ORIGINAL ARTICLE

# A novel ternary quantum-dot cell for solving majority voter gate problem

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**Abstract** Since the complementary metal-oxide semiconductor (CMOS) technology has experienced many serious problems in fulfilling the need for more robust and efficient circuits, some emerging nanotechnologies have been introduced as the candidates for replacing CMOS. Quantum-dot cellular automata (QCA) is one of the promising nanotechnology candidates with majority function as its fundamental logic element. It has one implementation in binary QCA and several implantations in ternary QCA, but none of the ternary QCA implementations are as efficient as the binary one. In this paper, a new cell configuration for ternary QCA is proposed which works as well as previous cell configuration. Also, a new design for ternary QCA majority function is proposed which performs faster and occupies less area.

## Introduction

Gordon Moore in 1965 predicted that the number of transistors on a chip will be doubled every 18 months. Scaling down of device size in microelectronics technology led to high-density circuits with obvious benefits such as faster

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performance and less power consumption. For two decades scaling down of CMOS technology has been sufficient for implementing very large scale integrated circuit (VLSI) systems with high-speed, high-density, small feature-size and low power (Taur et al. 1997; Frank et al. 2001). But after years, because of submicron effects in transistor's physical features, CMOS technology reached its scaling down limits, so other technologies needed to be explored to compensate for this limitation and to replace CMOS in some special cases. Nowadays, quantum cellular automata (QCA), single electron transistor (SET) and carbon nano tube (CNT) are being used as an alternative to CMOS.

This paper concentrates on quantum-dot cellular automata because of its special features which will be discussed. Various representation methods have been used in logic design, such as voltage level, electric-current and electrostatic charge. But logic states in QCA are described by configuration of the electrons in the quantum-dot cells. In fact, neither static nor dynamic current is available in QCA-based design. Because of this special logic representation, QCA devices dissipate low power. They consume about $10^{-10}$ W per input and it does not depend on circuit area and complexity (Lent et al. 1993a; Ma et al. 2008).

QCA is a promising nanotechnology that could be used for ultra-fast computing and processing with low power consumption and it would facilitate the fabrication of ultra-dense memory storage which could be used for constructing quantum computers, arithmetic logics and microprocessors (Vankamamidi et al. 2005; Azghadi et al. 2007; Tehrani and Navi 2009; Tehrani et al. 2011). One of the considerable properties of the QCA is the nanoscale integration which leads to improvement of functional density. There are several fabrication methods for QCA circuits such as semiconductor QCA, metal island QCA,

magnetic QCA and molecular QCA, in some of which integration of $10^{14}$ cells/cm$^2$ could be achieved (Snider et al. 1998; Imre 2005; Tougaw 1996; Toth and Lent 1999). QCA has many worthwhile points but it has some fabrication problems, such as quantum-dot patterning and ultra-low working temperature. However, many efforts have been made to develop this technology and to solve its problems.

The main concern of this paper is to present a new design for ternary QCA cell. A novel three-input ternary majority gate is also presented in which the output immediately shows the result.

In the next section an introduction to multiple-valued logic is presented. The explanation of QCA and tQCA, their functions and logical gates are discussed in chapter three. In chapter four, the implementation of majority function in QCA and tQCA is shown and the existing problem and solutions are provided. In section five, we propose our new solution for implementation of tQCA majority function.

## Multiple-valued logic

In the real world, there are more than two simple logic levels. Things could not simply be considered as true or false, black or white and one or zero. Therefore, for simplification, it is worth considering a multi-level model rather than a binary model.

In an assumed system with multi-value logic levels, more values could be saved in a single bit and therefore a larger amount of data can be stored in a multi-value memory compared to a binary one. Having more logic levels to represent facts, makes it possible to compact data in fewer bits. Hence, using more than two values will lead to more efficient communication protocols implemented with less cost. Most of the real problems are not in binary values, so multi-value logic could be a perfect solution to non-binary problems, because it provides faster numerical analysis and arithmetic operations and it can also solve each problem in its original radix (Hurst 1984; Navi and Etiemble 1995).

One of the possible approaches to multi-value logic is ternary QCA which uses ternary logic rather than the standard QCA which uses binary logic. This model has the ability of binary QCA and the advantages of multi-value logic at the same time.

## Binary QCA

Cellular automata (CA) are made of a set of cells which have mutual interaction in a uniform grid. In 1993, Lent

et al. (Lent et al. 1993a) implemented an automaton using quantum-dot cells called quantum-dot cellular automata (QCA) which consisted of elements of cellular automata and quantum information processing. Despite special charge representation and special logic approach of QCA, all logical and arithmetic designs can be implemented using QCA.

### Binary QCA cell

A schematic of the basic quantum-dot cell whose configuration consists of five dots is shown in Fig. 1a. Another reduced implementation of QCA has also been proposed with four quantum-dots. Notice that in the second design, the central dot is omitted. In a neutral environment all quantum-dots have same potential and tunneling between any of the neighboring quantum-dots is possible (Lent et al. 1993a, b).

There are two electrons in each cell that can tunnel freely between its dots. Because of the interaction between the electrons named coulomb repulsion, electrons are always positioned in antipodal sites. In order to have the system at ground state, each cell's electrons change their configuration according to the nearest neighbor's charges. This phenomenon causes the signal to propagate in the circuit from input to output and facilitate implementation of logical devices.

### Binary QCA logic devices

To start working with QCA, some basic elements could be implemented. Wire is the most primitive device of every circuit which is used for data transferring. Because of the coulomb repulsion between the neighbors' electrons, information propagates through the wire. Each cell's polarization is affected by the neighbors' polarization, so an array of cells transfer the information from the input to the output and make the wire function (Fig. 2a). Another model for the QCA wire is a rotated wire whose data ripple in cells and value of the output is dependent on the length of wire. Making a fan-out is as simple as shown in figure (2b) and there is no limitation on the number of fan-out wires. There are several kinds of QCA inverter and two examples are shown in Fig. 2c, d) (Tougaw and Lent 1994).
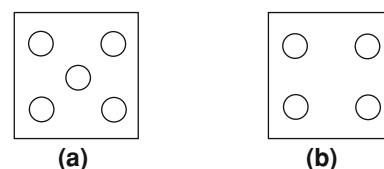


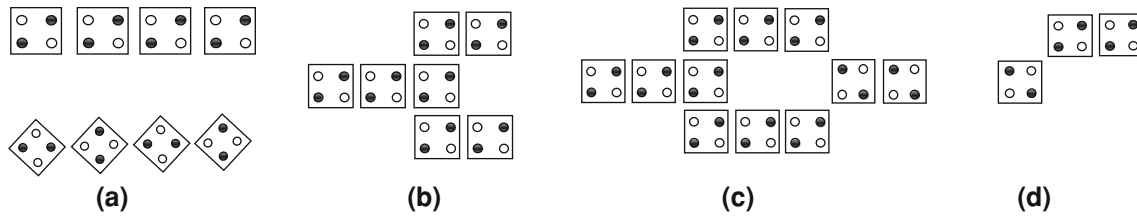Fig. 1 a QCA with five dots. b QCA with four dots

**Fig. 2** bQCA logic devices. **a** bQCA wire, 45° rotated wire; **b** fan-out; **c**, **d** inverter

Every QCA circuit and digital functions are implemented using majority gate and inverter. Designing digital devices with majority gates is more complicated than conventional AND/OR gates, but in some cases more simplified circuit can be obtained.

Majority gate is the fundamental logic gate in QCA. In majority with three inputs, output is true if two or three of its inputs are true and vice versa.

The function for majority logic is as below:

$$M (A, B, C) = AB + AC + BC$$

A majority gate can be used either as an AND or an OR gate. One of the inputs can be used as a selector to implement logical gates. By setting the selected input to 0 or 1, majority gate behaves like AND-logic function or OR-logic function (Figs. 3, 4).

### QCA clocking

QCA clocking is used for synchronizing the flow of information in the QCA circuits. Standard CMOS has two phases, high and low, but in QCA clock is multi-phase which is shown in Fig. 5. Using cyclic signal that has four phases (switch, hold, release, relaxed), interdot barriers are controlled by controlling the probability of tunneling of electrons. Controlled interdot barriers and cell's switching process results in adiabatic switching which guarantees the ground state at any time. In the switch phase, cell's state affected by the neighbor's state is gradually updated. In the hold phase with raised barriers, tunneling of electrons is impossible. In the hold phase the state is stabilized for transmitting to the cells that are in the switch state. Release
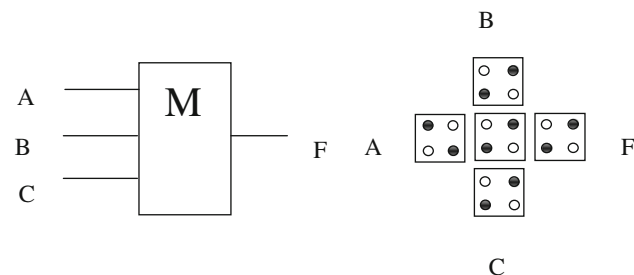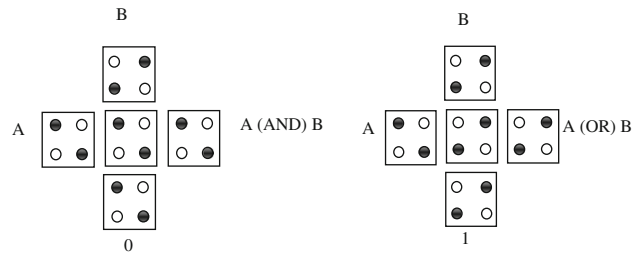


**Fig. 4** AND/OR gate with the use of majority gate

and relax phase is used for the preparation for the next switch (Orlov et al. 2000).

### Ternary QCA

For the first time, Lebar Bajec et al. (2006) extended the binary QCA that was proposed by Lent. The new model has four additional quantum-dots that makes this model suitable for implementing multi-value logic. Also a new terminology to distinguish between binary QCA (bQCA) and ternary QCA (tQCA) was proposed.

In tQCA dots are arranged in a circular model connected with low potential paths that allow the electrons to tunnel freely between them. Electrons configuration depends on the electrostatic repulsion as well as the interaction between them in the cell; therefore, they are placed as far apart as possible. There are four configurations for two electrons in eight quantum-dots which are shown in Fig. 6. State A is used for presenting logical value 1 and state B is
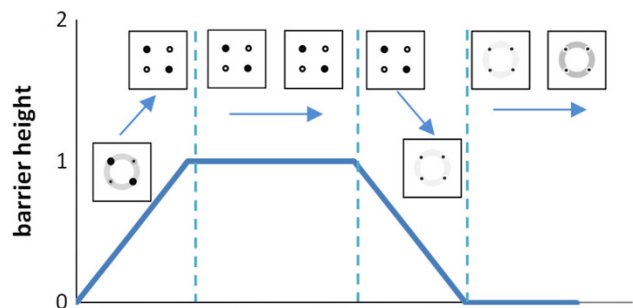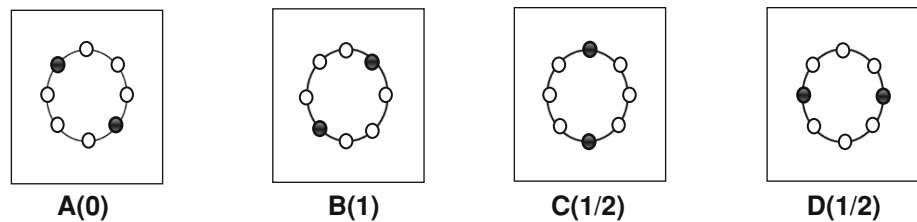


**Fig. 3** Binary QCA majority gate for (1-0-1) inputs



**Fig. 5** Four cyclic signal phases: switch, hold, release, relax (Kim et al. 2006)

A(0)  B(1)  C(1/2)  D(1/2)

for logical value 0. Both C and D configurations are known as ½ (Lebar Bajec et al. 2006).

## Ternary QCA cell

Quantum-dots in tQCA cell are placed in a circular arrangement. In tQCA there are two electrons in each cell and the internal tunnels between the dots are the same as bQCA. To minimize the system potential, the electrons must be as far apart as possible. With this limitation only four arrangements out of 28 possible states can be selected. The cells' centers are chosen to be separated by $r = 110$ nm, because the ratio between the maximum interdot distance in a cell and the inter-cell distance must be kept the same as in the bQCA cell (Lebar Bajec et al. 2006).

## Ternary QCA logic devices

Because the cell's basis is very similar to bQCA cell, implementation of all logical function that was provided in bQCA is also possible in tQCA. In Fig. 7, zero and one values propagate in tQCA wire as well as they do in bQCA, but ½ propagates rippling between C and D states then it is obvious that the bQCA's rotated wire does not have any equivalent in tQCA.

## Ternary QCA majority gates

A majority gate could not be built in tQCA with the same structure as in bQCA because there are two input states whose outputs are not correct. As is shown in Table 1,
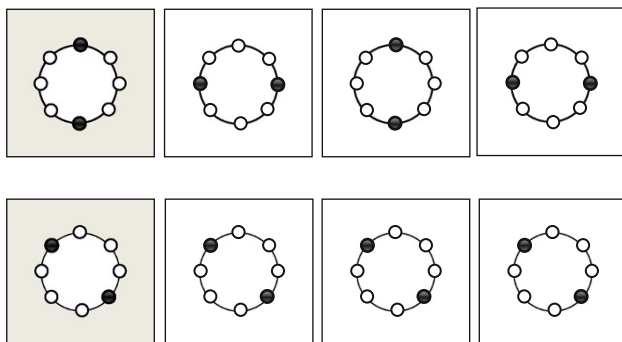


**Fig. 7** Wire in tQCA with the value of 1 and ½

M(0,1,0) is expected to be 0, but the output is ½ and the same mistake for M(1,0,1) which results in ½, whereas the correct answer is 1. With this majority-like gate, it is hard to design real circuits because the majority gate is the most important logic device in both bQCA and tQCA and it seems vital to solve the problem.

To resolve the problem, Bajec et al. introduced a design with some majority-like gates to make a real majority gate. The design is implemented in two layers as shown in Fig. 8; two majority-like gates make the partial products and the other one produces the final result (Lebar Bajec et al. 2006).

Using adiabatic pipelining is another method to overcome the problem offered by Pecar et al. (2008). In the bQCA a majority gate gives the output in just one clock phase but in the model the output is ready after three clock phases. As shown in Fig. 9, the input cells are in release phase when the internal cell (voter) is in the hold phase and the output is in switch phase so the inputs have minimum effect on the output. In this solution state, C and D should not be used as inputs simultaneously.

Afterward Janez et al. (2008) introduced another configuration whose output was produced in just two clocking phases. The layout of this configuration is shown in Fig. 10. Although this model is implemented with two clock phases, using less clock cycle is better to implement the AND/OR logic gate.

## Proposed tQCA model

There are two different cell configuration introduced for bQCA, one with four quantum-dots and the other with five. Both have four quantum-dots on the cell's corners and the latter one has a surplus quantum-dot at the center. In bQCA, all the facts of the logic design are the same in four-dot cell as well as the five-dot cell (Tougaw 1996). The mentioned tQCA cell configuration has many problems in implementing majority gate; however, another design of tQCA might overcome these issues.

### Proposed tQCA cell

For modeling the new tQCA cell shown in Fig. 11, we used a tight-binding Hubbard-type Hamiltonian. This method

**Table 1** AND/OR truth table in tQCA majority gate

| AND | | | | OR | | | |
|---|---|---|---|---|---|---|---|
| S | X | Y | M | S | X | Y | M |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1/2 | 0 | 1 | 0 | 1/2 | 1/2 |
| 0 | 0 | 1 | 0 | **1** | **0** | **1** | **1/2** |
| 0 | 1/2 | 0 | 0 | 1 | 1/2 | 0 | 1/2 |
| 0 | 1/2 | 1/2 | 1/2 | 1 | 1/2 | 1/2 | 1/2 |
| 0 | 1/2 | 1 | 1/2 | 1 | 1/2 | 1 | 1 |
| **0** | **1** | **0** | **1/2** | 1 | 1 | 0 | 1 |
| 0 | 1 | 1/2 | 1/2 | 1 | 1 | 1/2 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Bold values are unexpected values which should be corrected
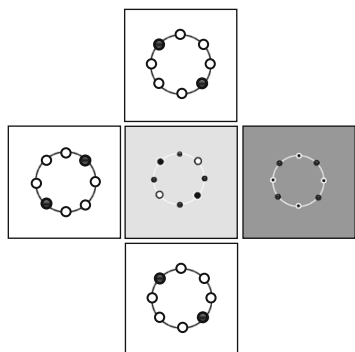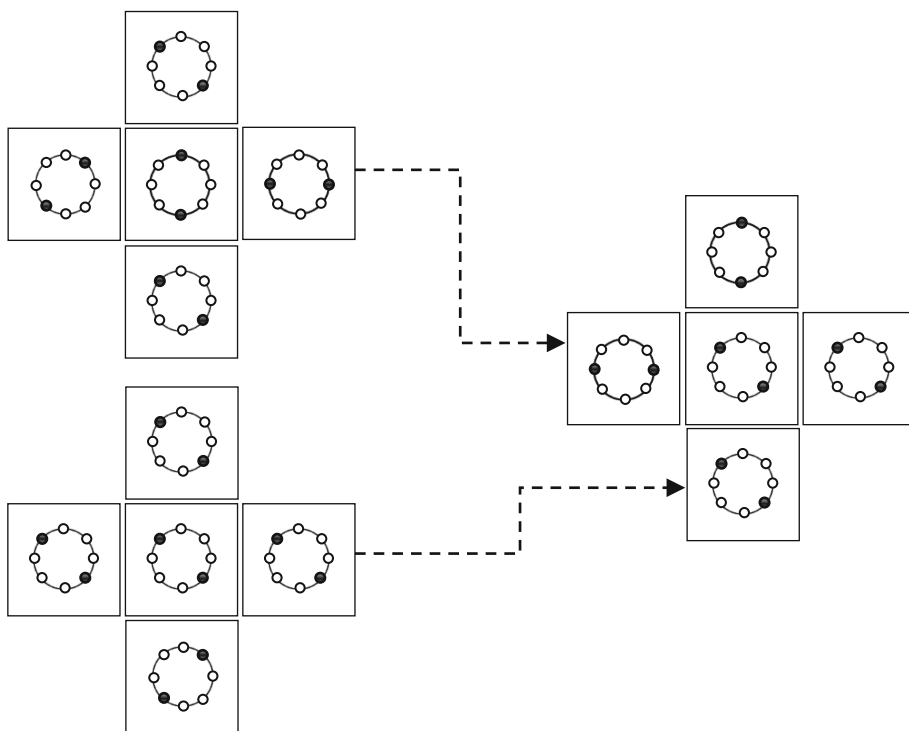
**Fig. 8** Cascaded majority gate
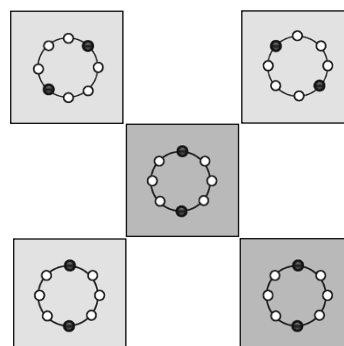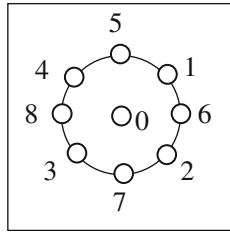




**Fig. 9** Majority gate with three clock phase



**Fig. 10** Two phase majority gate

was first used by Lent et al. (1993a) for bQCA and Lebar Bajec et al. (2006) for modeling tQCA. The corresponding Hamiltonian for an isolated cell $c$ could be written as Eq. 1 (Hamiltonian for an isolated cell):

$$\hat{H}^c = \sum_{i,\sigma} E_{0,i} \hat{n}_{i,\sigma}$$
$$+ \sum_{i>j>0,\sigma} t_{i,j} \left( \hat{a}_{i,\sigma}^\dagger \hat{a}_{j,\sigma} + \hat{a}_{i,\sigma} \hat{a}_{j,\sigma}^\dagger \right)$$
$$+ \sum_{i>j>0,\sigma} t'_{i,0} \left( \hat{a}_{i,\sigma}^\dagger \hat{a}_{0,\sigma} + \hat{a}_{i,\sigma} \hat{a}_{0,\sigma}^\dagger \right) \quad (1)$$
$$+ \sum_{i} E_Q \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$
$$+ \sum_{i>j,\sigma,\sigma'} V_Q \frac{\hat{n}_{i,\sigma} \hat{n}_{j,\sigma'}}{r_{i,j}}$$

The first term of equation is the on-site energy for the cell where; $E_{0,i}$ is the on-site energy for the $i$th site and $\hat{n}_{i,\sigma}$ represents the number operator for site $i (0 \le i \le 8)$ and spin $\sigma$. All the on-site energies are assumed to be equal and the degree of freedom internal to each dot is ignored. In the second line $\hat{a}_{i,\sigma}$ is the annihilation operator which destroys a particle at site $i$ of spin $\sigma$. $t_{i,j}$ is tunneling energy between dots $i$ and $j$ and $t'_{i,0}$ is tunneling energy between dot $i$ and the central dot. This line corresponds to the electron tunneling energy between sites. The third term computes the on-site charging energy needed to confine two electrons of opposite spin in one dot where $E_Q$ is the charging energy for a single dot. In the last term $r_{i,j}$ stands for the distance between sites $i$ and $j$ and $V_Q$ is the Coulomb coupling strength. For the GaAs/AlGaAs material system with uniform dielectric constant of 11.9, $V_Q$ is 120.9 meV (Lebar Bajec et al. 2006). So this term evaluates the mutual Coulomb interaction of electrons localized at different sites. Because in a reasonable approximation the distance of electrons in a circular quantum-dot can be assumed as one-third of the dot diameter $D$, then $E_Q = V_Q (D/3)^{-1}$ (Lent 1991).

The on-site energy $E_{0,i}$ has two factors, the overall energy constant of the environment $V_0$ which could be considered as zero because all the cells have the same design and the same number of electrons. $V_i^c$ the potential energy of electron at site $i$ in cell $c$. To maintain charge neutrality in the cell, a fixed positive charge $\tilde{\rho}$ is added to each dot whose magnitude is equal to (2/9) electron charge. So $V_i^c$ is represented as Eq. 2 (potential energy of electron at a site):

$$E_{0,i} = V_0 + V_i^c$$
$$V_i^c = \sum_{k \ne c,j} V_Q \frac{\rho_j^k - \tilde{\rho}}{r_{i,j}^{c,k}} \quad (2)$$

where $\rho_j^k$ stands for the electron density at site $j$ in cell $k$ and $r_{i,j}^{c,k}$ is the distance between site $i$ in cell $c$ and site $j$ in cell $k$. The physical parameters for the model correspond to a semiconductor implementation of QCA with GaAs/AlGaAs material.

According to Coulomb repulsion, the electrons situate diagonally in the cell minimizing the total energy. So there are four stationary states in which the electrons could be organized. The central dot does not contribute directly in stationary states and its main role is to emphasis the negative charges on the cell borders. So the states A and B could be written as Eq. 3 [A(0) and B(1) states]:

$$|A\rangle = \frac{1}{\sqrt{2}} \left( \hat{a}_{2,\uparrow}^\dagger \hat{a}_{4,\downarrow}^\dagger - \hat{a}_{2,\downarrow}^\dagger \hat{a}_{4,\uparrow}^\dagger \right) |0\rangle$$
$$|B\rangle = \frac{1}{\sqrt{2}} \left( \hat{a}_{1,\uparrow}^\dagger \hat{a}_{3,\downarrow}^\dagger - \hat{a}_{1,\downarrow}^\dagger \hat{a}_{3,\uparrow}^\dagger \right) |1\rangle \quad (3)$$

The parameter $P$ as shown in Eq. 4 (cell polarization) is the polarization of the cell whose value is between $-1$ and $+1$ where the logic 0 is represented by polarization $-1$ and logic 1 is shown by polarization $+1$. Here, polarization 0 represents the value ½ and stands for states C or D.

$$P = \frac{(\rho_1 + \rho_3) - (\rho_2 + \rho_4)}{\sum_i \rho_i} \quad (4)$$

The parameter $\rho_i$ stands for single particle density at site $i$. It should be noted that when all on-site energies are equal and the cell is isolated in the environment, $P$ is also equal to zero.
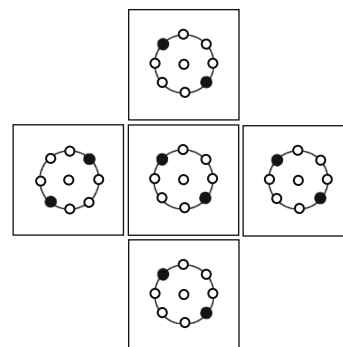


**Fig. 12** Proposed tQCA majority gate with nine-dot

**Table 2** AND/OR truth table in proposed tQCA majority gate

| AND | | | | OR | | | |
|---|---|---|---|---|---|---|---|
| S | X | Y | M | S | X | Y | M |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1/2 | 0 | 1 | 0 | 1/2 | 1/2 |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1/2 | 0 | 0 | 1 | 1/2 | 0 | 1/2 |
| 0 | 1/2 | 1/2 | 1/2 | 1 | 1/2 | 1/2 | 1/2 |
| 0 | 1/2 | 1 | 1/2 | 1 | 1/2 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1/2 | 1/2 | 1 | 1 | 1/2 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

**Table 3** Proposed model feature comparison

| Model | Area | Clock phase |
|---|---|---|
| Cascade model (Lebar Bajec et al. 2006) | 12[a] | 3 |
| Adiabatic (Pecar et al. 2008) | 4 | 3 |
| Diagonal cell (Janez et al. 2008) | 4 | 2 |
| Proposed model | 4 | 1 |

[a] In Lebar Bajec et al. (2006), the wiring area is ignored

## Proposed tQCA majority gate

Building a majority gate is the main problem for eight-dot tQCA cell. Adding a dot to the center of the cell changes the charge densities and corrects the faulty states of the majority gate constructed with eight-dot cell. With proposed cell (nine-dot cell) a majority voter gate could be constructed as in bQCA. Figure 12 illustrates the new tQCA model and the truth table is shown in Table 2. It should be noted that the model is similar to bQCA and no surplus cells or clock cycles are needed. Table 3 compares the proposed model features with the other models.

## Conclusion

By adding just one dot, a novel majority gate function for ternary QCA is proposed twice faster than the best one maintaining the same area.

The best state-of-the-art cell configuration ever reported in the literature has eight quantum-dots. The proposed cell configuration has eight dots on the corners as well, but there is one dot in the center of the cell. This surplus dot affects the charge distribution of the edge dots. All the ternary QCA basic elements function remains the same as in the case of eight-dot cell except majority gate's function which works faster and uses less area.

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