Virial Balance in Turbulent MHD Two Dimensional Numerical Simulations of the ISM [1]

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Abstract. We present results from a virial analysis of fully nonlinear two-dimensional (2D) simulations of the ISM. We discuss the Eulerian Virial Theorem in 2D, and describe preliminary results on the virial budget of clouds in the simulations. The clouds are far from a static equilibrium, and the Virial Theorem is dominated by the time-derivative terms, indicating the importance of flux through the cloud boundaries and mass redistributions. A trend towards greater importance of the gravitational term at larger scales is observed, although a few small clouds are strongly self-gravitating. The magnetic and kinetic terms scale linearly with each other.

I INTRODUCTION

Vázquez-Semadeni *et al.* (1995, hereafter Paper I) and Passot *et al.* (1995, hereafter Paper II) have produced a numerical model of the interstellar medium (ISM) including enough physical agents as to render it feasible to perform statistical studies of the clouds formed in the simulations. The simulations include self-gravity, magnetic fields, parameterized cooling and diffuse heating, the Coriolis force, large-scale shear, and localized stellar energy input. In the present work, we discuss the Virial Theorem (VT) as it applies to the simulations, and present preliminary statistical results from a two-dimensional (2D) simulation with a resolution of 800×800 grid points, performed specifically for this analysis. In § II we discuss the VT, applying the formalism developed by McKee & Zweibel (1992) to the 2D case. In § III we describe the cloud-identifying algorithm and show preliminary statistical results, and in § IV we present some remarks and future work.

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II VIRIAL THEOREM IN 2D

The VT is obtained by dotting the momentum equation (eq. (1b) in Paper I) with the position vector $\mathbf{x}$ and integrating over volume. McKee & Zweibel (1992) have discussed an Eulerian form of the VT, which is most appropriate for our simulations, since they are performed with an Eulerian code. Because the simulations are 2D (in order to reach a sufficiently large resolution), we must consider the VT in 2D as well. It reads:

$$\frac{1}{2}\frac{d^2I}{dt^2} = 2\Big(\tau_{\rm kin} + \tau_{\rm int}\Big) + M - W - E_{\rm cor} - \frac{1}{2}\frac{d\Phi}{dt} \tag{1}$$

where $\tau_{\rm kin} = 1/2(\int \rho u^2 dV - \oint_X x_i \rho u_i u_j \hat{n}_j dS)$ is the kinetic term, $\tau_{\rm int} = \int PdV - 1/2\oint_S Px_i\hat{n}_i dS$ is the thermal term, $M = 1/8\pi \oint_S x_i T_{ij}\hat{n}_j dS$ is the magnetic term, $W = \int x_i \rho\, g_i dV$ is the gravitational term, $E_{\rm cor} = 2\int x_i(\mathbf{\Omega}\times\mathbf{u})_i dV$ is the Coriolis term, $\Phi = \oint_S \rho u_i r^2 \hat{n}_i dS$ is the flux of moment of inertia through the surface S, and ρ, $\mathbf{u}$, P and g_i are the density, velocity, thermal pressure and self-gravitational acceleration, respectively. Because of two-dimensionality, we must replace volumes by areas and surfaces by contours in (1). However, we retain the above notation for generality. Since in 2D $\nabla\cdot\mathbf{x} = 2$, in equation (1) we note the three following interesting points: *a)* Although magnetic fields are present in the surface term $M = \int_S x_i\ T_{ij}\ \hat{n}_j\ dS$, (where the Maxwell stress tensor is defined as $T_{ij} \equiv 1/4\pi[B_iB_j - 1/2B^2\delta_{i,j}]$), the "classical" magnetic energy term $E_{\rm mag} = 1/8\pi\int B^2 dV$ does not enter the virial equation, so it does not provide support against gravity. *b)* The internal energy $E_{\rm int} \equiv \int PdV$ does not contain the 3/2 factor as in 3D. Nevertheless, in 2D this term still coincides with the total internal energy, because there are only two translational degrees of freedom. *c)* Additionally, it can be shown that the gravitational term $\int x_i\ \rho\ g_i\ dV$ does not coincide with the gravitational energy $E_{\rm grav} = 1/2\int \rho\ \phi\ dV$ as it does in 3D for isolated clouds. Essentially, this is due to the slower distance dependence of the gravitational potential in 2D.

III PRELIMINARY STATISTICS

In order to calculate the terms in equation (1), we have performed a 2D simulation similar to the one called "Run 28" in Paper II, but with a resolution of 800 × 800 grid points. In this run we analyze the data shortly after turning off star formation, in order to allow for the largest possible density gradients (see Vázquez-Semadeni, Ballesteros-Paredes & Rodríguez 1997, hereafter Paper III) while still retaining the structure induced by the stellar energy injection. We have developed a numerical algorithm to identify clouds and evaluate within them the various terms entering the VT, as well as their velocity dispersion and mean density. We define a cloud as a connected set of pixels whose densities are larger than an arbitrary threshold $\rho_{\rm t}$. Previous calculations (Paper III) have shown that the simulations exhibit similar scaling

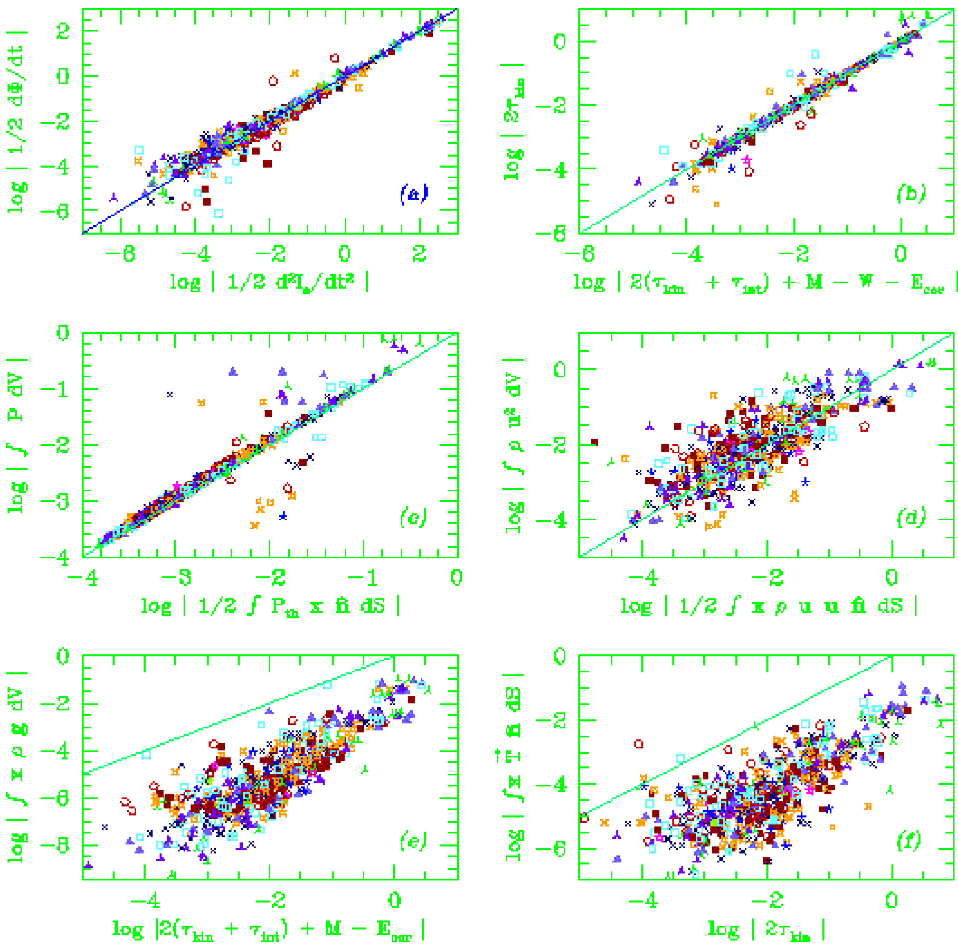

FIGURE 1. In all panels, the solid line is the identity. (a) $1/2||d\Phi/dt||$ vs. $1/2||d^2I/dt^2||$. Their near equality shows that the term $1/2 d\Phi/dt$ dominates the virial sum, indicating the importance of the variability of the mass flux through the clouds' borders for the total virial balance. (b) Volume-plus-surface kinetic terms vs. the virial sum neglecting the $1/2d\Phi/dt$ term. The near equality of both terms indicates the dominance of the kinetic terms over the remaining ones. This effect may be due to cloud bulk motion and should be eliminated by using an instantaneously-at-rest frame of reference for each cloud. (c) Volume vs. surface terms for internal energy (pressure) and (d) kinetic energy. The surface terms are seen to be comparable to the volume terms in general. The few points with large scatter in (c) are likely to correspond to regions of anomalous pressures due to recent star formation. (e) The gravitational term W vs. the sum of the remaining virial terms. A trend towards greater importance at larger scales is seen. However, a few points at near balance with gravity are seen at all scales. (f) Magnetic term M vs. the sum of the kinetic terms. An almost linear relation is observed. This is consistent with equipartition between kinetic and magnetic modes, if an offset is present, again due to the fact that clouds may have bulk velocities with respect to the integration volume.

properties as those observed in real interstellar clouds (Larson 1981), except for the density-size scaling relation, supporting the possibility that it may be the result of an observational effect (see also Larson 1981, Kegel 1989, Scalo 1990). With this motivation, we have now performed evaluations of the various terms in the VT. We have the following preliminary results: 1.- Both the second derivative of the moment of inertia and the last term in the equation (1) are dominant in the overall virial balance (fig. 1a). 2.-Comparing the remaining terms, the turbulent terms are seen to dominate (fig. 1b). 3.-The surface terms (which are often neglected under the assumption of vanishing fields outside the clouds) are in general of magnitude comparable to that of the volumetric ones (figs. 1c and d). 4.-The gravitational term is most important at large scales (fig. 1e). However, there are a few small (low energy content) clouds which have large values of the gravitational term. These may be the best candidates for collapse and star formation. Their scarcity appears consistent with the low efficiency of star formation. 5.-The magnetic term and the sum of the kinetic terms are proportional to each other (fig. 1f). This suggests there is equipartition between kinetic and magnetic modes, except for a constant factor, which may be due to the fact that clouds have bulk velocities with respect to the integration volume.

IV FINAL REMARKS

The dominance of the time-derivative and kinetic terms indicates the importance of flow through the volume boundaries, contrary to the cases considered by McKee & Zweibel (1992). In order to minimize this effect, it appears necessary to consider Eulerian volumes instantaneously at rest with respect to the center of mass of the clouds. However, preliminary attempts suggest that the flow through the boundaries cannot be eliminated completely, since the clouds are extremely amorphous and change shape rapidly. This work will be reported in a future paper (Ballesteros-Paredes & Vázquez-Semadeni 1997, in preparation).

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