Color superconductivity: Continuity of quark and hadron matter, the role of the strange quark mass, and perturbative results

Thomas Schäfer[a]

[a]School of Natural Sciences
Institute for Advanced Study
Princeton, NJ 08540

We summarize some recent results on the structure of QCD at very high baryon density.

1. Introduction

It was pointed out almost 20 years ago that asymptotic freedom and the presence of a Fermi surface imply that QCD at very large density is a color superconductor [1]. This result was largely forgotten until it was realized that, using interactions that reproduce the strength of chiral symmetry at zero density, one expects the superconducting gap to be quite large, on the order of $\Delta \simeq 100$ MeV at densities $\rho \sim 5\rho_0$ [2,3]. A lot of work on the superconducting phase of QCD has appeared over the past year. In this contribution we would like to summarize a number of interesting results.

2. Continuity of quark and hadron matter

For two flavors the order parameter has the structure $\langle \epsilon^{ab3}\psi^{a\,T} C\gamma_5\tau_2\psi^b\rangle$. This order parameter leaves the chiral $SU(2)_L \times SU(2)_R$ symmetry unbroken, but breaks $SU(3)$ color down to $SU(2)$. In the case of three flavors an interesting new possibility arises. The order parameter [4]

$$\langle \psi_i^{a\,T} C\gamma_5\psi_j^b\rangle = \Delta_1\delta_i^a\delta_j^b + \Delta_2\delta_j^a\delta_i^b \tag{1}$$

locks the local color orientation a, b to the flavor orientation i, j. In the color-flavor-locked state color $SU(3)$ is completely broken and all gluons acquire a mass. In addition to that, the chiral $SU(3)_L \times SU(3)_R$ is broken to the diagonal $SU(3)_{L+R+C}$. This provides a very unusual mechanism for chiral symmetry breaking. The chiral structure of the order parameter is $\psi_L\psi_L - \psi_R\psi_R$, so there is no direct coupling between left and right handed fields. Chiral symmetry is broken because color locks the residual flavor symmetry of the left handed quarks to the corresponding symmetry of the right handed quarks.

Not only is chiral symmetry broken, but also the spectrum of excitations in the color-flavor-locked (CFL) phase looks remarkably like the spectrum of QCD at low density [5]. The excitations can be classified according to their quantum numbers under the unbroken $SU(3)$, and by their electric charge. The modified charge operator that generates a true symmetry of the CFL phase is given by a linear combination of the original charge operator Q_{em} and the color hypercharge operator $Q = \mathrm{diag}(-2/3, -2/3, 1/3)$. Also, baryon number is only broken modulo 2/3, which means that one can still distinguish baryons from mesons. We find that the CFL phase contains an octet of Goldstone bosons associated with chiral symmetry breaking, an octet of vector mesons, an octet and a singlet of baryons, and a singlet Goldstone boson related to superfluidity. All of these states have integer charges.

With the exception of the $U(1)$ Goldstone boson, these states exactly match the quantum numbers of the lowest lying multiplets in QCD at low density. In addition to that, the presence of the $U(1)$ Goldstone boson can also be understood. The $U(1)$ order parameter is $\langle (uds)(uds) \rangle$. This order parameter has the quantum numbers of a $J^\pi = 0^+$ $\Lambda\Lambda$ pair condensate. In $N_f = 3$ QCD, this is the most symmetric two nucleon channel, and a very likely channel for superfluidity to occur in nuclear matter at low to moderate density. We conclude that in QCD with three degenerate light flavors, there is no fundamental difference between the high and low density phases. This implies that the low density nuclear phase and the high density quark phase might be continuously connected, without an intervening phase transition.

The order parameter (1) breaks the chiral $SU(3)$ symmetry but leaves a discrete Z_2 symmetry unbroken. This symmetry, however, is explicitly broken by instantons. The quark condensate $\langle \bar{\psi}\psi \rangle$ also violates Z_2 symmetry. In weak coupling, the quark condensate is therefore generated by instantons. This is easy to see from the structure of the instanton induced interaction between quarks. In the case $N_f = 3$, the instanton vertex is of the form $(\bar{\psi}_L \psi_R)^3$. In the CFL phase, we can absorb four of the external legs into the $\langle \bar{\psi}_L \bar{\psi}_L \rangle$ and $\langle \psi_R \psi_R \rangle$ condensates. The remaining $\bar{\psi}_L \psi_R$ vertex directly generates a quark condensate. The magnitude of the chiral condensate in the CFL phase was calculated in [6]. We found that the quark constituent mass $\Sigma \simeq 10$ MeV is significantly smaller than the gap, $\Delta \simeq 50$ MeV.

3. The role of the strange quark mass

So far, we have only considered the case of three degenerate quark flavors. In the real world, the strange quark is significantly heavier than the up and down quarks. The role of the strange quark mass in the high density phase was studied in [10,8]. The main effect is a purely kinematic phenomenon that is easily explained. The Fermi surface for the strange quarks is shifted by $\delta p_F = \mu - (\mu^2 - m_s^2)^{1/2} \simeq m_s^2/(2\mu)$ with respect to the Fermi surface of the light quarks. The condensate involves pairing between quarks of different flavors at opposite points on the Fermi surface. But if the Fermi surfaces are shifted, then the pairs do not have total momentum zero, and they cannot mix with pairs at others points on the Fermi surface. In a superfluid the Fermi surface is smeared out over a range Δ. This means that pairing between strange and light quarks can take place as long as the mismatch between the Fermi momenta is smaller than the gap,

$$\Delta > \frac{m_s^2}{2\mu}. \qquad (2)$$

This conclusion is supported by a more detailed analysis [7,8]. Since flavor symmetry is broken, we allow the $\langle ud \rangle$ and $\langle us \rangle = \langle ds \rangle$ components of the CFL condensate to be

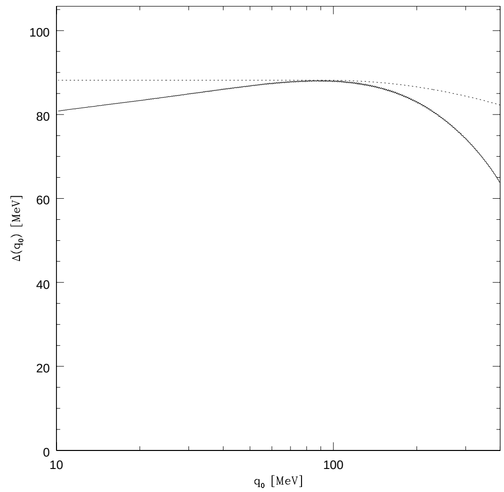

Figure 1. Solution of the Eliashberg equation with electric and magnetic gluon exchanges for $\mu = 400$ MeV.

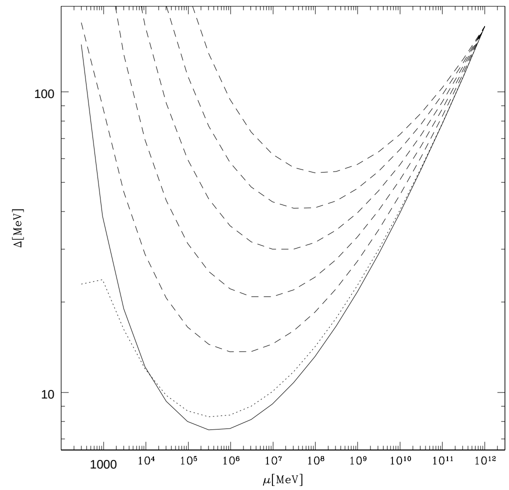

Figure 2. Dependence of the gap on the chemical potential, compared to a simple scaling form $g^{-k}\exp(-c/g)$, $(k = 1,\ldots 5)$.

different. The $N_f = 2$ phase corresponds to $\langle us\rangle = \langle ds\rangle = 0$. We find that there is a first order phase transition from the CFL to the $N_f = 2$ phase. The critical strange quark mass is in rough agreement with the estimate (2).

This brings up the question whether QCD is in the CFL phase for realistic values of the quark masses and for physically relevant densities $\rho =\simeq (5-10)\rho_0$. This question is difficult to answer since it requires an accurate estimate of the gap and of the dynamically generated strange quark mass in the high density phase. We will see in the next section that for asymptotically large densities the gap grows as a function of density. This means that QCD will eventually enter the CFL phase for any value of the strange quark mass.

4. Superconductivity from perturbative gluon exchanges

In our initial work the gap was calculated from an instanton induced interaction. This interaction reproduces the strength of chiral symmetry breaking at low density and leads to gaps on the order of 100 Mev at several times nuclear matter density. At very high density, instantons are suppressed and the gap is dominated by perturbative gluon exchanges. This problem was recently reexamined by Son [9], who pointed out that unscreened magnetic gluon exchanges lead to a gap that scales as $\Delta \sim \exp(-const./g)$, rather than the naive expectation $\Delta \sim \exp(-const./g^2)$. We have recently strengthened this result by deriving an Eliashberg equation for the gap in the weak coupling limit [10]. Including magnetic gluon exchanges only, the gap equation reads

$$\Delta(p_0) = \frac{g^2}{18\pi^2}\int dq_0 \log\left(1+\frac{64\pi\mu}{N_f g^2|p_0-q_0|}\right)\frac{\Delta(q_0)}{\sqrt{q_0^2+\Delta(q_0)^2}}. \tag{3}$$

This equation is independent of the gauge parameter in a class of covariant gauges. The logarithm arises from almost colinear magnetic gluon exchanges. The colinear singularity is regulated by dynamic screening. The effects of screening are taken into account by using the gluon propagator in the hard dense loop approximation. Magnetic gluon exchanges generate a gap that scales as $\Delta \simeq c\mu g^{-2}\exp(-3\pi^2/(\sqrt{2}g))$, where the coefficient in front of the exponent is on the order of 64π (for $N_f = 2$).

We have also included the effects of electric gluon exchanges. Electric gluons are less important than magnetic gluons because electric screening takes place at $q_E \sim g\mu$, while dynamic screening sets in at $q_M \sim (g^2\mu^2\Delta)^{1/3}$. As a consequence, electric gluons do not modify the coefficient in the exponent, but they change the overall magnitude of the gap. A typical solution of the Eliashberg equation with both electric and magnetic gluon exchanges included is shown in Fig. 1. The scaling of the gap with the chemical potential is shown in Fig. 2. Here, we have used the one-loop running coupling constant $g(\mu)$. The result is well described by

$$\Delta \simeq c\mu g^{-5}\exp(-3\pi^2/(\sqrt{2}g)) \tag{4}$$

with $c \simeq 256\pi^4$. We find that for densities that are of physical interest, $\mu < 500$ MeV, the gap reaches $\Delta \simeq 100$ MeV. We should caution that in this regime, $g \simeq (2-4)$ and the result may not be reliable. Nevertheless, it is gratifying to see that the result matches the number inferred from effective interactions at low density.

REFERENCES

1. D. Bailin and A. Love, Phys. Rept. **107**, 325 (1984).
2. R. Rapp, T. Schäfer, E. V. Shuryak, and M. Velkovsky, Phys. Rev. Lett. **81**, 53 (1998).
3. M. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. **B422**, 247 (1998).
4. M. Alford, K. Rajagopal, and F. Wilczek, Nucl. Phys. **B537**, 443 (1999).
5. T. Schäfer and F. Wilczek, Phys. Rev. Lett. **82**, 3956 (1999).
6. R. Rapp, T. Schäfer, E. V. Shuryak, and M. Velkovsky, hep-ph/9904353.
7. T. Schäfer and F. Wilczek, hep-ph/9903503.
8. M. Alford, J. Berges, and K. Rajagopal, hep-ph/9903502.
9. D. T. Son, Phys. Rev. **D 59**, 094019 (1999).
10. T. Schäfer and F. Wilczek, hep-ph/9906512.