Models and Techniques for Hotel Revenue Management using a Rolling Horizon

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Abstract

This paper studies decision rules for accepting reservations for stays in a hotel based on deterministic and stochastic mathematical programming techniques. Booking control strategies are constructed that include ideas for nesting, booking limits and bid prices. We allow for multiple day stays. Instead of optimizing a decision period consisting of a fixed set of target booking days, we simultaneously optimize the complete range of target booking dates that are open for booking at the moment of optimization. This yields a rolling horizon of overlapping decision periods, which will conveniently capture the effects of overlapping stays.

Keywords: Revenue Management, Yield Management, Mathematical Programming

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1. Introduction

Hotels offer the same rooms to different types of guests. While hotel managers would like to fill their hotels with highly profitable guests as much as possible, it is generally necessary to allow for less profitable guests in order to prevent rooms from remaining vacant. An important decision to be made, is whether to accept a booking request and generate revenue now, or to reject it in anticipation of a more profitable booking request in the future. Because this decision must be made at the time of the booking request and future demand is never certain, the booking control problem contains both dynamic and stochastic elements. Finding the right combination of guests in the hotel such that revenues are maximized, is the topic of revenue management.

Revenue management originates from the airline industry, where the seats on a plane can be sold to different types of passengers. In comparison to this problem, hotel revenue management has the distinct feature that booking requests can occur for different lengths of stay and can therefore overlap. Most models for hotel revenue management consider a fixed set of target booking days over which to maximize revenues. In general, such a fixed set of days can not be determined without missing some of the effects of the overlapping stays. In this paper, we study booking control policies based on a rolling horizon of decision periods. For each optimization, all types of stays that span the current decision period are considered. Because of the rolling nature of the decision periods, eventually no overlap between the stays will be left out.

The booking control policies we study in this paper, include nested booking limit and bid price methods. A deterministic as well as a stochastic model is used to derive the booking control policies. We assume that every guest has a strict preference for a specific type of stay. This means that whenever a booking request is rejected, it is lost forever and is not turned into a booking request for another type of stay. Further, we do not consider batch bookings or cancellations and no-shows.

The organization of this paper is as follows: In Section 2 we give a short overview of the related literature on hotel revenue management. The deterministic and stochastic mathematical programming models are presented in Section 3. Booking control policies based on the mathematical programming models and their application over a rolling horizon are presented in Section 4. In section 5 we sketch the environment of a test case. We use this environment as a basis to simulate arrival processes by which we study the performances of the different booking control policies. The results of the simulation studies are presented in section 6.

2. Literature

Hotel revenue management has received attention in a number of papers. Bitran and Mondschein (1995) and Bitran and Gilbert (1996) concentrate on the room allocation problem at the targeted booking day itself. The hotel manager has to decide whether or not to accept a guest that requests a room on the target day, taking into account the number of reservations made and the potential number of guests who will show up without reservations (walk-ins). They formulate this problem as a stochastic and dynamic programming model. Bitran and Gilbert also provide three simple heuristics to construct booking control policies that can be used during the booking period.

Weatherford (1995) concentrates completely on the booking control problem. He proposes a heuristic which is called the nested by deterministic model shadow prices (NDSP) method. He formulates a mathematical programming model to obtain booking limits, i.e. the number of rooms to reserve for each type of guest. These booking limits are nested such that a guest can always make use of the capacity reserved for any less profitable guest. A possible drawback of the model is that it considers demand to be deterministic. Weatherford allows for multiple day stays and maximizes the model for a decision period consisting of a fixed set of target booking days. He does not account for overlapping stays outside of the decision period. Nevertheless, he shows that taking into account multiple day stays produces better results than when only single day stays are considered.

Baker and Collier (1999) compare the performances of five booking control policies: two simple threshold approaches, Weatherford’s NDSP method, a NDSP method that includes overbooking, and a bid price method based on work by Williamson (1991) for the airline industry. Baker and Collier compare the performances of these solution techniques under 36 hotel operating environments by ways of simulation and advise on the best heuristic for each operating environment.

In this paper we concentrate on the booking control problem. This makes our work comparable to the work of Weatherford (1995) and Baker and Collier (1999). Unlike these previous researches, we use the booking control policies over a rolling horizon of decision periods, such that all overlap between the different types of stay can be accounted for. Also, next to the well-known deterministic model, we introduce a second mathematical programming model that accounts for the stochastic nature of demand. We consider both nested booking limit and bid price control policies. As Baker and Collier (1999), we compare the performances of the different methods by simulation.

3. Mathematical Formulations

In this section we present two mathematical programming models to find the optimal allocation of the rooms over the different types of guests. In Section 3.1 we discuss the deterministic model and in Section 3.2 the stochastic model. The models are defined for use over a fixed decision period. Booking control policies based on the models and the application these policies over a rolling horizon of decision periods are discussed in Section 4.

3.1. Deterministic Model

The deterministic model we consider in this paper is the same as Weatherford (1995) uses for his NDSP method. This model replaces demand for each type of stay by an estimation and obtains the optimal allocation of the rooms over the expected demand; i.e. it treats demand as if it were deterministic and equal to its expectation. To formulate the deterministic model, define a stay in the hotel by (a,L,k), where a is the first night of the stay, L the length of the stay and k the price class. Further, denote the set of stays that make use of night l by N_l, where $N_l = \{(a,L,k) : a = l = a+L-1\}$. The deterministic model is then formulated as follows:

$$\text{maximize} \quad \sum_{a,L,k} R_{a,L,k} X_{a,L,k} \tag{3.1}$$

$$\text{subject to} \quad \sum_{a,L,k \in N_l} X_{a,L,k} \le C_l \qquad \forall\, l$$

$$X_{a,L,k} \le d_{a,L,k} \qquad \forall\, a,L,k$$

$$X_{a,L,k} \ge 0 \quad \text{integer} \qquad \forall\, a,L,k$$

where:

$X_{a,L,k}$ = the number of rooms allocated to a stay of type (a,L,k)

$R_{a,L,k}$ = the revenue obtained from a stay of type (a,L,k)

$d_{a,L,k}$ = the expected demand for a stay of type (a,L,k)

C_l = the capacity (number of rooms) of the hotel available on night l.

The objective of the model is to maximize revenues under the restriction that the total number of reservations for a night does not exceed the capacity of the night. In order to prevent vacant rooms, the number of rooms allocated to each type of stay is restricted by the level of the demand, which in this model is replaced by its expectation.

Although no proof exists that the constraint matrix is totally unimodular, our experience and previous experiences (see Williamson (1992) en De Boer et al. (2002)) with the LP relaxation of this model show that when demand is integer the LP solutions are often integer. It can be expected that when the LP relaxation produces a fractional solution, it will not take much effort to produce an integer solution by applying branch-and-bound techniques.

3.2. Stochastic Model

The deterministic model never allocates more rooms to a type of stay than the hotel expects to book for that type of stay. However, because demand can deviate from its expectation, it can be more profitable to allocate more rooms to the more expensive types of stay. In order to consider this, the stochastic nature of demand has to be taken into account. We present here a stochastic model first introduced by De Boer et al. (2002) for the airline industry. For this model we suppose that the demand for a type of stay, $D_{a,L,k}$, can take on a limited number of different realizations, which we will denote by $d_{a,L,k,1} < d_{a,L,k,2} < ... < d_{a,L,k,N}$. The stochastic model is now formulated as follows:

$$\text{maximize} \quad \sum_{a,L,k}\sum_{j=1}^{N} R_{a,L,k} \Pr(D_{a,L,k} \geq d_{a,L,k,j}) X_{a,L,k,j} \tag{3.2}$$

$$\begin{aligned}
\text{subject to} \quad & \sum_{a,L,k\in N_l}\sum_{j=1}^{N} X_{a,L,k,j} \leq C_l && \forall\, l \\
& X_{a,L,k,1} \leq d_{a,L,k,1} && \forall\, a,L,k \\
& X_{a,L,k,j} \leq d_{a,L,k,j} - d_{a,L,k,j-1} && \forall\, a,L,k \text{ and } j = 2, 3, ..., N \\
& X_{a,L,k,j} \geq 0 \quad \text{integer} && \forall\, a,L,k \text{ and } j = 1, 2, ..., N
\end{aligned}$$

The decision variables, $X_{a,L,k,j}$, each represent the part of the demand that falls in the interval $(d_{a,L,k,j-1},\ d_{a,L,k,j}]$. Notice that $X_{a,L,k,j}$ will only be nonzero when $X_{a,L,k,j-1}$ has reached its upperbound of $d_{a,L,k,j-1}$, since $\Pr(X_{a,L,k} = d_{a,L,k,j-1}) = \Pr(X_{a,L,k} = d_{a,L,k,j})$. Summing the decision variables, $X_{a,L,k,j}$, over all j, yields the total number of rooms allocated to the stays of type (a,L,k). As for the deterministic model, we solve the LP relaxation of the stochastic model.

The deterministic model can be obtained from the stochastic model by considering only one demand scenario. The EMR model introduced by Wollmer (1986) for the airline industry, can be obtained by considering all possible demand scenarios. De Boer et al (2002) show that for the airline industry 3 or 4 demand scenarios will suffice to capture most of the extra revenue generated by considering the stochastic nature of the demand. We will follow their approach and consider only 3 scenarios; a low, an average and a high demand scenario. We say that a scenario occurs whenever the demand exceeds the level of the demand of the scenario. For all 3 scenarios, the level of the demand, i.e. $d_{a,L,k,j}$, and the probability that the scenario occurs, i.e. $\Pr(D_{a,L,k} = d_{a,L,k,j})$, have to be determined. In this research, we set the probabilities that the scenarios occur equal for all types of stay. We denote these probabilities by p_j for $j = 1, 2, 3$. Also to determine the level of a demand scenario, we use a uniform rule over all types of stay. We set the level of the average scenario equal to the expected demand, and define the levels of the low and high demand scenarios as a fixed number of times the standard deviation below and above the expected demand. We apply the model for a number of different demand levels for the scenarios and a number of different combinations for the scenario probabilities.

4. Booking Control Policies

In this section we discuss booking control policies based on the models presented in Section 3. Nested booking limits and bid price control policies are constructed. Further, we discuss how to use these booking control policies over a rolling horizon of decision periods.

4.1. Nested Booking Limits

The number of rooms allocated to each type of stay by the models from the previous section, can easily be interpreted as booking limits. These limits can be used as the maximum number of booking requests to accept for each type of stay during the booking period. It is never optimal, however, to reject a booking request when there are still rooms available for other less profitable types of stay, even if its own booking limit has been reached. Therefore, each type of stay should be allowed to tap into the rooms allocated to any less profitable type of stay. This is called nesting. In order to form nested booking limits, the different types of stay need to be ranked by their contribution to the overall revenue of the hotel. When such a ranking is determined, a nested booking limit for a type of stay can be set equal to the sum of the number of rooms allocated to that and every other, lower ranked type of stay.

It is not trivial what measurement to use to determine a nesting order of the different types of stay. Using the price class does not take into account the length of the stay. Such a measurement will rank guests who are willing to pay more for one night above guests who are willing to pay a little less for multiple nights, whereas the overall revenue generated by the multiple night stay will most likely be higher. Nesting by the complete revenue generated by the stay does take into account the length of the stay. But this measurement does not account for the load factors of the different nights. Certain nights can be very busy and always fully booked, whereas other nights can be mainly vacant. A stay that occupies many busy nights should be valued differently from a stay that uses mainly nights with a lot of vacant rooms. One way to take into account all of these aspects, is to use the shadow prices obtained from the underlying allocation model. The shadow price corresponding to the capacity restriction for a night, reflects the expected gain that can be obtained if one additional room were available on that night. It can be interpreted as the value of a room. Adding the shadow prices of all nights used by a stay, gives an indication of the opportunity costs of the stay. A measurement for nesting is then obtained by subtracting these opportunity costs from the revenue generated by the stay. Thus, a nesting order is based on:

$$\overline{R}_{a,L,k} = R_{a,L,k} - \sum_{a,L,k \in N_l} s_l \qquad (4.1)$$

where s_l denotes the shadow price of the capacity constraint for night l. Nested booking limits can now easily be constructed.

4.2. Bid Prices

The second type of booking control policy we study in this paper is the bid price policy. This method directly links the opportunity costs of a stay to the acceptance/rejection decision. Bid prices are constructed for every night to reflect the opportunity costs of renting a room on that night. As before, we estimate the bid price of a night by the shadow price of the capacity constraint corresponding to that night. A booking request is only accepted if the revenue it generates is above the sum of the bid prices of the nights it uses. Thus, if its revenue is more than its opportunity costs.

4.3. Rolling Horizon

The mathematical programming models we presented in Section 3 provide an allocation of the rooms for a fixed decision period. We will use them over a rolling horizon of decision periods. Assume that booking requests can not be made more than F days in advance, and that the longest possible stay in the hotel consists of M days. The stays corresponding to the booking requests that come in at day t can then start at day t at the earliest and at day $t+F$ at the latest. The latest possible booking request will end at day $t+F+M$. Therefore, if a booking control policy is determined at day t, the decision period we consider, is given by the time interval $[t, t+F+M]$. Within this decision period all overlap between the different types of stay are taken into account, except for the overlap at the end of the interval corresponding to the stays that fall partly outside of the decision period. But only the types of stay for which booking has just opened, fall into this category. It can be expected that the total level of booking requests for these types of stay will not yet be such that booking requests will have to be rejected. By the time critical decisions have to be made for these types of stay, the decision period will have rolled forward and capture all overlap for these types of stay.

The booking control policy is constructed at different points in time. Every time a new policy is constructed, the decision period rolls forward. The booking limits and bid prices for the types of stay already open for booking are adjusted and new booking limits and bid prices for the types of stay that have just opened up for booking are added.

5. Test Case

The performances of the different booking control policies are tested by ways of simulation. In this section, we discuss the simulation environment which is chosen such that it reflects the situation described to us by a hotel in the Netherlands. We consider a hotel with a total capacity of 150 identical rooms. These rooms can be rented out in 10 different price classes, described in Table 5.1. We consider that the maximal length of a stay is 7 days and that a booking request can come in at most 90 days in advance. We do not consider cancellations or no-shows.

	Class	Price[1]
1	Tourist Rate Tours & Groups	$ 50
2	Tourist Rate Low Budget	$ 75
3	Tourist Rate Packages	$ 110
4	Tourist Rate Medium Budget	$ 120
5	Rack Rate	$ 250
6	Corporate Rate, liaison corporation	$ 75
7	Corporate Rate, management	$ 125
8	Corporate Rate, salesperson	$ 100
9	Corporate Rate, MCI	$ 175
10	Corporate Rate, other	$ 150

Table 5.1 Price Classes

We simulate the arrivals of booking requests by a non-homogeneous Poisson process with intensities dependent on the price class, the starting day of the stay (e.g. Monday, Tuesday, ...) and the time until the target booking day. We allow for different booking patterns for the different price classes to account for low tourist

[1] Originally, all revenues in this research were measured in Dutch guilders. For sake of simplicity we substitute this currency for the US dollar on a one-to-one rate

classes to book early in the booking process and high corporate classes to book at the end of the booking process among others. Further, we let some days, e.g. Friday, be more busy than other days, e.g. Thursday. In order to let the arrival intensities fluctuate over time, we divide the booking period into 10 smaller periods of 9 days, each with a constant arrival intensity. Just as Baker and Collier (1999) and Bitran and Mondschein (1994), we do not consider the length of the stay to be of influence on the arrival intensity. Instead, we model the length of the stay of each arrival by a logistic distribution with a parameter dependent on the price class and the starting day of the stay. The arrival intensities and the parameters for the logistic distribution we use for our simulation will be made available to the interested reader upon request. It should be noted that the parameters are chosen to reflect a busy period in the hotel in which on average the total demand exceeds the capacity of the hotel. This is the situation in which revenue management produces the highest gains in revenue.

We compare the performances of the different booking control policies over a 6 week period. However, because the hotel is empty at the start of simulation and we also want to consider the overlap of the stays already in the hotel, we make use of a start-up period. To make sure that no stay that could have arrived before the start-up period will overlap with any stay considered for the evaluation, we choose the start-up period to consist of 2 weeks. Likewise, we also use a cool-down period of 2 weeks. The first day of the start-up period is denoted by $t = 1$. Because a booking request can be made 90 days in advance, the process starts at $t = -89$. At that moment, booking control policies are derived for the decision period $t = 1$ until $t = 14$. A new booking control policy is constructed weekly, such that a next optimization takes place at $t = -82$, which produces booking limits and bid prices for the decision period $t = 1$ until $t = 21$. This way, every week the decision period is extended until it eventually encompasses the maximum number of 104 days. However, because in this simulation we are only interested in a period of 6 weeks plus two times 2 weeks to start-up and cool-down, the maximum length of the decision period we will work with, will be 70 days. A graphical illustration of the rolling decision periods is given in Figure 5.1. In this figure, the start-up and cool-down periods are colored light and the actual evaluation period is colored dark.

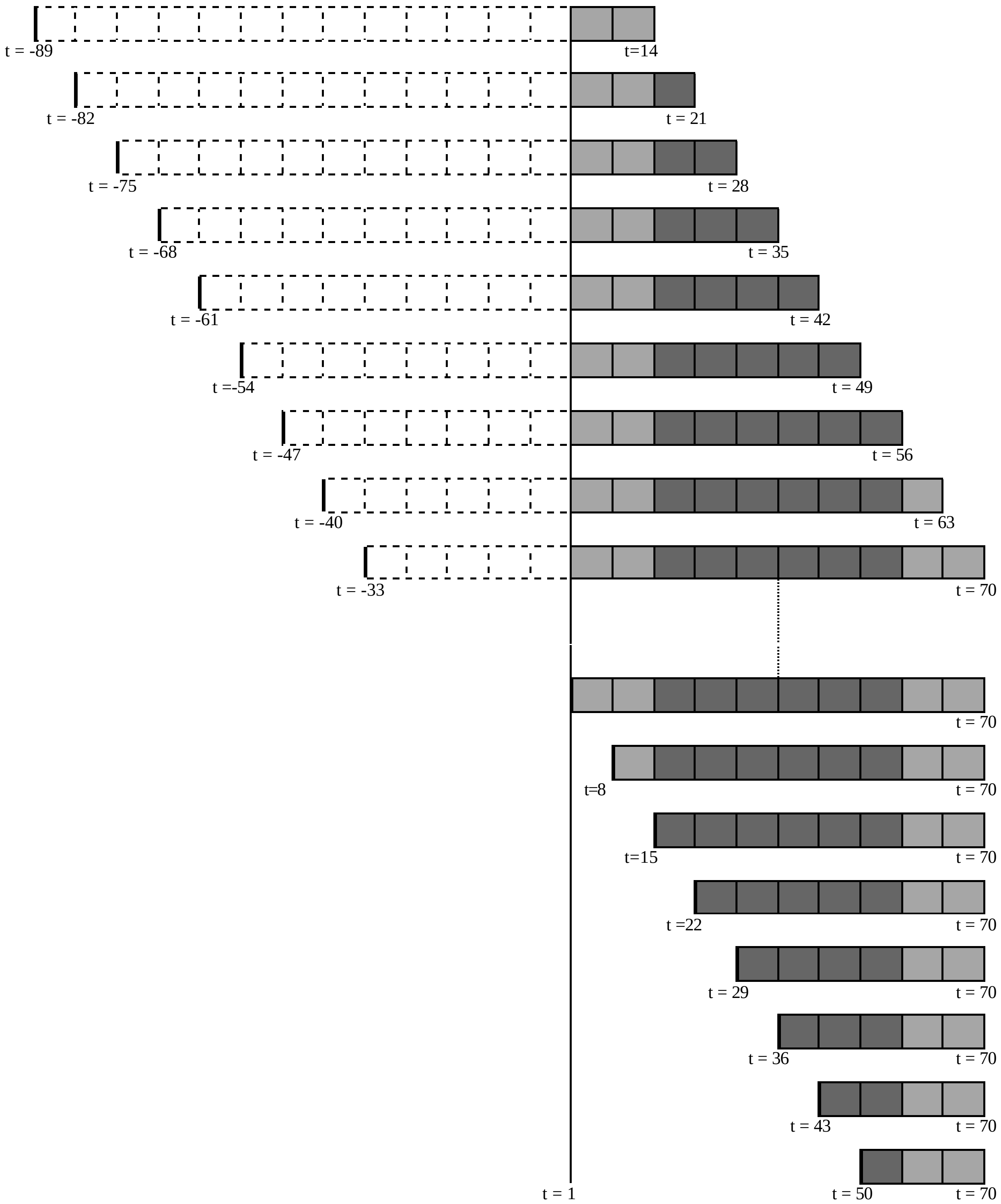

Figure 5.1 Illustration of the rolling decision periods in the test case.

6. Results

Combining the deterministic and stochastic programming models with the two methods to construct booking control policies from the models, we obtain the following four booking control policies:

- Deterministic Nested Booking Limits (DNBL)
- Deterministic Bid Prices (DBP)
- Stochastic Nested Booking Limits (SNBL)
- Stochastic Bid Prices (SBP)

In this section, we evaluate the performances of these four methods when they are applied to the simulated environment discussed above. We measure the performances of the booking control policies over 100 simulated arrival processes, and compare the results with the performances of a simple first-come-first-serve (FCFS) policy and with the optimal acceptance policy which can be determined with hindsight. The results for the optimal, FCFS, DNBL and DBP booking control policies, are presented in Table 6.1.

	Average Revenue ($)	Standard Deviation	Percentage Optimal
Optimal	727,477	8,485	100 %
FCFS	606,115	6,553	83.3 %
DNBL	665,816	11,098	91.5 %
DBP	537,186	10,308	73.8 %

Table 6.1 Performances of the optimal, FCFS, DNBL and DBP policies.

Table 6.1 shows that the DNBL policy performs better on average then the FCFS and DBP policies. On average, the DNBL policy obtains a revenue of 91.5% of the maximum revenue that can be obtained. The DBP does not seem to perform very well. Even a simple FCFS policy outperforms the DBP policy. From this we can suspect that the deterministic model does not provide the right bid prices for the problem. In Figure 6.1 the average number of booking requests accepted by the four policies during the 6 week period, is shown for each price class. The FCFS policy obviously accepts too many booking requests for the lower price classes. The number of booking requests accepted by the DNBL and DBP policies for the price classes with a high revenue, i.e. classes 5, 9 and 10, is near optimal. For the lower price classes, however, these policies reject too many booking requests. Especially the DBP policy does not accept enough booking requests for the price classes with a low revenue, i.e. classes 1, 2, 3, 4, 6 and 8. This can be explained by the fact that the deterministic model does not take into account the stochastic nature of demand. In the deterministic model, the probability that an extra booking request for a certain type of stay arrives, is considered to be 1 if the number of booking requests is below the expected level, and 0 if the number of booking requests exceeds the expected level. In reality, however, the probability that an extra booking request arrives will diminish smoothly. Therefore, the estimate of the opportunity costs of a room by the deterministic model is not correct. This results in poor bid prices.

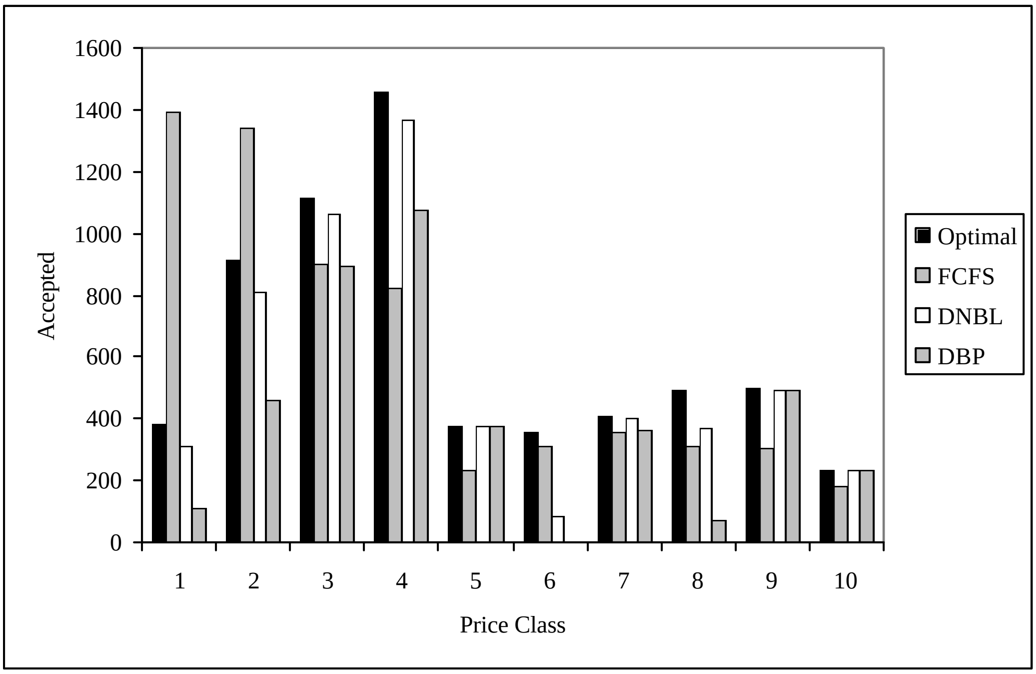

Figure 6.1 The average number of booking requests accepted for each price class by the optimal, FCFS, DNBL and DBP policies.

For the stochastic model, we consider two different spreads for the levels for the high and low demand scenarios. We consider a small spread between the scenarios for which we define the levels of the low and high scenarios as one times the standard deviation away from the average demand. Further, we consider a large spread for which we define the levels of the low and high scenarios as two times the standard deviation away from the average demand. For the probabilities that the low, average and high scenarios occur, i.e. p_1, p_2 and p_3, we consider 6 combinations. This means that, in total, the stochastic model is optimized 12 times for each of the two methods that use the model. The performances of the SNBL and SBP policies, along with the spread of the scenarios and the scenario probabilities that are used for each policy, are shown in Tables 6.2 and 6.3.

Spread	Scenario Probabilities $p_1/p_2/p_3$	Average Revenue ($)	Standard Deviation	Percentage Optimal
small	.8/.6/.4	678,664	11,625	93.3 %
small	.7/.6/.5	641,930	12,562	88.2 %
small	.7/.5/.3	689,421	9,831	94.8 %
small	.6/.5/.4	657,370	12,242	90.4 %
small	.6/.4/.2	681,736	7,216	93.7 %
small	.5/.4/.3	673,570	10,914	92.6 %
large	.8/.6/.4	634,776	12,416	87.3 %
large	.7/.6/.5	558,073	12,178	76.7 %
large	.7/.5/.3	658,488	11,586	90.5 %
large	.6/.5/.4	592,170	12,552	81.4 %
large	.6/.4/.2	674,511	9,081	92.7 %
large	.5/.4/.3	628,373	12,120	86.4 %

Table 6.2 Performances of the SNBL policy.

Spread	Scenario Probabilities $p_1/p_2/p_3$	Average Revenue ($)	Standard Deviation	Percentage Optimal
small	.8/.6/.4	681,812	9,615	93.7 %
small	.7/.6/.5	678,437	10,255	93.3 %
small	.7/.5/.3	673,603	7,695	92.6 %
small	.6/.5/.4	666,449	7,139	91.6 %
small	.6/.4/.2	633,551	6,391	87.1 %
small	.5/.4/.3	630,584	6,447	86.7 %
large	.8/.6/.4	683,845	9,896	94.0 %
large	.7/.6/.5	668,384	10,876	91.9 %
large	.7/.5/.3	672,793	7,670	92.5 %
large	.6/.5/.4	680,135	8,526	93.5 %

large	.6/.4/.2	623,585	6,493	85.7 %
large	.5/.4/.3	630,293	6,397	86.6 %

Table 6.3 Performances of the SBP policy.

The results in Table 6.2 show that for every combination of the scenario probabilities the SNBL policy produces better results when it is applied with a small spread then when it is applied with a larger spread for the scenarios. Of all the booking control policies that we consider in this research, the SNBL policy, when it is applied with a small spread of the scenarios and scenario probabilities given by .7/.5/.3, performs best. On average it yields a revenue which consists if 94.8% of the maximum revenue that can be obtained. The SBP policy comes nearest to the performance of the SNBL policy. Table 6.3 shows that when the right parameters are chosen, its average performance reaches up to 94.0% of the optimal revenue. It is important to notice that both booking control policies based on the stochastic model perform better that their deterministic counterparts for various combinations of the parameters of the stochastic model. Especially the bid price policy seems to benefit from the use of the stochastic model. This comes forth from the fact that the stochastic model pays more attention to modeling the probability that an extra booking request arrives for a type of stay. This results in better estimates for the opportunity costs of a room and better bid prices.

In Figure 6.2 we show the average number of booking requests accepted for each price class by the optimal, DNBL, DBP, SNBL and SBP policies. The parameters of the stochastic model, for which the results of the SNBL and SBP policies are presented, are the parameters for which the policies perform best. Figure 6.2 shows that there is little difference between the average number of booking requests accepted by the optimal, the deterministic and the stochastic policies in the high revenue price classes, i.e. classes 5, 7, 9 and 10. For the low revenue price classes, i.e. classes 1, 2, 3, 4, 6 and 8, the stochastic policies accept more booking requests than their deterministic counterparts. Still, the average number of booking requests accepted by the SNBL and SBP policies is generally less than the average number of booking requests accepted in the optimal policy. Only for price class 2 do the two stochastic policies accept more than the optimal policy.

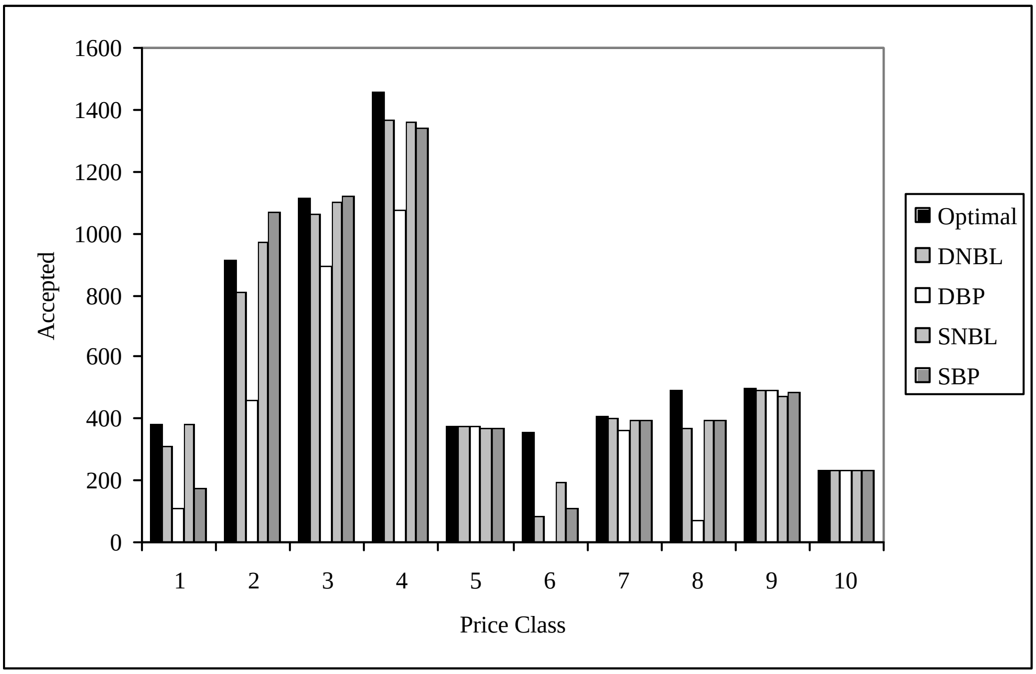

Figure 6.2 The average number of booking requests accepted for each price class by the optimal, DNBL, DBP, SNBL and SBP policies.

7. Conclusion

In this paper we studied four booking control policies for hotel revenue management and show how to apply them over a rolling horizon of decision periods. Next to the well known deterministic model, we also looked at a stochastic model to construct nested booking limits and bid prices. The performances of the different booking control policies are evaluated in a simulated environment. The results show that when the parameters of the stochastic model are chosen right, the booking control policies based on the stochastic model perform better than those based on the deterministic model. Especially the bid price policy benefits from the use of the stochastic model. Nevertheless, when the right parameters are set for the stochastic model, the nested booking limits policy based on this model performs better on average then any of the other policies studied in this research.

8. References

Baker, T.K. and Collier, D.A. (1999), A Comparative Revenue Analysis of Hotel Yield Management Heuristics, Decision Sciences, 30, 239-263.

Bitran, G.R. and Gilbert, S.M. (1996), Managing Hotel Reservations with Uncertain Arrivals, Operations Research, 44, 35-49.

Bitran, G.R. and Mondschein, S. (1995), An Application of Yield Management to the Hotel Industry Considering Multiple Day Stays, Operations Research, 43, 427-443.

De Boer, S.V., Freling, R., Piersma, N. (2002), Stochastic Programming for Multiple-Leg Network Revenue Management, European Journal of Operational Research, 137, 72-92.

Weatherford, L.R. (1995), Length of Stay Heuristics: Do They Really Make a Difference, Cornell Hotel and Restaurant Administration Quarterly, 70-79.

Williamson, E.L. (1992), Airline Network Seat Inventory Control: Methodologies and Revenue Impacts, Ph.D. Thesis, Flight Transportation Laboraty, Massachusetts Institute of Technology, Cambridge, MA.

Wollmer, R.D. (1986), A Hub-Spoke Seat Management Model, unpublished company report, Douglas Aircraft Company, McDonnell Douglas Corporation, Long Beach, CA.