

Gauge Symmetries and Vector-Boson Self Interactions[†]

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I review why we believe the electromagnetic, strong, and weak interactions are gauge theories, and what this implies for the self interactions of the gauge bosons. The modern point of view regarding non-renormalizable effective field theories is emphasized.

INTRODUCTION

The discovery of the W and Z bosons at the CERN $Spp\bar{p}S$ in 1983 (1) began the era of the weak vector boson. It opened with a bang, earning a Nobel prize for the discovery of the particles and the development of the machine that made it possible (2). That era is now in its maturity, with precision studies of Z bosons taking place at the CERN LEP and SLAC SLC e^+e^- colliders, and studies of both Z and W bosons taking place at the Fermilab Tevatron $p\bar{p}$ collider.

The era of weak boson pair production began more quietly about two years ago with the first WZ event at the Tevatron, shown in Fig. 1. We will hear at this meeting of the first direct evidence for the WWZ interaction from the CDF Collaboration (3,4). Soon we will see the production of W^+W^- pairs at the CERN LEP II e^+e^- collider (5), and large numbers of weak boson pairs will be provided by the CERN LHC (6). Future e^+e^- colliders will further contribute to the study of W^+W^- and ZZ pairs at high energy (7).

Given the present situation, this is an appropriate time to ask two questions:

- What have we learned from the weak-boson era?
- What can we learn from the era of weak-boson pair production?

The language for this discussion will be quantum field theory. As far as we know, quantum field theory is the only possible way to wed quantum mechanics and special relativity.¹ More precisely, it is the only formalism capable of simultaneously implementing the constraints of Lorentz invariance, unitarity, analyticity, and cluster decomposition (8).² Due to the well-known ultraviolet divergences of quantum field theory, it is unlikely that it is a valid description of nature to arbitrarily high energies. Thus we believe that at the energies currently available to us, nature must be described by an “effective” quantum field theory, even though we do not believe that quantum field theory is truly fundamental (8,9).

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¹There is also string theory, but at low energies this reduces to quantum field theory.

²Cluster decomposition is the requirement that scattering amplitudes factorize when two particles are separated by a large spacelike distance.

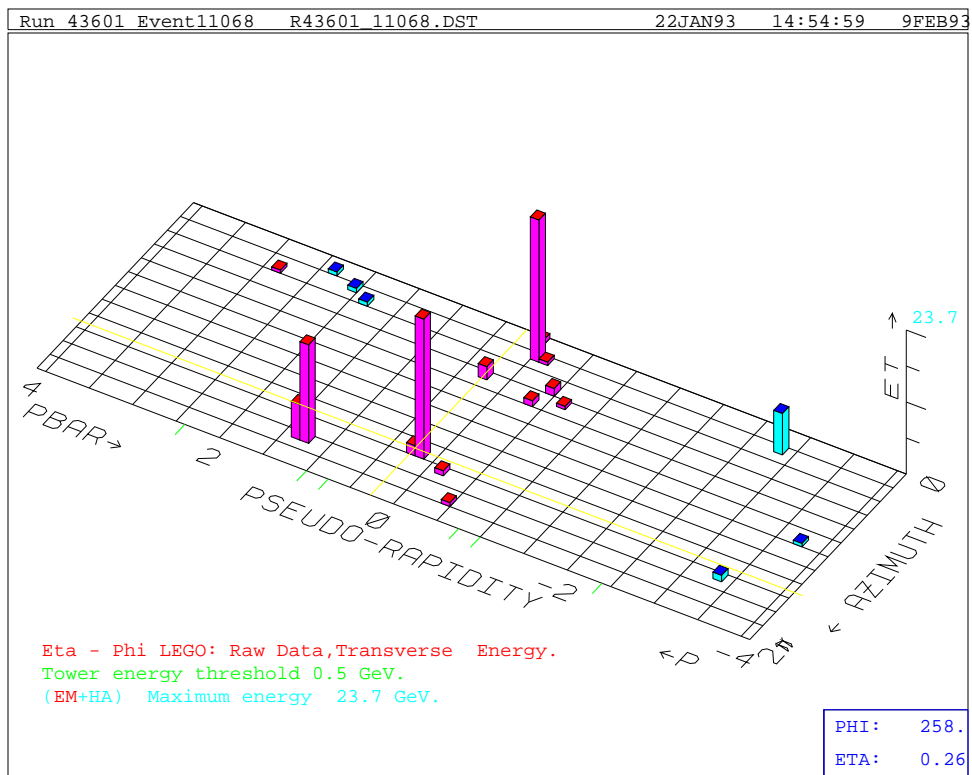


FIG. 1. The first WZ event at the Fermilab Tevatron, with leptonic decay of both weak bosons. The two tallest towers are the e^+e^- decay products of the Z boson, and the third tallest tower is an e^+ from W^+ decay.

In this talk I present a discussion of vector-boson self interactions from a modern point of view. The presentation is ahistorical, although I occasionally make remarks pertinent to the historical development of the theory. In particular, the modern point of view regarding non-renormalizable effective field theories figures prominently in the discussion. My goal is to present a theoretical overview of the subject, and to point to subsequent speakers who will develop various subtopics in more detail. In keeping with this style, I will often leave the citation of the literature to these speakers.³

Although the subject of this talk is mostly of interest for the weak interaction, it is instructive to also consider the electromagnetic and strong interactions. The order of presentation is as follows:

- Quantum Electrodynamics
- Quantum Chromodynamics
- Weak Interaction:
 - Higgs model
 - No-Higgs model

In a final section I reflect upon what we have learned from our deliberations.

QUANTUM ELECTRODYNAMICS

Let us begin by building the theory of quantum electrodynamics from two experimental facts:

1. The photon is massless.⁴
2. The photon has spin one.

From these experimental facts, the challenge is to construct a consistent quantum field theory of photons and electrically-charged fermions. The simplest field which contains spin one is the vector field,⁵ so we begin by associating the photon with a field $A^\mu(x)$. In so doing, we immediately encounter two difficulties:

1. The photon has only two degrees of freedom, corresponding to helicity ± 1 , while the vector field A^μ has four degrees of freedom.
2. The temporal component of the vector field has negative energy.

To see the latter point, consider the following Lagrangian for a vector field,

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2}\partial^\mu A^\nu \partial_\mu A_\nu \\ &= -\frac{1}{2}\left[\left(\frac{\partial A^0}{\partial t}\right)^2 - \left(\frac{\partial \mathbf{A}}{\partial t}\right)^2 + \dots\right] \end{aligned} \tag{1}$$

³Some of the observations made in this talk are also made in Ref. (10).

⁴The experimental upper bound on the photon mass is 3×10^{-27} eV. For the sake of argument, let us regard the photon as being exactly massless.

⁵Tensor fields also contain spin one, but do not reproduce Maxwell's equations in the classical limit (11).

which shows that in order for the spatial components of the vector field to have positive energy, the temporal component must have negative energy.

The resolution of these difficulties is well known. To eliminate the negative-energy component, we add an additional term to the Lagrangian which cancels the offending term above,

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}(\partial^\mu A^\nu \partial_\mu A_\nu - \partial^\mu A^\nu \partial_\nu A_\mu) \\ &= -\frac{1}{2} \left[\left(\frac{\partial A^0}{\partial t} \right)^2 + \dots - \left(\frac{\partial A^0}{\partial t} \right)^2 + \dots \right] \\ &= -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}\end{aligned}\tag{2}$$

where the last line casts the Lagrangian in the familiar form in terms of the electromagnetic field-strength tensor

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu .\tag{3}$$

The field A^0 has been eliminated as a dynamical degree of freedom. We now notice that this Lagrangian is invariant under the transformation

$$A^\mu \rightarrow A^\mu - \partial^\mu \lambda\tag{4}$$

which allows us to eliminate another degree of freedom from the theory, bringing us down to the desired two degrees of freedom (12).

We recognize Eq. 4 as the familiar gauge invariance of QED. What the above argument shows in a heuristic way, and has been proven rigorously (11,13), is that gauge invariance is *mandatory*; it can be derived from the assumption of a massless spin one particle.⁶ Gauge invariance is necessary to reconcile Lorentz invariance (the four-vector field A^μ) and unitarity (two degrees of freedom).⁷

The necessity of gauge invariance in the formulation of QED implies that photon self interactions of the form

$$\mathcal{L}_{int} = c_1 A^\mu A_\mu A^\nu A_\nu + c_2 \partial^\mu A^\nu A_\mu A_\nu\tag{5}$$

are strictly forbidden. Such terms are not gauge invariant, and their presence would destroy the consistency of the theory.

This does not mean that there cannot be photon self interactions, however. Let's write down the most general Lagrangian for the interaction of photons and fermions allowed by Lorentz invariance and gauge invariance:

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i\bar{\psi} \not{D}\psi - m\bar{\psi}\psi \\ &+ \frac{c_1}{M^2} m\bar{\psi} \sigma^{\mu\nu} \psi F_{\mu\nu} + \frac{c_2}{M^2} \bar{\psi} \gamma^\mu \psi \bar{\psi} \gamma_\mu \psi \\ &+ \frac{c_3}{M^4} (F^{\mu\nu} F_{\mu\nu})^2 + \dots\end{aligned}\tag{6}$$

⁶We now regard gauge invariance as fundamental, and use it to *explain* the masslessness of the photon, the reverse of the above logic. This point of view is largely a consequence of our realization that the strong and weak interactions are also gauge theories.

⁷An alternative point of view is that A^μ is not a four vector, because under Lorentz transformations it undergoes a gauge transformation as well. Again, gauge invariance is mandatory to ensure Lorentz invariance (11,14).

where the terms are arranged in increasing powers of dimension,⁸ and M is a mass scale introduced to make the constants c_i dimensionless. The first line above is the familiar Lagrangian of QED, and it describes the interaction of photons and fermions with remarkable success. The (infinite number of) additional terms are unnecessary; there is no experimental observation which requires any of them. In the past, such terms would have been dismissed on the grounds that they are non-renormalizable; they have coefficients with inverse powers of mass, the hallmark of non-renormalizable interactions. However, we no longer regard renormalizability as a fundamental requirement of a field theory, since we do not demand that a given field theory (or even field theory itself) be valid to arbitrarily high energy. Instead, we recognize that these additional terms are suppressed by inverse powers of M , which we regard as the energy scale at which ordinary QED ceases to be a valid description of the interaction of photons and fermions. The presence of such terms would be revealed to us by performing experiments at sufficiently high energy or with sufficient accuracy. The success of QED implies that M is a very large mass, at least 1 TeV. The renormalizability of ordinary QED ensures that these terms are not needed to cancel divergences, to all orders in perturbation theory, so the scale M can be arbitrarily large. However, the renormalizability of QED is just a consequence of the fact that M is much larger than the currently accessible energy and accuracy.

The last term in Eq. 6 represents a gauge-invariant four-photon interaction.⁹ The observation of such an interaction would be evidence for new physics beyond QED, but would be consistent with what we already know about QED.

QUANTUM CHROMODYNAMICS

Let us now approach QCD in a manner analogous to our approach to QED. We again begin with a list of “experimental facts”:

1. The gluon is massless.¹⁰
2. The gluon has spin one.¹¹
3. The gluon interacts with itself.¹²

Of course, these facts cannot be gleaned directly from experiment, which is the reason it took so many years to realize that QCD is the theory of the strong interaction. Let’s construct a consistent theory which incorporates the above facts.

As with QED, we attempt to construct a theory based on the vector field $G^\mu(x)$. We encounter the same difficulties as in QED (too many degrees of freedom, one of which has negative energy), with the same resolution (gauge invariance). However, we argued that

⁸QED possesses a global chiral symmetry, $\psi \rightarrow \exp[i\theta\gamma_5]\psi$, in the limit $m \rightarrow 0$, so we expect the coefficient of the term $\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}$, which violates this symmetry, to contain an explicit power of the fermion mass.

⁹The term $F_\nu^\mu F_\rho^\nu F_\mu^\rho$ vanishes since $F^{\mu\nu}$ is antisymmetric.

¹⁰Since gluons (and quarks) are confined, their masses cannot be measured directly. There is no evidence for a bare gluon mass, so let us assume it is exactly massless. Gluons behave as if they have a dynamically-generated mass of order 300 MeV, in the same sense that quarks have a dynamically-generated “constituent” mass of the same order; this should not be confused with the bare mass.

¹¹As evidenced, for example, by the angular distribution of three-jet events in e^+e^- collisions (15).

¹²This is necessary to explain confinement, asymptotic freedom, and other phenomena.

gauge invariance forbids vector-field self interactions, such as those in Eq. 5, so we run into a new problem: how do we allow the gluon to interact with itself and not spoil gauge invariance?

The resolution of this problem is also well known. Instead of a single gluon, we introduce a multiplet of gluons, eight to be exact. We expand the gauge transformation of QED, Eq. 4, to include a rotation of the eight gluons into each other under the group SU(3). The result is the familiar Yang-Mills theory of QCD, with eight self-interacting gluons. The essential point is that, as in QED, gauge invariance is *mandatory* for the consistency of the theory.¹³

The gluon self interaction is believed to be responsible for much of the physics of QCD which sets it apart from QED, such as confinement and asymptotic freedom. The gluon self interaction has been tested via $Z \rightarrow 4j$ events at LEP, where the decay $Z \rightarrow q\bar{q}gg$ involves the three-gluon interaction. Since gauge invariance is mandatory for the consistency of the theory, it is not acceptable to arbitrarily vary the three-gluon interaction when comparing theory with experiment. Instead, the analysis by the LEP experiments leaves the Yang-Mills gauge symmetry intact, but varies the gauge group (leaving the fermions in the fundamental representation, as in QCD) (20). Fig. 2 shows the result of an analysis of $Z \rightarrow 4j$, comparing the expectation of various gauge groups (boxes) and QCD (circle) with the data (star); the axes identify the gauge group, and are explained in the figure caption. The agreement of the data with the SU(3) prediction is impressive.

As with QED, gauge symmetry does not mean there cannot be anomalous vector-boson self interactions. The most general Lagrangian for gluons and quarks, consistent with Lorentz invariance and SU(3) gauge symmetry, is

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}\text{Tr} G^{\mu\nu}G_{\mu\nu} + i\bar{\psi}\not{D}\psi - m\bar{\psi}\psi \\ & + \frac{c_1}{M^2}\bar{\psi}\gamma^\mu\psi\bar{\psi}\gamma_\mu\psi + \frac{c_2}{M^2}\text{Tr} G_\nu^\mu G_\rho^\nu G_\mu^\rho + \dots \end{aligned} \quad (7)$$

where $G^{\mu\nu}$ is the non-Abelian field-strength tensor. The first line is the Lagrangian of ordinary QCD, and the (infinite number of) additional terms correspond to new physics associated with a mass scale M , as in QED. The first such term corresponds to a four-quark contact interaction, and is searched for in high- p_T jet events at the Tevatron, resulting in a lower bound on M of about 1 TeV (17). The second such term yields an anomalous three-gluon interaction,¹⁴ and is best sought in top-quark production at the LHC (18).¹⁵ It also yields an anomalous four-, five-, and six-gluon interaction.

As with QED, there is no reason not to expect these additional terms in the Lagrangian to be present, but there is also nothing which tells us at what mass scale, M , we should expect them to manifest themselves. The renormalizability of ordinary QCD ensures that these terms are not necessary to cancel divergences, to all orders in perturbation theory. However, as with QED, the renormalizability of the theory is simply a consequence of the fact that M is much greater than the currently accessible energy and accuracy.

¹³To the best of my knowledge, it has never been rigorously shown that Yang-Mills gauge theory is the unique theory of massless, interacting, spin-one particles, based on vector fields. The necessity of gauge symmetry is suggested by the Weinberg-Witten theorem on massless charged particles (16). Of course, we now regard the masslessness of the gluons to be a *consequence* of gauge invariance, just as in QED.

¹⁴As mentioned in a previous footnote, the analogous term vanishes in QED. It does not vanish in QCD because $G^{\mu\nu}$ is an SU(3) matrix.

¹⁵This term may also be sought in $Z \rightarrow 4j$, which yields a weak bound (19).

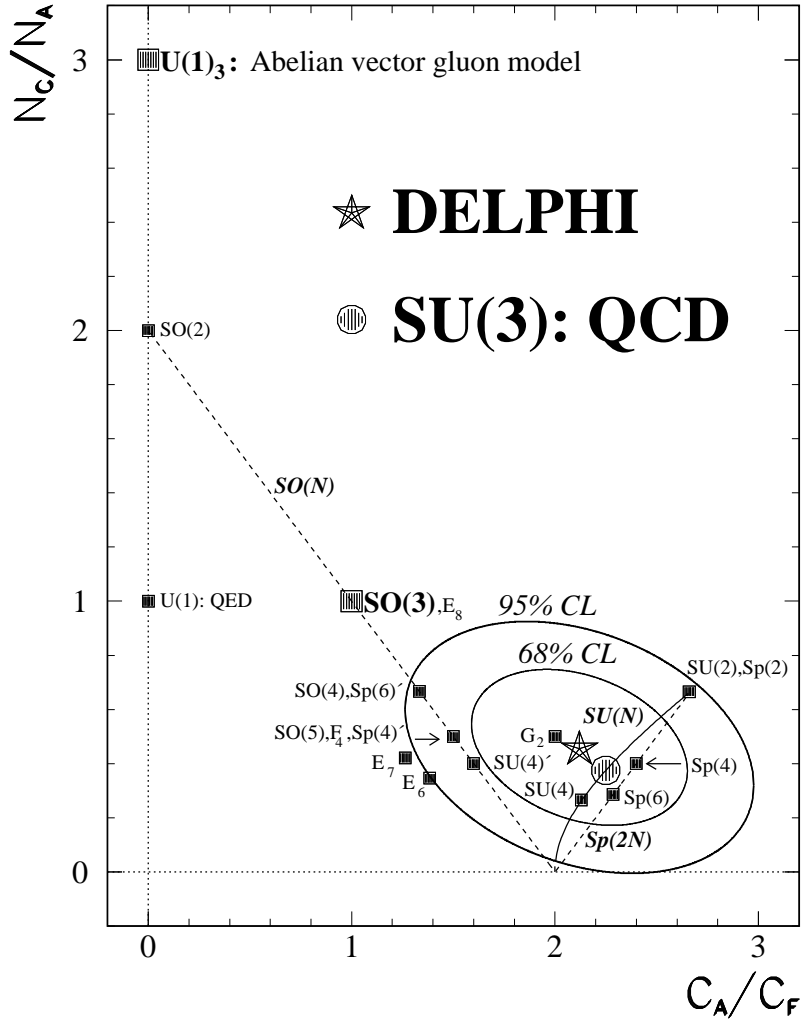


FIG. 2. Comparison of $Z \rightarrow 4j$ (\star) with the expectation of a Yang-Mills gauge theory based on various gauge groups (\square) and $SU(3)$ (\odot). N_C/N_A is the ratio of the number of quark colors to the number of gluons, and C_A/C_F is the ratio of the strength of the three-gluon interaction to that of gluon bremsstrahlung from quarks. Figure from Ref. (20).

WEAK INTERACTION

Let's move on to the weak interaction. As with QED and QCD, we begin with experimental facts:

1. The weak bosons are massive.
2. The weak bosons have spin one.

The big difference between the weak interaction and both QED and QCD is that the vector bosons are massive. Let's construct a consistent theory of massive vector bosons, associated with a field $W^\mu(x)$. We'll add more experimental facts later.

As with QED and QCD, we immediately run into the problem that the vector field W^μ has too many degrees of freedom.¹⁶ This problem is less severe for a massive spin-one particle because it has three degrees of freedom, corresponding to helicities $\pm 1, 0$, rather than the two degrees of freedom of the massless case. As with QED and QCD, the temporal component of the vector field corresponds to a state of negative energy, so we must eliminate it as a dynamical degree of freedom.

Consider the following Lagrangian for a non-interacting vector field W^μ of mass M_W :

$$\mathcal{L} = -\frac{1}{4}W^{\mu\nu}W_{\mu\nu} + \frac{1}{2}M_W^2W^\mu W_\mu \quad (8)$$

where

$$W^{\mu\nu} = \partial^\mu W^\nu - \partial^\nu W^\mu . \quad (9)$$

The kinetic part of the Lagrangian is written in terms of the field-strength tensor $W^{\mu\nu}$ in order to remove W^0 as a dynamical field, just as we did for QED and QCD. However, for the case of a massive vector field one can do even better; the extra degree of freedom can be removed in a manifestly Lorentz-invariant manner (21–24). Consider the equation of motion of the field W^μ , derived from the Lagrangian in Eq. 8:

$$\partial_\nu W^{\nu\mu} + M_W^2 W^\mu = 0 . \quad (10)$$

Now apply ∂_μ to this equation. The first term vanishes since $W^{\mu\nu}$ is antisymmetric, so we find

$$\partial_\nu W^\nu = 0 . \quad (11)$$

This is a constraint equation on the field W^μ , and allows us to remove one degree of freedom. Since it is a Lorentz-invariant condition, Lorentz invariance remains manifest.¹⁷

Although Eq. 11 is reminiscent of the familiar Lorentz gauge condition of QED and QCD, it is not a gauge condition at all. The massive vector theory has no gauge invariance whatsoever; gauge invariance is *non-existent* and *unnecessary*. This is in striking contrast to QED and QCD. Because we are so used to working with gauge theories, this simple point is sometimes forgotten. The quantization of a massless vector theory such as QED or QCD is a difficult task, and one tends to forget how easy it is to quantize a massive vector theory.¹⁸

¹⁶At this point, I use W^μ to denote a generic massive vector field.

¹⁷Alternatively, one can simply impose this constraint on the field as an auxiliary condition (22).

¹⁸If the massive vector theory is a spontaneously-broken gauge theory, quantization is as complicated as in QED and QCD, of course.

This construction is not upset by the introduction of interactions with other fields, or even self interactions. One way to see this is to consider the propagator of the vector field. The free field equation, Eq. 10, written in terms of the vector field, is

$$\square W^\mu - \partial_\nu \partial^\mu W^\nu + M_W^2 W^\mu = 0 . \quad (12)$$

This yields the momentum-space propagator

$$D^{\mu\nu}(p) = i \frac{-g^{\mu\nu} + \frac{p^\mu p^\nu}{M_W^2}}{p^2 - M_W^2} . \quad (13)$$

The numerator of the propagator contains the sum over the three polarization states corresponding to the three helicity states of a massive spin-one particle, and nothing more.¹⁹ Hence there is no concern about interactions potentially coupling to unphysical polarization states, as there is in QED and QCD (14,22,23).²⁰

Given that gauge invariance has nothing to do with a generic massive vector boson theory, one must wonder why we believe the weak interaction is described by a gauge theory. The answer lies in a third experimental fact:

3. The couplings of the weak bosons to the three generations of quarks and leptons are, to high precision, those of an $SU(2)_L \times U(1)_Y$ gauge theory.

But if the weak interaction is a gauge theory, why aren't the weak bosons massless, as appears to be required of gauge bosons? The well-known solution to this puzzle is that the gauge symmetry is spontaneously broken (25). This means that while the Lagrangian *is* invariant under $SU(2)_L \times U(1)_Y$ gauge transformations, the solution to the Lagrangian is not.

A skeptic might ask if the (local) gauge symmetry is really necessary. Wouldn't it be enough to impose *global* $SU(2)_L \times U(1)_Y$ symmetry on the Lagrangian to reproduce the observed couplings of the weak bosons to fermions? The answer is no; one needs the local gauge symmetry to explain the universality of the weak interaction, i.e., to explain why the weak bosons couple the same to quarks as to leptons, and to all three generations (as far as we know).²¹ To see this, consider the Lagrangian for the coupling of the weak bosons to fermions,²²

$$\mathcal{L} = i\bar{\psi}_L \gamma^\mu (\partial_\mu + ig \frac{1}{2} \tau \cdot W_\mu) \psi_L . \quad (14)$$

Each term is separately invariant under global $SU(2)_L$ transformations, regardless of the value of g . However, both terms are needed to ensure invariance under *local* $SU(2)_L$ transformations $U(x)$,

¹⁹This is the familiar “unitary gauge” propagator of a spontaneously-broken gauge theory, meaning it contains only the physical polarization states. Since we are not (yet) treating the massive vector field as a gauge field, we avoid this language.

²⁰In QED and QCD, the propagator must couple to a conserved current, or gauge invariance (and hence Lorentz invariance) is lost (12).

²¹It is particularly important that we test the universality of the weak interaction with respect to the recently-discovered top quark (26).

²²Here and throughout I consider only the $SU(2)_L$ part of the weak interaction, and ignore the hypercharge interaction. The field W^μ represents an $SU(2)_L$ triplet of gauge fields, and τ are the usual Pauli matrices. As usual, $\psi_L \equiv \frac{1}{2}(1 - \gamma_5)\psi$ denotes the left-chiral fermion field.

$$\psi_L \rightarrow U\psi_L \quad (15)$$

$$\tau \cdot W^\mu \rightarrow U\tau \cdot W^\mu U^\dagger + \frac{2i}{g}(\partial^\mu U)U^\dagger \quad (16)$$

and they must be present exactly as shown in Eq. 14, with the same coupling g for all fermions (27).

The skeptic might counter that, while willing to accept local gauge invariance as the explanation of the universality of the coupling of weak bosons to fermions, this universality need not extend to the gauge-boson self interactions. Couldn't one imagine that the gauge symmetry is present only in the fermionic sector of the theory? The answer to this is also negative. The one-loop correction to the coupling of weak bosons to fermions involves the weak-boson self interaction, and unless this interaction is of the Yang-Mills form, it will generally destroy the gauge-theory form of the fermionic coupling. While the couplings may be "readjusted" to their experimentally-observed values, the explanation of universality is lost. This problem is especially severe in light of the fact that the quarks experience the strong interaction, while the leptons do not, so the amount of "readjustment" necessary will generally differ for the two types of fermions. Thus we conclude that in order for gauge symmetry to explain the universality of the weak interaction, it must be a symmetry of the full Lagrangian, not just part of it.

Just as in QED and QCD, anomalous vector-boson self interactions may be introduced via higher-dimension terms in the Lagrangian, suppressed by inverse powers of some mass scale, M . However, in the weak interaction, the implementation of this differs depending on whether or not a fundamental Higgs field is introduced in the Lagrangian. Below we pursue these possibilities separately.

Higgs model

Consider including the Higgs-doublet field, ϕ , to break the electroweak symmetry in the standard way. The Lagrangian is

$$\begin{aligned} \mathcal{L} = & -\frac{1}{8}\text{Tr} W^{\mu\nu}W_{\mu\nu} + i\bar{\psi}_L \not{D}\psi_L \\ & + (D^\mu\phi)^\dagger D_\mu\phi - V(\phi^\dagger\phi) \\ & + \frac{c_1}{M^2}(D^\mu\phi)^\dagger W_{\mu\nu}D^\nu\phi + \frac{c_2}{M^2}\text{Tr} W_\nu^\mu W_\rho^\nu W_\mu^\rho + \dots \end{aligned} \quad (17)$$

where $W^{\mu\nu}$ is the full non-Abelian field-strength tensor,

$$W^{\mu\nu} = \tau \cdot (\partial^\mu W^\nu - \partial^\nu W^\mu - gW^\mu \times W^\nu). \quad (18)$$

The first two lines are the standard electroweak Lagrangian. When the Higgs field ϕ acquires a vacuum-expectation value, the first term in the third line produces additional three- and four- W interactions. The last term contributes additional three-, four-, five-, and six- W interactions (27–29). These anomalous vector-boson self interactions are suppressed by inverse powers of some mass scale, M , which is the scale at which the ordinary electroweak theory ceases to be a valid description of nature. As with QED and QCD, we have no reason not to expect that such terms are there. Since the standard electroweak theory is renormalizable, these terms are not necessary to cancel divergences, to all orders in perturbation theory, so M can be arbitrarily large. However, radiative corrections to the Higgs vacuum-expectation value diverge quadratically,²³ so the value

²³An equivalent argument is usually presented in terms of the Higgs mass.

$v \approx 250$ GeV is natural only if there is new physics which cuts off the divergence at or below 1 TeV (30). Thus naturalness of the Higgs model suggests that M should not be greater than about 1 TeV.²⁴

No-Higgs model

Although we believe that the electroweak interaction is a spontaneously-broken gauge theory, we do not know if the spontaneous symmetry breaking is the result of the vacuum-expectation value of a fundamental Higgs field. If we insist that the theory be renormalizable and perturbative (weak coupling), then the only option is indeed the standard Higgs model (31) and generalizations thereof, such as a two-Higgs-doublet model as employed in the supersymmetric standard model (32). However, we have no guarantee that nature is so kind as to provide us with a symmetry-breaking mechanism that can be analyzed perturbatively.

Whatever the symmetry-breaking mechanism, it must provide the three Goldstone bosons which are absorbed by the W^\pm and Z bosons to become massive. A generic approach to the symmetry-breaking physics is then to introduce only these three Goldstone bosons into the Lagrangian, but no other fields (27–29,33,34). Although the resulting theory is non-renormalizable, it should be a valid effective field theory at energies below the mass scale of the symmetry-breaking physics responsible for the Goldstone bosons.

Let us introduce the three Goldstone-boson fields π via the field²⁵

$$\Sigma \equiv \exp[i\tau \cdot \pi/v] \quad (19)$$

where $v = 2M_W/g$. The Lagrangian is

$$\begin{aligned} \mathcal{L} = & -\frac{1}{8}\text{Tr} W^{\mu\nu}W_{\mu\nu} + i\bar{\psi}_L \not{D}\psi_L \\ & + \frac{v^2}{4}\text{Tr} (D^\mu\Sigma)^\dagger D_\mu\Sigma \\ & + c_1 \frac{v^2}{M^2}(\text{Tr} (D^\mu\Sigma)^\dagger D_\mu\Sigma)^2 + c_2 \frac{v^2}{M^2}\text{Tr} W^{\mu\nu}(D_\mu\Sigma)^\dagger D_\nu\Sigma + \dots \end{aligned} \quad (20)$$

where

$$D^\mu\Sigma = (\partial^\mu + i\frac{g}{2}\tau \cdot W^\mu)\Sigma \quad (21)$$

is the gauge-covariant derivative. The Σ field transforms under $\text{SU}(2)_L$ as

$$\Sigma \rightarrow U\Sigma. \quad (22)$$

The first line in Eq. 20 is the usual weak-interaction Lagrangian. The second line is responsible for the W mass, which is evident when it is expanded in terms of the π fields:

$$\frac{v^2}{4}\text{Tr} (D^\mu\Sigma)^\dagger D_\mu\Sigma = \frac{1}{2}\frac{g^2}{4}v^2W^\mu W_\mu + \frac{1}{2}\partial^\mu\pi \cdot \partial_\mu\pi + \frac{1}{2v^2}(\pi \cdot \partial^\mu\pi)(\pi \cdot \partial_\mu\pi) + \dots \quad (23)$$

The physical content of the theory is manifest in the unitary gauge, $\Sigma = 1$, in which the Goldstone bosons are completely absorbed by the weak vector bosons, and disappear from the Lagrangian. However, it is convenient for our discussion (and for calculational purposes) to consider a gauge in which the Goldstone bosons are present.

²⁴One possibility for new physics which cuts off the quadratic divergence is supersymmetry.

²⁵The choice of the symbol π to denote the Goldstone bosons is by analogy with the physical pion field, which is an (approximate) Goldstone boson of spontaneously-broken chiral symmetry in QCD.

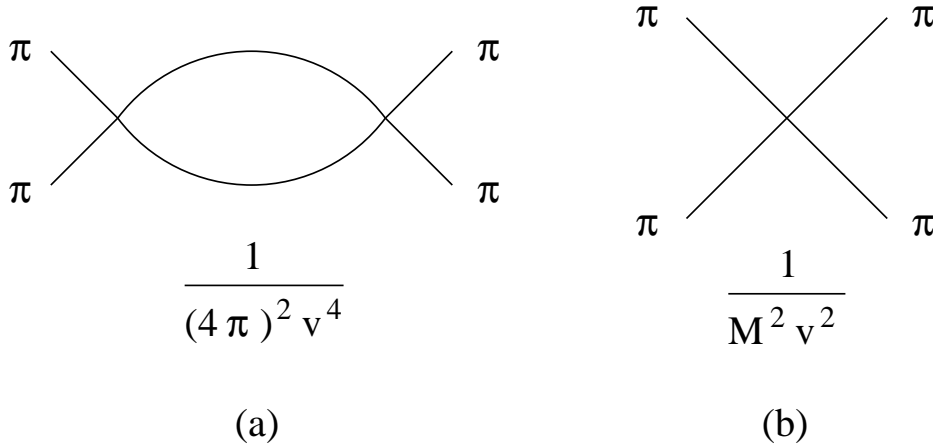


FIG. 3. (a) One-loop amplitude for four Goldstone bosons. (b) Tree-level interaction of four Goldstone bosons required to cancel the divergence from the one-loop amplitude.

The first term in the third line contributes an anomalous four- W interaction, and the second term an anomalous three- and four- W interaction. These terms are suppressed by inverse powers of a mass scale, M , which is the scale at which the physics responsible for spontaneous symmetry breaking resides. At first sight this mass scale can be made arbitrarily large, as in QED, QCD, and the Higgs model. However, the term responsible for the vector-boson masses, Eq. 23, contains a non-renormalizable four- π interaction with a coefficient proportional to $1/v^2$, as shown. This coefficient sets the scale for the other non-renormalizable terms. A one-loop four- π amplitude constructed from two of these four- π interactions, shown in Fig. 3(a), is of order $1/(4\pi)^2 v^4$, where the factor $1/(4\pi)^2$ arises from the loop integration. This has the same dimensions as the contribution to the four- π amplitude from the terms in the last line of Eq. 20, shown in Fig. 3(b), of order $1/M^2 v^2$. Since the one-loop amplitude is divergent, these terms must be there to cancel the divergence. Thus M must be of order $4\pi v \approx 3$ TeV or less. The physics responsible for electroweak symmetry breaking must therefore manifest itself by at least 3 TeV.

There is one last experimental fact we can add to our discussion:

$$4. M_W \approx M_Z \cos \theta_W$$

This is embodied in the ρ parameter,

$$\rho \equiv M_W^2 / (M_Z^2 \cos^2 \theta_W) \approx 1. \quad (24)$$

In other words, not only are the W and Z bosons massive, but their masses are related. This can be explained by hypothesizing that the symmetry-breaking sector of the theory possesses a global $SU(2)$ symmetry, called a “custodial” symmetry (30,34–36). Although it is not always made explicit, the standard Higgs model contains this symmetry. Models of dynamical electroweak symmetry breaking, such as Technicolor (30,35), must contain this symmetry even if Eq. 24 is satisfied at tree level, since the corrections are potentially large (strong coupling). The custodial symmetry is further evidence that the properties of the weak interaction are dictated by symmetry.

OUTLOOK

Of the electromagnetic, strong, and weak interactions, only the last guarantees new physics at an accessible mass scale. This is the physics associated with electroweak symmetry breaking. All we know for sure about this physics is that it must manifest itself by at least $4\pi v \approx 3$ TeV. Furthermore, the fact that the W and Z masses are related by $M_W \approx M_Z \cos \theta_W$ suggests that the symmetry-breaking sector contains a “custodial” global $SU(2)$ symmetry.

The electroweak-symmetry-breaking sector could be the source of anomalous weak-vector-boson self interactions. We have considered two scenarios for the electroweak-symmetry-breaking physics:

1. Higgs model
2. No-Higgs model: only Goldstone bosons up to $4\pi v \sim 3$ TeV

These two models are so commonly studied that one begins to think they are the only possibilities. This is not the case. The symmetry-breaking physics could be very rich, containing resonances, new fermions, new gauge bosons, etc. The fact that nature makes use of gauge theories for the three known low-energy forces leads one to guess that the symmetry-breaking sector is also a gauge theory. Examples which implement this idea are fixed-point Technicolor (37), walking Technicolor (38), two-scale Technicolor (39), fermions in large representations of the gauge group (40,41), etc. From a theoretical point of view, these models receive less attention because they are neither amenable to a perturbative analysis (like the Higgs model) nor to a “model-independent” analysis (like the No-Higgs model). They also generically run into difficulty with precision electroweak experiments (42–44), something we have learned from the vector-boson era. Nature may not care about any of these objections. We should probe higher energies and keep an open mind regarding the manner in which the physics of electroweak symmetry breaking reveals itself.

After this long discussion, we are now prepared to go back and answer a question we posed at the beginning: What can we learn from the era of weak boson pair production? At the very least, we will see a confirmation of the universality of the weak interaction, extended to the weak-boson self interaction, as depicted schematically in Fig. 4(a). It is the universality of the fermionic couplings of the weak bosons which led us to the electroweak theory in the first place, so this confirmation will be a crowning achievement. However, we hope for much more from this era; we anticipate at least the first signs of the physics responsible for electroweak symmetry breaking, and, at best, the complete revelation of this physics. One possible manifestation of this physics is a $J = 1$ resonance which couples to the weak bosons, as depicted in Fig. 4(b). Although we would not usually regard this as an “anomalous vector-boson self interaction”, there is no reason why we should not. If we observe such a resonance, it would be a *very* anomalous vector-boson self interaction.

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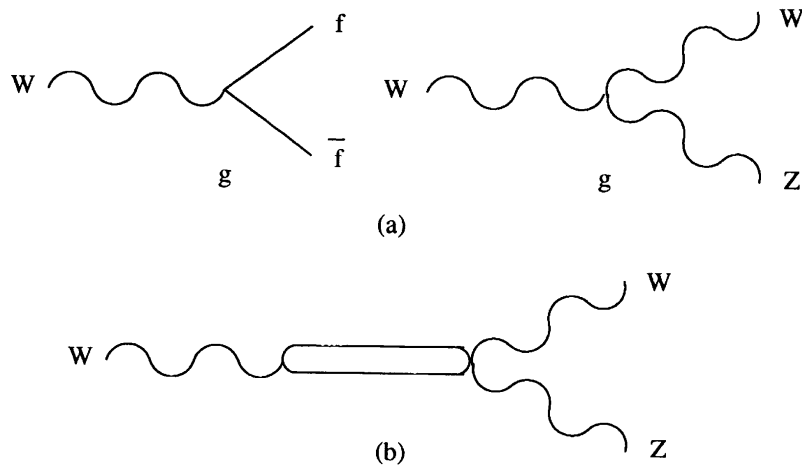


FIG. 4. (a) Schematic illustration of the universality of the coupling of weak bosons to fermions and to themselves. (b) An anomalous weak-boson self interaction produced by a $J = 1$ resonance.

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