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Two Firms is Enough for Competition, but Three or More is Better

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Two Firms Is Enough for Competition, But Three or More Is Better.

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Abstract

We present an oligopoly model where a certain fraction of consumers engage in costly non-sequential search to discover prices. There are three distinct price dispersed equilibria characterized by low, moderate and high search intensity, respectively. We show that the effects of an increase in the number of firms active in the market are *sensitive* (*i*) to the equilibrium consumers' search intensity, and (*ii*) to the *status quo* number of firms. For instance, when consumers search with low intensity, increased competition does not affect expected price, leads to greater price dispersion and welfare declines. In contrast when consumers search with high intensity, increased competition results in lower prices when the number of competitors in the market is low to begin with, but in higher prices when the number of competitors is large. Moreover, duopoly yields identical expected price and price dispersion but higher welfare than an infinite number of firms.

Keywords: consumer search, expected price, fixed-sample-size search, oligopoly, price dispersion

JEL Classification: D40, D83, L13

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1 Introduction

One of the questions that has received much attention in economic theory is how the number of competitors in a market and market outcomes are related. Despite the existence of work showing the contrary (see e.g. Rosenthal, 1980; Satterthwaite, 1979), the prediction that emerges from a market where firms interact in a Cournot fashion has come to dominate economic thought, namely, that an increase in the number of competitors leads to larger aggregate output, lower market prices and improved market performance measured in terms of some social welfare criterion (Ruffin, 1971).[1]

One of the branches in the economics literature that has seriously challenged the generality of this belief is the consumer search literature. Stahl (1989) and Stiglitz (1987) have shown that the impact of increasing the number of firms in a market characterized by consumer search can very well be the opposite and thus lead to higher prices and poorer market performance. Our paper is in this tradition; we consider a search model and compare market outcomes when the number of firms that are active in the market varies.

We present a model with two types of consumers: 'fully informed' consumers, who can costlessly search for prices, and 'less-informed' consumers, who must pay a fixed search cost for each price quotation they observe.[2] Consumers have a common willingness to pay for the good and buy at most a single unit. On the supply side of the market, there are N firms producing a homogeneous good and they set prices so as to maximize profits. Firms produce at constant marginal cost. We normalize this cost to zero without loss of generality. The game is a one-shot *simultaneous* move game: firms set prices and consumers decide how many searches to make at the same moment in time. Our model thus brings the demand side of Varian (1980) and Stahl (1989) into an *oligopolistic* version of Burdett and Judd (1983), where consumers search in a non-sequential way using a fixed-sample-size search strategy.

Depending on the parameters of the model, and irrespective of the number of firms, there is a maximum of three types of price dispersed equilibria. These equilibria are characterized by the intensity with which less-informed consumers search. These consumers may search with high, moderate and low intensity in equilibrium. In the *high search intensity equilibrium*, less-informed consumers randomize between making one and two searches. This is the type of equilibrium that Burdett and

[1] Both stability conditions and decreasing best-replies are needed for this result to hold. For a discussion on these issues see Vives (1999).

[2] In the Internet-age, one can think of the fully informed consumers as those who use electronic agents to search the web for prices (cf., Janssen and Moraga, 2001). More traditionally, fully informed consumers are consumers who read consumer reports, acquire newspapers or happen to see advertisements from all firms in the market.

Judd (1983) focussed upon most intensely, with the important difference that we study an oligopolistic market structure. In the *moderate search intensity equilibrium*, less-informed consumers make one search. This is similar to the situation analyzed by Varian (1980), with the modification that we have an endogenous search rule. Finally, in the *low search intensity equilibrium*, less-informed consumers randomize between searching for one price and not searching at all. This low search intensity equilibrium has been overlooked in previous research. In this equilibrium, less-informed consumers expect prices to be so high –relative to the difference between product value and search cost– that they are indifferent between searching for one price and not searching at all. In equilibrium, these expectations turn out to be correct.

For any type of search intensity, a symmetric price equilibrium exists only in mixed strategies.[3] The intuition behind the fact that firms randomize over a set of prices in any equilibrium is found in the observation that firms intend to extract surplus from the two different groups of consumers that co-exist in the economy. A firm has an incentive to charge low prices in an attempt to attract the well-informed consumers. We will refer to this force as a *business-stealing* effect. However, the fact that in any equilibrium a fraction of less-informed consumers search for only one price implies that firms always hold monopoly power over this fraction of less-informed consumers. This gives firms an incentive to charge higher prices. We will refer to this force as a *surplus-appropriation* effect. A firm's mixed strategy balances these two forces so as to maximize profits.

The three types of equilibria are studied in turn. For each type of equilibrium, we are interested in existence results, and in the effects of an increase in the number of firms ('increased competition') on expected price, price dispersion and welfare. We first study the moderate search intensity equilibrium. Here we obtain three results. Our *first* observation is that the equilibrium price distributions for N and $N+1$ firms cannot be ranked according to the criterion of first-order stochastic dominance. The reason is that the two effects shaping the distribution of prices strengthen as N rises. In other words, firms respond to increased competition by decreasing the frequency with which they charge intermediate prices in benefit of more extreme prices.[4] Our *second* finding is that, in a moderate search intensity equilibrium, expected price increases in the number of firms. That is, the surplus appropriation effect dominates the business stealing effect for all parameters. Moreover,

[3] We concentrate on symmetric equilibria. Baye, Kovenock and de Vries (1992) study existence of asymmetric equilibria in Varian's model. They argue, however, that only the symmetric equilibrium survives an appealing equilibrium refinement consideration when the model of Varian is embedded in a larger game.

[4] In a similar model, Rosenthal (1980) finds that the distribution of prices with N firms dominates in a first-order stochastic sense the distribution of prices with $N+1$ firms. This difference in results reveals that the model of Varian and the model of Rosenthal are not as closely related as they seem to be. We discuss the differences between these two models in Section 3.

since in this equilibrium all less-informed consumers search for one price and acquire the product surely, welfare turns out to be insensitive to N. *Finally*, and most importantly we note that, for given values of the other parameters, the moderate search intensity equilibrium fails to exist if the market accommodates a large enough number of competitors. Thus, under endogenous fixed-sample-search, Varian's assumption that less-informed consumers search surely for one price cannot be supported in equilibrium if the market hosts sufficiently many firms. When N becomes large, less-informed consumers find it beneficial to search less intensively, which motivates an examination of the comparative statics of increased competition in a low search intensity equilibrium.

When less-informed consumers search with low intensity in equilibrium, we obtain the following results. *First,* we find that this type of equilibrium always exists, even when search costs are arbitrarily small, provided that the number of firms is sufficiently large. Hence, there are good reasons to investigate the properties of this type of equilibrium. A *second* result pertains to the comparative statics of an increase in N. We find that if the number of competitors rises, (i) expected price remains constant, (ii) price dispersion increases and (iii) fewer less-informed consumers decide to search and thus welfare declines. We now provide some intuition for these observations. We observe that if the less-informed consumers did not change their search intensity, expected price would be increasing in the number of firms. This would in turn make searching less attractive for these buyers, and thus consumers respond by economizing on search. As a result, fewer less-informed consumers –over which firms can extract quite some surplus– remain in the market as N rises. Interestingly, it turns out that when N increases consumers adjust their search behavior in such a way that the strengthening of the business-stealing effect is *proportional* to the strengthening of the surplus-appropriation effect, and thus increased competition leads to the same expected price but greater price dispersion. Indeed, in the limiting economy when N goes to infinity, firms randomize between marginal cost pricing and monopoly pricing.

We finally focus on the high search intensity equilibrium. In this equilibrium consumers randomize between obtaining one price quotation and two price quotations. Two effects are important to understand the impact of an increase in the number of firms. First, keeping the search behavior of the less-informed consumers fixed, expected price rises as more competitors enter the market. This partial result stems from the dominating influence of the surplus-appropriation effect as mentioned before in the context of the other types of equilibria. A second effect has to do with the optimal search strategy of the less-informed consumers. We show that in this case the *search incentives are non-monotonic* in the number of firms. In particular, we note that less-informed consumers search more intensively when an additional firm enters the market and the number of competitors is small to begin with. By contrast, when the number of firms is already large enough, less-informed consumers decrease their intensity of search as a result of increased competition. An endogenous high search intensity equilibrium of course gathers these two effects together and, interestingly, we find that in equilibrium *expected price is non-monotonic with respect to the number of firms.*

One important result in this respect is that in an endogenous high search intensity equilibrium, expected price and price dispersion in the cases of duopoly and an infinite number of firms are identical. We establish this result by showing that firms' incentives to set prices are similar under both market structures. We notice first that an individual firm sells to a consumer either (*i*) because the consumer is less-informed and does not compare prices, or (*ii*) because he/she is less-informed, does compare prices and the firm's price is lower than the price of just *one* competitor, or (*iii*) because the consumer is informed and the firm quotes the lowest price in the market. With two firms in the market, being the lowest priced firm is, however, identical to having a price lower than just *one* competitor. When there are infinitely many firms, the chance of being the lowest priced firm is negligibly small and firms disregard the possibility of selling to the fully informed consumers. Thus, in the limit economy firms see themselves as competing exclusively for a fraction of less-informed consumers with just *one* other firm. Consumers understand the firms' incentives to set prices and tune their search behavior in a way such that expected prices coincide in those two settings. For any other number of firms, by contrast, a firm can effectively sell to the fully informed consumers when it happens to quote the lowest price in the market. Seen together, the results obtained when consumers search with high intensity suggest that *two firms is enough for competition, but three or more firms is better.*

As to the welfare implications increased competition has in a high search intensity equilibrium, we note that welfare is intimately related to the search activity of the consumers. The reason is that all consumers acquire the product in this type of equilibrium and thus welfare is higher the lower consumers' search activity. This suffices to show that welfare is also non-monotonic in the number of firms, first declining and at some point increasing. Moreover, since consumers search is more intensive under triopoly than under duopoly, and since the same holds under an infinite number of firms and duopoly, it follows that, among these three cases, welfare attains its maximum under duopoly.

As already stated, our model falls under the heading of the consumer search literature. The great bulk of this literature has analyzed *competitive* models, in the sense that it is assumed that a continuum of firms compete in the market (e.g., Benabou, 1993; Bester, 1994; Burdett and Coles, 1997; Burdett and Judd 1983; Fershtman and Fishman, 1992, 1994; MacMinn, 1980; McAfee, 1995; Reinganum, 1979; Rob, 1985). Perhaps the main contribution of our paper to this body of work is to show that an *oligopoly* version of the widely-used non-sequential search model of Burdett and Judd (1983) offers quite interesting and new insights.

Some papers have dealt with consumer search in oligopolistic markets but they have used the arguably unsatisfactory assumption that consumers know the realized market distribution of prices before they search, even though they cannot tell which firm charges a particular price (e.g., Braverman, 1980; Salop and Stiglitz, 1982; Stiglitz, 1987). As Stahl (1996) argues, the use of this 'Stackelberg-type' assumption gives consumers quite a bit of information before they engage in actual search, without further theoretical justification.

The only papers we are aware of studying consumer search in oligopolistic markets under the 'Nash paradigm' are Stahl (1989, 1996). The difference between the approach taken by Stahl and ours concerns the search behavior of less-informed consumers. While his papers consider costly sequential search, we deal with costly fixed-sample-size search *a la* Burdett and Judd (1983). As is well-known, sequential and non-sequential search each have their own advantages and disadvantages.[5] In this sense, our paper is complementary to the work of Stahl and Burdett and Judd. Compared to the results obtained by Stahl, we derive the following interesting and new results: *first*, depending on the parameter values, different degrees of search intensity are possible in equilibrium; second, whereas in Stahl's model expected price rises with the number of firms, and approaches monopoly price (Diamond, 1971) and price dispersion vanishes when N converges to infinity, in our model price dispersion may increase at constant expected prices (as in a low search intensity equilibrium), or expected prices may be non-monotonic (as in a high search intensity equilibrium), not converge to the monopoly price, and remain dispersed in the limit economy. These differences are important because they imply that two seemingly identical markets can exhibit remarkable different outcomes and welfare distributions between firms and consumers because certain consumer search behavior is more appealing in one market than in the other.

The remainder of the paper is organized as follows. Section 2 describes the model and shows that only the three behavioral rules described above regarding consumers' search can be part of Nash equilibria. Existence of distinct equilibria and the comparative statics analysis of an increase in the

[5] Morgan and Manning (1985) show that, in general, *optimal* search strategies combine features of the fixed-sample-size search strategy and the sequential search strategy. Thus, the attractiveness of one search protocol over the other is sensitive to the context in which consumers operate.

number of firms when consumers search with moderate, low and high intensity are given in Sections 3, 4 and 5, respectively. We conclude in Section 6.

2 The Model and Preliminary Results

Consider a market for a homogeneous good. On the demand side of the market, there is a mass of consumers, which we normalize to one without loss of generality, who wish to purchase at most a single unit of the good. A fraction $\lambda > 0$ of the consumers search for prices costlessly. We will refer to these consumers as *informed* consumers. The rest of the consumers, a fraction $1 - \lambda$, must pay search cost $c > 0$ to observe a price quotation. These consumers, referred to as *less-informed* consumers, may decide to obtain several price quotations, say n, in which case they incur search cost equal to nc. All consumers are fully rational, i.e., informed consumers buy the good from the lowest priced store, while less-informed consumers acquire it from the store with the lowest price in their sample, provided they obtain no negative surplus.[6] The maximum price any consumer is willing to pay for the good is $v > c$.

On the supply side of the market there are $N \geq 2$ firms.[7] These firms produce the good at constant returns to scale and their identical unit production cost is normalized to zero, without loss of generality. In this economy, welfare W (total surplus) is simply measured by the multiplication of v and the number of consumers who acquire the product in equilibrium minus actually incurred search costs.

Firms and consumers play a *simultaneous* move game. An individual firm chooses its price taking price choices of the rivals as well as consumers' search behavior as given. A firm's strategy is denoted by a distribution of prices $F(p)$. Less-informed consumers form conjectures about the distribution of prices in the market and decide how many price observations to pay for. A less-informed consumer's strategy is thus a probability distribution over the set $\{0, 1, 2, ..., N\}$. After observing the prices requested, these consumers buy from the observed lowest-price store. Let μ_n denote the probability with which a less-informed consumer searches for n price quotations. We will only consider symmetric equilibria (cf., footnote 3). A symmetric equilibrium is a pair of strategies $\{F(p), \{\mu_n\}_{n=0}^N\}$ such that (a) $\pi(p) = \overline{\pi}$ for all p in the support of $F(p)$, (b) $\pi(p) \leq \overline{\pi}$ for all p, and (c) $\{\mu_n\}_{n=0}^N$ is an optimal search behavior for the less-informed consumers given that their conjectures

[6] We assume that, once price information has been processed by the consumers, 'ordering' the good involves no additional costs.

[7] We shall refer to the case $N \to \infty$ as the 'limit economy,' or, the 'fully competitive' case.

about the price distribution actually charged by the firms are correct.

Our *first* result shows that only three possible search behavioral hypotheses can be part of an equilibrium. Namely, less-informed consumers may search (*i*) with low intensity, (*ii*) with moderate intensity, or (*iii*) with high intensity. We mean by *low search* intensity a situation where less-informed consumers randomize between searching for one price quotation and not searching at all, that is $0 < \mu_0 < 1$ and $\mu_0 + \mu_1 = 1$. *Moderate search* intensity refers to the case where less-informed consumers surely search for one and only one price quotation, that is $\mu_1 = 1$. Finally, we say that less-informed consumers search with *high intensity* when they randomize between observing one price quotation and two price quotations, that is $0 < \mu_2 < 1$ and $\mu_1 + \mu_2 = 1$. Next we show that any other search rule cannot be part of an equilibrium.

Lemma 1 *Equilibria where (i)* $\mu_0 = 1$, *or (ii)* $\mu_n = 1$, *for some* $n = 2, 3, ..., N$, *or (iii)* $\mu_n > 0$, *for some* $n = 3, 4, ..., N$, *or (iv)* $\mu_0 + \mu_1 + \mu_2 = 1$ *with* $\mu_0 > 0$, *and* $\mu_2 > 0$ *do not exist.*

Proof. (*i*) Suppose $\mu_0 = 1$. Then, only fully informed consumers would remain in the market and firms would charge Bertrand prices, i.e., $p_i = 0$, $i = 1, 2, ..., N$. As a consequence, less-informed consumers would find it beneficial to search at least once. Therefore, $\mu_0 = 1$ cannot be part of an equilibrium.

(*ii*) Suppose $\mu_n = 1$, for a single $n = 2, 3, 4, ..., N$. Then, all consumers would exercise price comparisons at least once, and thus all firms would charge Bertrand prices again. But if this is so, less-informed consumers would search only once. Therefore $\mu_n = 1$, for some $n = 2, 3, 4, ..., N$ cannot be part of an equilibrium either.

(*iii*) The same reasoning as before selects away equilibria where $\mu_1 = 0$ and some $\mu_n > 0$, $n = 3, 4, ..., N$. It thus remains to show that $\mu_1 > 0$ together with some $\mu_n > 0$, $n = 3, 4, ..., N$ cannot be part of an equilibrium either. Suppose, to the contrary, that an equilibrium of this type exists. Then, it must be the case that $v - E[p] - c = v - E[\min\{p_1, p_2, ..., p_n\}] - nc$, for some $n = 3, 4, ..., N$; or, in words, less-informed consumers must be indifferent between acquiring one price and n prices. This equality can be rewritten as $E[p] - E[\min\{p_1, p_2, ..., p_n\}] = (n-1)c$. Note now that the expected value of the minimum of a random sample of n observations is a decreasing function of n, and, further, that such a minimum decreases at a decreasing rate (Stigler, 1961; p. 215). This implies that $E[p] - E[\min\{p_1, p_2, ..., p_{n-1}\}] > (n-2)c$. But this can be rewritten as $v - E[\min\{p_1, p_2, ..., p_{n-1}\}] - (n-1)c > v - E[p] - c$, which implies that the proposed search strategy is not optimal.

(iv) Similar considerations as in (ii) select away equilibria where $\mu_1 = 0$ and $\mu_0 + \mu_2 = 1$. So, let us consider the case where $\mu_1 > 0$. In this case, the following two conditions must hold:

$$v - E[p] - c = 0 = v - E[\min\{p_i, p_j\}] - 2c, \tag{1}$$

i.e., less-informed consumers must be indifferent between searching for one price, not searching at all, and searching for two prices. We now show that both equalities cannot hold together. We note first that

$$E[\min\{p_i, p_j\}] = 2\int_{\underline{p}}^{v} p(1 - F(p))f(p)dp = 2E[p] - \int_{\underline{p}}^{v} 2pF(p)f(p)dp.$$

Thus, the RHS of (1) can be written as

$$\begin{aligned} v - E[\min\{p_i, p_j\}] - 2c &= v - 2E[p] + \int_{\underline{p}}^{v} 2pF(p)f(p)dp - 2c = \\ \int_{\underline{p}}^{v} 2pF(p)f(p)dp - E[p] - c &= \int_{\underline{p}}^{v} 2pF(p)f(p)dp - v, \end{aligned}$$

where the second equality follows from $v - E[p] - c = 0$. Integrating by parts, we can show that

$$\int_{\underline{p}}^{v} pF(p)f(p)dp = v - \int_{\underline{p}}^{v} [F(p)]^2 dp - \int_{\underline{p}}^{v} pF(p)f(p)dp.$$

Thus,

$$v - E[\min\{p_i, p_j\}] - 2c = -\int_{\underline{p}}^{v} [F(p)]^2 dp < 0,$$

which constitutes a contradiction. The proof is now complete. ■

We are interested (i) in the different equilibria that may emerge in this economy and (ii) in the comparative statics effects of increased competition on firms' prices, on price dispersion, and on welfare. A primary feature common to the three possible equilibria that can arise in our economy is that less-informed consumers search for one price with positive probability, i.e., $\mu_1 > 0$. This implies that any equilibrium in this economy necessarily exhibits price dispersion.

Lemma 2 *Suppose less-informed consumers search with low, moderate or high intensity. Then, if $F(p)$ is an equilibrium price distribution, it must be atomless. Hence, there is no equilibrium where* *firms employ a pure strategy.*

Proof. Suppose firms charged a particular price with positive probability. Then, the probability that such a price was the minimum price charged in the market would be positive. This implies that a small reduction in that price by one of the rivals would be beneficial as it would attract all informed consumers with positive probability. As a result, the only price that could be proposed as having positive probability equals marginal cost, i.e., $p = 0$. However, since in any equilibrium $\mu_1 > 0$, then for any firm there is a positive probability that it attracts a less-informed consumer who has obtained only such a price quotation, and thus a single firm would make a positive (expected) profit by raising its price. ■

The intuition behind the fact that firms randomize over a set of prices in any equilibrium is found in the observation that firms intend to extract surplus from the two different groups of consumers. A firm has an incentive to charge low prices in an attempt to attract the well-informed consumers. We will refer to this force as a *business-stealing* effect. However, the fact that in any equilibrium some less-informed consumers search for only one price implies that firms always hold monopoly power over some less-informed consumers. This gives firms an incentive to charge higher prices. We will refer to this force as a *surplus-appropriation* effect. A firm's mixed strategy is intended to balance the benefits accruing from low and high prices.

In the following sections we will consider the cases of low, moderate and high search intensity separately. We will characterize when these equilibria exist and analyze the impact of increased competition on market variables. An increase in the number of firms has, presumably, two effects. First, it directly impacts the supply side of the market by bringing more competitors together. Second, it affects the demand side of the market by possibly altering the manner in which surplus maximizing consumers search. As a presentation strategy, we have chosen to consider our setting first in a context where consumer search rules are exogenous. Then, we incorporate endogenous consumers search. This strategy is useful for two reasons: First, it helps disentangle the different mechanisms at work when considering the effects of increased competition on the relevant market variables. This is interesting because it allows us to show the detrimental effects of non-optimal search behavior. Second, it enables us to easily compare our results against two branches of the literature: earlier papers with exogenous search rules, as well as later articles with endogenous consumers search.

3 Moderate search intensity equilibrium

We first consider the case of moderate search intensity as this provides a convenient reference point vis-a-vis some earlier work, most notably Varian (1980), Rosenthal (1980) and Stahl (1989). Varian and Rosenthal consider more general demand functions than we do here, but they restricts their analyses to exogenous search rules.[8] Stahl studies a model similar to Varian's where consumers search sequentially. We shall see here that, when consumers search non-sequentially using a fixed-sample-size search rule, the moderate search intensity equilibrium of Varian's model collapses for a sufficiently large number of competitors. As a by-product of our analysis, we provide a different perspective on the relationship between the analyses of Rosenthal (1980) and Varian (1980). We shall see that, in contrast to a common belief –see e.g. Stahl (1989, p. 701)– the model of Rosenthal (1980) is different than the model of Varian (1980), and therefore the comparative statics results of the former do not apply to the latter.

To analyze the moderate search equilibrium, consider that less-informed consumers search for one price quotation. The expected payoff to firm i of charging price p_i when competitors choose a random pricing strategy according to the cumulative distribution function $F(p)$ is

$$\pi_i(p_i, F(p)) = p_i \left[\frac{1-\lambda}{N} + \lambda (1 - F(p_i))^{N-1} \right]. \tag{2}$$

Equation (2) is interpreted as follows. Firm i obtains a per consumer profit of p_i. The expected demand faced by firm i stems from the two different groups of buyers. Firm i attracts the fully informed consumers only when it is the lowest priced store, which happens with probability $(1 - F(p_i))^{N-1}$. The less-informed consumers only observe one price with probability 1. Firm i attracts these consumers when it happens that they observe firm i's price, which occurs with probability $1/N$.

In equilibrium, a firm must be indifferent between charging any price in the support of F. The maximum price a firm will ever charge is v since no buyer who observed a price above his/her reservation value would acquire the good. Moreover, the upper bound of the price distribution cannot be lower than v because a firm charging a different upper bound would gain by slightly raising its price. Thus, it must hold that $F(v) = 1$, and $F(p) < 1$, for all $p < v$. Characterization of a mixed strategy equilibrium requires that any price in the support of F must satisfy $\pi_i(p_i, F(p)) = \pi_i(v)$,

[8] Towards the end of the paper, Varian discusses some issues pertaining to the consumer incentives to engage in newspaper search.

that is,

$$p_i \left[\frac{(1-\lambda)}{N} + \lambda(1-F(p_i))^{N-1} \right] = \frac{(1-\lambda)v}{N}.$$

This equation can be solved for the equilibrium price distribution

$$F(p_i) = 1 - \left(\frac{(1-\lambda)(v-p_i)}{N\lambda p_i} \right)^{\frac{1}{N-1}}.$$

The lower bound of the price support is easily found by setting $F(\underline{p}) = 0$ and solving for $\underline{p}$. It follows that $\underline{p} = (1-\lambda)v/[\lambda N + (1-\lambda)]$. Note that $F(p)$ is well defined on the interval $[\underline{p}, v]$, and that it is a continuous and strictly increasing function of p, valued between 0 and 1. The next result summarizes.

Lemma 3 *Suppose less-informed consumers search with moderate intensity. Then, there exists an equilibrium price distribution*

$$F(p;N) = 1 - \left(\frac{(1-\lambda)(v-p)}{N\lambda p} \right)^{\frac{1}{N-1}}$$

with support $\left[\frac{(1-\lambda)v}{\lambda N + (1-\lambda)}, v \right]$.

Holding the search intensity of the less-informed consumers constant, consider next the effects of an increase in the number of rival firms on the equilibrium price distribution described in Lemma 3. Our first observation is that the equilibrium price distributions $F(p;N)$ and $F(p;N+1)$ cannot be ranked according to the first-order stochastic dominance criterion. To establish this result it is useful to observe, first, that the lower bound of the price distribution decreases as N increases, that is, $\underline{p}(N) > \underline{p}(N+1)$. This means that the business-stealing effect strengthens as N rises. Second, we note that there exists a single price

$$\widehat{p} = \frac{(1-\lambda)v}{\left(\frac{N}{N+1}\right)^{N-1} \lambda N + (1-\lambda)}$$

that satisfies $F(p;N) = F(p;N+1)$. By inspection, it is also readily seen that $\underline{p}(N) < \widehat{p} < v$ for all N. Finally we note that $F(p;N) > F(p;N+1)$ for all $p \in (\widehat{p}, v)$. This means that the surplus-appropriation effect also strengthens as N rises. These observations together, shown in Figure 1, imply that no relationship in terms of first-order stochastic dominance can be established between

$F(p; N)$ and $F(p; N+1)$.

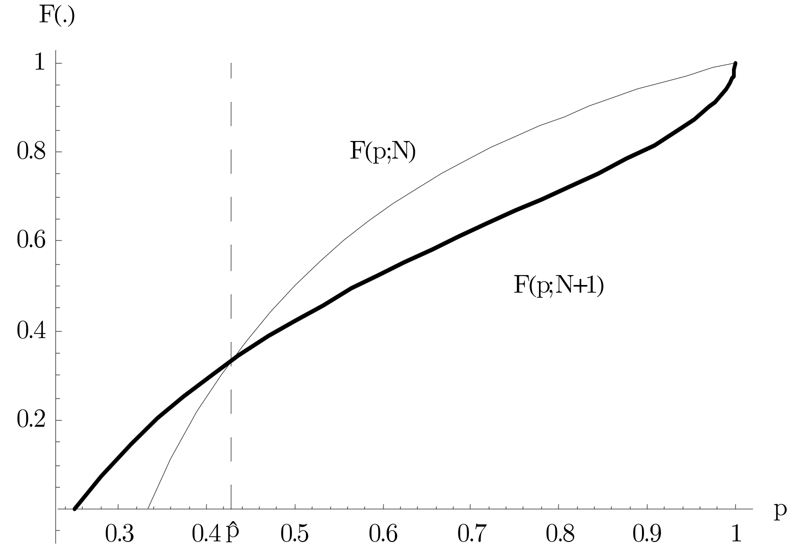

Figure 1: Equilibrium price distributions for N and $N+1$ firms ($\lambda = 0.5$).

This result contrasts sharply with the analysis in Rosenthal (1980). In his paper an increase in the number of firms shifts the entire price distribution downwards, and thus, the distribution of prices with $N+1$ firms dominates in a first-order stochastic sense the distribution of prices with N firms. We now explain the difference between Rosenthal's model and the present model –and by implication the models of Varian and Stahl–. Both models deal with competition in a market which is segmented in terms of the information consumers have about the prices charged by the firms. As a result the entire market can be regarded as divided into a 'contestable' segment and a 'captive' segment. In Rosenthal's model, the size of the segment of captive consumers per firm does not change when the number of active firms varies. This implies that the size of the contestable market relative to the entire market decreases as N increases. Therefore, it is not surprising that the business stealing effect is less important in Rosenthal's model. In the present setting (as well as in Varian's and Stahl's), by contrast, the size of the contestable market relative to the entire market is invariable with respect to the number of firms. This explains why, holding the search behavior of the consumers fixed, a result in terms of first-order stochastic dominance in the spirit of Rosenthal's cannot be established in our setting.

Once we have shown that a relationship of first-order stochastic dominance cannot be established between $F(p; N)$ and $F(p; N+1)$, a natural question to ask is whether expected price increases or decreases when the number of firms in the market increases. The next result answers:

Proposition 1 *Let $F(p;N)$ be the equilibrium price distribution under moderate search intensity, given by Lemma 3. Then, keeping the search behavior of the consumers fixed, the expected price increases in N, and approaches v when N converges to infinity.*

Proof. The expression for the expected price can be written as

$$E[p;N] = \int_{\underline{p}(N)}^{v} \frac{(1-\lambda)v}{\lambda p N(N-1)} \left(\frac{(1-\lambda)(v-p)}{\lambda N p}\right)^{\frac{2-N}{N-1}} dp.$$

Consider the following variable change:

$$z = \left(\frac{(1-\lambda)(v-p)}{\lambda N p}\right)^{\frac{1}{N-1}}$$

Then we have

$$\begin{aligned} p &= \frac{v}{1+\alpha N z^{N-1}} \\ dp &= \frac{v\alpha N(N-1)z^{N-2}}{\left(1+\alpha N z^{N-1}\right)^2} dz \end{aligned}$$

where $\alpha = \lambda/\mu_1(1-\lambda) > 0$. This enables us to rewrite the expected price as follows:

$$E[p;N] = \int_0^1 \frac{v}{1+\alpha N z^{N-1}} dz$$

Then we can compute the difference

$$\begin{aligned} & E[p;N+1] - E[p;N] \\ = & \; v\int_0^1 \left[\frac{1}{1+\alpha(N+1)z^N} - \frac{1}{1+\alpha N z^{N-1}}\right] dz \\ = & \; v\int_0^1 \left[\frac{\alpha z^{N-1}(N-(N+1)z)}{\left(1+\alpha(N+1)z^N\right)\left(1+\alpha N z^{N-1}\right)}\right] dz. \end{aligned}$$

To complete the argument it is enough to show that this last integral is positive. To show this, note that the expressions $1+\alpha(N+1)z^N$ and $1+\alpha N z^{N-1}$ are both positive and strictly increasing in z.

Then,

$$
\begin{aligned}
& v\int_0^1 \left[\frac{\alpha z^{N-1}(N-(N+1)z)}{(1+\alpha(N+1)z^N)\,(1+\alpha N z^{N-1})}\right]dz \\
= \;& v\int_0^{\frac{N}{N+1}} \left[\frac{\alpha z^{N-1}(N-(N+1)z)}{(1+\alpha(N+1)z^N)\,(1+\alpha N z^{N-1})}\right]dz \\
& -v\int_{\frac{N}{N+1}}^1 \left[\frac{\alpha z^{N-1}((N+1)z-N)}{(1+\alpha(N+1)z^N)\,(1+\alpha N z^{N-1})}\right]dz \\
\geq \;& v\int_0^{\frac{N}{N+1}} \left[\frac{\alpha z^{N-1}(N-(N+1)z)}{\left(1+\alpha(N+1)\left(\frac{N}{N+1}\right)^N\right)\left(1+\alpha N\left(\frac{N}{N+1}\right)^{N-1}\right)}\right]dz \\
& -v\int_{\frac{N}{N+1}}^1 \left[\frac{\alpha z^{N-1}((N+1)z-N)}{\left(1+\alpha(N+1)\left(\frac{N}{N+1}\right)^N\right)\left(1+\alpha N\left(\frac{N}{N+1}\right)^{N-1}\right)}\right]dz \\
= \;& \frac{\alpha v}{\left(1+\alpha\frac{N^N}{(N+1)^{N-1}}\right)}\int_0^1 z^{N-1}((N+1)z-N)dz = 0
\end{aligned}
$$

To establish that the expected price approaches v when N converges to infinity, it is enough to note that $F(p, N)$ converges to zero as N goes to infinity. The proof is now complete. ■

From an economic point of view, Proposition 1 shows that, relative to the *business-stealing* effect, the *surplus-appropriation* effect strengthens as the number of firms in the market increases. In other words, as N rises, firms attach less importance to the consumers who are well-informed than to the consumers who are less-informed. This is because a firm understands that the probability of attracting a consumer falls less for a less-informed consumer than for a well-informed consumer, as N increases. Remarkably, this is true irrespective of the fraction of well-informed consumers and the status quo number of firms in the market. The question that arises now is whether less-informed consumers find it optimal to keep searching for one price when N increases.

Endogenous consumer search:

We now turn to consider optimal search behavior of the less-informed consumers. A mixed strategy according to the cumulative distribution function $F(p; N)$ specified in Lemma 3 is an actual equilibrium with moderate search intensity if and only if the following two conditions hold:

Condition 3.1: $v - E[p; N] - c > 0$

Condition 3.2: $v - E[\min\{p_i, p_j\}; N] - 2c < v - E[p; N] - c$, for all $i, j \in N$.

Condition 3.1 simply states that less-informed consumers must obtain, *ex-ante*, a positive expected surplus from searching for one price and acquiring the good. Condition 3.2 requires that these consumers do not want to search more intensively.

Condition 3.1 can be rewritten as

$$\Phi(1; N) = 1 - \int_0^1 \frac{1}{1 + \frac{\lambda}{1-\lambda} N z^{N-1}} dz > \frac{c}{v}. \tag{3}$$

Condition 3.2 requires that $E[p; N] - E[\min\{p_i, p_j\}; N] < c$. Or, in other words, that

$$\int_{\underline{p}}^{v} 2pF(p)f(p)dp - E[p] < c.$$

This last inequality can be rewritten as

$$E[p] - 2\int_{\underline{p}}^{v} pf(p)\left(\frac{(1-\lambda)(v-p)}{\lambda N p}\right)^{\frac{1}{N-1}} dp < c.$$

We can introduce the variable change

$$z = \left(\frac{(1-\lambda)(v-p)}{\lambda N p}\right)^{\frac{1}{N-1}}$$

and proceed as in the proof of Proposition 1 to rewrite Condition 3.2 as follows:

$$\int_0^1 \frac{1-2z}{1 + \frac{\lambda}{1-\lambda} N z^{N-1}} dz < \frac{c}{v}$$

We are now ready to state the following result:

Proposition 2 *Let* $1 - \int_0^1 \frac{1}{1+\frac{\lambda}{(1-\lambda)} N z^{N-1}} dz > \frac{c}{v} > \int_0^1 \frac{1-2z}{1+\frac{\lambda}{1-\lambda} N z^{N-1}} dz$. *Then an endogenous moderate search intensity equilibrium exists where the less-informed consumers search surely for one price and firms randomly select prices from the set* $\left[\frac{(1-\lambda)v}{\lambda N + (1-\lambda)}, v\right]$ *according to the cumulative distribution function*

$$F(p) = 1 - \left(\frac{(1-\lambda)(v-p)}{N\lambda p}\right)^{\frac{1}{N-1}}.$$

There is at most one such equilibrium.

We now provide a discussion on the *existence* of a moderate search intensity equilibrium. For this type of equilibrium to exist, the set of parameters given above in Proposition 2 must be non-empty. There are two alternative ways of looking at this question. *First*, one may wonder whether for any given number of firms N, there exist values of c, v and λ such that an equilibrium with moderate search intensity exists. To answer this question, it suffices to show that

$$\int_0^1 \frac{1-2z}{1+\frac{\lambda}{1-\lambda}Nz^{N-1}}dz < 1 - \int_0^1 \frac{1}{1+\frac{\lambda}{1-\lambda}Nz^{N-1}}dz$$

Or, rearranging, that

$$2\int_0^1 \frac{1-z}{1+\frac{\lambda}{1-\lambda}Nz^{N-1}}dz < 1, \tag{4}$$

which is always satisfied because the function $1-z$ is an upper bound of the integrand in (4), for all $N < \infty$.

A *second,* and somewhat more interesting, manner to consider the existence question is whether for given values of c, v and λ, there exists a moderate search intensity equilibrium for all N. It turns out that the answer to this question is negative; when N becomes sufficiently large, this equilibrium fails to exist. What happens as N increases is that, if the less-informed consumers keep searching for one price with probability one, expected prices tend to the monopoly price (cf., Proposition 1) and eventually, the condition $v - E[p; N] - c > 0$ is violated. This is easily seen upon inspection of (3) and noting that $\Phi(1; N)$ declines monotonically with N and converges to zero as N approaches infinity.

Theorem 1 *(i) For any parameters c, v and λ there exists a critical number of firms $\widetilde{N}$ such that for all $N \geq \widetilde{N}$ an equilibrium with endogenous moderate search intensity as described in Proposition 2 does not exist. (ii) Let $\{F(p; N), \mu_1 = 1\}$ be an equilibrium with endogenous moderate search intensity. Then, provided that $\{F(p; N+1), \mu_1 = 1)$ is also an equilibrium with endogenous moderate search intensity, $E[p; N+1] > E[p; N]$ and welfare is constant with respect to N.*

The welfare result in Theorem 1 deserves an explanation. Notice that in our model every consumer who acquires the good generates a total surplus of v, which is somehow distributed between firms and consumers. Since in a moderate search intensity equilibrium all consumers acquire the product, and since their search intensity does not change when the number of firms increases, it follows that welfare is insensitive to the number of rivals.

Theorem 1 illustrates that Varian's model of sales may collapse when the number of firms in the market is large and consumers search for prices non-sequentially in an optimal fashion. Provided that an equilibrium with moderate search intensity exists, this result also shows that an increase in the number of firms brings about an increase in expected price. Since adding a firm to a market with N competitors in a manner such that $N+1<\widetilde{N}$ does not alter the search behavior of the less-informed consumers, there is no difference between the exogenous search case (Proposition 1) and the endogenous search one (Theorem 1).[9] However, we shall see in the next section that as N grows above $\widetilde{N}$, consumers will respond by searching less intensively than in a moderate search intensity equilibrium. Moreover, we shall see that in a low search intensity equilibrium, expected price will be always bounded below monopoly price. These remarks together imply that, starting from a situation where consumers search with moderate intensity, expected prices will never converge to the monopoly price as the number of firms grows due to the economizing behavior of the consumers. This is in sharp contrast with the findings of Stahl (1989).

4 Low search intensity equilibrium

When the less-informed consumers randomize between searching for one price quotation and not searching at all, the expected payoff to firm i can be written as

$$\pi_i(p_i, F(p)) = p_i\left[\frac{(1-\lambda)\mu_1}{N} + \lambda(1-F(p_i))^{N-1}\right]. \tag{5}$$

Equation (5) has an economic interpretation analogous to equation (2). The only difference between them is that the number of less-informed consumers who are now active is $(1-\lambda)\mu_1$, rather than $1-\lambda$. A similar analysis as in the case of moderate search yields the following result.

Lemma 4 *Suppose less-informed consumers search with low intensity. Then, for any search intensity $0<\mu_1<1$ there exists an equilibrium price distribution*

$$F(p) = 1 - \left(\frac{\mu_1(1-\lambda)(v-p)}{N\lambda p}\right)^{\frac{1}{N-1}}$$

with support $\left[\frac{(1-\lambda)\mu_1 v}{\lambda N+(1-\lambda)\mu_1}, v\right]$.

[9] We have been unable to analytically characterize the behaviour of price dispersion with respect to the number of firms in this setting. However, a numerical analysis we have conducted reveals that the influence of N on price dispersion is sensitive to the size of the fraction of fully informed consumers λ.

As before, a relationship of first-order stochastic dominance cannot be established between $F(p;N)$ and $F(p;N+1)$ because both the business-stealing and the surplus-appropriation effect strengthen as N rises. However, holding the search intensity of the less-informed consumers μ_1 constant, it is readily seen that the expected price increases in the number of firms.[10]

Endogenous consumer search:

We now turn to consider optimal search behavior of the less-informed consumers. A mixed strategy according to the cumulative distribution function $F(p;N)$ specified in Lemma 4 is indeed an equilibrium with low-search intensity if and only if the following two conditions are satisfied:

Condition 4.1: $v - E[p;N] - c = 0$

Condition 4.2: $v - E[\min\{p_i, p_j\}; N] - 2c < 0$, for all $i, j \in N$.

Condition 4.1 states that the less-informed consumers must be indifferent between searching for one price and not searching at all. Condition 4.2 requires that no less-informed consumer finds it profitable to search more intensively. An argument similar to the one given in the proof of Lemma 1(iv) can be used to show that condition 4.2 is implied by condition 4.1. To show existence of a low search intensity equilibrium, it is convenient to rewrite Condition 4.1 as follows:

$$1 - \int_0^1 \frac{1}{1 + \frac{\lambda}{\mu_1(1-\lambda)} N z^{N-1}} dz = \frac{c}{v} \tag{6}$$

Denote the LHS of equation (6) as $\Phi(\mu_1; \lambda, N)$. We can easily establish the following facts:

Fact 1: $\frac{d\Phi(\cdot)}{d\mu_1} = -\int_0^1 \frac{\lambda N z^{N-1}}{\left(1+\frac{\lambda}{\mu_1(1-\lambda)} N z^{N-1}\right)^2 (1-\lambda)\mu_1^2} dz < 0$

Fact 2: $\lim_{\mu_1 \to 0} \Phi(\mu_1) = 1$. This fact follows from the observation that $F(p;N) \to 1$ as $\mu_1 \to 0$. The economic interpretation of this is that when $\mu_1 \to 0$, only fully informed consumers are left over in the market and firms must employ marginal cost pricing.

[10] To see this, it is enough to observe that the expression for the expected price can now be written as (cf., Proposition 1)

$$E[p;N] = \int_{\underline{p}(N)}^{v} \frac{\mu_1(1-\lambda)v}{\lambda p N(N-1)} \left(\frac{\mu_1(1-\lambda)(v-p)}{\lambda N p}\right)^{\frac{2-N}{N-1}} dp.$$

The following variable change

$$z = \left(\frac{\mu_1(1-\lambda)(v-p)}{\lambda N p}\right)^{\frac{1}{N-1}}$$

can be used to apply the same proof as in Proposition 1.

Fact 3: $\Phi(1) = 1 - \int_0^1 \frac{1}{1+\frac{\lambda}{(1-\lambda)}Nz^{N-1}}dz > 0.$

Facts 1 to 3 imply that the LHS of (6) is a smooth positive-valued declining function of μ_1. In Figure 1 we have represented $\Phi(\cdot)$ as a function of μ_1, for a particular parametrical point. The flat line is just the RHS of (6). An endogenous low search intensity equilibrium is thus given by some intensity of search $\mu_1 \in (0,1)$ for which the curve $\Phi(\cdot)$ and the line c/v intersect. The Facts above imply that this occurs at most once.

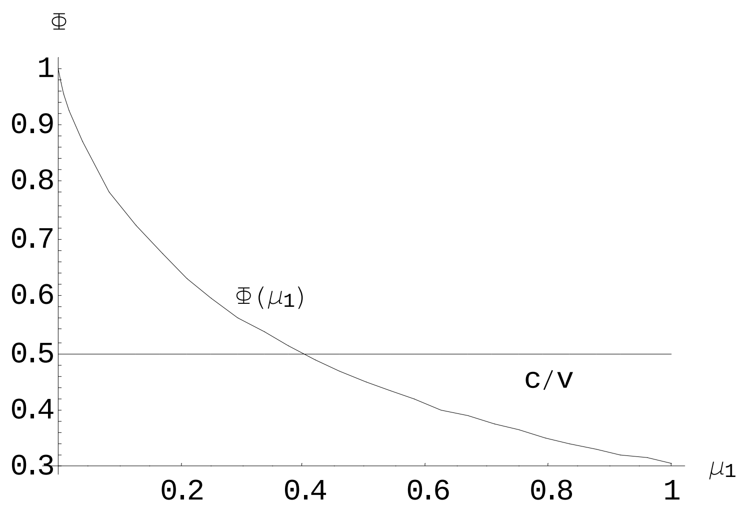

Figure 2: Low search intensity equilibrium ($\lambda = 1/3$; $N = 2$)

The following proposition summarizes these findings:

Proposition 3 *Let* $1 \geq \frac{c}{v} \geq 1 - \int_0^1 \frac{1}{1+\frac{\lambda}{(1-\lambda)}Nz^{N-1}}dz$. *Then an equilibrium of the game described above exists where the less-informed consumers search with a low intensity given by* $\mu_1^* \in (0,1)$ *solution to*

$$1 - \int_0^1 \frac{1}{1 + \frac{\lambda}{\mu_1(1-\lambda)}Nz^{N-1}}dz = \frac{c}{v},$$

and firms randomly select prices from the set $\left[\frac{(1-\lambda)\mu_1^* v}{\lambda N + (1-\lambda)\mu_1^*}, v\right]$ *according to the cumulative distribution function*

$$F(p) = 1 - \left(\frac{\mu_1^*(1-\lambda)(v-p)}{N\lambda p}\right)^{\frac{1}{N-1}}.$$

There is at most one such equilibrium.

It is straightforward to analyze now the comparative statics effects of an increase in the number of firms N when less-informed consumers search optimally. Our *first* observation pertains to the *existence* of a low search intensity equilibrium. The following two facts prove useful for such aim:

Fact 4: $\Phi(1; N) > \Phi(1; N+1)$. This follows from the observation above that, holding constant the search intensity of the less-informed consumers, $E[p; N] < E[p; N+1]$.

Fact 5: $\lim_{N\to\infty} \Phi(1; N) = 0$. This follows from the fact that $Nz^{N-1} \to 0$ as $N \to \infty$, since $0 < z < 1$.

Facts 1 to 5 enable us to state that for any set of parameters v, c, and λ, there is a critical number of firms $\widetilde{N}$ such that for all $N > \widetilde{N}$ an endogenous low search intensity equilibrium exists. We note that this equilibrium has been disregarded by the search literature, and in the present setting it certainly exists for arbitrarily low search costs. This equilibrium with low search intensity is a natural extension of the model studied in Varian (1980) when consumers search optimally in a non-sequential fashion.

Our *second* observation is that expected price is insensitive to N in an endogenous low search intensity equilibrium. To see this, note that condition 1 above must be met in equilibrium. This implies that expected price must be equal to $v - c$ for all N. As we have argued above, holding the search behavior of the less-informed consumers constant, expected price would rise. To keep expected price constant as N increases, consequently, the probability μ_1 with which less-informed consumers search for one price must decrease. This is easily seen either upon using Fact 4, or by looking at the expression for the expected price in this case

$$E[p; N] = \int_0^1 \frac{v}{1 + \frac{\lambda}{\mu_1(1-\lambda)} N z^{N-1}} dz$$

and noting that, holding N constant, $E[p]$ falls as μ_1 decreases.

Our *third* remark is that $F(p; N+1)$ *is a mean-preserving spread of* $F(p; N)$ when consumers search optimally. This follows from the following two remarks. First, since μ_1 falls as a result of an increase in N, inspection of the lower bound of the equilibrium price distribution

$$\underline{p}(N) = \frac{v}{1 + \frac{\lambda N}{(1-\lambda)\mu_1^*}}$$

reveals that an increase in the number of firms enlarges the set of prices over which firms mix. Second, there exists a unique value of p, denoted by $\widetilde{p}$, such that $F(p;N) = F(p;N+1)$. This value is implicitly defined by the equality

$$\left(\frac{1-\lambda}{\lambda}\frac{v-\widetilde{p}}{\widetilde{p}}\right)^{\frac{1}{N(N-1)}} = \frac{\left(\frac{\mu_1^*(N+1)}{N+1}\right)^{\frac{1}{N}}}{\left(\frac{\mu_1^*(N)}{N}\right)^{\frac{1}{N-1}}},$$

where $\mu_1^*(N)$ and $\mu_1^*(N+1)$ are the equilibrium values of μ_1 when there are N and $N+1$ firms in the market, respectively. As expected price remains constant, these two observations imply that $F(p;N+1)$ *is a mean-preserving spread of* $F(p;N)$. A remark on the intuition behind this result is in line. We have noticed above that, for a given search behavior of the consumers, both the business-stealing effect and the surplus-appropriation effect strengthen as N rises and that, relative to the first effect, the second effect is stronger. The interesting issue about this result is that the economizing behavior of the less-informed consumers makes the business-stealing effect and the surplus-appropriation effect become *proportionally* important as N rises.

Finally, we note that the limiting equilibrium price distribution $\lim_{N\to\infty} F(p;N)$ is a discrete distribution where firms randomize over marginal cost pricing (with probability c/v) and monopoly pricing v (with probability $(v-c)/v$). To see this, let us first investigate the behavior of the density function when $N \to \infty$ for a fixed μ_1 and any $0 < p < v$. the density function can be written as

$$f(p;N) = \frac{1}{N-1}\left(\frac{\mu_1(1-\lambda)}{N\lambda}\right)^{\frac{1}{N-1}}\left(\frac{v-p}{p}\right)^{\frac{2-N}{N-1}}\frac{v}{p^2}.$$

It is easy to see that, for a fixed μ_1 and any $0 < p < v$, $f(p) \to 0$ as $N \to \infty$. From the observations above, we also know that the probability μ_1 with which less-informed consumers search for one price decreases as $N \to \infty$, which strengthens this effect. Only for $p = 0$ and $p = v$, this argument does not hold. As we also know that $E[p;N] = v - c$ for all N, it follows that the probabilities with which the firms randomize over $p = 0$ and $p = v$ must be equal to c/v and $(v-c)/v$, respectively. These observations are summarized in the following result:

Theorem 2 *(i) For any parameters c, v and λ there exist a critical number of firms $\widetilde{N}$ such that for all $N \geq \widetilde{N}$ an equilibrium with endogenous low search intensity as described by Proposition 3 exists. (ii) Let $\{F(p;N), \mu_1(N)\}$ be an equilibrium with endogenous low search intensity. Then, $\mu_1(N) > \mu_1(N+1)$ and $F(p;N+1)$ second order stochastically dominates $F(p;N)$, for all N.*

Moreover, welfare is declining in the number of firms. (iii) The limiting equilibrium price distribution $\lim_{N\to\infty}\{F(p;N),\mu_1(N)\}$ *is a discrete distribution where firms randomize over a price equal to 0 (with probability* c/v*) and a price of* v *(with probability* $(v-c)/v$*).*

Theorem 2 highlights the beneficial effects of optimizing behavior of the less-informed consumers. If these consumers did not search optimally expected price would increase as a response of increased competition. When less-informed consumers search optimally, their behavior fully offsets the firms' incentives to raise expected prices and increased competition only results in an increase in price dispersion.

For the purpose of illustration, Figure 3a below shows how the function $\Phi(\cdot) = (v - E[p;N])/v$ shifts downwards as the number of rivals increases (Fact 4). As a consequence, equilibrium search intensity μ_1 decreases with N. Figure 3b depicts the equilibrium price distributions when low search intensity is endogenous for different values of N. This graph shows how increased competition results in greater price dispersion. It also shows that the distribution for $N = 100$ is already close to the limiting distribution.

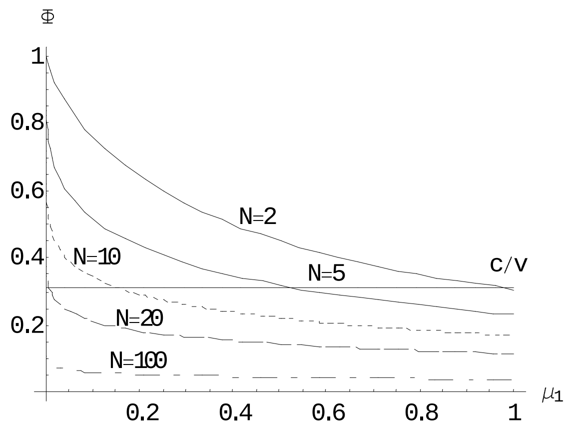

Figure 3a

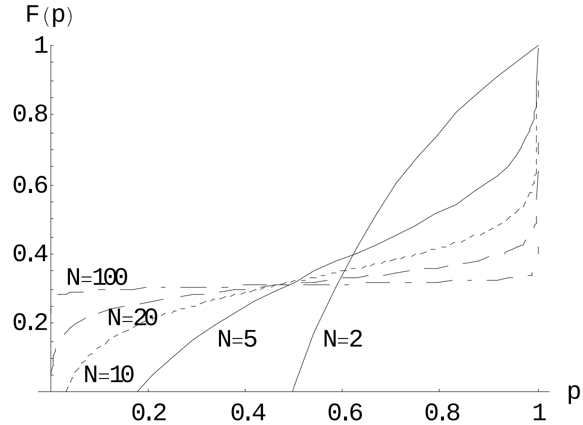

Figure 3b

Before closing this Section, we would like to point out the detrimental effects of entry in a low search intensity equilibrium. To see this, observe that welfare equals $W = \lambda v + \mu_1(1-\lambda)(v-c)$. Since the equilibrium low search intensity μ_1 declines with N, it is readily seen that social welfare falls as a result of increased competition: as the number of firms rises, the economy saves on search costs but this implies that desirable transactions are not realized and consequently welfare declines.

5 High search intensity equilibrium

Consider now the case where less-informed consumers randomize between searching for one price and searching for two prices. In this case, the expected payoff to firm i is

$$\pi_i(p_i, F(p)) = p_i \left[\lambda(1-F(p_i))^{N-1} + \frac{2(1-\lambda)(1-\mu_1)}{N}(1-F(p_i)) + \frac{(1-\lambda)\mu_1}{N} \right].$$

Notice that, unlike before, $1-\mu_1$ denotes now the probability with which less-informed consumers search for two prices. Again, this profit function is easily interpreted. Note first that the consumers can now be grouped into three subsets, in regard to the information they possess: less-informed consumers who observe a single price, less-informed consumers who observe two prices, and fully informed consumers. Firm i attracts the latter consumers only when it quotes the lowest price, which happens with probability $(1-F(p_i))^{N-1}$. There is a fraction $1-\mu_1$ of the less-informed buyers who search twice; these consumers buy from firm i whenever they are aware of firm i's price, which occurs with probability $2/N$, and when firm i happens to set the lowest price among the sampled ones, which occurs with probability $1-F(p_i)$. Finally, a fraction μ_1 of the less-informed consumers search for only one price; firm i attracts these consumers when it happens that they observe firm i's price, which occurs with probability $1/N$.

As before, the highest price a firm would ever charge is v; thus $F(v)=1$ and $F(p)<1$, $p<v$. In equilibrium, a firm must be indifferent between charging any price in the support of F and charging v. Hence, it must be the case that

$$p\left[\frac{(1-\lambda)\mu_1}{N} + \frac{2(1-\lambda)(1-\mu_1)}{N}(1-F(p)) + \lambda(1-F(p))^{N-1}\right] = \frac{\mu_1(1-\lambda)v}{N}. \tag{7}$$

Unfortunately, an explicit solution of equation (7) for $F(p)$ does not exist for general values of N. However, by setting $F(\underline{p})=0$ and solving for $\underline{p}$, the lower bound of the price distribution can be found: $\underline{p}=\mu_1(1-\lambda)v/[(1-\lambda)(2-\mu_1)+N\lambda]$. To show existence of an equilibrium price distribution it is convenient to rewrite equation (7) as

$$\lambda(1-F(p))^{N-1} + \frac{2(1-\lambda)(1-\mu_1)}{N}(1-F(p)) = \frac{\mu_1(1-\lambda)(v-p)}{Np}, \tag{8}$$

Note that the LHS of this equation is continuously increasing in $1-F(p)$, while the RHS is a positive constant with respect to $1-F(p)$. Therefore, these two functions always cross at a single point. Note further that the RHS of (8) decreases with p and is valued in $[0, 2(1-\lambda)(1-\mu_1)/N+\lambda]$. Then it is clear that $1 - F(p)$ is valued in $[0, 1]$ and decreases with p. The next result summarizes:

Lemma 5 *Suppose less-informed consumers search with high intensity. Then, for any search intensity* $0 < \mu_1 < 1$, *there exists an equilibrium price distribution* $F(p)$ *with support* $\left[\frac{\mu_1(1-\lambda)v}{(1-\lambda)(2-\mu_1)+N\lambda}, v\right]$. *Such a price distribution is given by the unique solution to equation (7).*

Our next observations pertain to the comparative statics of increased competition, holding the search intensity of the consumers constant. We note that the behavior of the price distribution with respect to N exhibited in Figure 1 above also holds in this case of high search intensity. To see this, note *first* that (as in the previous cases) the lower bound of the equilibrium price distribution also decreases as N increases in a high search intensity equilibrium. The reason is that increased competition strengthens the business-stealing effect. Second, observe that, from equation (7), it is easy to see that there exists some unique $\widetilde{p} \in (\underline{p}(N), v)$ for which $F(\widetilde{p}; N) = F(\widetilde{p}; N+1) = 1/(N+1)$. These two remarks imply that firms respond to increased competition by decreasing the frequency with which they set intermediate prices in favor of more extreme prices.

As earlier in the paper, these observations call for an examination of equilibrium expected prices under the price distributions $F(p; N)$ and $F(p; N+1)$ when consumers search with high intensity. Our next result states that expected price increases monotonically with N. This again highlights the dominating influence of the surplus-appropriation effect. In addition, we show that the expected price is bounded away from v for the limiting case $N \to \infty$, even if the search behavior of the less-informed consumers is held constant. This result is in sharp contrast with the cases of moderate and low search intensity analyzed above.

Proposition 4 *Let* $F(p; N)$ *be the equilibrium price distribution when less-informed consumers search with high intensity, given by Lemma 5. Then, holding the search behavior of the less-informed consumers constant, the expected price increases in* N *and approaches*

$$E[p; \infty] = \frac{\mu_1 v \ln\left[\frac{2-\mu_1}{\mu_1}\right]}{2(1-\mu_1)} < v \text{ for all } \mu_1 \in (0, 1)$$

as N *converges to infinity.*

Proof. We note that $E[p; N] = v - \int_{\underline{p}(N)}^{v} F(p; N)dp$. As argued above, there exists a unique solution to equation (7) that is monotonically increasing in p. Thus, we can invert the function

$F(p;N)$ to obtain:

$$p(z;N) = \frac{v}{g(z;N)}$$

where

$$g(z;N) = 1 + 2(a-1)(1-z) + abN(1-z)^{N-1},$$

with $a = 1/\mu_1$, $b = \lambda/(1-\lambda)$ and $z \in [0,1]$. We note that $E[p;N] = \int_0^1 p(z;N)dz$; therefore $E[p;N+1] \geq E[p;N]$ if and only if

$$\int_0^1 \left(p(z;N+1) - p(z;N)\right) dz \geq 0 \tag{9}$$

The LHS of (9) can be written as

$$\begin{aligned}
&\int_0^1 \frac{abv\left(N(1-z)^{N-1} - (N+1)(1-z)^N\right)}{g(z;N+1)g(z;N)}dz \\
= &\int_{\frac{1}{N+1}}^1 \frac{abv\left(N(1-z)^{N-1} - (N+1)(1-z)^N\right)}{g(z;N+1)g(z;N)}dz - \int_0^{\frac{1}{N+1}} \frac{abv\left((N+1)(1-z)^N - N(1-z)^{N-1}\right)}{g(z;N+1)g(z;N)}dz \\
\geq &\int_{\frac{1}{N+1}}^1 \frac{abv\left(N(1-z)^{N-1} - (N+1)(1-z)^N\right)}{g(\frac{1}{N+1};N+1)g(\frac{1}{N+1};N)}dz - \int_0^{\frac{1}{N+1}} \frac{abv\left((N+1)(1-z)^N - N(1-z)^{N-1}\right)}{g(\frac{1}{N+1};N+1)g(\frac{1}{N+1};N)}dz \\
= &\frac{abv}{g(\frac{1}{N+1};N+1)g(\frac{1}{N+1};N)}\left[\int_{\frac{1}{N+1}}^1 \left(N(1-z)^{N-1} - (N+1)(1-z)^N\right) dz\right. \\
&\left. - \int_0^{\frac{1}{N+1}} \left((N+1)(1-z)^N - N(1-z)^{N-1}\right) dz\right] \\
= &\frac{abv}{g(\frac{1}{N+1};N+1)g(\frac{1}{N+1};N)}\int_0^1 \left(N(1-z)^{N-1} - (N+1)(1-z)^N\right) dz = 0,
\end{aligned}$$

where the inequality follows from the fact that $g(z;N)$ is a decreasing function of z.

It remains to show that expected price does not converge to the monopoly price as N goes to infinity, when consumers search with high intensity. We note that a solution to (7) can be found for $N \to \infty$. The reason is that, for any given value of the other parameters, as N increases, the term $\lambda(1-F(p))^{N-1}$ in (7) approaches zero at a much greater speed than the other terms. In words, this means that as the number of firms goes to infinity, the probability that a price quoted by a firm is undercut by some other firm converges to 1. As a result, in the limiting case $N \to \infty$ firms ignore the informed consumers and we can find the limit price distribution by setting $\lambda = 0$ in (7) and solving for $F(p)$. It follows that

$$F(p;\infty)=\frac{2-\mu_1}{2(1-\mu_1)}-\frac{\mu_1}{2(1-\mu_1)}\frac{v}{p};\text{ with } p\in\left[\frac{\mu_1 v}{2-\mu_1},v\right]$$

We note that this solution is similar to the one obtained by Burdett and Judd (1983), for competitive markets without fully informed consumers. A little algebra yields the expected price $E[p;\infty]=\mu_1 v\ln\left[(2-\mu_1)/\mu_1\right]/(2(1-\mu_1))<v$, for all $\mu_1\in(0,1)$. ■

Endogenous consumers search:

We now turn to consider optimal search behavior of the less-informed consumers. A mixed strategy $F(p;N)$ solution to (7) is indeed an equilibrium with high search intensity if and only if the following two conditions are satisfied:

Condition 5.1: $v-E[\min\{p_i,p_j\};N]-2c=v-E[p;N]-c$, for all $i,j\in N$.

Condition 5.2: $v-E[p;N]-c>0$.

Condition 5.1 states that the less-informed consumers must be indifferent between obtaining one price quotation and obtaining two price quotations. Condition 5.2 says that less-informed consumers' expected surplus must be positive, that is, less-informed consumers always enter the market.

Our *first* observation is that in any endogenous high search intensity equilibrium, Condition 5.2 is irrelevant. We show this by proving that if Condition 5.1 is satisfied, then Condition 5.2 holds as well. To show this suppose the contrary, i.e., that $v-E[p;N]-c=0$.[11] If this is so, then (using the derivations in Lemma 1)

$$\begin{aligned}0&=v-E[\min\{p_i,p_j\};N]-2c=v-2E[p;N]+\int_{\underline{p}(N)}^{v}2pF(p)f(p)-2c\\&=\int_{\underline{p}(N)}^{v}2pF(p)f(p)-E[p;N]-c=\int_{\underline{p}(N)}^{v}2pF(p)f(p)-v<\frac{v}{2}-v=-\frac{v}{2},\end{aligned}$$

which constitutes a contradiction.

Our *second* observation pertains to the *existence* of an endogenous high search intensity equilibrium.

Lemma 6 *Let* $0<c<\max_{\mu_1\in(0,1)}\int_{\underline{p}(N,\mu_1)}^{v}\left[F(p;N,\mu_1)(1-F(p;N,\mu_1)\right]dp$. *Then, there exist at least one and at most two endogenous high search intensity equilibria.*

[11] Note that the case $v-E[p;N]-c<0$ is of no interest since no less-informed consumer would enter the market.

Proof. Condition 5.1 requires that

$$2\int_{\underline{p}(N,\mu_1)}^{v} pf(p;N,\mu_1)(1-F(p;N,\mu_1))dp - \int_{\underline{p}(N,\mu_1)}^{v} pf(p;N,\mu_1)dp = c.$$

Integrating by parts yields

$$\int_{\underline{p}(N,\mu_1)}^{v} \left[F(p;N,\mu_1)(1-F(p;N,\mu_1)\right] dp = c.$$

We can write this equation using the inverse function $p(z;N,\mu_1)$ described above as follows:

$$\int_0^1 \left[p(\sqrt{z};N,\mu_1) - p(z;N,\mu_1)\right] dz = c.$$

Or

$$\int_0^1 p(z;N,\mu_1)(2z-1)dz = c. \tag{10}$$

Denote the LHS of (10) as $\rho(N,\mu_1)$. Differentiation with respect to μ_1 yields

$$\rho'_{\mu_1}(\cdot) = \int_0^1 \frac{a^2 v(2z-1)\left[2(1-z)+bN(1-z)^{N-1}\right]}{\left[1+2\left(a-1\right)\left(1-z\right)+abN(1-z)^{N-1}\right]^2}dz.$$

Differentiating again with respect to μ_1 yields:

$$\rho''_{\mu_1}(\cdot) = -\int_0^1 \frac{2a^3 v(2z-1)^2\left[2(1-z)+bN(1-z)^{N-1}\right]}{\left[1+2\left(a-1\right)\left(1-z\right)+abN(1-z)^{N-1}\right]^3}dz < 0,$$

where the inequality follows from noting that the integrand is a positive number. Therefore, the function $\rho(z;N,\mu_1)$ is strictly concave in μ_1.

We now note that $\rho(N,1) > 0$.

$$\begin{aligned}
\rho(N,1) &= \int_0^1 \frac{v(2z-1)}{1+abN(1-z)^{N-1}}dz \\
&= \int_{\frac{1}{2}}^1 \frac{v(2z-1)}{1+abN(1-z)^{N-1}}dz - \int_0^{\frac{1}{2}} \frac{v(1-2z)}{1+abN(1-z)^{N-1}}dz \\
&> \int_{\frac{1}{2}}^1 \frac{v(2z-1)}{1+ab\frac{N}{2^{N-1}}}dz - \int_0^{\frac{1}{2}} \frac{v(1-2z)}{1+ab\frac{N}{2^{N-1}}}dz \\
&= \frac{v}{1+ab\frac{N}{2^{N-1}}}\int_0^1 (2z-1)\,dz = 0.
\end{aligned}$$

On the other hand, it is easily seen that $\rho(N,0) = 0$. Since $\rho(N,\mu_1)$ is strictly concave in μ_1 and $\rho(N,1) > 0$, it follows that the equation (10) may have a single solution or two solutions at most. ■

We now provide a discussion on this existence result. As in previous cases, one can ask whether for any given N, there exist parameter values c, v and λ such that a high search intensity equilibrium exists. This amounts to show that

$$\max_{\mu_1\in(0,1)} \int_{\underline{p}(N,\mu_1)}^{v} [F(p;N,\mu_1)(1-F(p;N,\mu_1)]\,dp > 0 \text{ for all } N. \tag{11}$$

To prove that this inequality holds for all N we shall make use of the envelope theorem and the proof of Lemma 7 presented below. The argument is as follows. Using the envelope theorem, it is easily seen that the behavior of the LHS of (11) with respect to N is given in Lemma 7. This Lemma shows that this function is first increasing and then decreasing, and that in the limiting case $N \to \infty$ is bounded away from zero. This observation also enables us to argue that for given values of v and λ, when c is small, a high search intensity equilibrium exists for all N.

Lemma 6 also shows that there may be a single equilibrium or two equilibria at most, depending on parameters. However, we argue below that, if it is the case that there are two equilibria, only one of them seems reasonable. This Lemma is illustrated in Figure 4 below. The curved schedule depicts the function $\rho(N,\mu_1)$. For sufficiently high search costs, for example c_3, a high search intensity equilibrium fails to exist. For low enough search costs, for example c_1, there exists a unique equilibrium high search intensity. This equilibrium is given by the intersection point between the curve $\rho(N,\mu_1)$ and the line c_1/v (point A in the graph). By contrast, when the search cost is higher, say c_2, there are two equilibria (points B and C in the graph). We now argue (in line with Fershtman and Fishman, 1992) that if there are two equilibria, only the equilibrium with a higher search intensity (lower μ_1) is a stable equilibrium. In a neighborhood to the left of point C, the expected gains to buyers from searching for two prices instead of searching for one price are larger than the cost of an extra search. Therefore, a small perturbation around point C so that $\mu_1 < \mu_1^*$ would lead consumers to search more intensively, a movement away from point C. Similarly, a small perturbation so that $\mu_1 > \mu_1^*$ would lead consumers to search less intensively. These observations suggest that the equilibrium represented by the point C is not stable. A similar argument shows that the equilibrium depicted by point B is a stable equilibrium. In what follows, for our comparative statics results, we will concentrate on this stable equilibrium with endogenous high search intensity.

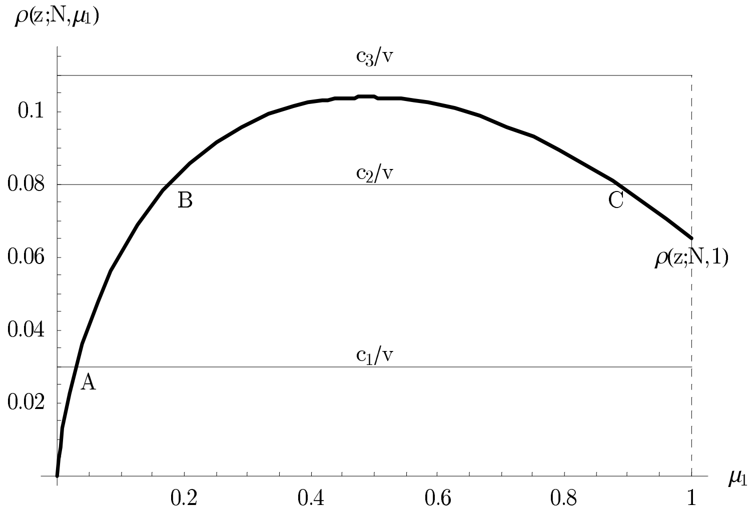

Figure 4

We now focus on the comparative statics effects of increased competition. We first observe that, keeping the price strategy of the firms given in Lemma 5 fixed, the incentives to search of the less-informed consumers are *non-monotonic* in the number of firms. In addition, we see that there exists a simple relationship between the incentives to search under duopoly and the same incentives under a fully competitive market.

Lemma 7 *For any given* c, v *and* λ, $\mu_1(2) > \mu_1(3)$, *and there exists some* $\widehat{N}$ *such that for all* $N \geq \widehat{N}$, $\mu_1(N) < \mu_1(N+1)$. *Moreover,* $\mu_1(\infty) = (1-\lambda)\mu_1(2)$.

Proof. We first prove that consumers search more under triopoly than under duopoly. To show this, we compare $\rho(N+1, \mu_1)$ with $\rho(N, \mu_1)$:

$$\begin{aligned}
&\frac{\rho(N+1,\mu_1)-\rho(N,\mu_1)}{\mu_1(1-\lambda)v}\\
=\;& \frac{1}{\mu_1(1-\lambda)v}\int_0^1 (2z-1)[p(z;N+1,\mu_1)-p(z;N,\mu_1)]dz\\
=\;& \int_0^1 (2z-1)\left[\frac{1}{(N+1)g(z;N+1,\mu_1)}-\frac{1}{Ng(z;N,\mu_1)}\right]dz\\
=\;& \int_0^1 (2z-1)\left[\frac{Ng(z;N,\mu_1)-(N+1)g(z;N+1,\mu_1)}{N(N+1)g(z;N+1,\mu_1)g(z;N,\mu_1)}\right]dz\\
=\;& \int_0^1 \frac{\lambda(1-z)^{N-1}((N+1)z-1)(2z-1)}{N(N+1)g(z;N+1,\mu_1)g(z;N,\mu_1)}dz = \xi_1+\xi_2,
\end{aligned}$$

where

$$\xi_1 = \int_0^{\frac{1}{N+1}} \frac{\lambda(1-z)^{N-1}((N+1)z-1)(2z-1)}{N(N+1)g(z;N+1,\mu_1)g(z;N,\mu_1)}dz$$

$$\xi_2 = \int_{\frac{1}{2}}^1 \frac{\lambda(1-z)^{N-1}((N+1)z-1)(2z-1)}{N(N+1)g(z;N+1,\mu_1)g(z;N,\mu_1)}dz - \int_{\frac{1}{N+1}}^{\frac{1}{2}} \frac{\lambda(1-z)^{N-1}((N+1)z-1)(1-2z)}{N(N+1)g(z;N+1,\mu_1)g(z;N,\mu_1)}dz$$

We note that since the integrand is positive, $\xi_1 > 0$. Moreover,

$$\xi_2 > \frac{\lambda}{N(N+1)g(\frac{1}{2};N+1,\mu_1)g(\frac{1}{2};N,\mu_1)} \int_{\frac{1}{N+1}}^1 (1-z)^{N-1}((N+1)z-1)(2z-1)dz$$

Note now that

$$\int_{\frac{1}{N+1}}^1 (1-z)^{N-1}((N+1)z-1)(2z-1)dz = 2(N+1)\int_{\frac{1}{N+1}}^1 z^2(1-z)^{N-1}dz +$$

$$+\int_{\frac{1}{N+1}}^1 (1-z)^{N-1}dz - (N+3)\int_{\frac{1}{N+1}}^1 z(1-z)^{N-1}dz = \frac{\left(\frac{N}{N+1}\right)^N (2+N(3-N))}{(N+1)^2(N+2)},$$

where the second equality follows from integration by parts. Then, $\xi_2 > 0$, and by implication $\rho(N+1,\mu_1) > \rho(N,\mu_1)$, for all λ and μ_1, and $N \leq 3$. This proves that less-informed consumers search more under triopoly than under duopoly.

To complete the argument, we show that there exists a number of firms $\widehat{N}$ sufficiently large such that for all $N \geq \widehat{N}$, $\rho(N+1,\mu_1) < \rho(N,\mu_1)$ and therefore less-informed consumers' search incentives decline for all $N \geq \widehat{N}$. To show this, we study the derivative of $\rho(N,\mu_1)$ with respect to N. We first rewrite $\rho(N,\mu_1)$ as follows:

$$\rho(N,\mu_1) = v\int_0^1 \frac{(2z-1)}{1+2(a-1)(1-z)+abN(1-z)^{N-1}}dz$$

where $a = 1/\mu_1$ and $b = \lambda/(1-\lambda)$. Differentiation yields

$$\frac{1}{vab}\frac{\rho(N,\mu_1)}{dN} = \int_0^1 \frac{(1-2z)(1-z)^{N-1}(1+N\ln(1-z))}{\left(1+2(a-1)(1-z)+abN(1-z)^{N-1}\right)^2}dz \quad (12)$$

We need to show that the RHS of (12) is negative for N large. We note that this is equivalent to show that $I_1 - I_2 + I_3 < 0$, where

$$\begin{aligned}
I_1 &= \int_0^{1-e^{-\frac{1}{N}}} \frac{(1-2z)(1-z)^{N-1}(1+N\ln(1-z))}{\left(1+2(a-1)(1-z)+abN(1-z)^{N-1}\right)^2} dz > 0,\\
I_2 &= \int_{1-e^{-\frac{1}{N}}}^{\frac{1}{2}} \frac{(2z-1)(1-z)^{N-1}(1+N\ln(1-z))}{\left(1+2(a-1)(1-z)+abN(1-z)^{N-1}\right)^2} dz > 0,\\
I_3 &= \int_{\frac{1}{2}}^{1} \frac{(1-2z)(1-z)^{N-1}(1+N\ln(1-z))}{\left(1+2(a-1)(1-z)+abN(1-z)^{N-1}\right)^2} dz > 0.
\end{aligned}$$

We now observe that, for N large, the integral

$$\widehat{I_2} = \int_{\frac{\ln N}{2N}}^{\frac{\ln N}{N}} \frac{(2z-1)(1-z)^{N-1}(1+N\ln(1-z))}{\left(1+2(a-1)(1-z)+abN(1-z)^{N-1}\right)^2} dz < I_2.$$

Therefore, showing that $\widehat{I_2} > I_1 + I_3$ for large N suffices. Notice that

$$\begin{aligned}
I_1 &< \frac{1}{\left(1+2(a-1)e^{\frac{-1}{N}}+abNe^{\frac{-(N-1)}{N}}\right)^2} \int_0^{1-e^{-\frac{1}{N}}} (1-2z)(1-z)^{N-1}(1+N\ln(1-z))dz\\
&< \frac{1}{\left(1+2(a-1)e^{\frac{-1}{N}}+abNe^{\frac{-(N-1)}{N}}\right)^2} \int_0^{1-e^{-\frac{1}{N}}} (1-z)^{N-1}(1+N\ln(1-z))dz\\
&= \frac{1}{\left(1+2(a-1)e^{\frac{-1}{N}}+abNe^{\frac{-(N-1)}{N}}\right)^2} -(1-z)^N\ln(1-z)\Big|_0^{1-e^{-\frac{1}{N}}}\\
&= \frac{1}{Ne\left(1+2(a-1)e^{\frac{-1}{N}}+abNe^{-1+\frac{1}{N}}\right)^2} \underset{for\ large\ N}{<} \frac{e}{a^2b^2N^3}
\end{aligned}$$

Moreover,

$$\begin{aligned}
I_3 &< \int_{\frac{1}{2}}^{1} (1-2z)(1-z)^{N-1}(1+N\ln(1-z))dz\\
&< \int_{\frac{1}{2}}^{1} -(1-z)^{N-1}(1+N\ln(1-z))dz = \frac{\ln 2}{2^N}
\end{aligned}$$

Finally, we note that

$$
\begin{aligned}
\widehat{I}_2 &> \frac{1}{\left(1+2(a-1)(1-\frac{\ln N}{2N})+abN(1-\frac{\ln N}{2N})^{N-1}\right)^2}\int_{\frac{\ln N}{2N}}^{\frac{\ln N}{N}}(2z-1)(1-z)^{N-1}(1+N\ln(1-z))dz \\
&> \frac{(\frac{\ln N}{N}-1)}{\left(1+2(a-1)(1-\frac{\ln N}{2N})+abN(1-\frac{\ln N}{2N})^{N-1}\right)^2}\int_{\frac{\ln N}{2N}}^{\frac{\ln N}{N}}(1-z)^{N-1}(1+N\ln(1-z))dz \\
&= \frac{(\frac{\ln N}{N}-1)\left[\left(1-\frac{\ln N}{2N}\right)^N\ln\left(1-\frac{\ln N}{2N}\right)-\left(1-\frac{\ln N}{N}\right)^N\ln\left(1-\frac{\ln N}{N}\right)\right]}{\left(1+2(a-1)(1-\frac{\ln N}{2N})+abN(1-\frac{\ln N}{2N})^{N-1}\right)^2} \\
&\underset{for\ large\ N}{>} \frac{(1-\frac{\ln N}{N})\left[-\frac{1}{N}\frac{\ln N}{N}+\frac{1}{\sqrt{N}}\frac{\ln N}{2N}\right]}{\left(1+2(a-1)+ab\sqrt{N}\right)^2} \underset{for\ large\ N}{>} \frac{\ln N}{2a^2b^2N^2\sqrt{N}}
\end{aligned}
$$

Now, since

$$\frac{\ln N}{2a^2b^2N^2\sqrt{N}} > \frac{e}{a^2b^2N^3}+\frac{\ln 2}{2^N}$$

for large N, the result follows.

It remains to prove that $\mu_1(\infty)=(1-\lambda)\mu_1(2)$. Setting $N=2$ in (10), integrating and rearranging yields

$$\frac{(1-\lambda)\mu_1(2)}{2(1-(1-\lambda)\mu_1(2))}\left[\frac{1}{1-(1-\lambda)\mu_1(2)}\ln\left(\frac{2-(1-\lambda)\mu_1(2)}{(1-\lambda)\mu_1(2)}\right)-2\right]=\frac{c}{v}. \tag{13}$$

Similarly, setting $N=\infty$ in (10) yields

$$\frac{\mu_1(\infty)}{2(1-\mu_1(\infty))}\left[\frac{1}{1-\mu_1(\infty)}\ln\left(\frac{2-\mu_1(\infty)}{\mu_1(\infty)}\right)-2\right]=\frac{c}{v}. \tag{14}$$

Inspection of (13) and (14) reveals the result. ■

Lemma 7 shows that consumers respond to increased competition by searching more intensively when the number of firms is small to begin with, and by searching less intensively when the number of competitors is already high. Thus, *search incentives are non-monotonic in the number of firms.* Moreover, this result shows the existence of a simple relationship between consumers' search intensity when there are infinitely many firms and search intensity when there are only two firms. We have noted above that when the number of firms in the market grows without limit, the probability that a firm is undercut by some other firm goes to 1. It is precisely for this reason that in the limit economy firms ignore the λ percent of informed consumers and only consider attracting the remaining $1-\lambda$

percent of consumers. Note that these consumers search for one price with probability μ_1 and for two prices with probability $1-\mu_1$. So in effect, the limit economy can be seen as a duopoly competing for only $1-\lambda$ consumers. This is the reason behind the similarity between the two cases. What consumers do in their search behavior is to internalize the 'practical' inexistence of the fully informed consumers by searching more intensively in the limit economy, which explains the relationship $\mu_1(\infty) = (1-\lambda)\mu_1(2)$. These remarks together enable us to argue that, provided that consumers' optimal search intensity exhibits a smooth relationship with respect to the number of firms, consumers search activity attains its minimum under duopoly.

Our final and most important result borrows from these observations and brings together Proposition 4 and Lemma 7. We have seen above that holding the search intensity of the consumers constant, firms have an incentive to raise prices as a response to an increase in the number of firms operating in the market (Proposition 4). Moreover, holding the pricing strategy of the firms constant, the response of less-informed consumers to increased competition is to search more intensively when the number of firms is small to begin with, and less intensively when the number of firms is large to begin with (Lemma 7). Therefore, when the number of firms increases and the *status quo* number of firms is large, both firm and consumer responses influence expected price in the same direction, which yields the result that expected price increases without ambiguity. By contrast, when the *status quo* number of firms is small to begin with, firms incentives to raise prices and consumers incentives to search more influence expected price in opposite directions. Thus, the comparative statics analysis of increased competition in the case of a initial small number of firms requires to balance these two effects. This task has proven to be very difficult because we do not have explicit solutions to equations (7) and (10) for arbitrary N at our disposal. However, the next result shows that expected prices (and price dispersion) in the case of duopoly and the case of a very large number of firms are equal, which, together with the observations above, suffices to show that expected price is *non-monotonic* in the number of firms when consumers search with high intensity in equilibrium.

Theorem 3 *Suppose less-informed consumers search with high-intensity in equilibrium. Then, expected price is non-monotonic in the number of firms and duopoly yields the same expected price and price dispersion than the competitive case $N=\infty$. Moreover, welfare is non-monotonic in the number of firms and duopoly yields a higher welfare than the competitive case.*

Proof. Setting $N = 2$ in (7) and solving for $F(p; 2)$ yields

$$F(p; 2, \mu(2)) = \frac{2 - (1-\lambda)\mu(2)}{2(1 - (1-\lambda)\mu(2))} - \frac{(1-\lambda)\mu(2)}{2(1 - (1-\lambda)\mu(2))}\frac{v}{p}.$$

Similarly, setting $N = \infty$ in (7) and solving for the limiting price distribution yields

$$F(p; \infty, \mu(\infty)) = \frac{2 - \mu_1(\infty)}{2(1 - \mu_1(\infty))} - \frac{\mu_1(\infty)}{2(1 - \mu_1(\infty))}\frac{v}{p}$$

Since $\mu_1(\infty) = (1-\lambda)\mu_1(2)$ as shown in Lemma 7, it follows that $F(p; 2, \mu(2)) = F(p; \infty, \mu(\infty))$. The second part of the result follows from Proposition 4 and Lemma 7. Since total surplus is maximized when incurred search costs are lower, the welfare result follows from Lemma 7. ■

We would like to elaborate on three aspects of this Theorem. The first is that expected prices and price dispersion with two firms and an infinite number of firms are equal. This result seems reminiscent to Bertrand type of competition but it does not hold in other articles dealing with consumer search (see, e.g., Varian, 1980 and Stahl, 1989). It is interesting to see how economizing search behavior of less-informed consumers fully internalizes the pressure that fully informed consumers exert on the firms in a duopoly setting (Lemma 7). This explains why the two market settings generate the same equilibrium price distribution.

The second observation pertains to the behavior of expected price when one moves from duopoly to triopoly. Theorem 3 suggests that, presumably, expected price is first declining and then increasing in the number of firms. We have not been able to show this analytically. However, we have solved the model numerically,[12] for various levels of search cost c and percentages of informed consumers λ. Tables 5.1 and 5.2 show the outcome of this exercise. It can be seen that expected prices are always lower under triopoly than under duopoly.[13] This proves the dominating influence of the consumers' search behavior when we move from duopoly to triopoly. We view this observation as giving support to the conjecture made above.

[12] To this end, we have used the software Mathematica 4.1.

[13] In Janssen and Moraga (2001) we show that when $N = 2$ expected price is constant when λ changes. The numerical results also show that price dispersion (measured by the variance of the equilibrium price distribution and omitted from Tables 5.1 and 5.2 to save on space) is greater under triopoly than under duopoly. This suggests that price dispersion exhibits a non-monotonic relationship with respect to the number of rivals.

	$c = 0.02$				$c = 0.04$			
	$N = 2$		$N = 3$		$N = 2$		$N = 3$	
	μ_1	$E[p]$	μ_1	$E[p]$	μ_1	$E[p]$	μ_1	$E[p]$
$\lambda = 0.05$	0.0133175	0.032399	0.012642	0.0319147	0.0377135	0.074395	0.0357142	0.0729443
$\lambda = 0.15$	0.0148848	0.032399	0.0126536	0.0309735	0.0421507	0.074385	0.0356206	0.0701803
$\lambda = 0.25$	0.0168695	0.032399	0.0127142	0.0300587	0.0477708	0.074395	0.0357218	0.0675558
$\lambda = 0.35$	0.0194648	0.032399	0.0128375	0.0291591	0.0551193	0.074395	0.0360631	0.0650308
$\lambda = 0.45$	0.0230038	0.032399	0.0130476	0.0282625	0.065142	0.074395	0.0367291	0.0625661
$\lambda = 0.55$	0.0281157	0.032399	0.0133889	0.0273543	0.079618	0.074395	0.0378821	0.0601194
$\lambda = 0.65$	0.0361487	0.032399	0.0139534	0.0264139	0.102366	0.074395	0.0398653	0.0576378
$\lambda = 0.75$	0.0506076	0.032399	0.0149675	0.0254111	0.143312	0.074395	0.0435392	0.0550435
$\lambda = 0.85$	0.0843425	0.032399	0.0171903	0.0242782	0.238853	0.074395	0.051902	0.0521913
$\lambda = 0.95$	0.0253042	0.032399	0.0266565	0.0228223	0.716562	0.074395	0.0900745	0.0486904

Table 5.1. Comparison of search intensities and expected prices under duopoly and triopoly.

	$c = 0.06$				$c = 0.08$			
	$N = 2$		$N = 3$		$N = 2$		$N = 3$	
	μ_1	$E[p]$	μ_1	$E[p]$	μ_1	$E[p]$	μ_1	$E[p]$
$\lambda = 0.05$	0.0761624	0.128013	0.0717813	0.124774	0.138738	0.201257	0.129286	0.194051
$\lambda = 0.15$	0.0851227	0.128013	0.07104	0.118765	0.15506	0.201257	0.125723	0.181443
$\lambda = 0.25$	0.964723	0.128013	0.0708497	0.113248	0.175735	0.201257	0.123822	0.170569
$\lambda = 0.35$	0.111314	0.128013	0.071285	0.108093	0.202771	0.201257	0.123497	0.160894
$\lambda = 0.45$	0.131553	0.128013	0.072508	0.103193	0.239639	0.201257	0.124949	0.152057
$\lambda = 0.55$	0.160787	0.128013	0.0749205	0.0984449	0.292891	0.201257	0.128802	0.14379
$\lambda = 0.65$	0.206725	0.128013	0.079278	0.0937415	0.376564	0.201257	0.136529	0.135849
$\lambda = 0.75$	0.289417	0.128013	0.0876271	0.0889406	0.527205	0.201257	0.152047	0.12798
$\lambda = 0.85$	0.482362	0.128013	0.107198	0.0838095	0.878673	0.201257	0.189519	0.11983
$\lambda = 0.95$	Existence	fails	0.200199	0.077776	Existence	fails	0.372971	0.110658

Table 5.2. Comparison of search intensities and expected prices under duopoly and triopoly.

Finally, the welfare result of this Theorem deserves an explanation. In a high search intensity equilibrium, welfare is maximized when actual search intensity is lowest, since all consumers acquire the good in equilibrium. In this sense, Lemma 7 suffices to show that welfare is non-monotonic with respect to the number of firms, and that welfare is higher under duopoly than under an infinite number of firms because actual search is more modest under the former market structure. Moreover, provided that search intensity is a smooth function of the number of firms, welfare attains its maximum under duopoly.

6 Conclusions

In this paper we have presented an oligopoly model where a certain fraction of consumers engage in costly non-sequential search to discover prices. The economy may present three distinct price dispersed equilibria. These equilibria can be characterized in terms of how intensively consumers search. Consumers search with low intensity when they randomize between searching for one price and not searching at all. Consumers search with moderate intensity when they search for one price quotation surely. Finally, consumers search with high intensity when they mix between searching for one price quotation and searching for two price quotations. We show that the impact of an increase in the number of firms active in the market is *sensitive (i)* to the equilibrium consumers' search intensity, and *(ii)* to the *status quo* number of firms. In the case the economy is in a low search intensity equilibrium, increased competition does not influence expected price and results in greater price dispersion, while total surplus falls. In the case of a moderate search intensity equilibrium, the impact of bringing more competitors together is an unambiguous increase in expected price, but this type of equilibrium fails to exist when N becomes large. Finally, when consumers search with high intensity, increased competition leads to a lower expected price when the number of competitors in the market is low to begin with, but in a higher expected price when the number of competitors is large to begin with. Thus, expected price exhibits a *non-monotonic* relationship with respect to the number of firms in the case of high search intensity.

Two more results are worth emphasizing. First, we have seen how economizing search behavior prevents firms from charging monopoly prices, even when the market accommodates a very large number of competitors. This is explained by the following observations. First, in a high search intensity equilibrium, consumers keep comparing prices with positive probability, for any number of firms, which prevents from monopoly pricing. Second, if the economy is initially in a moderate search intensity equilibrium, as the number of firms grows consumers find it profitable to economize on search and step out of the market with positive probability, which also prevents from monopoly pricing. This result, which is at odds with previous research on search and oligopoly, is important because it shows that price dispersion does not vanish as the number of firms grows without limit.

The second result we wish to emphasize is that in case the economy is in a high search intensity equilibrium, arguably the most interesting case, *two firms is enough for competition and three or more firms is better.* The reason why the limit economy and the duopoly case are alike is that in the former case an individual firm is in effect also competing with just one other firm, since when there are so many firms in the market the probability of being undercut by some other firm is close to one and firms ignore the fully informed consumers. Less-informed consumers adjust their search intensity and search more actively in the limit economy so as to compensate for the negligible impact that the fully informed consumers have in such case. When the market accommodates three or more competitors, firms find themselves competing in effect for three groups of consumers (those comparing all prices, those comparing just two prices and those making only one search), which leads to a more competitive outcome. This result is also at odds with previous research on search and oligopoly and highlights the strong influence of consumer search on firms' pricing behavior.

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