	4772.08	.0001
Health status	1.5	0.6	2.2	0.9	29.60	.0001
Years of education	13.9	0.9	15.4	3.2	13.08	.005
Years since retirement	—	—	8.6	7.6	—	—
Gender (% male)	42		32		1.28	.25

Note. Health status was rated on a 1 (*excellent*) to 5 (*bad*) Likert-type scale. The *dfs* for the *F* tests were 1, 118.

Procedure

Each subject was tested individually and in a quiet room. The 40 experimental problems were presented one at a time and were preceded by 8 practice problems. The instructions encouraged the subjects to use whatever strategy made it easiest for them to obtain the answer, although equal emphasis was placed on speed and accuracy.

After each trial, subjects were asked to describe how they got the answer and based on this description the trial was classified by the experimenter as one of the earlier described strategies: (a) verbal counting, (b) decomposition, or (c) memory retrieval. In studies with children, strategies can typically be classified by an independent observer, because children frequently count to solve the problem, and counting is easily observable (e.g., children typically move their lips when implicitly counting). Several previous studies have demonstrated that children's descriptions of their problem-solving strategies in arithmetic are consistent with the classifications of independent observers and with the associated RTs, if the children are asked to describe the strategy immediately after the problem is solved (Siegler, 1987, 1989b).

For adults, however, implicit counting, for example, cannot be easily classified by an independent observer, because most adults do not move their lips when implicitly counting to solve an arithmetic problem. Thus, these verbal reports must be interpreted with caution (Hamann & Ashcraft, 1985), because the experimenter could not directly observe indications of different strategy usage. Nevertheless, having some information on reported problem-solving strategies seems preferable to no information at all. In support of the validity of the adult reports, the descriptions provided for the different strategies were very similar to those reported by children (e.g., Siegler, 1987). For retrieval trials, subjects typically reported "just knowing" or "remembering" the answer. For verbal counting trials, for instance, one young adult reported solving the problem "7 + 2" by counting "7, 8, 9." Reports for decomposition trials were also consistent with developmental studies but indicated subtly different ways of decomposing the problem. For example, for the problem "8 + 7," one elderly subject reported subtracting 1 from the 8 and then adding "7 + 7 + 1," whereas another elderly subject reported solving this same problem by subtracting 2 from the 7 and then adding "8 + 2 + 5." Analyses designed to more explicitly assess the validity of these reports are described in the Results section.

Results

The results with brief discussion are presented in three major sections, followed by a more general discussion of the results and their implications. In the first section, analyses of group differences in strategy choices are presented. The second section presents results for the Addition Test. The final section presents a componential analysis of the RT data designed to assess the validity of the self-reports and potential group differences in the rate of executing various arithmetical processes, for instance, rate of retrieving facts from long-term memory.

---

[2] The excluded stimuli were 2 + 4, 2 + 6, 3 + 2, 3 + 4, 4 + 7, 4 + 8, 5 + 3, 5 + 8, 6 + 5, 6 + 9, 7 + 5, 7 + 6, 8 + 3, 8 + 9, 9 + 2, and 9 + 7.

Table 2
Characteristics of Addition Strategies

	Mean % of trials on which strategy used		Mean % of errors		Mean RT (ms)[a]			
					Young		Elderly	
Strategy	Young	Elderly	Young	Elderly	*M*	*SD*	*M*	*SD*
Verbal counting	5	0[b]	8	0	1,703	697	—	—
Decomposition	7	2	7	0	1,189	255	1,271	175
Retrieval	88	98	2	2	833	96	930	95

Note. Overall (across strategies) mean reaction time (RT) was 885 ms ($SD = 142$) for the young group and 939 ms ($SD = 102$) for the elderly group. The difference in overall mean RT was marginally reliable ($p = .054$). The overall error rates for the young and elderly groups were 2.5% and 1.5%, respectively.
[a] Mean RT excluded error and spoiled trials. [b] The actual percentage was 0.2.

Strategy Choices

The group-level characteristics of addition strategies are displayed in Table 2. Retrieval was the primary strategy choice for both the young and elderly adults, although the decomposition and verbal counting strategies were used occasionally. Univariate analyses of variance (ANOVAs) with group as the between-subjects variable indicated reliable differences across groups for the frequency with which the retrieval, $F(1, 118) = 19.07$, $p < .0001$, decomposition, $F(1, 118) = 8.66$, $p < .01$, and verbal counting, $F(1, 118) = 13.19$, $p < .001$, strategies were used for problem solving. In all, the elderly subjects used the retrieval strategy more frequently, and the backup decomposition and verbal counting strategies less frequently, than did the young subjects.

Examination of individual protocols indicated that 10% and 50% of the elderly and young adults, respectively, used more than one problem-solving strategy. The group difference in the proportion of subjects using more than one strategy was reliable, $z = 4.78$, $p < .01$. Examination of individual strategy distributions indicated that retrieval was the primary problem-solving process for 5 of the 6 elderly subjects who reported using a backup strategy. These 5 subjects reported using retrieval to solve, on average, 34 of the 40 addition problems (the range was 27 to 39 problems). The sixth subject reported using decomposition to solve 21 problems, and retrieval to solve 18 problems (one trial was spoiled). A similar pattern emerged for the young group. Here, 28 of the 30 subjects who reported using a backup strategy relied primarily on retrieval for problem solving. These subjects reported using retrieval to solve, on average, 31 of the 40 addition problems (the range was 21 to 38 problems). One of the remaining subjects reported using the verbal counting and retrieval strategies equally often, whereas the final subject reported using retrieval to solve 13 problems, decomposition to solve 16 problems, and verbal counting to solve the remaining problems.

A second series of univariate ANOVAs revealed no reliable group difference in the frequency of retrieval errors, $F(1, 118) < 1$, but the young group committed significantly more decomposition, $F(1, 118) = 5.76$, $p < .05$, and verbal counting, $F(1, 118) = 5.09$, $p < .05$, errors than did the elderly group.

The final set of columns in Table 2 displays the mean solution times across strategies and groups. No mean RT is displayed for the elderly group for the verbal counting strategy because there were almost no verbal counting trials from the elderly subjects. Inspection of mean RTs from the young and elderly subjects (averaged across problems to be consistent with the later described componential analyses) for the decomposition and retrieval strategies reveals a group difference favoring the young subjects of 82 ms and 97 ms, respectively. The mean difference for the retrieval strategy was highly reliable, $F(1, 78) = 20.87$, $p < .0001$, but was not reliable for the decomposition strategy, $F(1, 78) < 1$. Note that the solution time data for the decomposition strategy were based on a relatively small number of trials and therefore might not be a reliable estimate of the rate with which young and elderly adults execute this strategy. A componential analysis of the retrieval trial solution times is presented in the final section.

The final set of analyses in this section sought to determine if the use of the backup strategies was related to problem difficulty, as is the case for children (Geary, 1990; Geary & Brown, 1991; Siegler & Shrager, 1984). To achieve this end, the frequency (across problems) with which the backup strategies were used for problem solving was correlated with the value of the problem's correct sum. The sum value was used because it provides a straightforward index of problem difficulty (Washburne & Vogel, 1928) and is highly correlated with other indexes of problem difficulty (e.g., Geary & Burlingham-Dubree, 1989). Consistent with studies in which children were used as subjects, the results indicated that the frequency of backup trials increased with an increase in the value of the correct sum for both the young, $r = .73$, $p < .001$, and elderly, $r = .80$, $p < .0001$, groups. The frequency of decomposition trials was also reliably correlated with the presence of a "9" in the problem for both the young, $r = .82$, $p < .001$, and elderly, $r = .41$, $p < .01$, adults. The value of these two correlation coefficients (i.e., .82 and .41) differed reliably, $z = 3.10$, $p < .01$. Although only 25% of the problems contained a "9," 70% and 45% for the young and elderly groups, respectively, of the decomposition trials occurred when either the augend or the addend was a "9." Finally, for the young group, the frequency of decomposition and verbal counting trials was not reliably correlated, $r = .19$, $p > .20$. This result suggests that decomposition and counting tended to be used to solve different types of problems. In all, the results indicate that the backup strategies tended to be used for the solution of more difficult problems for both young and elderly adults, and that the decomposition strategy tended to be used, rather than counting, if the problem contained a "9."

Ability Test Performance

The mean number of problems solved correctly on the Addition Test (Ekstrom et al., 1976) was 21.4 ($SD = 5.7$) and 24.0 ($SD = 7.6$) for the young and elderly groups, respectively. A univariate ANOVA (with group as the between-subjects variable) indicated that the performance advantage of the elderly adults was reliable, $F(1, 118) = 4.25$, $p < .05$. Across groups, performance on the Addition Test was unrelated to reported health status, $r = .04$, $p > .50$, but was marginally correlated with age, $r = .16$, $p < .10$. The frequency with which the backup strategies were used for problem solving was inversely correlated with performance on the Addition Test, $r = -.19$, $p < .05$. This result is consistent with the argument that decomposition and verbal counting are developmentally immature strategies; that is, the use of these strategies indicates that the subject has not yet achieved the level of expert in simple addition. Finally, a multiple regression analysis revealed that partialing the frequency of correct retrieval trials from the group variable reduced the elderly advantage on the Addition Test to nonsignificance, $F(1, 117) = 1.23$, $p > .25$. Thus, the advantage of the elderly adults on the ability test appears to be related to the fact that nearly all of the elderly subjects had memorized nearly all basic addition facts, whereas many of the young subjects had not.

Componential Analyses

The componential analyses served two purposes. First, the pattern of solution times was examined across strategies to determine if they were consistent with the self-reports. Second, these analyses were designed to determine the locus of the group difference in rate of executing the memory retrieval strategy. Both sets of analyses were based on correct trials, but excluded trials on which the voice-operated relay was triggered by responses other than the answer (e.g., coughing) or if the initial response did not trigger the relay.[3] Process models for addition were fitted to average RT data by using regression techniques for both groups. Because not all subjects used the same strategy to solve all problems, the matrix of RTs from which these averages were computed necessarily contained missing data. In this circumstance, the resulting regression equation could be biased in some way. This bias would probably result in some increase in the variability of the regression estimates and attenuate the goodness of fit of the overall model. Nevertheless, this procedure seems preferable to the alternative of averaging all RTs across different strategies (Siegler, 1987, 1989b).

The min and product (prod) variables were used to model RTs (Geary, 1990; Siegler, 1987). Because the min variable represents a process whereby the subject implicitly counts to solve the problem, the RTs from verbal counting trials should increase linearly with the min value, and the associated regression weight should be consistent with estimates for the rate of counting, when counting is used to solve simple addition problems (Groen & Parkman, 1972). The prod variable can be used to represent the long-term memory network of addition facts (Geary et al., 1986; Miller et al., 1984). The raw regression weight for the prod variable multiplied by the product of the augend and addend provides an estimate of the rate of retrieving the sum from long-term memory. The raw regression estimate for the prod variable for retrieval trials should be similar to the 3.0 value obtained by Miller et al. for a similar addition task.

Validity of self-reports. Two approaches can be used to assess the validity of the self-reports. The first approach involves comparing the magnitude of the correlation between RTs and the variables thought to represent the processes underlying the solution of addition problems. For example, the prod variable should show a higher correlation coefficient, than the min variable, with retrieval trial RTs. Although this approach seems logical, Pellegrino and Goldman (1989) showed that relying on the value of the best fitting variable (i.e., the variable that is most strongly correlated with RTs) to make inferences about the underlying processes is not always appropriate because variables representing different processes are often correlated with RTs to the same degree. Nevertheless, we report the results for this type of comparison.

Second, if the subjects' reports of their problem-solving strategies are valid, then the regression weights for the min and prod variables should differ across strategies. Moreover, the value of the regression weights should be consistent with theoretical expectations and previous studies. The min variable should produce a regression estimate for verbal counting trials that is consistent with the rate of implicit counting but a much lower estimate for decomposition and retrieval trials. The regression estimate for the prod variable, as noted earlier, should be about 3.0 for retrieval trials but much larger for decomposition and verbal counting trials (cf. Siegler, 1987). No predictions can be made regarding the expected regression weights for decomposition trials, because there are no appropriate comparisons available in the literature and because, as noted earlier, different subjects used subtly different decomposition strategies.

As noted in Table 3, the min variable was correlated .45 with retrieval trial RTs for both the young and elderly groups, whereas for these same RTs the prod variable showed a correlation coefficient of .55 for the young group and .63 for the elderly group (for all *r*s, $p < .05$). For the young group, however, the min and prod variables were both correlated .80 with verbal counting trial RTs. No correlations were computed for the elderly group for the verbal counting strategy because the elderly subjects almost never used this strategy. Table 3 also displays the regression weights for the min and prod variables across groups and strategies. As expected, the largest estimate for the min variable was obtained for the verbal counting trials. The associated regression weight of 372 ms is consistent with the 400 ms estimate, reported by Groen and Parkman (1972), for the counting rate associated with the min process. In a similar manner, the lowest estimates for the prod variable were obtained for retrieval trials, followed in turn by the decomposition and verbal counting trials. For the retrieval trials the prod variable regression estimates of 3.2 ms and 3.6 ms for the young

---

[3] A total of 6.0% and 6.2% of the responses were spoiled for the young and elderly groups, respectively.

Table 3
Regression Coefficients for Counting and Memory Retrieval Models Across Strategies

	Verbal counting				Decomposition		Retrieval			
Group	Min	r^a	Prod	r	Min	Prod	Min	r	Prod	r
Young	372	.80	38	.80	83	7.7	25	.45	3.2	.55
Elderly	—	—	—	—	71	8.2	25	.45	3.6	.63

Note. Min = the cardinal value of the smaller integer. Prod = the product of the augend and the addend. Regression coefficients were estimated from mean reaction times, which excluded error and spoiled trials.
[a] The correlations are between the variable and mean reaction times.

and elderly groups, respectively, were highly consistent with the estimate reported by Miller et al. (1984). For the retrieval trials, both the correlation values and the regression estimates are consistent with the self-reports. For the counting trials, however, the correlation values do not allow for a differentiation of the prod and min models, although the regression estimate for the min variable is consistent with a counting process. Moreover, although the prod variable was strongly correlated with counting trial RTs, the associated regression estimate of 38 ms is very inconsistent with a retrieval process (Miller et al., 1984). Overall, the pattern of RTs across strategies seems to support the validity of the self-reports.

Memory retrieval strategy. The final set of analyses in this section sought to determine the locus of the group difference, favoring young adults, in mean retrieval trial RTs. To achieve this end, mean RTs from both groups were combined into a single data set and were analyzed by means of regression techniques. The prod variable was used as one of the independent measures. The second independent measure was a dummy coded variable, group (coded 0 for the young group and 1 for the elderly group). The partial F ratio for the group variable tested the intercept difference between groups. The Group × Prod interaction tested the slope difference for the prod variable between groups.

The resulting regression equation is displayed in Table 4. Inspection of the table reveals reliable group, $F(1, 76) = 5.90$, $p < .05$, and prod, $F(1, 76) = 18.08$, $p < .01$, variables, but a nonreliable Group × Prod interaction, $F(1, 76) < 1$. The results indicate that the young and elderly adults did not differ in the rate of executing the substantive addition process—the rate of retrieving facts from long-term memory. The intercept term (i.e., 740 for the young adults and 740 + 86 for the elderly adults) theoretically represents a combined estimate for the rate of encoding digits and for the rate of the strategy selection and the answer production processes (Ashcraft & Battaglia, 1978; Campbell & Clark, 1988). The reliable group variable indicated that the elderly adults required an additional 86 ms, relative to the young adults, to execute the processes subsumed by the intercept term.

Table 4
Statistical Summaries of Regression Analyses: Retrieval Trials

Equation	R^2	df	F	RMS_e
RT = 740 + 86 (Group)* + 3.2 (Prod)* + 0.4 (Group × Prod)	.49	3, 76	23.97	78
Partial Fs: 5.90, 18.08, 0.13				

Note. RT = mean reaction time; Prod = the product of the augend and addend. Group was coded 0 for the young adults and 1 for the elderly adults.
* $p < .05$.

Discussion

The first purpose of our study was to provide normative information on strategy choices in addition in adulthood. The second purpose was to compare young and elderly adults on strategy choice and speed-of-processing characteristics associated with the cognitive addition task. Both of these issues are discussed within the context of the strategy choice model. According to the model, the combination of variations in the peakedness of the distributions of associations parameter and the rigor of the confidence criterion can accommodate developmental changes in strategy choices, the proportion of retrieval errors, and the use of backup strategies (Geary et al., in press; Siegler, 1988). Peaked distributions of associations, where the associative strength is concentrated on a single answer, should produce a high proportion of retrieval trials, independent of the rigor of the confidence criterion. Conversely, for less peaked distributions the rigor of the confidence criterion can have a substantial impact on strategy choices. For these problems a stringent confidence criterion should produce many trials on which a backup strategy is used for problem solving, because the associative strength of the retrieved answers would not often exceed the value of the confidence criterion, making necessary the use of a backup strategy. Less peaked distributions of associations combined with a less rigorous confidence criterion, however, should result in fewer backup strategy trials and a relatively high proportion of retrieval errors (Siegler, 1988).

The results for the young subjects do not support strong assumptions about the exclusive use of memory retrieval processes for the solution of mental arithmetic problems in adulthood (Ashcraft, 1982; Campbell, 1987a; Geary et al., 1986; Miller et al., 1984; Widaman et al., 1989). For the young adults, 12% of the rather simple problems used in the experimental task were apparently solved by means of either the verbal counting or decomposition backup strategies. Moreover, 50% of the young subjects reported using at least one of the backup strategies. Thus, the development of skill in simple addition, for many individuals, is not complete until well into adulthood. In other words, many of the young adults had not yet achieved the level of expert in simple addition. Within the context of the strategy choice model, the low proportion of retrieval errors combined with the relatively high proportion of backup strategy trials by the young adults suggests a rather rigorous confidence criterion (Geary & Brown, 1991; Siegler, 1988). The finding that the use of the backup strategies increased with increases in the difficulty of the problem indicates that the backup strategies were probably used when an answer was not readily retrievable from long-term memory. In other words, the relatively immature mix of problem-solving strategies for the young group appeared to reflect a distributions of associations parameter that was relatively flat, for many of these subjects, for more difficult problems.

Indeed, a group difference in the peakedness of the distributions of associations parameter can easily accommodate the advantage of the elderly adults in the distribution of strategy choices. In this study, the advantage of the elderly adults in the proportion of retrieval trials, combined with no group difference in the proportion of retrieval errors, suggests rather more peaked distributions of associations for the elderly adults relative to the young adults. The finding of an equivalent proportion of retrieval errors for the two groups suggests that the age difference in the frequency of retrieval trials was not due to group differences in the rigor of the confidence criterion. The data in fact suggest that both groups had a rather stringent confidence criterion. In other words, no group difference in the proportion of retrieval errors suggests that the young and elderly subjects did not differ in speed–accuracy trade-offs. In all, the data argue that the strength of the underlying memory representation for basic addition facts is not adversely affected by the aging process (Smith, 1980), assuming good health (Manton, Siegler, & Woodbury, 1986).

It is not clear from these data, however, whether the age-related difference in the distribution of problem-solving strategies is primarily due to developmental or cohort differences. A developmental interpretation would argue that the elderly subjects might have relied on backup strategies more frequently as younger adults. With the continued use of the backup strategies, the associative strength between the correct answer and the problem eventually reached a point where retrieval became the dominant problem-solving strategy for nearly all simple addition problems. An equally plausible explanation is that the group difference in strategy choices is related to cohort differences. Indeed, Schaie (1983) reported reliable cohort differences for basic numerical skills, with later cohorts generally showing relatively poor basic skills. These cohort differences would presumably be related to a greater emphasis on rote memorization of basic facts during the elementary school years for earlier cohorts (Washburne & Vogel, 1928). From this perspective, the elderly adults have probably been experts in basic addition since the elementary school years. Either way, the data suggest that the elderly subjects showed better developed basic addition skills than the college students and that this advantage can be understood in terms of the strategy choice model. Within the context of the strategy choice model, however, the attainment of expert status in addition might be more closely related to level of practice than to age per se (Charness, 1987).

Consistent with many other cognitive aging studies (e.g., Cerella, 1985; Salthouse & Somberg, 1982), the young adults showed an advantage relative to the elderly adults in the rate of information processing. However, contrary to generalized slowing models of age-related changes in rate of information processing (Cerella, 1990; Myerson et al., 1990), the componential analysis of the retrieval trial RTs suggested that the group difference in speed of processing might not have been uniform across basic processes. The young and elderly adults showed no reliable difference in the rate with which addition facts could be retrieved from long-term memory, but the young adults did show an advantage in the rate of executing the processes subsumed by the intercept term, that is, the processes of encoding, strategy selection, and verbal production of the answer. This basic finding is similar to a finding by Charness (1987) for a somewhat similar task.

The finding that the elderly subjects nearly always used retrieval suggests that the speed-of-processing difference found in this study was probably not due to a strategy selection process. Nor do these data support the strategy-shift hypothesis of an age-related slowing in information processing. In fact, the elderly were more likely to use the less time-consuming retrieval strategy than were the younger adults. The Nebes (1978) and Salthouse and Somberg (1982) findings of no age difference in verbal production RTs suggests that the intercept difference might not be related to age differences in the rate of verbally producing an addition answer, although Charness (1987) found that young adults showed about a 28-ms advantage over elderly adults on a similar verbal production task. Thus, it is possible that a portion of the intercept difference is related to age differences in rate of verbally producing an answer, although the 28-ms estimate (Charness, 1987) would not explain the entire intercept difference. The remainder of the intercept difference could be parsimoniously explained by age differences in the rate of encoding integers (Hunt, 1978; Smith, 1980). In all, the pattern of results supports Charness's argument that "much of the age-related slowing is attributable to encoding and response processes" (pp. 229–230), and not attributable to more central processes.

Future studies will, of course, need to explicitly test this speed-of-encoding/response hypothesis. Regardless, the finding of no age difference in the rate of retrieving simple addition facts from long-term memory is consistent with the finding of Cerella and Fozard (1984). Recall that Cerella and Fozard found no age difference in the rate of retrieving the meaning of single words from long-term memory. In fact, data from converging areas, such as psychometrics, mental chronometry, learning disabilities, and neuropsychology, support the position that the long-term memory representation of arithmetic facts is a semantic language-like system (Ashcraft & Battaglia, 1978; Biller & Grafman, 1983; Geary, 1990; Geary & Brown, 1991; Geary et al., in press; Horn, 1968; Luria, 1980; Richman, 1983). Thus, the finding of no age difference in the rate of addition fact retrieval might be interpreted as a replication, albeit with a different content, of the Cerella and Fozard finding.

Although the elderly adults did not appear to be at a disadvantage in the rate of fact retrieval, their overall solution times for retrieval trials were slower relative to the young adults. Yet, consistent with Schaie's (1989) findings, the elderly adults outperformed the young adults on the pencil-and-paper test of numerical facility (i.e., the Addition Test). The elderly adults' advantage on the ability measure in this study, and perhaps in Schaie's study, appeared to be related to a mature distribution of strategy choices (i.e., less frequent use of backup strategies). Because the elderly adults were able to correctly retrieve nearly all basic addition facts from long-term memory and therefore rarely use a backup strategy, they were able to solve more addition problems in the allotted time than were the young adults. For the young adults the relatively frequent use of backup strategies appeared to have lowered their overall performance on the ability measure. This finding supports Charness's (1981) argument that decreases in the overall rate of information processing typically associated with aging can be compensated for by a shift to more efficient problem-solving strategies, at least in domains in which elderly individuals have considerable knowledge.

Finally, our study demonstrated the utility of developmental models for guiding the study of age-related differences on basic cognitive tasks and suggests that the development of seemingly simple cognitive skills, such as addition, might continue well into adulthood for many individuals. The study also illustrated the need to consider age differences in the distribution of problem-solving strategies when comparing young and elderly adults on information-processing tasks. The finding of an age difference in the mix of problem-solving strategies for everyday, well-practiced addition problems suggests that strategic differences might also emerge on other basic information-processing tasks. As has been aptly demonstrated by Siegler (1987, 1989b) for children's arithmetic, the averaging of RTs across different strategies can seriously distort estimates of the speed of executing basic information processes and therefore distort any group comparisons in rate of executing these processes if the groups differ in strategy approaches to the task. Finally, this study indicated that the strategy choice model (Siegler, 1986) can provide a fruitful theoretical context for the study of cognitive aging in that the model can, and should, be extended into adulthood.

References

Ashcraft, M. H. (1982). The development of mental arithmetic: A chronometric approach. *Developmental Review, 2,* 213–236.

Ashcraft, M. H., & Battaglia, J. (1978). Cognitive arithmetic: Evidence for retrieval and decision processes in mental addition. *Journal of Experimental Psychology: Human Learning and Memory, 4,* 527–538.

Ashcraft, M. H., & Fierman, B. A. (1982). Mental addition in third, fourth, and sixth graders. *Journal of Experimental Child Psychology, 33,* 216–234.

Biller, F., & Grafman, J. (1983). Acalculia: Historical development and current significance. *Brain and Cognition, 2,* 205–223.

Birren, J. E. (1965). Age changes in speed of behavior: Its central nature and physiological correlates. In A. T. Welford & J. E. Birren (Eds.), *Behavior, aging, and the nervous system: Biological determinants of speed of behavior and its changes with age* (pp. 191–216). Springfield, IL: Charles C Thomas.

Birren, J. E., & Botwinick, J. (1951). Rate of addition as a function of difficulty and age. *Psychometrika, 16,* 219–232.

Brainerd, C. J. (1983). Young children's mental arithmetic errors: A working memory analysis. *Child Development, 54,* 812–830.

Campbell, J. I. D. (1987a). Network interference and mental multiplication. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 13,* 109–123.

Campbell, J. I. D. (1987b). Production, verification, and priming of multiplication facts. *Memory & Cognition, 15,* 349–364.

Campbell, J. I. D., & Clark, J. M. (1988). An encoding-complex view of cognitive number processing: Comment on McCloskey, Sokol, and Goodman (1986). *Journal of Experimental Psychology: General, 117,* 204–214.

Campbell, J. I. D., & Graham, D. J. (1985). Mental multiplication skill: Structure, process, and acquisition. *Canadian Journal of Psychology, 39,* 338–366.

Cerella, J. (1985). Information processing rates in the elderly. *Psychological Bulletin, 98,* 67–83.

Cerella, J. (1990). Aging and information-processing rate. In J. E. Birren and K. W. Schaie (Eds.), *Handbook of the psychology of aging* (3rd ed., pp. 201–233). New York: Academic Press.

Cerella, J., & Fozard, J. L. (1984). Lexical access and age. *Developmental Psychology, 20,* 235–243.

Charness, N. (1981). Aging and skilled problem solving. *Journal of Experimental Psychology: General, 110,* 21–38.

Charness, N. (1987). Component processes in bridge bidding and novel problem solving tasks. *Canadian Journal of Psychology, 41,* 223–243.

Charness, N., & Bieman-Copland, S. (in press). The learning perspective: Adulthood. In R. J. Sternberg & C. A. Berg (Eds.), *Intellectual development.* New York: Cambridge University Press.

Charness, N., & Campbell, J. I. D. (1988). Acquiring skill at mental calculation in adulthood: A task decomposition. *Journal of Experimental Psychology: General, 117,* 115–129.

Ekstrom, R. B., French, J. W., & Harman, H. H. (1976). *Manual for kit of factor-referenced cognitive tests.* Princeton, NJ: Educational Testing Service.

Geary, D. C. (1990). A componential analysis of an early learning deficit in mathematics. *Journal of Experimental Child Psychology, 49,* 363–383.

Geary, D. C., & Brown, S. C. (1991). Cognitive addition: Strategy choice and speed-of-processing differences in gifted, normal, and mathematically disabled children. *Developmental Psychology, 27,* 398–406.

Geary, D. C., Brown, S. C., & Samaranayake, V. A. (in press). Cognitive addition: A short longitudinal study of strategy choice and speed-of-processing differences in normal and mathematically disabled children. *Developmental Psychology.*

Geary, D. C., & Burlingham-Dubree, M. (1989). External validation of the strategy choice model for addition. *Journal of Experimental Child Psychology, 47,* 175–192.

Geary, D. C., Widaman, K. F., & Little, T. D. (1986). Mental addition and multiplication: Evidence for a single memory network. *Memory & Cognition, 14,* 478–487.

Geary, D. C., Widaman, K. F., Little, T. D., & Cormier, P. (1987). Cognitive addition: Comparison of learning disabled and academically normal elementary school children. *Cognitive Development, 2,* 249–269.

Gelman, R., & Meck, E. (1983). Preschoolers' counting: Principles before skill. *Cognition, 13,* 343–359.

Greeno, J. G., Riley, M. S., & Gelman, R. (1984). Conceptual competence and children's counting. *Cognitive Psychology, 16,* 94–143.

Groen, G. J., & Parkman, J. M. (1972). A chronometric analysis of simple addition. *Psychological Review, 79,* 329–343.

Hamann, M. S., & Ashcraft, M. H. (1985). Simple and complex mental addition across development. *Journal of Experimental Child Psychology, 40,* 49–72.

Hartley, A. A., & Anderson, J. W. (1983). Task complexity, problem representation, and problem-solving performance by younger and older adults. *Journal of Gerontology, 38,* 78–80.

Hertzog, C. (1989). Influences of cognitive slowing on age differences in intelligence. *Developmental Psychology, 25,* 636–651.

Hertzog, C., Raskind, C. L., & Cannon, C. (1986). Age-related slowing in semantic information processing speed: An individual differences analysis. *Journal of Gerontology, 41,* 500–502.

Horn, J. L. (1968). Organization of abilities and the development of intelligence. *Psychological Review, 75,* 242–259.

Hunt, E. (1978). Mechanics of verbal ability. *Psychological Review, 85,* 109–130.

Kail, R. (1988). Developmental functions for speeds of cognitive processes. *Journal of Experimental Child Psychology, 45,* 339–364.

Kaye, D. B. (1986). The development of mathematical cognition. *Cognitive Development, 1,* 157–170.

Luria, A. R. (1980). *Higher cortical functions in man* (2nd ed.). New York: Basic Books.

Madden, D. J. (1985). Age-related slowing in the retrieval of information from long-term memory. *Journal of Gerontology, 40,* 208–210.

Manton, K. G., Siegler, I. C., & Woodbury, M. A. (1986). Patterns of intellectual development in later life. *Journal of Gerontology, 41,* 486–499.

Miller, K., Perlmutter, M., & Keating, D. (1984). Cognitive arithmetic: Comparison of operations. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 10,* 46–60.

Mueller, J. H., Kausler, D. H., & Faherty, A. (1980). Age and access time for different memory codes. *Experimental Aging Research, 6,* 445–449.

Myerson, J., Hale, S., Wagstaff, D., Poon, L. W., & Smith, G. A. (1990). The information-loss model: A mathematical theory of age-related cognitive slowing. *Psychological Review, 97,* 475–487.

Nebes, R. D. (1978). Vocal versus manual response as a determinant of age difference in simple reaction time. *Journal of Gerontology, 33,* 884–889.

Pellegrino, J. W., & Goldman, S. R. (1989). Mental chronometry and individual differences in cognitive processes: Common pitfalls and their solutions. *Learning and Individual Differences, 1,* 203–255.

Richman, L. C. (1983). Language-learning disability: Issues, research, and future directions. In M. Wolraich & D. K. Routh (Eds.), *Advances in developmental and behavioral pediatrics* (Vol. 4, pp. 87–107). Greenwich, CT: JAI Press.

Salthouse, T. A., Mitchell, D. R. D., Skovronek, E., & Babcock, R. L. (1989). Effects of adult age and working memory on reasoning and spatial abilities. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 15,* 507–516.

Salthouse, T. A., & Somberg, B. L. (1982). Skilled performance: Effects of adult age and experience on elementary processes. *Journal of Experimental Psychology: General, 111,* 176–207.

Schaie, K. W. (1983). The Seattle longitudinal study: A 21-year exploration of psychometric intelligence in adulthood. In K. W. Schaie (Ed.), *Longitudinal studies of adult psychological development* (pp. 64–135). New York: Guilford Press.

Schaie, K. W. (1989). Perceptual speed in adulthood: Cross-sectional and longitudinal studies. *Psychology and Aging, 4,* 443–453.

Siegler, R. S. (1983). Five generalizations about cognitive development. *American Psychologist, 38,* 263–277.

Siegler, R. S. (1986). Unities across domains in children's strategy choices. In M. Perlmutter (Ed.), *Perspectives for intellectual development: Minnesota Symposia on Child Psychology* (Vol. 19, pp. 1–48). Hillsdale, NJ: Erlbaum.

Siegler, R. S. (1987). The perils of averaging data over strategies: An example from children's addition. *Journal of Experimental Psychology: General, 116,* 250–264.

Siegler, R. S. (1988). Individual differences in strategy choices: Good students, not-so-good students, and perfectionists. *Child Development, 59,* 833–851.

Siegler, R. S. (1989a). How domain-general and domain-specific knowledge interact to produce strategy choices. *Merrill-Palmer Quarterly, 35,* 1–26.

Siegler, R. S. (1989b). Hazards of mental chronometry: An example from children's subtraction. *Journal of Educational Psychology, 81,* 497–506.

Siegler, R. S., & Jenkins, E. (1989). *How children discover new strategies.* Hillsdale, NJ: Erlbaum.

Siegler, R. S., & Shrager, J. (1984). Strategy choice in addition and subtraction: How do children know what to do? In C. Sophian (Ed.), *Origins of cognitive skills* (pp. 229–293). Hillsdale, NJ: Erlbaum.

Smith, A. D. (1980). Age differences in encoding, storage, and retrieval. In L. W. Poon, J. L. Fozard, L. S. Cermak, D. Arenberg, & L. W. Thompson (Eds.), *New directions in memory and aging: Proceedings of the George A. Talland Memorial Conference* (pp. 23–45). Hillsdale, NJ: Erlbaum.

Svenson, O. (1985). Memory retrieval of answers of simple additions as reflected in response latencies. *Acta Psychologica, 59,* 285–304.

Washburne, C., & Vogel, M. (1928). Are any number combinations inherently difficult? *Journal of Educational Research, 17,* 235–255.

Widaman, K. F., Geary, D. C., Cormier, P., & Little, T. D. (1989). A componential model for mental addition. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 15,* 898–919.

Widaman, K. F., Little, T. D., Geary, D. C., & Cormier, P. (in press). Individual differences in the development of skill in mental addition: Internal and external validation of chronometric models. *Learning and Individual Differences.*

Received November 14, 1990
Revision received February 19, 1991
Accepted February 20, 1991 ■