Essays on Internet and Information Economics

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Essays on Internet and Information Economics

Essays over internet en informatie economie

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born at Schiedam

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As can be read in the chapters to come, I have elaborated on the fact that different messages are best communicated in different ways. It therefore appears to me that I should attempt to find better ways than this preface for communicating my appreciation to all those that are closer to me, for the inexplicable encouragement and simply making these years pleasant years.

Contents

Chapter 1

Introduction

The last decade we have witnessed advances in information and communication technologies. These ICTs play a central role in the world economy (OECD 2004): the rise of the Internet has facilitated the dissemination of information, while at the same time technological progress enhances the acquisition of information, for example in health care. Nevertheless, information processing is still bounded by human nature, and the ease by which one economic agent can nowadays contact another may overload the latter. Other limitations are that not everything of value can be digitalized, exchange via Internet is indirect and disclosure of personal information may enable discrimination. The impact of these technologies on society is therefore equivocal. Indeed, OECD (2004) identifies the following as major policy issues of ICTs: lack of trust in the online environment, privacy concerns and combating spam.

This book consists of five essays that each discusses some of the above issues and analyzes the impact of an information or communication technology on the functioning of markets. The thesis consists of two parts. Part I discusses how and to what extent the Internet can be used for retailing. Access to the Internet by households has been rising almost everywhere, and ranged in 2001 between 40% and 60% of households in Northern Europe and North America (OECD 2002). The Internet offers new ways for supply and demand to meet and trade. A distinctive feature of retailing electronically is that transactions no longer require the physical coordination of buyers and sellers: market participants find each other at their screens. There are many aspects of market interaction which are affected by this online nature of trade (see also Borenstein and Saloner 2001). On the supply side, we may think of all kinds of cost reductions, resulting from new ways of organizing production and sales processes. On the demand side, the major impact of the Internet is on consumers' ability to acquire information about product characteristics and prices. Moreover, due to the global nature of the Internet, there is a potentially dramatic increase in market size: a firm selling online may reach customers worldwide, round the clock. Yet, there are also drawbacks associated to online trading. In this part of the book the trade-offs that are involved are investigated thoroughly in order to explain the importance of online retailing.

Part II concerns information acquisition. In many markets, information is not being held symetrically: the buyer for example does not now the characteristics of a good as well as the seller does. Since the seminal paper by Akerlof (1970) many authors have shown how taking such an informational structure of a market into account can lead to dramatic predictions, like the foreclosure of trade. My approach instead is to analyze how the uninformed party can improve upon his disadvantage, by employing some technology to uncover what is unknown. This information acquisition is commonplace; a buyer of a car tries the car, an employer interviews a job candidate and an insurer asks numerous questions to an insuree. Also organizations such as assessment centers and certification agencies specialize in this task and are widespread. The essays study this use of information in the insurance market and when receiving large numbers of applications for a prize or contract. The technology based solution is explicitly set aside from earlier studied solutions that are incentive based, such as signaling and screening.
Each essay is self-contained and discusses the relevant literature in detail. I now discuss both parts in more detail.

Part I: Retailing on the Internet

Borenstein and Saloner (2001) and Bakos (2001) sketch far-reaching transformative powers to online retailing and motivate the need for economic analysis. While electronic commerce is now indeed an established retail channel, its share of total sales does not exceed 2% in 2000, which contrasts with the high level of Internet use (OECD 2002). The expectation that the Internet would prevail in a 'New Economy', that was held by many observers a decade a go, has clearly not materialized. Yet Bakos (2001) quotes a projection for the year 2005 that 7.8% of sales will be online. On a more skeptical tone, Ellison and Ellison (2005) suggest that the online environment does not eliminate all frictions as predicted, thus not creating the rather utopian "frictionless commerce".

The question of the importance of Internet retailing is thus an open one. In this part of the thesis the frictions in commerce are analyzed in detail and additionally, we suggest that online trading may create new frictions, such as the inability to evaluate intangible product attributes and the indirect nature of payment and delivery. OECD (2002) and Hummerston (2001) report that these barriers are indeed empirically significant. We incorporate these barriers in theoretical analysis, thereby contributing to understanding the gap between Internet access and sales, and between the promising prospect of e-commerce and the reality of its low share.

Another intriguing discrepancy on the research agenda is between the prices set by different online retailers for one and the same product (Ellison and Ellison 2005 and Borenstein and Saloner 2001). In many e-commerce environments, a consumer checking prices is exposed to offerings by rival firms on the same webpage, and the price dispersion is therefore surprising at first sight. Some explanations have been given in an overview by Ellison and Ellison (2005). We contribute to understanding observed price differences by characterizing firms' online entry and pricing strategies. For firms, online retailing requires an investment and their presence online may be far from profitable when it only cannibalizes on their own customer base. Various industry structures emerge in our analysis and we attribute some of the anomalies in price dispersion to these structures and to market conditions.

This part of the thesis thus extensively models the incentives to use the Internet, for consumers and firms. The results provide insights about the organization of the industry, the market share of e-commerce and the prices that are charged.

In Chapter 2, a model is devised in which there are two firms, both selling offline as well as online. We consider the question to what extent consumers will substitute the conventional retail channel for the Internet. On the Internet, product attributes that can be communicated digitally can easily be checked and compared, such as prices. However, intangible attributes such as fit cannot be evaluated. The trade-off a consumer faces is thus between price information that can be found online, and product information that can be found offline. On the supply side, a firms' optimal pricing strategy depends on the number of prices a consumer compares. The pricing policy for a firm that sells mostly through his conventional shop is less competitive than one that sells mainly through Internet, where potential consumers are informed about the rivals' offer. This interaction is analyzed in a game and relates the share of Internet sales to the degree of confidence in the online environment. Surprisingly, prices are dispersed, and the consumer that is lured to the electronic shop by the expectation to find low prices often discovers that the price is not low enough for compensating the lack of confidence and does not purchase at all.

Chapter 3 contributes to the understanding of e-commerce by analyzing the stage where firms decide whether to sell online or not. Initially, there are two offline firms, and various entry decisions are analyzed, including one in which a third player enters with an online channel only. The latter possibility provides scope for entry deterrence. Firms are differentiated with respect to both physical location and their products, but on the Internet only the latter kind plays a role. When entering online, the incumbent firms have the advantage of bringing their product reputation with them. Additionally, in this model consumers may find the Internet either more or less convenient than conventional shopping. Equilibrium market structure and prices for either case are being analyzed, allowing us to explain for example the low viability of pure online players and why prices can be higher online than offline.

Part II: Using information acquisition technology

The second part of the book is related to the literature on asymmetric information. From a methodological viewpoint, this literature enriches the prevailing perfect competition model (Maki, 2001). The fact that the Nobel Prize 2001 was awarded to George Akerlof, Michael Spence and Joseph Stiglitz shows that information economics now fully belongs to the core of economic science. Most existing work has discussed the consequences of a given situation of asymmetric information for the market outcome. My essays rather analyze how the uninformed party can improve upon his disadvantage by employing some technology and I discern such an approach from earlier studied solutions that are incentive based and often costly to implement, such as signaling and screening. In an overview article on signaling and screening, Riley (2001) supports this approach:

"there is a strong incentive for the market to seek alternative means of information transmission [for signaling or screening devices]. It is likely that in environments where this is the case, there will be evidence of direct testing, early monitoring, etc. -all provided to greatly reduce, if not eliminate, asymmetric information." (p. 474)

and emphasizes "the need for further discussion of equilibrium in which screening/signaling costs are not perfectly negatively correlated with quality." (p. 475).

In Chapters 4 and 5, adverse selection in insurance is taken as the starting point. Insurers try to overcome their disadvantage by acquiring information about the applicants' health. Some practices, such as genetic screening, are highly controversial as such disclosure of personal information inflicts upon privacy, yet health-related biotechnology is developing at unprecedented speed (OECD 2005). In Chapter 4, I look at the situation where insurers screen applicants before signing a contract. With this knowledge, they can offer different terms to high and low risks. This constitutes a loss for the high risks and a gain for the low risks, since adverse selection is softened. To contribute to the debate on the desirability of such discrimination, I show when aggregate surplus increases and when this is not the case. Interestingly, this depends on the risk attitude of insurees, when for example risk aversion is low, the discrimination mechanism I study saves the market from being eradicated by adverse selection.

For many contracts, insurees are required to provide information to the insurer themselves. Insurance law provides for contestability, that is, after a claim has been filed the insurer may dispute a claim by showing that the insuree provided false information. This is the subject of Chapter 5: information use by the insurer after the contract has been signed. Smoking behaviour in the application for life insurance is the leading example. Again, there are counterforcing welfare effects: the highest risks are being discouraged to purchase non-smokers insurance, softening adverse selection, but the non-smokers face the risky dispute and the smokers pay a higher premium. I devise a model that allows me to compute the aggregate welfare effects and it turns out that the distinction between non-smokers contracts and smokers contracts (as we observe in the U.S., among other countries) generates less welfare than a one-contract economy.

Chapter 6 is the final essay, where in addition to information acquisition, I analyze the consequences of the ease with which individuals and firms can contact one another. The difficulties of processing on one hand and the ease of sending on the other, together create problems such as spam, that accounts for as much as 60% of all e-mail (OECD 2004). More generally, the challenge of allocating attention to the desired sources encompasses recognizing the ever-changing unwanted sender, while at the same time avoiding desired senders being blocked.

I present a model in which individuals compete for a prize by choosing to apply or not. Abilities are private information and in an attempt to select the best candidate, the committee compares applicants with an imperfect technology. Imperfection is incorporated by letting the accuracy of the ranking of candidates decrease in the number of applicants being processed. A committee therefore sometimes suffers from overload and an optimal strategy appears to be to randomly ignore some applicants. The choice of application cost, size of the prize and use of information technology are being studied. To this end, I model the incentives to apply extensively and it appears that the number of individuals that has access to the contest is an important variable. The emergence of online job search (as well as searches for grants etc.) suggests that the use of Internet increases this number substantially. However, since this competition is taken into account by each individual, I show in this chapter that the actual number of applying candidates might decrease in market size. The question being addressed is whether an increase in the market size (due to e.g. the Internet), urges a capacity-constrained committee to raise the utility cost of applying or invest in its ranking technology. One of the dangers turns out to be the possibility that no one applies. The optimal strategy that I characterize strikes a balance between this danger and overload, which is a challenge faced by any communication technology.

Finally, Chapter 7 summarizes the findings of the thesis and concludes.

Chapter 2

Electronic Commerce and Retail Channel Substitution

2.1 Introduction

The importance of the Internet as a marketplace has substantially grown the past few years, even though expectations have tempered after March 2000. A distinctive feature of doing business electronically is that transactions no longer require the physical coordination of buyers and sellers: market participants find each other at their screens. There are many aspects of market interaction which are affected by this online nature of trade. On the supply side, we may think of all kinds of cost reductions, resulting from new ways of organizing production and sales processes. On the demand side, the major impact of the Internet is on consumers' ability to acquire information about product characteristics and prices. Moreover, due to the global nature of the Internet, there is a potentially dramatic increase in market size: a firm selling online may reach customers worldwide, round the clock. Finally, as far as information goods are concerned, the distribution process can be made entirely electronically.

While electronic commerce has an enormous potential, it is good to realize that even for books and CDs it is still of minor importance compared to other retail channels.[1] It is

[0] This chapter is a version of an article co-authored by Maarten Janssen.

[1] According to a study by the Boston Consulting Group (2000) in the United States and Europe, e-commerce accounts for less than 5 percent of total sales in the books and CDs categories.

important, however, to consider the question to what extent consumers will substitute the Internet for the conventional retail channel. This paper provides a theoretical framework that sheds some light on the relative importance of different factors that determine the market share of electronic sales. In doing so, we also investigate the way in which the Internet may promote competition in markets.

The basic point of our paper is to account for the relative strengths and weaknesses of the online retail channel compared to conventional stores. On the Internet, it is easy to find firms and compare their prices, but the online nature of the transaction leads to uncertainties. For example, it is more difficult to assess how well a particular product fits the specific needs of the costumer. Other uncertainties and inconveniences are related to the payment method and the delivery.[2] We refer to the uncertainty associated with an online purchase as *'online uncertainty'*. In a conventional store, physical inspection of goods is much easier, but in order to compare firms' offers one has to make more than one costly visit. We will refer to this disadvantage of a visit in the offline channel as *'best-price uncertainty'*. It is therefore the purpose of this paper to contribute to the understanding of the functioning of electronic markets by analyzing the trade-off between (i) online uncertainty on the Internet and (ii) best-price uncertainty in the conventional store.

The view that some consumers may derive a lower value from an online purchase, as compared to a conventional purchase, can be expressed as follows:

> "Why do users decide not to shop?
>
> The reasons why more online users decide not to shop online despite the fact that they have considered it, than those who actually do, may be several: ... E-commerce web sites must also carry part of the responsibility, as many are not good at making the shopping experience easy and trustworthy for the online users. Lack of privacy statements, difficult navigation, poor product declarations, slow delivery of goods, limited price discounts, uncertainty

[2] A survey by Hummerston (2001) indicates that the most important reason for not purchasing online is related to payment security and the second most important reason is given by 'You don't know what you get.' and 'Easier/more fun to buy goods/services in a store.'

> around security, and unclear redemption policies are examples of issues faced by the users." Andersen (2000).

We present a duopoly model where firms sell in both the conventional and the electronic retail channel. Firms sell homogeneous products and the price at which a firm sells the product is the same in both retail channels. There are two types of consumers. *Experienced* consumers do not incur purchase uncertainties online. As a result they always search on the Internet for the lowest price of the product. *Inexperienced* consumers, in contrast, incur an online uncertainty. For example, they do not know which version of a product they like best, prior to physical inspection. They therefore face a trade-off between best-price uncertainty, if they purchase in the store, and online uncertainty, if they purchase online. We shall refer to the percentage of experienced consumers in the economy as *market maturity.* The pricing decisions by firms and the shopping decisions by buyers are modelled as a simultaneous move game.

By means of this model, we arrive at the following insights. First, searching consumers do not always buy. We find that inexperienced consumers who search the Internet may not purchase, if the online uncertainty is not compensated by a low price. As was pointed out above by the survey of Andersen (2000), this so-called *drop-out* seems relevant for electronic markets.

Second, the share of the market captured by the electronic channel crucially depends on the importance of the online uncertainty and the maturity of the market. The more *non-digital* product attributes matter, such as in the clothing industry, the more important is physical inspection, hence the online uncertainty is large in that case. As a result, more buyers frequent the conventional store and channel substitution is small. Market maturity has the following surprising impact: if some inexperienced consumers use the Internet, the total market share of this channel decreases in market maturity. The reason for this result is that when a market matures, the gain from comparing prices decreases.

Third, we show that the equilibrium exhibits price dispersion. The reason for this result is as follows. Firms have monopoly power over those consumers who visit the physical store and observe only one price. This gives firms incentives to charge high prices. However, firms also aim at attracting the consumers who browse their Internet homepages searching for the best price. These consumers give firms incentives to charge lower prices. The interplay of these two forces results in price dispersion: some firms aim at selling to a few consumers in the conventional store at higher prices, others aim at selling to more consumers at lower prices. In equilibrium, the two different strategies result in the same profit.

Fourth, the equilibrium price distribution is such that marginal cost pricing never occurs. Interestingly, the expected price increases in the number of price comparing consumers, if some inexperienced consumers also use the Internet. Finally, if no inexperienced consumer uses the Internet, the price distribution is not depending on online uncertainty, while if this channel does receive some inexperienced consumers, the expected price is increasing in online uncertainty.

Our theoretical framework is built around three crucial assumptions. Firstly, both firms are present on both retail channels. The relevance of these so-called multichannel retailers has widely been recognized. In Europe, multichannel retailers accounted for two-thirds of the online retail sales in 1999 (Boston Consulting Group 2000). A 'clicks-and-mortar' approach seems to be good business practice, as multichannel retailers have a brand advantage over their 'pure-play' counterparts. Internet retailers start recognizing the importance of a physical presence for customer satisfaction. The two other other important features of our model are that, for a particular product, a firm sets the same price on the Internet as in the physical store, and that consumers search only once. Section 5 discusses these assumptions extensively.

There is an emerging theoretical and empirical literature on the implications of the Internet for the competitiveness of markets. On the empirical side, a number of studies compare average prices and price dispersion in electronic marketplaces against those in conventional markets. Some studies find that *prices* in electronic markets are lower than corresponding prices in conventional markets (see, e.g., Brynjolfsson and Smith (1999)), while others report that prices in electronic markets are approximately equal to (see, e.g., Clay *et al.* (2000)) or even higher than prices in conventional stores (see, e.g., Lee (1997) and Bailey (1998)). Some of these studies also find that price dispersion is not lower on-line than in traditional outlets.

On the theoretical side, there are papers by Bakos (1997) and Janssen and Moraga (2000), among others, studying the implications of a reduction in search cost due to the emergence of Internet on the competitiveness of markets. Bakos presents a model of circular product differentiation where consumers search for prices and product features. In his model, consumers can get to know all product characteristics if they engage in costly search. In our model, in contrast, consumers cannot get around some of the uncertainties associated with buying on the Internet. Commenting on Bakos' paper, Harrington, Jr. (2001) questions the validity of some of Bakos' results.

Janssen and Moraga present a consumer search model showing that a reduction in consumer search cost does not unambiguously imply increased competition.

There are a few papers addressing the issue of channel substitution. Zettelmeyer (2000) focuses on the incentives firms have to increase the ease with which consumers can evaluate their products. Consumer search cost is in this way a strategic variable of firms. He shows that firms may provide selected groups of consumers with different information (level of search cost), thereby providing monopoly power to the firms and reducing market competition. The focus of our model is different. The source of monopoly power in our model lies in the fact that some consumers visit one conventional store only and, hence, do not compare prices. In other words, a major difference is that in our model consumers have to search to learn prices, whereas prices are known in Zettelmeyer's world.

Lal and Sarvary (1999) pose a different question: When is the Internet likely to *decrease* price competition? They study a model in which a consumer needs to evaluate the product, like clothing, physically to learn about its non-digital attributes. Firms are also present on two retail channels. Since digital attributes can be communicated over the web, the Internet changes the effective search cost structure: without the Internet, the cost of evaluating an unfamiliar brand of clothing is the cost of visiting an additional store. With the Internet however, consumers do not have to visit the store at all to buy their familiar brand, and thus, the cost of trying another brand is to undertake the entire shopping trip. In this way, the Internet may increase the effective cost of search and customer loyalty, which decreases price competition.

Finally, Mazón and Pereira (2000) analyse whether firms have incentives to open electronic retail channels and the different price equilibria these incentives generate. They show that, depending on variables as possible retail cost reductions, established firms may or may not compete with new virtual shops.

The paper is organized as follows. Section 2 presents the basic model. Section 3 analyzes the equilibrium properties. Section 4 discusses the model's predictions when parameters values change. A discussion about the robustness of the model and conclusions are contained in Section 5 and proofs are contained in an Appendix.

2.2 The Model

On the supply side there are two vertically integrated firms, each offering a homogeneous good in a conventional physical store and on the Internet. Firms produce the good at constant returns to scale at zero production cost.

On the demand side of the market, there is a mass of consumers, normalised to one. The consumer has a maximum willingness to pay for the good equal to v and purchases at most one unit. There are two types of consumers. The first type incurs an online uncertainty such that when she buys online, the value of her purchase is equal to λv, where $\lambda \leq 1$. We will refer to this type as the *inexperienced* consumer. The second type does not incur this uncertainty. We will call this second type the *experienced* consumer. The uncertainty can be seen as resulting from the inability to physically inspect a good online and hence, to select the most preferred version of a good. In this interpretation, the novel feature is that some consumers do not know what they want to have prior to inspection, that is, they are uncertain about their preferences, while some do know their preferences.[3]

There are $1-\mu$ inexperienced consumers and μ experienced consumers, with $0 \leq \mu \leq 1$. Both types have a restricted time endowment such that consumers can either visit the Internet, where they can observe all prices, or they can visit one store.[4]

[3] As mentioned above, another model that comes close to this feature is Zettelmeyer (2000). Above we discussed the major differences.

[4] Here we recognize that, although often suggested otherwise, searching on the Internet does take time. For simplicity, and without loss of generality, we assume that it takes the same amount of time to visit two sites electronically or one shop physically. Section 2.5 contains more discussion on this point.

In our model, firms and consumers play a simultaneous move game. An individual firm i sets its price p^i, taking price behavior of the rival, firm j, as well as consumers' search behavior as given. Consumers form expectations about their individual valuations and the prices in the market and decide which channel to search. Experienced consumers do not incur online uncertainty and they will therefore just look for the best price. Inexperienced consumers however face a trade-off and may either search the store or the Internet. Specifically, we will say that a fraction α searches the store and a fraction $(1-\alpha)$ searches the Internet, with $0 \leq \alpha \leq 1$. If this consumer searches the Internet, she either buys at the lowest price, or if this lowest price is too high, she exits without making a purchase. If she searches the conventional store, she observes only the price charged by the particular firm she visits.

The pay-offs to the players of this game can be expressed as follows. For the firms:

$$\Pi^i\left(p^i, p^j\right) = \begin{cases} \left[\frac{\alpha(1-\mu)}{2} + \mu + (1-\alpha)(1-\mu)\right] p^i & if \quad p^i < p^j \text{ and } p^i \leq \lambda v \\ \left[\frac{\alpha(1-\mu)}{2} + \mu\right] p^i & if \quad p^i < p^j \text{ and } \lambda v < p^i \leq v \\ \left[\frac{\alpha(1-\mu)}{2}\right] p^i & if \quad p^j < p^i \leq v \end{cases} . \tag{2.1}$$

This can be understood as follows. Firm i obtains a per consumer profit of p^i. In the store, the firm sells to the inexperienced consumers who (i) search the store, which happens with probability α and (ii) who find the store of i, which happens with probability one half. This results in a demand of $\alpha(1-\mu)/2$. On the Internet, the firm attracts all μ experienced consumers when it asks a lower price than the rival, since they observe all prices and buy from the cheapest firm. Additionally, the $(1-\alpha)(1-\mu)$ inexperienced consumers who search the Internet, also go for the lowest price, but purchase only if the lowest price does not exceed their maximum willingness to pay of λv.

The expected value a consumer derives is straightforward to determine. For the inexperienced consumer:

$$\begin{array}{ll} v - E\left(p^i\right) & \textit{if she searches the store of firm } i, \\ \Pr\left[\min\left[p^1, p^2\right] \leq \lambda v\right] \times & \\ \left\{\lambda v - E\left(\min\left[p^1, p^2\right] \mid \min\left[p^1, p^2\right] \leq \lambda v\right)\right\} & \textit{if she searches the Internet.} \end{array} \tag{2.2}$$

The surplus for the experienced consumer can be described as follows:

$$\begin{array}{ll} v - E\left(\min\left[p^i, p^j\right]\right) & \textit{if he searches the Internet,} \\ v - E\left(p^i\right) & \textit{if he searches the store of firm } i. \end{array}$$

Since we will illustrate below that the model exhibits price dispersion, we have $E(p) > E(\min[p])$. It is then easy to see that the experienced consumer will choose to visit the Internet, since he can obtain the same value v in both channels.

2.3 Equilibrium Analysis

In this section we will analyze possible equilibrium configurations and their properties. An *equilibrium* is a pricing strategy, one for each firm, and a set of decisions where to shop, one for each consumer, such that no one has an incentive to change his behavior. A first observation is that there is no pure strategy equilibrium. To see why, suppose firm i adopts the strategy to set p^{i*}, its rival will have an incentive to set its price just below p^{i*}, such that it can capture all experienced consumers. If we continue this reasoning we end up with Bertrand prices, $p^{i*} = p^{j*} = 0$. However, these prices do not constitute an equilibrium either: firm i will raise price so as to collect some revenues from the inexperienced consumers in its store.[5]

Thus, firms play mixed strategies and randomly pick a price from a set of prices, with each price generating the same expected pay-off. We will denote the mixed strategy of firms by $F(p)$: the probability that a firm charges a price smaller than p.

The following result shows that there are no equilibria in which all inexperienced consumers prefer the Internet, i.e. $\alpha = 0$. Therefore, full retail channel substitution does not occur. Additionally, we have shown above that experienced consumers never use the physical store. The lemma proves.

Lemma 2.1 *An equilibrium where (i) $\alpha = 0$ or (ii) experienced consumers use the physical store does not exist.*

[5] A similar argument establishes that there are no mass points in the mixed strategy of firms.

Proof. (i) Suppose $\alpha = 0$, then all consumers of both types would use the Internet and therefore, observe all prices. This would lead firms to charge Bertrand prices, i.e. both prices equal to marginal cost, normalized to zero $p^i = p^j = 0$. If this were so, inexperienced consumers would prefer the store, since there they can avoid online inconveniences, and hence, make a better purchase. Thus, $\alpha = 0$ cannot be part of an equilibrium.

(ii) Experienced consumers obtain valuation v in both channels, but pay a strictly lower price on the Internet than in the store, since $E(\min[p^i, p^j]) < E(p)$.[6] ■

We now proceed by investigating the pricing behavior of firms for the case that $\alpha > 0$. We will distinguish two situations: $\alpha = 1$, i.e., all inexperienced consumers visit the store and hence, no channel substitution occurs, and $0 < \alpha < 1$, i.e., inexperienced consumers randomize between visiting the store and visiting the Internet. This second case can be interpreted as a situation of partial channel substitution, with some inexperienced consumers visiting the store, while others visit the Internet.

2.3.1 No channel substitution

In this case all inexperienced consumers use the conventional channel, i.e. $\alpha = 1$, and all experienced consumers use the Internet. By substituting $\alpha = 1$ in (2.1) we can write expected profits as:

$$\Pi^i\left(p^i, F(p)\right) = \left[\frac{(1-\mu)}{2} + \mu\left(1 - F\left(p^i\right)\right)\right] p^i,$$

where $(1 - F(p^i))$ represents the probability that i is the cheapest firm.

Firms are indifferent between all prices in a certain support. It is easily seen that the upperbound of the price support equals v. Charging prices higher than v cannot be optimal as no consumer will buy. Also, by charging a price equal to the upperbound, a firm will only make profits from consumers who visit the shop. But, if the highest price a firm would ever charge is strictly smaller than v, then the firm could raise profits

[6]The combination of pure strategies in which firms charge the monopoly price v and both types of consumers use the store can be disregarded. As it is well known in the search literature, this equilibrium has the informed (here: experienced) consumer playing a weakly dominated strategy.

on these consumers by charging higher prices. Hence, the highest price a firm will ever charge equals v and the profit the firm makes in this case is $\frac{(1-\mu)}{2}v$. Since the firm must be indifferent between setting any price in the support of F, the following equality must hold:

$$\Pi^i(\cdot) = \left[\frac{(1-\mu)}{2} + \mu\left(1 - F\left(p^i\right)\right)\right] p^i = \frac{(1-\mu)}{2}v.$$

By rewriting this equality, we can find the mixed strategy price distribution[7]

$$F(p) = \frac{1+\mu}{2\mu} - \frac{1-\mu}{2\mu}\frac{v}{p}. \tag{2.3}$$

Now we want to investigate whether the proposed pricing behavior by firms and search behavior by consumers indeed constitutes an equilibrium. To this end, we now verify when the inexperienced consumer indeed will find it optimal to use the conventional channel. If she visits the physical shop, she will obtain her maximum willingness to pay, equal to v, but she is not able to make a price comparison between firms. In contrast, if she decides to browse the Internet, she will obtain only λv and buy at the best price if this price is low enough. Therefore, to let the physical store be the optimal channel for the inexperienced consumer, the following has to hold:

$$\begin{aligned} v - E(p) \geq\; & \Pr\left[\min\left[p^1, p^2\right] \leq \lambda v\right] \times \\ & \left\{\lambda v - E\left(\min\left[p^1, p^2\right] \mid \min\left[p^1, p^2\right] \leq \lambda v\right)\right\}, \end{aligned} \tag{2.4}$$

which easily follows from (2.2). Since v drops out of the above expression and the parameter μ enters via the expected price and minimum price, the conventional channel is indeed optimal for the inexperienced consumer for values of λ and μ satisfying inequality (2.4), see the Appendix for a detailed derivation. In that case there exist equilibria that exhibit no channel substitution, in the sense that all inexperienced consumers will shop at the conventional store.

When the online uncertainty is high, λ is low. Now the following Proposition states that for λ low enough, the trade-off between online uncertainty and best-price uncertainty

[7] There must be some $\underline{p}$ for which $F\left(\underline{p}\right) = 0$, from (2.3) we obtain that the lowerbound of the support is $\underline{p} = \frac{(1-\mu)v}{1+\mu}$.

is made in favor of the store. Put differently, for low λ, we have that physical aspects are more important than price comparisons, no matter what the value of μ is. Additionally, for low λ, the probability of a purchase on the Net is lower, since the best price is less likely to be low enough. Thus, for λ low enough (2.4) may be satisfied.

The result also states that (2.4) will be satisfied for μ low or high enough. This can be understood as follows. The difference between the prices the duopolists set, approaches zero for μ close to zero or close to one. Intuitively this makes sense, for when everybody is experienced ($\mu = 1$) and buys on the Internet, competition is strong and Bertrand prices result. Similarly, when everybody is inexperienced ($\mu = 0$), monopoly prices may result and there is no price difference to be expected either. If the price difference is small, inexperienced consumers have no reason to search the Internet. Thus for μ small or large enough, we may have an equilibrium where inexperienced consumers buy in the store. The proposition summarizes our findings.

Proposition 2.2 (no channel substitution) *An equilibrium of the game described above in which inexperienced consumers do not use the Internet and all experienced consumers do use the Internet exists when inequality (2.4) is met. Firms randomly select prices from the set* $\left[\frac{(1-\mu)v}{1+\mu}, v\right]$ *according to the cumulative distribution function* $F(p)$ *given by (2.3). Furthermore, there exists* $\overline{\lambda}$ *such that:*

(i) if $\lambda \leq \overline{\lambda}$ *the no channel substitution equilibrium exists for all* μ*;*

(ii) if $\lambda > \overline{\lambda}$ *there exist* $\underline{\mu}(\lambda)$ *and* $\overline{\mu}(\lambda)$ *with* $\underline{\mu}(\lambda) < \overline{\mu}(\lambda)$ *such that the no channel substitution equilibrium exists for all* $\mu < \underline{\mu}(\lambda)$ *and all* $\mu > \overline{\mu}(\lambda)$*. Moreover,* $\lim_{\lambda\to 1} \underline{\mu}(\lambda) = 0$ *and* $\lim_{\lambda\to 1} \overline{\mu}(\lambda) = 1$*.*

2.3.2 Partial channel substitution

When inequality (2.4) is not satisfied, inexperienced consumers do not prefer the store, implying some buyers substitute the online for the offline retail channel. In this case, $\alpha \in (0, 1)$ and the profits per firm are as in (2.1). Observe that if an inexperienced consumer chooses the Internet, her maximum willingness to pay is λv and this leads to a discontinuity of demand at λv. The derivation of firm's behavior must account for this discontinuity. Since we have shown that there is no equilibrium in pure strategies, we will investigate how a mixed strategy can be described. Firstly, observe that for a given $\widehat{\alpha}$, firms will never be indifferent between a price slightly above λv and a price of λv, since they prefer a price of λv in order to attract also the mass of $(1-\widehat{\alpha})(1-\mu) > 0$ inexperienced consumers. Therefore, in equilibrium, prices in a right neighbourhood of λv will never be charged. Hence, the price support can be of one of the following forms:

1. the lowerbound of the price support is larger than or equal to λv;
2. the upperbound of the price support is smaller than or equal to λv;
3. the support is nonconvex and contains a 'hole': prices not greater than λv are charged and prices strictly higher than λv are charged, leaving open an interval of prices that will not be set in equilibrium.

The first case implies that $\alpha = 1$ as illustrated in Section 2.3.1: since the probability that the best price is low enough is zero, no inexperienced consumer will search the Internet. The analysis is provided above. The second case cannot be an equilibrium since in this case, charging a price equal to v dominates charging the upperbound.

Therefore, for an equilibrium with $\alpha < 1$, the price support must be as described in the third case. In equilibrium, prices in a certain range are charged with zero probability. For convenience, we denote by $x \equiv \alpha(1-\mu)$ the share of all consumers going to the shop. Hence, x is the mass of consumers on which firms exercise market power. In the Appendix, we show that

Lemma 2.3 *In equilibrium, prices in the interval* $\left(\lambda v, \frac{(1-x)v}{1-x+\frac{\mu(1-\lambda)}{\lambda}}\right)$ *are set with zero probability.*

Let us denote the upperbound of this interval by $p' \equiv \frac{(1-x)v}{1-x+\frac{\mu(1-\lambda)}{\lambda}}$. Now we want to describe the cumulative distribution function for prices above p', denoted $F^u(p)$, and for prices below λv, denoted $F^l(p)$. For $p \geq p'$, the following must hold:

$$\left(\frac{x}{2} + \mu\left[1 - F^u(p)\right]\right) p = \frac{x}{2} v,$$

since, as was argued in Section 2.3.1, the maximum price a firm will charge is $p = v$ and at any price in the support of F, expected profits must equal profits at $p = v$. It follows that:

$$F^u(p) = \frac{2\mu + x}{2\mu} - \frac{x}{2\mu}\frac{v}{p}.$$

For prices $p \leq \lambda v$, we must have that

$$\left(\frac{x}{2} + [1 - x]\left[1 - F^l(p)\right]\right) p = \frac{x}{2} v$$

and we obtain:

$$F^l(p) = \frac{2 - x}{2 - 2x} - \frac{x}{2 - 2x}\frac{v}{p}.$$

Summarizing, we can describe the strategy of the firm as follows:[8]

$$F(p) = \begin{cases} \frac{2-x}{2-2x} - \frac{x}{2-2x}\frac{v}{p} & \text{if } \underline{p} \leq p \leq \lambda v \\ 1 - \frac{x(1-\lambda)}{2\lambda(1-x)} & \text{if } \lambda v \leq p \leq p' \\ \frac{2\mu+x}{2\mu} - \frac{x}{2\mu}\frac{v}{p} & \text{if } p' < p \leq v \end{cases} . \tag{2.5}$$

Note that when $\alpha = 1$, the 'hole' disappears and the case of no channel substitution analyzed above emerges.

To find an equilibrium, we must now turn to the demand side to describe how α is determined. Since $\alpha \in (0, 1)$ the consumer must be indifferent between the two retail channels. The following equality must therefore be satisfied:

$$\begin{aligned} v - E(p) &= \Pr\left[\min\left[p^1, p^2\right] \leq \lambda v\right] \times \\ &\quad \left\{\lambda v - E\left(\min\left[p^1, p^2\right] \mid \min\left[p^1, p^2\right] \leq \lambda v\right)\right\}. \end{aligned} \tag{2.6}$$

The above expression therefore implicitly determines α. The left-hand side of this expression represents the expected surplus of visiting the physical shop and the right-hand side represents the expected surplus of visiting the Internet. We let $\pi \equiv \Pr\left[\min\left[p^1, p^2\right] \leq \lambda v\right]$ denote the probability of purchase for an online inexperienced consumer. [9]

[8] To find the lowerbound of the support, we solve $\frac{2-x}{2-2x} - \frac{x}{2-2x}\frac{v}{p} = 0$, resulting in $\underline{p} = \frac{xv}{2-x}$.

[9] Consumers might be tempted to continue searching and visit the store after the search on the Internet. We discuss this possibility in Section 3.5.

To obtain the expression above explicitly, we derive the expected price and the conditional expected minimum price. In the Appendix the reader can check that

Lemma 2.4 *The expected price is given by*

$$E(p) = \frac{xv}{2(1-x)} \ln\left[\frac{\lambda(2-x)}{x}\right] + \frac{xv}{2\mu} \ln\left[1 + \frac{(1-\lambda)\mu}{\lambda(1-x)}\right],$$

and the conditional expected minimum price is given by[10]

$$E\left(\min\left[p^i, p^j\right] \mid \min\left[p^i, p^j\right] \le \lambda v\right) = \frac{1}{\pi} \frac{x^2 v}{2(1-x)^2} \left[\frac{-1}{\lambda} + \frac{2-x}{x} - \ln\left[\frac{(2-x)\lambda}{x}\right]\right],$$
$$\textit{for } 0 \le x \le \overline{x} \equiv \frac{2\lambda}{\lambda+1}.$$

Now that we have determined the expected price and the relevant minimum price, we consider again the equation that determines α. Rewriting the equilibrium condition given by (2.6), we define $\Gamma(x; \lambda, \mu)$ by

$$\Gamma(x; \lambda, \mu) = \pi\left\{\lambda v - E\left(\min\left[p^1, p^2\right] \mid \min\left[p^1, p^2\right] \le \lambda v\right)\right\} + E(p) - v.$$

The value of $\Gamma(x; \lambda, \mu)$ represents the surplus that can be obtained by visiting the Internet, in excess of the surplus in the store. Therefore, $\Gamma(x; \lambda, \mu) > 0$ implies that consumers prefer the Internet and $\Gamma(x; \lambda, \mu) < 0$ implies that they prefer the physical store. By employing Lemma 2.4 we can write this function explicitly as:

$$\Gamma(x; \lambda, \mu) = \frac{x}{(1-x)^2}\left(\frac{x}{4}\left(4 - \lambda + \frac{1}{\lambda}\right) - 1\right) + \frac{x}{2(1-x)^2} \ln\frac{\lambda(2-x)}{x} +$$
$$+ \frac{x}{2\mu} \ln\left[1 + \frac{(1-\lambda)\mu}{\lambda(1-x)}\right] - 1 + \lambda = 0, \ \textit{for } 0 \le x \le \overline{x}.$$

For any given λ and μ, this equality determines the equilibrium value of $\alpha \in (0,1)$ if it exists. In the Appendix we prove the following useful properties:

[10] Note that the conditional expected minimum price is defined up to a certain upperbound on x. This restriction on the domain of x arises because for high enough x, the lowerbound of the price support exceeds λv. Below we will discuss this in a more intuitive way.

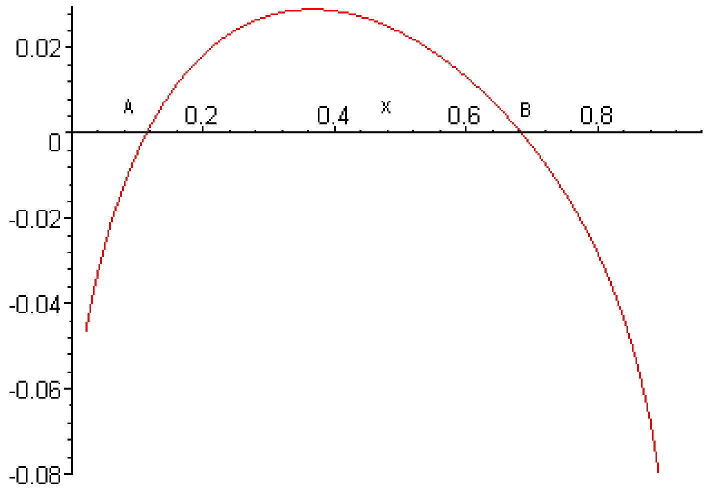


Figure 2-1: The equilibrium condition $\Gamma(x; .93, .5)$ for $0 < x < \overline{x}$. $\Gamma(\cdot) > 0$ implies consumers have an incentive to search the Internet. As $x \equiv \alpha(1-\mu)$, the function is in this case defined only for $0 \leq x \leq 0.5$.

Fact 1 $\Gamma(0; \lambda, \mu) < 0$ for all $\lambda < 1$ and all μ; $\Gamma(0; 1, \mu) = 0$ for all μ.

Fact 2 $\Gamma(\overline{x}; \lambda, \mu) < 0$ for all $\mu > 0$ and all $\lambda < 1$; $\Gamma(\overline{x}; \lambda, 0) = 0$ for all λ.

Fact 3 $\frac{\partial\Gamma(x;\lambda,\mu)}{\partial x}|_{x=0} > 0$.

Fact 4 There exist at most two values of $x \in [0, \overline{x}]$ such that $\Gamma(x; \lambda, \mu) = 0$.

Fact 5 $\frac{\partial^2\Gamma(x;\lambda,\mu)}{\partial x^2}|_{x=0} < 0$.

By using these facts, we may plot $\Gamma(x; \lambda, \mu)$ as a function of x as in Figure (2-1).[11] The first three facts and the shape of this curve can be understood as follows. Recall that $\Gamma(x; \lambda, \mu) > 0$ implies that consumers prefer the Internet and $\Gamma(x; \lambda, \mu) < 0$ implies that they prefer the physical store. An alternative interpretation is that the curve shows how beneficial it is to compare prices. Furthermore, x represents the segment of the market over which firms exercise market power. Now when $x = 0$, firms only sell to price comparing buyers and are induced to price competitively. There is no benefit to comparing prices and inexperienced consumers prefer to avoid online inconveniences and

[11] In the Figure we have $\frac{\partial^2\Gamma(\cdot)}{\partial x^2} < 0$ but this is not in general the case.

go to the physical shop. This is reflected in Fact 1. As x increases, we see that $\Gamma(x; \lambda, \mu)$ first increases. This is because it becomes more beneficial to compare prices, when firms gain market power and price dispersion emerges. Hence, it becomes more beneficial to search the Internet. Fact 3 states this formally. For x sufficiently large, we see that $\Gamma(x; \lambda, \mu)$ starts to decrease. This can be explained by the fact that when firms have a lot of market power, they price closer to the monopoly price, and then the price dispersion decreases in x. Additionally, as firms price closer to the monopoly price, it becomes less likely that the best price is low enough. Hence, comparing prices on the Internet becomes less rational. Finally, as x exceeds $\overline{x}$, π becomes zero. The surplus of buying on the Internet becomes zero as the consumer expects that even the lowest of the two prices she will observe will be too high. This is given by Fact 2.

For an equilibrium in which both retail channels receive some inexperienced consumers, it must be that $\Gamma(x; \lambda, \mu) = 0$. Since $x \in (0, 1-\mu]$ we may infer that for small μ, the curve is first increasing and then decreasing. Hence, there may be two intersection points of the line with the horizontal axis. It may be questioned whether both these points reflect stable equilibria.

To address the issue of stability, recall that $\Gamma(x; \lambda, \mu)$ represents the surplus that can be obtained in excess of the surplus attainable in the store. Consider now the two points of intersection A and B in Figure (2-1). To the right of point B, consumers expect to derive more value from searching the store than from searching the Internet. They will therefore prefer the store so that α (and x) has a tendency to rise. Between points A and B, the consumer derives more value from searching the Internet than from searching the store. This is because the expected price difference is high. Consumers will prefer to search the store less intensely, i.e., α (and x) decreases. A similar argument establishes that to the left of point A, α has a tendency to increase. Point A is therefore a stable equilibrium and point B an unstable one. We have thus esthablished that a stable equilibrium with $\alpha < 1$ must be such that the curve $\Gamma(x; \lambda, \mu)$ is increasing, as in point A. The following proposition summarises the case of partial channel substitution.

Proposition 2.5 (partial channel substitution) *A stable equilibrium of the game with* $\alpha < 1$ *has a share* α *of inexperienced consumers searching the conventional store and a* *share* $1-\alpha$ *searching the Internet, where* $\alpha \in (0,1)$ *is the solution to* $\Gamma(x;\lambda,\mu) = 0$. *Inexperienced consumers searching the Internet buy whenever* $\min[p^i, p^j] \leq \lambda v$, *that is, with probability* π. *Firms randomly select prices from the set* $\left[\frac{xv}{2-x}, v\right]$ *according to the cumulative distribution function given in (2.5).*

2.3.3 Characterization of Equilibria

After having introduced the following three facts, we are ready to provide a complete characterization of stable equilibria in our model:

Fact 6 $\frac{\partial\Gamma(x;\lambda,\mu)}{\partial\lambda} > 0$.

Fact 7 $\Gamma(x;1/2,\mu) < 0$.

Fact 8 For any $0 < \mu < 1, \Gamma(x;1,\mu) > 0$ for $0 < x < 1$ and $\Gamma(1;1,\mu) = \Gamma(0;1,\mu) = 0$

These facts tell us that as λ increases, the incentives to search the Internet increase. This is easily understood if one considers the fact that λ describes the confidence in the online purchase environment. Furthermore, when $\lambda \leq 1/2$ there is no incentive to visit the Internet, since the importance of a tangible purchase environment outweighs the importance of price comparison in that case, regardless of μ or x. When $\lambda = 1$ there is no incentive to visit the store since the advantage of physical interaction is absent in that case.

The following statement is proved in the Appendix:

Theorem 2.6 *There exists* $\bar{\mu}$ *such that*

(A) for all $\mu > \bar{\mu}$, *there exist a* $\widetilde{\lambda}(\mu)$ *such that the unique equilibrium is characterized by* $\alpha = 1$ *for all* $\lambda \leq \widetilde{\lambda}(\mu)$ *and the unique equilibrium is characterized by* $\alpha \in (0,1)$ *for all* $\lambda > \widetilde{\lambda}(\mu)$.

(B) for all $\mu \leq \bar{\mu}$, *there exist* $\widetilde{\lambda}(\mu)$ *and* $\widetilde{\widetilde{\lambda}}(\mu)$, *with* $\widetilde{\lambda}(\mu) \leq \widetilde{\widetilde{\lambda}}(\mu)$, *such that*

(i) the unique equilibrium is characterized by $\alpha = 1$ *for* $\lambda \leq \widetilde{\lambda}(\mu)$,

(ii) for $\widetilde{\lambda}(\mu) < \lambda < \widetilde{\widetilde{\lambda}}(\mu)$, *there are two stable equilibria, one in which* $\alpha = 1$ *and one in which* $\alpha \in (0,1)$ *and*

(iii) for $\lambda > \widetilde{\widetilde{\lambda}}(\mu)$, *the unique stable equilibrium is such that* $\alpha \in (0,1)$.

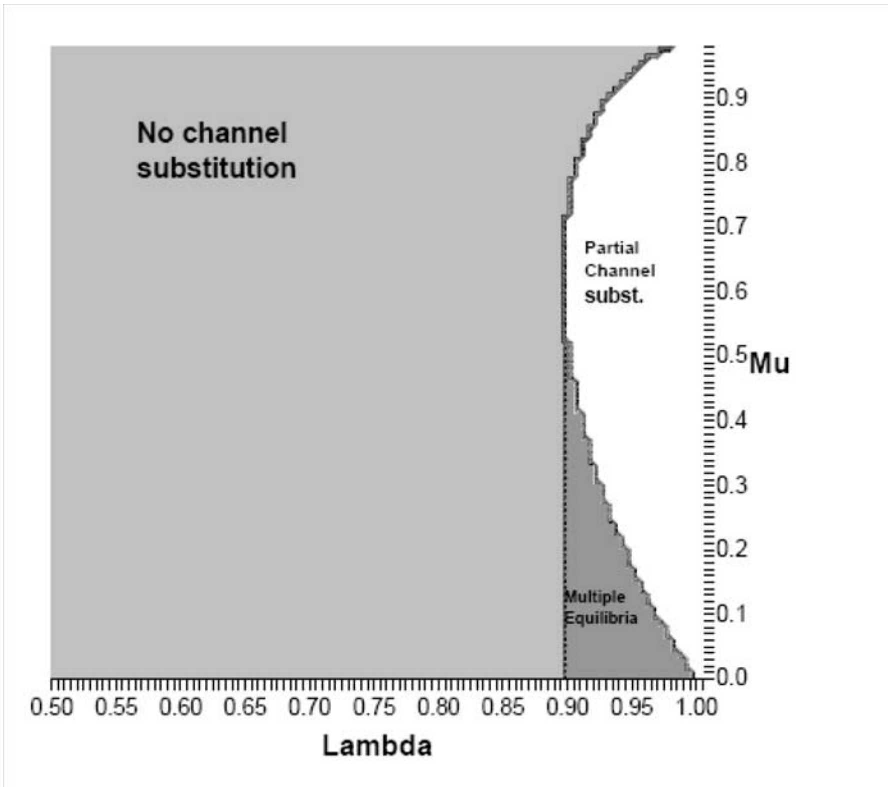


Figure 2-2: Characterization of equilibria.

The Theorem is illustrated in Figure 2-2 for $\lambda \geq 1/2$, which shows simulation results obtained by using Microsoft Excel. Basically, there are three regions of parameters: one in which no channel substitution occurs, one in which partial channel substitution occurs and one region of multiple equilibria in which either partial or no substitution occurs. We know by Fact 7 that for all $\lambda < 1/2$ the unique equilibrium is such that channel substitution does not occur.

The two possibilities A and B introduced in the Theorem are illustrated in Figures (2-3) and (2-4), respectively. Figure (2-3) exhibits a mature market where $\mu = .93 > \bar{\mu}$. In this case, the segment of the market on which firms exercise market power, $x \equiv \alpha(1-\mu)$, is necessarily small. Therefore $\Gamma(x; \lambda, \mu)$ is only increasing; x cannot become large enough to allow monopolistic pricing to diminish price dispersion in the market. In this case the equilibrium is unique; depending on λ either no channel substitution occurs or partial channel substitution occurs. In the picture, an equilibrium with partial channel substitution is found where the curve intersects the horizontal axis.

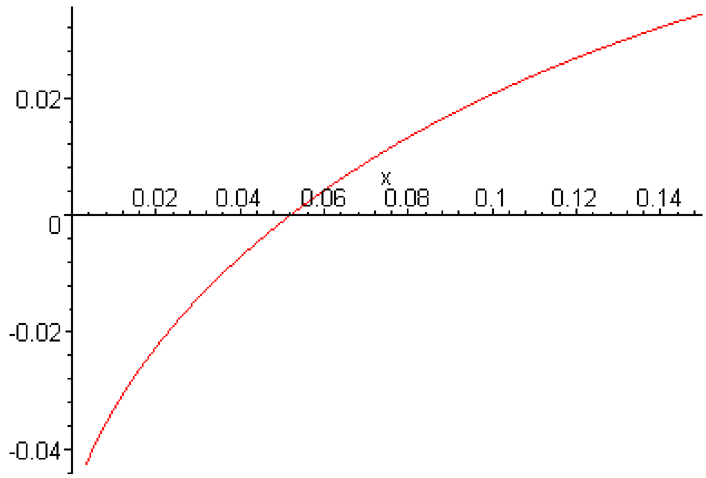


Figure 2-3: $\Gamma(x; 0.9, .93)$ for $x \in [0, 1-\mu]$. Determination of equilibrium for a mature market.

On the other hand, for an immature market ($\mu = 0.3$), x may become large and then the shape is as in Figure (2-4). Depending on the degree of online uncertainty λ, there may be one or two equilibria: if λ is low enough, $\Gamma(x; \lambda, \mu) < 0$ for all α and then the only equilibrium is $\alpha = 1$, hence no channel substitution occurs. For higher λ, we obtain one equilibrium with $\alpha \in (0, 1)$ where $\Gamma(x; \lambda, \mu) = 0$ and one with $\alpha = 1$ where $\Gamma(1 - \mu; \lambda, \mu) < 0$. Finally, for λ high enough, the unique equilibrium has $\alpha \in (0, 1)$.

The properties of the equilibria will be discussed in detail below. At this point we already would like to mention the following observations to which the Theorem points. Firstly, for all degrees of market maturity, if the online inconvenience is sufficiently high (low λ), and hence physical aspects are important, inexperienced consumers will not substitute the online for the offline channel. Secondly, for relatively immature markets, another reason for channel substitution not to occur is the following. In an immature market, the decisions of inexperienced consumers are important for pricing behavior. If all consumers coordinate on using the physical store, this shopping behavior induces monopolistic pricing, and hence, consumers infer that they cannot make a purchase on the electronic retail channel (since $\pi = 0$) and an equilibrium of no channel substitution may prevail.

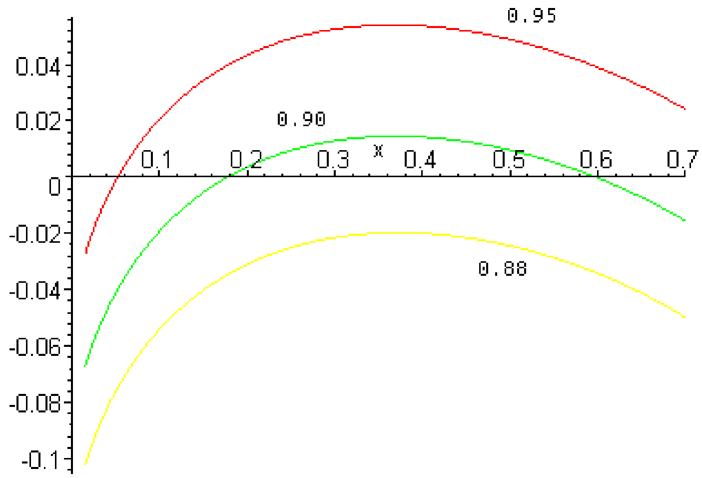


Figure 2-4: $\Gamma(x;\lambda,0.3)$ in $x\in[0,1-\mu]$ for different levels of λ. Illustration of equilibrium for an immature market.

2.4 Comparisons across Markets

We will now discuss our analysis of the issues raised in the Introduction. Specifically, we will analyze the market sizes of the two channels, the total market size and the expected price resulting from market interaction. We will argue that if we specify online inconvenience as the inability to inspect a product, the parameter λ describes a product market.

2.4.1 Retail Channel substitution

In this subsection we address the question how market circumstances affect the extent to which consumers will substitute the online for the offline retail channel. In doing so, we gain insight in the functioning of electronic markets. Furthermore, we will explain empirical differences in online penetration across markets.

To determine the relative importance of the electronic retail channel we have to look at the decisions by the $(1-\mu)$ inexperienced consumers, since the experienced consumers are already conquered by the electronic channel. A share α of them frequents the store, therefore the relative size of the conventional and the electronic channel is x and $(1-x)$, respectively. As we have shown in Section 2.3, the variable x is determined by two factors.

The valuation parameter λ, reflecting the confidence in an online purchase environment, and the maturity of the market, μ. We will discuss the impact of both determinants in turn, in each case distinguishing between partial and no channel substitution.

Online uncertainty: λ.

The greater the uncertainty or inconvenience online, the more consumers are tempted to use the conventional store. Indeed, we have already demonstrated in Theorem 2.6 that all inexperienced consumers will use the store for λ sufficiently low, that is, $\lambda \leq \widetilde{\lambda}(\mu)$. For these levels of λ there occurs no channel substitution and then the market share of the online channel is simply given by $1 - x \equiv \mu$. For $\lambda > \widetilde{\lambda}(\mu)$ partial channel substitution may prevail, with a fraction of inexperienced consumers using the online channel. We expect that as λ increases, this fraction increases. To investigate this hypothesis, we can use Fact 6 introduced above and the fact that in a stable equilibrium, $\frac{\partial \Gamma(\cdot)}{\partial x} > 0$ combined with the Implicit Function Theorem to obtain the result $\frac{\partial x}{\partial \lambda} < 0$ if $\alpha < 1$.[12] Thus if channel substitution is partial and λ decreases, α increases. This result supports our hypothesis: as λ decreases less buyers frequent the online retail channel.

If we interpret the online inconvenience as resulting from the inability to physically inspect a product before purchase, we can relate the value of λ to the degree of *non-digital* product variety: the higher the non-digital variety, the more important it is to inspect non-digital product attributes, hence, the lower λ.

In this framework, we can infer from our analysis that product markets which are characterised by a high degree of non-digital product variety, are likely to have relatively small electronic retail channels. Vice versa, we expect more intensive use of the electronic channel for products that show little non-digital variety. We illustrate this result in Figure 2-5, where the sales in the conventional channel are depicted.

Our characterization is supported empirically. The electronic channel makes up a small share of retailing in product categories as clothing, food, flowers, cards, home/gardening supplies and gifts (0.1% or less in the US and Europe), while online penetration is biggest for categories such as computer hardware/software and books (in the US respectively

[12] The Implicit Function Theorem implies in the case under consideration that $\frac{dx}{d\lambda} = -\frac{\partial \Gamma(\cdot)/\partial \lambda}{\partial \Gamma(\cdot)/\partial x} < 0$.

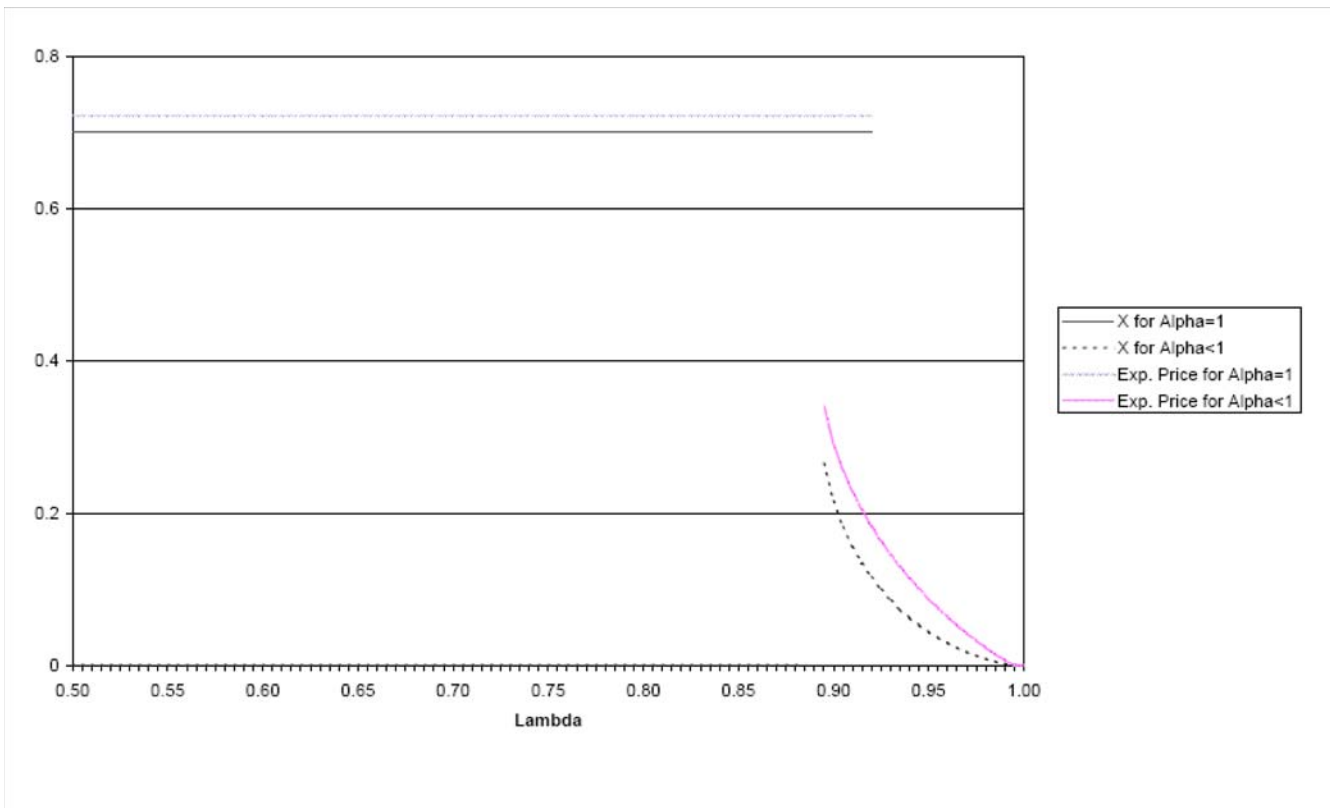


Figure 2-5: Sales in conventional channel (x) and expected price, μ=0.3.

9.2% and 5.1%, in Europe these figures are 3.5% and 1.6% resp.).[13] We may note that these figures show an enormous gap in the extent of online penetration for the different product categories. These observations are in line with our predictions, specifically, one of our results is that there occurs a substantial jump in market shares, when consumers switch to partial channel substitution, see Figure 2-5. Furthermore, since in the United States electronic commerce is more widely developed, it is convincing to observe that even there the online penetration for the highly differentiated categories is very low compared to the less differentiated categories.

The maturity of the market: μ.

The maturity of the market influences the expected price difference and therefore the best-price uncertainty for offline shoppers. We demonstrate in the Appendix that

Fact 9 $\frac{\partial x}{\partial \mu} > 0$ if $\alpha < 1$.

Clearly, there are two effects of μ at work. Recall that $x \equiv \alpha (1 - \mu)$ so that the direct effect of an increase in μ is to decrease x. However, there is also an indirect effect through the impact on α. This indirect effect is positive as an increase in μ increases α. This can be understood by taking into account that the more experienced consumers there are in the market, more price comparisons are being made, and the less interested inexperienced consumers are in making price comparisons themselves. Surprisingly, the latter indirect effect dominates the former! Therefore, when a market matures, inexperienced consumers infer that the increased magnitude of experienced consumers reduces the price dispersion in the market. Hence, they frequent the Internet less often. This effect dominates the fact that there are less inexperienced consumers in total and x increases. This process continues untill α becomes one, when only the first effect remains. In Figure 2-6 we illustrate this effect.

The figure shows that even though we can show that the impact of μ on x is positive when $\alpha < 1$, the impact is quite small. The figure also shows that when μ is small, there exist two equilibria. The important lesson here is that the presence of relatively many experienced consumers does not imply that expected prices are relatively low.

[13] The observations concern the year 1999, Boston Consulting Group 2000.

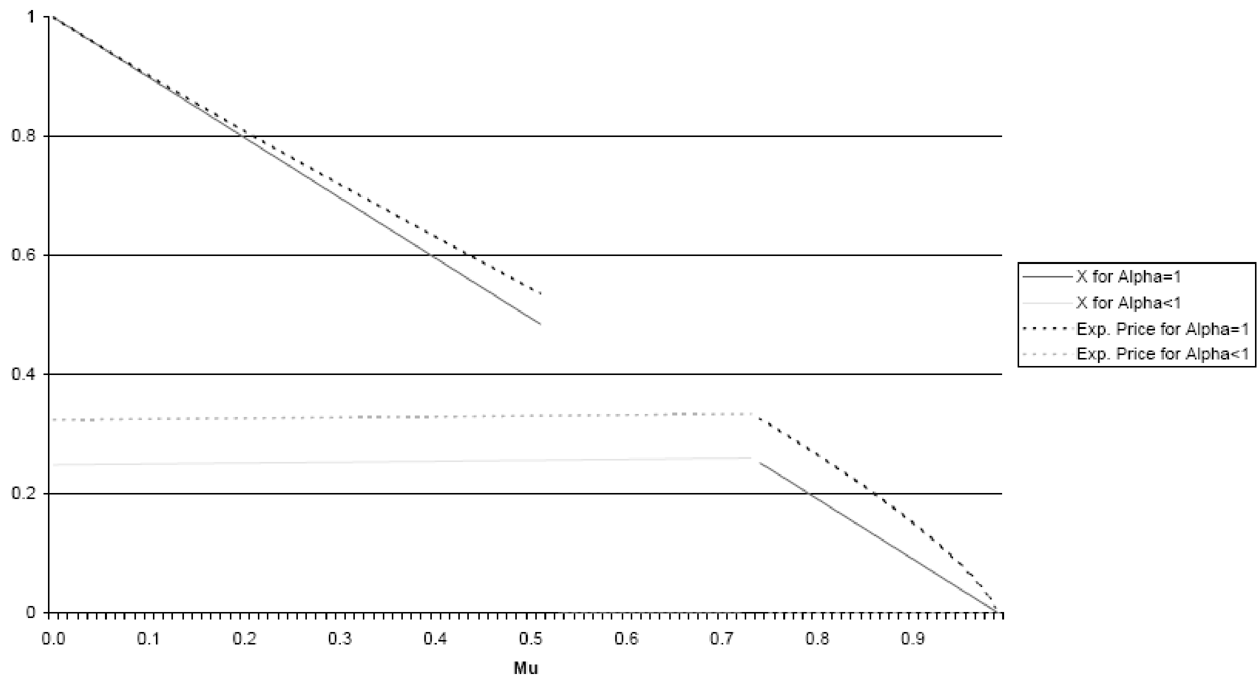


Figure 2-6: Sales in conventional channel (x) and expected price, λ=0.9.

2.4.2 Size of the market

In the Introduction it was pointed out that some consumers who consider to buy online actually do not make a purchase. This phenomenon of *drop-out* must therefore be included in a realistic model of electronic commerce. Above we illustrated that the online consumers do not necessarily make a purchase. In Section 2.3.2 we showed that the $(1-\alpha)(1-\mu)$ inexperienced consumers who search the Internet only make the purchase if $\min\left[p^i, p^j\right] \leq \lambda v$. Therefore, the total number of transactions in the market is given by

$$S(x;\lambda,\mu) \equiv 1-(1-\pi)(1-\alpha)(1-\mu).$$

Below we will investigate the role the two market characteristics of our model play in determining market size S.

Online uncertainty: λ.

We first focus on the role of the parameter λ. A few observations can be made. First, when λ is small enough, all inexperienced consumers visit the store ($\alpha = 1$) and there is no drop-out, i.e. $S(\cdot) = 1$. Second, when $\lambda = 1$ everybody visits the Internet ($\alpha = 0$), so that in equilibrium prices drop to zero and there is, again, no drop-out. In between, when $0 < \alpha < 1$, there will always be some drop-out as firms also randomize over prices larger than λv. Hence, there is a non-monotonic relation between S and λ. The reason for this non-monotonicity can be explained by looking at the expression for $S(x; \lambda, \mu)$ and realizing that there are two forces at work. When λ increases, the number of inexperienced consumers visiting the Internet increases, increasing the number of consumers that are prone to the drop-out phenomenon. However, when more consumers visit the Internet, the more competitive the firms' pricing behavior, thereby decreasing the chance that individual consumers drop out.

Our model thus predicts that the size of the market depends on the importance of physical interaction: for λ large, physical interaction plays an insignificant role in the consumer purchase. As a consequence, many consumers use the Internet, leading to competitive prices. In turn, competitive pricing ensures that almost all inexperienced consumers indeed make the purchase, so that the entire demand side is active in the market. As λ decreases, some consumers will prefer to search the conventional store in order to avoid online inconveniences. These consumers do not compare prices and therefore induce firms to price more monopolistically. In this way the consumers in the store impose a negative externality on the consumers searching the Internet: consumers comparing prices on the Net will more often drop-out. Because of this reason, the size of the market starts to decline. As λ becomes small enough, all inexperienced consumers prefer to search the store, and therefore all consumers make the purchase: the marketsize returns to its higher, initial level.

The impact of Market Maturity μ.

It is easily checked that $\frac{\partial S(x;\lambda,\mu)}{\partial \mu} = (1-\pi)\frac{\partial x}{\partial \mu} + (1-\alpha)(1-\mu)\frac{\partial \pi}{\partial \mu} + (1-\pi)$. When $\alpha = 1$ we have that $\frac{\partial x}{\partial \mu} = -1$ and hence, $\frac{\partial S(\cdot)}{\partial \mu} = 0$. For the case of $\alpha < 1$ we will consider each term. Firstly, experienced consumers always buy, and hence, there are more transactions in the market if there are more consumers of this type. This is reflected by the third term. Secondly, the second term shows us that the inexperienced online shoppers will more often make a purchase, since $\frac{\partial \pi}{\partial \mu} > 0$, and thirdly, the first term tells us that more inexperienced consumers will use the store, that is, the channel where they will make the purchase for sure. Therefore, the three terms are positive and the number of transactions increases in market maturity when channel substitution is partial.

2.4.3 Prices

We finally investigate the impact of market circumstances on price setting behavior. Since our model exhibits price dispersion, we will look at the expected price. In Lemma 2.4 we have shown that the expected price equals:

$$E(p) = \frac{xv}{2(1-x)} \ln\left[\frac{\lambda(2-x)}{x}\right] + \frac{xv}{2\mu} \ln\left[1 + \frac{(1-\lambda)\mu}{\lambda(1-x)}\right].$$

As before, we will investigate the impact of market determinants λ and μ.

Online uncertainty: λ

In the appendix, we establish the following property:

Fact 10 $\frac{dE(p)}{d\lambda} < 0$ if $\alpha < 1$ and $\frac{dE(p)}{d\lambda} = 0$ if $\alpha = 1$

Fact 10 tells us that the expected price is nondecreasing in the importance of tangible interaction. This result can be understood by considering that if $\alpha < 1$, $\frac{dE(p)}{d\lambda} = \frac{\partial E(p)}{\partial x}\frac{\partial x}{\partial \lambda} + \frac{\partial E(p)}{\partial \lambda} < 0$. The first term tells us that as λ increases, less consumers use the store and therefore, firms price more competitively, hence, this term is negative. The second term, $\frac{\partial E(p)}{\partial \lambda}$, is also negative, illustrating that an increase in λ causes π to increase, thereby making it more beneficial to have the best price in the market as it attracts more purchases. See also Figure 2-5 and Figure 2-7.

Market maturity μ

For an equilibrium of no channel substitution, we can easily infer from (2.3) that $\frac{\partial F(p)}{\partial \mu} > 0$, hence the expected price decreases in market maturity in such an equilibrium. If channel substitution is partial, we obtain that $\frac{dE(p)}{d\mu} = \frac{\partial E(p)}{\partial x}\frac{\partial x}{\partial \mu} + \frac{\partial E(p)}{\partial \mu}$ and from Fact 9

Expected price[1]					
μ	$\lambda = 0.80$	$\lambda = 0.85$	$\lambda = 0.90$	$\lambda = 0.95$	$\lambda = 0.99$
0.1	0.903018	0.903018	0.903018 0.325928	0.903018 0.099393	0.007025112
0.2	0.810930	0.810930	0.810930 0.327219	0.099400	0.007025113
0.3	0.722212	0.722212	0.722212 0.328531	0.099406	0.007025113
0.4	0.635473	0.635473	0.635473 0.329865	0.099413	0.007025114
0.5	0.549306	0.549306	0.549306 0.331223	0.099419	0.007025115
0.6	0.462098	0.462098	0.332608	0.099426	0.007025115
0.7	0.371700	0.371700	0.334021	0.099432	0.007025116
0.8	0.274653	0.274653	0.274653	0.099438	0.007025117
0.9	0.163580	0.163580	0.163580	0.099445	0.007025117
0.99	0.026734	0.026734	0.026734	0.026734	0.007025118

[1] For v=1. The white entries refer to no channel substitution and the grey entries refer to partial channel substitution.

Figure 2-7: Expected price for different values of λ and μ.

and the proof of Fact 10 we know that the first, indirect, effect is positive. We can also show that the direct effect is negative. However, by simulation we can obtain that the direct effect is offset by the positive indirect one (see Figure 2-7). This overall effect is small, but positive, which is a quite remarkable result: an increase in the proportion of price comparing shoppers leads to an increase in expected price! The intuition is the same as for Fact 9: an increase in μ causes more inexperienced consumers to use the store, such that the total number of consumers in the store, x, rises. For higher x firms are induced to set higher prices.

2.5 Discussion and Conclusion

In this paper we have set up a model of electronic commerce, describing how consumers choose between offline and online retail channels. Our analysis shows how the degree of retail channel substitution is determined, by considering the consumer's trade off between price comparisons and a tangible, immediate transaction.

We took the need to physically compare different versions of a product as the leading example of an online inconvenience. In this interpretation, we demonstrated that for a particular product market, the sales in each channel depend on the importance of physical inspection and on the number of consumers already familiar with the product. Our main results are as follows. Firstly, we expect online penetration to be low for the product categories that are characterised by the presence of non-digital product variety. More specifically, for product variety high enough, the consumers that need to learn product attributes do not use the online channel at all and for lower levels of product variety, the share of online sales is decreasing in product variety. Secondly, we found that online consumers do not always buy because of the reason that their uncertainty about the product match is not always compensated by a low enough observed price. Thirdly, we investigated the pricing behavior of firms and we found that firms price above the competitive level because they have market power over the consumers in their store. Additionally, the tension between having monopoly power in the store and competition on the Internet yields price dispersion in the market. The expected price in the market is increasing in the importance of the online uncertainty if some inexperienced use the online channel and constant if no inexperienced consumer uses the online channel.

As was mentioned in the Introduction, other reasons than physical inspection may give rise to the perception of online uncertainty. Examples are the payment method or the slow delivery of goods. In these cases, the parameter λ could describe the infrastructural and institutional state of affairs of Internet transactions or cultural determined attitudes towards electronic commerce. In this wider interpretation our results can be described as follows: If the perceived inconvenience is large enough, electronic commerce will only be used by the users in the economy who do not incur it. If some inexperienced consumers do shop online, the market share of the Internet is decreasing in the inconvenience.

All the above results are in line with observations that have been made on Internet purchases. We also found two quite unexpected results, namely that the market share of the electronic channel is decreasing and the expected price is increasing in the proportion of price comparing experienced consumers in the market. Finally, the fact that the expected price may rise in the online uncertainty is somewhat surprising at first sight: one would expect that firms would try to compensate the consumer for uncertainties, by lower prices. However, a firm rather prefers the consumer to feel uncertain indeed, and to shop where she is not exposed to the rival's offers.

In the remainder of this section we discuss the importance of two main features of our model: firms set the same price on both retail channels and consumers search only once.

Concerning the pricing policy, there are different ways to defend this assumption. Firstly, one may think that a firm will be punished by consumers if they find out that a firm discriminates between the two channels.[14] Secondly, one could also argue that the presence of administrative *menu cost* of having two prices prohibits firms to price discriminate between retail channels. Finally, suppose that firms choose two different prices, one for each channel. Then the equilibrium is such that firms charge $p = 0$ on the Internet, $p = (1 - \lambda) v$ in the store and all inexperienced consumers would search the store. For a set of values of λ and μ, firms obtain lower expected profits, however, than in the case we studied. One may therefore expect to find a Prisoner's Dilemma type of problem: both firms are better off if both pursue equal pricing than if both differentiate, but they individually have an incentive to differentiate. The reason for this is that equal pricing limits price competition on the electronic channel, which is in the interest of both firms. In a repeated setting, there exist then 'cooperative' type of equilibria where firms concentrate on the strategies we have considered in the main body of the paper.

The framework that we have studied has the consumer searching only once. However, it seems likely that consumers are tempted to use both channels for their purchase. They might for instance first select the firm with the best price by browsing the Internet, and subsequently visit the store of that firm to inspect the product line and make the purchase. Similarly, a consumer that has evaluated products in a particular store might decide to buy the preferred product version online, at the best price. In the remainder we discuss how our model could be extended to allow for more than one search, again taking product match uncertainty as an example of an online inconvenience. We will argue that our qualitative results will hold in such an extension.

Multiple search can be accounted for by allowing consumers to search sequentially, introducing different search costs c_s and c_I for the store and the Internet, where $0 < c_I \leq c_s$. Consumers first decide which channel to search. After having observed what there is

[14] One could oppose to this argument by saying that consumers understand that the two channels differ in their transaction costs, both on the demand and supply side, and would therefore accept a price differential. However, we do not consider transaction costs on the demand side and we think consumers do not have that much consideration towards the efficiency of the firm.

to be observed (prices (p^1, p^2) of different stores on the Internet, all product characteristics and one price in the store), consumers may decide to enter a second stage by engaging in an additional search. Therefore, if search costs are not prohibitive, the inexperienced consumer may find it optimal to eliminate both best-price uncertainty as well as product match uncertainty, by performing more than one search. If this were the case, however, all consumers would make price comparisons and Bertrand prices would result. Clearly, when $(1-\lambda)v > (c_s - c_I)$, this cannot be an equilibrium, as argued in Lemma 2.1: since the expected price difference is zero, consumers prefer to visit the store and buy the product that best matches their preferences (even it may be somewhat more costly in terms of search time) so that they will have no reason to search the Internet.

To give a sketch of possible equilibrium configurations, consider the following four strategies of the inexperienced consumer: (i) "go to the Internet and buy from the cheapest site", (ii) "go to one of the stores and buy the product that you like best", (iii) "go to the Internet first and afterwards go to the store that has the lowest price and buy the product you like best in the store" and, finally, (iv) "go to one of the stores and buy the product that you like best if the price is below a certain upperbound, otherwise continue searching on the Net and buy from the lowest priced site". If the consumer can make a purchase on the Internet, then strategy (i) and (iii) have a clear dominance relation to each other, namely, if $c_s > (1-\lambda)v$, then strategy (i) dominates strategy (iii). If the reverse inequality holds, then strategy (iii) dominates strategy (i). This easily follows from the fact that the cost and expected benefits of an additional search in the conventional channel are c_s and $(1-\lambda)v$, respectively. On the other hand, a version of strategy (iv), namely "go to one of the stores and buy the product that you like best if the price is smaller than $E(p) + c_I$, otherwise continue searching on the Net and buy from the lowest priced site" clearly dominates strategy (ii) and other strategies in class (iv). This is easily seen from the fact that the expected price the other firm charges equals $E(p)$ and one first has to incur the search cost c_I.

Given these observations on the optimal search strategy of the consumers, it follows that inexperienced consumers will either randomize over strategy (i) and the '$E(p) + c_I$' version of (iv), namely if $c_s > (1-\lambda)v$, or randomize over this last strategy and strategy (iii). It is also clear that some inexperienced consumers will not choose strategy (i) or

(iii) for a reason similar to the one argued in Lemma 2.1. Moreover, in equilibrium the upper bound of the price distribution equals $\min\left[E\left(p\right)+c_{I},v\right]$ as a firm who charges the maximal price will only attract costumers who visit the conventional store and if he would charge a price above $E\left(p\right)+c_{I}$ he will also loose these costumers. Hence, given this reaction by the firm to the threat to continue searching on the Net, in equilibrium no consumer will first visit the store to inquire which product fits his tastes best and then go and look for the lowest price on the Internet. So, even if we allow for sequential search, firms have some market power (bounded above by the search cost) and some inexperienced consumers will not use the sequential search possibility, but simply buy at the conventional store. Most qualitative results of the paper will therefore most likely continue to hold, although the analysis will become more complicated.

2.6 Appendix

Proof of Facts

The equilibrium condition for $\alpha\in(0,1)$ and $0\leq x\leq\overline{x}$ is given by:

$$\begin{aligned}\Gamma\left(x;\lambda,\mu\right)=\frac{x}{\left(1-x\right)^{2}}\left(\frac{x}{4}\left(4-\lambda+\frac{1}{\lambda}\right)-1\right)+\frac{x}{2\left(1-x\right)^{2}}\ln\frac{\lambda\left(2-x\right)}{x}+\\+\frac{x}{2\mu}\ln\left[1+\frac{\left(1-\lambda\right)\mu}{\lambda\left(1-x\right)}\right]-1+\lambda=0.\end{aligned}$$

Fact 1: $\Gamma\left(0;\lambda,\mu\right)<0$ for all $\lambda<1$ and all μ; $\Gamma\left(0;1,\mu\right)=0$ for all μ.

Proof of Fact 1: It is easy to see that $\Gamma\left(0;\lambda,\mu\right)=-1+\lambda$, the result then follows.

Fact 2: $\Gamma\left(\overline{x};\lambda,\mu\right)<0$ for all $\mu>0$ and all $\lambda<1$; $\Gamma\left(\overline{x};\lambda,0\right)=0$ for all λ.

Proof of Fact 2: It is easy to see that

$\Gamma\left(\overline{x};\lambda,\mu\right)=-1+\frac{\lambda}{\mu(\lambda+1)}\ln\left[1+\frac{\mu(1+\lambda)}{\lambda}\right]$. Now substituting $z=\frac{\mu(\lambda+1)}{\lambda}$, we have to show that

$$-1+\frac{\ln\left(1+z\right)}{z}\leq0.$$

For the case $\mu>0$ we have that $z>0$. As $\frac{\ln(1+z)}{z}<1$ for all $z>0$, $\Gamma\left(\overline{x};\lambda,\mu\right)<0$ for $\mu>0$. For the case $\mu=0$ it is enough to observe that $\lim_{z\rightarrow0}\frac{\ln(1+z)}{z}=1$.

Fact 3: $\frac{\partial\Gamma(x;\lambda,\mu)}{\partial x}\mid_{x=0}> 0$.

Proof of Fact 3: This is easily verified by first substituting $\mu = 1$ and taking the derivative:

$$\begin{aligned}\frac{\partial\Gamma(x;\lambda,1)}{\partial x} = & \frac{x}{2\lambda(1-x)^3}\left(2\lambda+2-(1-\lambda)^2\right) - \frac{1+x}{(1-x)^3} + \\ & + \frac{1+x}{2(1-x)^3}\ln\frac{\lambda(2-x)}{x} - \frac{1}{(2-x)(1-x)^2} + \\ & + \frac{1}{2}\ln\left[1+\frac{(1-\lambda)}{\lambda(1-x)}\right] + \frac{x}{2}\left[\frac{(1-\lambda)}{\lambda(1-x)^2+(1-\lambda)(1-x)}\right].\end{aligned}$$

Evaluating this expression at $x = 0$ gives $+\infty$. If we then take into account that $\frac{\Gamma^2(x;\lambda,\mu)}{\partial x\partial\mu} < 0$, shown in observation (ii), step 2 of Fact 4, it easily follows that the claim holds for all μ.

Fact 4: There exist at most two values of $x \in [0, \overline{x}]$ such that $\Gamma(x; \lambda, \mu) = 0$.

Proof of Fact 4: We proceed in 2 steps; first we concentrate on $\mu = 0$. The second step is to establish that the claim holds also for $\mu > 0$.

1 For $\mu = 0$ the equilibrium condition is

$$\begin{aligned}\Gamma(x;\lambda,0) = & \frac{x^2\left(4\lambda+1-\lambda^2\right)}{4\lambda(1-x)^2} - \frac{x}{(1-x)^2} - 1 + \lambda + \\ & + \frac{x}{2(1-x)^2}\ln\frac{\lambda(2-x)}{x} + \frac{(1-\lambda)x}{2\lambda(1-x)} = 0.\end{aligned}$$

We have that

$$\begin{aligned}\frac{\partial\Gamma(x;\lambda,0)}{\partial x} = & \frac{x\left(4\lambda+1-\lambda^2\right)}{2\lambda(1-x)^3} - \frac{1+x}{(1-x)^3} + \\ & + \frac{1+x}{2(1-x)^3}\ln\frac{\lambda(2-x)}{x} - \frac{1}{(2-x)(1-x)^2} + \frac{(1-\lambda)}{2\lambda(1-x)^2},\end{aligned}$$

and

$$\frac{\partial^2 \Gamma(x;\lambda,0)}{\partial x^2} = \frac{(1+2x)\left(4\lambda+1-\lambda^2\right)}{2\lambda(1-x)^4} - \frac{4+2x}{(1-x)^4} + $$
$$+\frac{2+x}{(1-x)^4}\ln\frac{\lambda(2-x)}{x} + \frac{5-3x}{(2-x)^2(1-x)^3} - \frac{1+x}{x(2-x)(1-x)^3} + \frac{(1-\lambda)}{\lambda(1-x)^3}.$$

Furthermore, $\frac{\partial^2 \Gamma(x;\lambda,0)}{\partial x^2} > 0$ iff $g(x,\lambda) \equiv \frac{2+x}{1-x}\frac{\left(1-\lambda^2\right)}{2\lambda} - 2 + \frac{2+x}{(1-x)}\ln\frac{\lambda(2-x)}{x} - \frac{2(1-x)^2}{x(2-x)^2} > 0.$

Now the following observations (i) to (iii) establish that there exists exactly one $\widehat{x}$ such that $\frac{\partial^2 \Gamma(x;\lambda,0)}{\partial x^2}\mid_{x=\widehat{x}}= 0$. This fact, in combination with Facts 1 and 2 ensures that the curve $\Gamma(x;\lambda,0)$ intersects the horizontal axis at most twice on the relevant interval $\left[0,\frac{2(1+\lambda)}{3+\lambda}\right]$. (Note that, moreover, if there are two intersection points, $\partial^2\Gamma(\cdot)/\partial x^2 < 0$ whenever $\partial\Gamma(\cdot)/\partial x > 0$ as otherwise $\Gamma(\cdot)$ will be only increasing).

(i) $g(0,\lambda) < 0$.

We obtain $\lim_{x\to 0} g(x,\lambda) = \frac{\left(1-\lambda^2\right)}{\lambda} - 2 + \lim_{x\to 0}\{\frac{2+x}{(1-x)}\ln\frac{\lambda(2-x)}{x} - \frac{2(1-x)^2}{x(2-x)^2}\}$. Now it is enough to observe that the second term decreases faster to $-\infty$ than the first increases to $+\infty$.

(ii) $g(\overline{x},\lambda) > 0$.

We have that: $g(\overline{x},\lambda) = \frac{8\lambda^2+4\lambda+4-(1-\lambda)\left(1-\lambda^2\right)}{4\lambda} > 0.$

(iii) There exists at most one $\widehat{\widehat{x}}$ such that $\frac{\partial g(x,\lambda)}{\partial x}\mid_{x=\widehat{\widehat{x}}}= 0.$

We proceed as follows. We construct a function that has the sign of $\frac{\partial g(x,\lambda)}{\partial x}$ and then we show that this function is monotone. We have:
$\frac{\partial g(x,\lambda)}{\partial x} = \frac{3\left(1-\lambda^2\right)}{2\lambda(1-x)^2} + \frac{3}{(1-x)^2}\ln\frac{\lambda(2-x)}{x} + \frac{2(1-x)\left(2-x+x^2\right)}{x^2(2-x)^3} - \frac{2(2+x)}{(1-x)x(2-x)}$,so that
$\frac{\partial g(x,\lambda)}{\partial x} > 0$ iff
$h(x,\lambda) \equiv \frac{3\left(1-\lambda^2\right)}{2\lambda} + 3\ln\frac{\lambda(2-x)}{x} + \frac{2(1-x)^3\left(2-x+x^2\right)}{x^2(2-x)^3} - \frac{2(2+x)(1-x)}{x(2-x)} > 0.$
Now we will show that $\frac{\partial h(x,\lambda)}{\partial x} < 0$. Consider:
$\frac{\partial h(x,\lambda)}{\partial x} = \frac{-6}{x(2-x)} + \frac{2(1-x)^2}{x^3(2-x)^4}\left[\left(-5x^2+6x-7\right)x(2-x) - (4-5x)(1-x)\left(2-x+x^2\right)\right] + \frac{2+4x}{x(2-x)} + \frac{4(2+x)(1-x)^2}{x^2(2-x)^2} = \frac{-4(1-x)}{x(2-x)} + \frac{2(1-x)^2}{x^2(2-x)^2}\left[\frac{-5x^3+x^2-4+2x}{x(2-x)} + 2(2+x)\right] - \frac{2(4-5x)(1-x)^3}{x(2-x)^4}$ (A1)

We can show that (A1) is negative. However, the proof is somewhat messy, but straightforward. It is available on request. Therefore, the proof of observations (i) to (iii) is now complete. They show that $\Gamma(x;\lambda,0)$ intersects the horizontal axis at most twice.

2 $\mu > 0$.

In step 1 we have shown that $\Gamma(x;\lambda,0) = 0$ occurs for at most two values of x. Now we will show that the claim holds for all μ. We prove this by considering partial derivatives of $\Gamma(\cdot)$ with respect to μ, for fixed x. The argument, based on the following three observations, will be discussed below.

(i) Keeping x fixed, $\frac{\partial \Gamma(x;\lambda,\mu)}{\partial \mu} < 0$.

We have that $\frac{\partial \Gamma(x;\lambda,\mu)}{\partial \mu} = -\frac{x}{2\mu^2} \ln\left[1+\frac{(1-\lambda)\mu}{\lambda(1-x)}\right] + \frac{x}{2\mu}\frac{(1-\lambda)}{\lambda(1-x)+(1-\lambda)\mu} < 0$ if $\ln\left[1+\frac{(1-\lambda)\mu}{\lambda(1-x)}\right] > \frac{(1-\lambda)\mu}{\lambda(1-x)+(1-\lambda)\mu}$.

Note that $LHS(\mu=0) = RHS(\mu=0)$ and that $\frac{\partial LHS}{\partial \mu} = \frac{1-\lambda}{\lambda(1-x)+(1-\lambda)\mu}$, $\frac{\partial RHS}{\partial \mu} = \frac{(1-\lambda)[\lambda(1-x)+(1-\lambda)\mu]-(1-\lambda)^2\mu}{[\lambda(1-x)+(1-\lambda)\mu]^2}$ and thus $\frac{\partial LHS}{\partial \mu} > \frac{\partial RHS}{\partial \mu}$. Therefore, $\frac{\partial \Gamma(x;\lambda,\mu)}{\partial \mu} < 0$.

(ii) The second derivative $\frac{\partial^2 \Gamma(x;\lambda,\mu)}{\partial x \partial \mu} < 0$.

We obtained under (i) that $\frac{\partial \Gamma(x;\lambda,\mu)}{\partial \mu} = -\frac{x}{2\mu^2} \ln\left[1+\frac{(1-\lambda)\mu}{\lambda(1-x)}\right] + \frac{x}{2\mu}\frac{1-\lambda}{\lambda(1-x)+(1-\lambda)\mu}$. Taking the derivative with respect to x we have

$2\mu\frac{\partial^2 \Gamma(x;\lambda,\mu)}{\partial \mu \partial x} = -\frac{1}{\mu} \ln\left[1+\frac{(1-\lambda)\mu}{\lambda(1-x)}\right] - \frac{x(1-\lambda)}{\lambda(1-x)^2+(1-\lambda)(1-x)\mu} + \frac{1-\lambda}{\lambda(1-x)+(1-\lambda)\mu} + \frac{x(1-\lambda)\lambda}{[\lambda(1-x)+(1-\lambda)\mu]^2}$.

Under (i) we have shown that the first and third term together are negative. Therefore we only consider the second and fourth terms. Rewriting yields

$\frac{-x(1-\lambda)}{\lambda(1-x)+(1-\lambda)\mu}\left[\frac{1}{1-x} + \frac{\lambda}{\lambda(1-x)+(1-\lambda)\mu}\right]$, which is obviously negative.

(iii) The third derivative, $\frac{\partial^3 \Gamma(x;\lambda,\mu)}{\partial x^2 \partial \mu} < 0$.

It is convenient to consider the term of $\Gamma(\cdot)$ that depends on μ and then take the second derivative to x. Then we obtain the following: $A \equiv \left[\frac{(1-\lambda)}{(1-x)^2\lambda+(1-\lambda)(1-x)\mu}\right] + \frac{x(1-\lambda)[2\lambda(1-x)+(1-\lambda)\mu]}{2\left[(1-x)^2\lambda+(1-\lambda)(1-x)\mu\right]^2}$. The first term is decreasing in μ, therefore it suffices to demonstrate that the second term is decreasing in μ. The second term can be rewritten as

$\frac{x(1-\lambda)}{(1-x)}\left[2-\frac{(1-\lambda)\mu}{(1-x)(\lambda)+(1-\lambda)\mu}\right]$ and the sign of the derivative with respect to μ is the sign of $\frac{-(1-\lambda)\lambda(1-x)}{[\lambda(1-x)+(1-\lambda)\mu]^2}$, which is clearly negative.

We will now describe how the above three facts can be used to complete the argument for $\mu > 0$. Firstly, on the domain where $\Gamma(x;\lambda,0) < 0$ we have $\Gamma(x;\lambda,\mu) < 0$ because of observation (i). Hence, no intersections arise. Secondly, on the domain where $\Gamma(x;\lambda,0)$ is decreasing, $\Gamma(x;\lambda,\mu)$ is decreasing as well because of observation (ii). Therefore on this domain there can emerge at most one intersection. Thirdly, on the domain where $\Gamma(x;\lambda,0) \geq 0$ and increasing, we have that $\frac{\partial^2\Gamma(x;\lambda,0)}{\partial x^2} < 0$. Then because of observation (iii) given in step 2 above it must be that $\frac{\partial^2\Gamma(x;\lambda,\mu)}{\partial x^2} < 0$ as well on this domain. This concavity property implies that at most two intersections arise and if two intersections arise, there does not arise any intersection on the domain mentioned secondly.

Therefore, $\Gamma(x;\lambda,\mu) = 0$ occurs for at most two values of x.

Fact 5: $\frac{\partial^2\Gamma(x;\lambda,\mu)}{\partial x^2}\mid_{x=0} < 0$.

Proof of Fact 5: This property is established for $\mu = 0$ in the Proof of Fact 4, step 1, observation (i). Now since we demonstrated in step 2 of that proof that $\frac{\Gamma^3(x;\lambda,\mu)}{\partial x^2\partial\mu} < 0$ the claim holds for all μ.

Fact 6: $\frac{\partial\Gamma(x;\lambda,\mu)}{\partial\lambda} > 0$.

Proof of Fact 6. We can compute that

$$\frac{\partial\Gamma(\cdot)}{\partial\lambda} = \frac{x}{(1-x)^2}\left[\frac{2\lambda - x\lambda^2 - x}{4\lambda^2}\right] + \frac{x(1-x-\mu)}{2\mu[\lambda(1-x)+(1-\lambda)\mu]} + \frac{\mu 2\lambda - x}{\mu 2\lambda}.$$

This is larger than zero if

$$k(x;\lambda,\mu) \equiv \frac{1}{(1-x)^2}\left[\frac{2\lambda - x\lambda^2 - x}{4\lambda^2}\right] + \frac{(1-x-\mu)}{2\mu[\lambda(1-x)+(1-\lambda)\mu]} + \frac{1}{x} - \frac{1}{2\mu\lambda} > 0.$$

We can show that, for fixed x:
$\frac{\partial k(\cdot)}{\partial\mu} = \frac{-2\mu[\lambda(1-x)+(1-\lambda)\mu]-(1-x-\mu)[2\lambda(1-x)+4(1-\lambda)\mu]}{4\mu^2[\lambda(1-x)+(1-\lambda)\mu]^2} + \frac{1}{2\mu^2\lambda} = \frac{2(1-\lambda)\mu^2}{4\mu^2[\lambda(1-x)+(1-\lambda)\mu]^2} > 0$. We therefore concentrate on $\mu = 0$. In this case, the expression reduces to:

$$k(x;\lambda,0) = \frac{1}{(1-x)^2}\left[\frac{2 - x\left(\lambda + \frac{1}{\lambda}\right)}{4\lambda}\right] + \frac{1}{x}.$$

Now we will argue that this expression is positive for all $x \leq \bar{x}$. This is certainly the case if $2 - x\left(\lambda + \frac{1}{\lambda}\right) > 0$. As the latter expression decreases in x, we may evaluate it at $x = \bar{x}$ to obtain $2\left(1 - \frac{\lambda^2+1}{\lambda+1}\right) \geq 0$ and conclude $\frac{\partial\Gamma(x;\lambda,\mu)}{\partial\lambda} > 0$.

Fact 7: $\Gamma(x; 1/2, \mu) < 0$.

Proof of Fact 7: We have

$$\Gamma(x; 1/2, \mu) = \frac{x}{(1-x)^2}\left(\frac{11x-8}{8} + \frac{\ln\frac{2-x}{2x}}{2}\right) + \frac{x}{2\mu}\ln\left[1 + \frac{\mu}{(1-x)}\right] - \frac{1}{2}.$$

We have shown in Fact 4, step 2 that $\frac{\partial\Gamma(x;\lambda,\mu)}{\partial\mu} < 0$. Therefore $\Gamma(x; 1/2, 0) < 0$ implies $\Gamma(x; 1/2, \mu) < 0$. We obtain $\Gamma(x; 1/2, 0) = \frac{x}{(1-x)^2}\left(\frac{11x-8}{8} + \frac{\ln\frac{2-x}{2x}}{2}\right) + \frac{2x-1}{2(1-x)}$. We want to show that this expression is negative for $x \leq \frac{2}{3}$. This is the case iff $\frac{1}{(1-x)}\left(\frac{3x}{8} + \frac{\ln\frac{2-x}{2x}}{2}\right) - \frac{1}{2x} < 0$, or iff $\frac{3x^2-4+4x}{4x} + \ln\frac{2-x}{2x} < 0$. Taking the derivative of LHS to x we obtain $\frac{\partial LHS}{\partial x} = \frac{12x^2+16}{16x^2} - \frac{2}{x(2-x)}$. We show that this derivative is positive. This is true if $12x^2 + 16 > \frac{32x}{2-x}$ and certainly if $12x^2 - 32x + 16 > 0$. Since this polynomial has a root at $x = \frac{2}{3}$ and at $x = 2$, it is positive for $x \leq \frac{2}{3}$. Therefore, the expression is increasing and we may evaluate at $x = \frac{2}{3}$ to conclude that the inequality holds.

Fact 8: For any $0 < \mu < 1$ and $0 < x < 1$, $\Gamma(x; 1, \mu) > 0$ and $\Gamma(1; 1, \mu) = \Gamma(0; 1, \mu) = 0$.

Proof of Fact 8: We obtain $\Gamma(x; 1, \mu) = \frac{-x}{(1-x)} + \frac{x}{2(1-x)^2}\ln\frac{(2-x)}{x}$. This expression is positive for all $0 < x < 1$ if $-1 + \frac{1}{2(1-x)}\ln\frac{2-x}{x} > 0$ or, $\ln\frac{2-x}{x} \geq 2 - 2x$. To verify the latter inequality, check that it holds with equality at $x = 1$ and that $\frac{\partial LHS}{\partial x} = \frac{-2}{x(2-x)} < \frac{\partial RHS}{\partial x} = -2$, since $x(2-x) < 1$.

The last equality of the statement is easily verified:

$$\Gamma(0; 1, \mu) = \lim_{x\to 0}\frac{x}{2(1-x)^2}\ln\frac{(2-x)}{x} = 0.$$

Fact 9: $\frac{\partial x}{\partial\mu} > 0$ if $\alpha < 1$.

Proof of Fact 9: We have that $\frac{d\Gamma(\cdot)}{d\mu} = \frac{\partial\Gamma(\cdot)}{\partial\mu} - \alpha\frac{\partial\Gamma(\cdot)}{\partial x}$. Above in Fact 4 (step 2) we have shown that the first term, that is, the direct effect, is negative. Furthermore, we know that in equilibrium $\frac{\partial\Gamma(\cdot)}{\partial x} > 0$, therefore $\frac{\partial\Gamma(\cdot)}{\partial\mu} < 0$. Now by employing the Implicit Function Theorem we obtain: $\frac{\partial x}{\partial\mu} = -\frac{\partial\Gamma(\cdot)}{\partial\mu} / \frac{\partial\Gamma(\cdot)}{\partial x} > 0$.

Fact 10: $\frac{\partial E(p)}{\partial\lambda} < 0$ if $\alpha < 1$ and $\frac{\partial E(p)}{\partial\lambda} = 0$ if $\alpha = 1$.

Proof of Fact 10: To prove the first statement, it is sufficient to show that $F(p)$ as specified in (2.5) increases in λ. Recall that $\frac{\partial x}{\partial\lambda} < 0$ if $\alpha < 1$ and consider first the lower segment $F^l(p)$. We can show that

$\frac{\partial F^l(p)}{\partial x} = \frac{1}{2(1-x)}\left[\frac{1}{1-x}\left(2 - x - \frac{xv}{p}\right) - 1 - \frac{v}{p}\right] < \frac{1}{2(1-x)}\left[\frac{1}{1-x}(2-2x) - 2\right] = 0$, where the inequality follows from the fact that $\frac{v}{p} > 1$. Hence, $\frac{dF^l(p)}{d\lambda} > 0$. Secondly, consider now the part of $F(\cdot)$ on the domain $[\lambda v, p']$. It can be checked that both terms in $\frac{dF^m(\cdot)}{d\lambda} = \frac{\partial F^m(\cdot)}{\partial\lambda} + \frac{\partial F^m(\cdot)}{\partial x}\frac{\partial x}{\partial\lambda}$ are positive. Finally, consider the upper segment $F^u(p)$. It is easily checked that $\frac{\partial F^u(\cdot)}{\partial x} < 0$, hence $\frac{dF^u(p)}{d\lambda} > 0$.

The second statement, $\frac{\partial E(p)}{\partial\lambda} = 0$ if $\alpha = 1$, can easily be verified by observing that (2.3) does not depend on λ.

Proposition 2.2: Equilibrium for the case of no channel substitution.

Proof. The proof is included in the proof of Theorem 2.6. ■

Lemma 2.3: Prices in the interval $\left(\lambda v, \frac{(1-x)v}{1-x+\frac{\mu}{\lambda}-\mu}\right)$ are set with zero probability.

Proof. Let us denote the upperbound by p'. To see why the lowerbound of this open interval is λv, suppose first that a slightly higher price is asked with some probability. The firm will then not sell to the inexperienced consumers on the Internet, so it can improve by not charging that price and charging λv instead. Suppose now that λv has zero density in equilibrium and that the lowerbound of the hole is $\hat{p} < \lambda v$. We will show that firm i can improve by charging λv with positive probability, i.e. by shifting mass from $\hat{p}$ to λv. The profit of charging $\hat{p}$ is given by $\left(\frac{x}{2} + [1-x]\left[1 - F_j(\hat{p})\right]\right)\hat{p}$ while the profits of charging λv are given by $\left(\frac{x}{2} + [1-x]\left[1 - F_j(\lambda v)\right]\right)\lambda v$. Now since the rival j has zero probability over the interval $(\hat{p}, \lambda v]$, $F_j(\hat{p}) = F_j(\lambda v)$ and thus i prefers to charge λv in stead of $\hat{p}$. Thus, the gap consists of the interval $(\lambda v, p')$.

The next step is to find an expression for p' by using the fact that $F(\lambda v) = F(p')$. Since the firm must be indifferent between λv and p' it must be the case that

$$\left(\frac{x}{2} + [1-x]\left[1 - F(\lambda v)\right]\right)\lambda v = \left(\frac{x}{2} + \mu\left[1 - F(p')\right]\right)p'.$$

From this we can obtain that

$$1 - F(\lambda v) = 1 - F(p') = \frac{\left(1 - \frac{\lambda v}{p'}\right)\frac{x}{2}}{[1-x]\frac{\lambda v}{p'} - \mu}.$$

We know that the firm can always attain the profit $\frac{x}{2}v$, by charging $p = v$. We can use this fact to obtain an expression for p':

$$p' = v\frac{1-x}{1-x+\frac{\mu}{\lambda}-\mu}.$$

■

Lemma 2.4: Expressions for expected price and conditional expected minimum price.

Proof. The expected price can be calculated as follows:

$$E(p) = \int_{\underline{p}}^{\lambda v} pf^l(p) \quad dp + \int_{p'}^{v} pf^u(p) \quad dp,$$

where f^l and f^u denote the density functions corresponding to the lower and upper parts of the cumulative distribution function, respectively. The first term can be derived by noting that

$$f^l = \frac{dF^l}{dp} = \frac{xv}{(2-2x)p^2}.$$

It then follows that

$$\int_{\underline{p}}^{\lambda v} f^l p \; dp = \frac{vx}{2-2x}\ln p\Big]_{\underline{p}}^{\lambda v} = \frac{xv}{2(1-x)}\ln\left[\frac{\lambda(2-x)}{x}\right].$$

The second term can be derived in a similar way. We have that:

$$f^u = \frac{dF^u}{dp} = \frac{2\mu xv}{4\mu^2 p^2}$$

and

$$\int f^u p \; dp = \frac{xv}{2\mu}\ln p\Big]_{p'}^{v} = \frac{xv}{2\mu}\ln\left[1 + \frac{(1-\lambda)\mu}{\lambda(1-x)}\right].$$

The expected price can therefore be expressed as follows:

$$E(p) = \frac{xv}{2(1-x)} \ln \left[\frac{\lambda(2-x)}{x} \right] + \frac{xv}{2\mu} \ln \left[1 + \frac{(1-\lambda)\mu}{\lambda(1-x)} \right].$$

Now we turn to the minimum price. We have to determine the conditional expected value of the minimum of the two prices, that is, the expected minimum price given the fact that it is not greater than λv. First we will derive the cumulative distribution function of the minimum price. It is defined as follows:

$$F_{\min}(p) \equiv \Pr\left[\min\left[p^1, p^2\right] \leq p\right] = 1 - [1 - F(p)]^2.$$

Using $F(p)$ found above:

$$F_{\min}(p) = \begin{cases} 0 & if\ p < \underline{p} \\ 1 - \left(\frac{x}{2(1-x)} \left[\frac{v}{p} - 1 \right] \right)^2 & if\ \underline{p} \leq p \leq \lambda v \\ 1 - \left(\frac{x(1-\lambda)}{2(1-x)\lambda} \right)^2 & if\ \max\left[\underline{p}, \lambda v\right] \leq p \leq p' \\ 1 - \left(\frac{(v-p)x}{p2\mu} \right)^2 & if\ \max\left[\underline{p}, p'\right] < p \leq v \\ 1 & if\ p > v \end{cases}.$$

The inclusion of the expression $\max\left[\underline{p}, \lambda v\right]$ ensures that $F_{\min}(p)$ is nonnegative. It furthermore leads to a restriction on the domain of x for the usage of the explicit expression for $F_{\min}(p)$. Consider the value $\pi \equiv F_{\min}(\lambda v)$ and note that $\underline{p} = \frac{xv}{2-x} < \lambda v$ if $x < \overline{x} \equiv \frac{2\lambda}{\lambda+1}$. Therefore we obtain

$$\pi = \begin{cases} 1 - \left(\frac{x(1-\lambda)}{2(1-x)\lambda} \right)^2 & \text{if } x < \overline{x} \\ 0 \text{ otherwise} \end{cases}.$$

The conditional expected value can be expressed as follows:

$$E\left(\min\left[p^1, p^2\right] \mid \min\left[p^1, p^2\right] \leq \lambda v\right) = \frac{\int_{\underline{p}}^{\lambda v} p\, dF_{\min}(p)}{F_{\min}(\lambda v)} \text{ for } x < \overline{x}.$$

For the integral part (the numerator) we have that

$$\begin{aligned}\int_{\underline{p}}^{\lambda v} p\frac{dF_{\min}}{dp}dp &= \int_{\underline{p}}^{\lambda v} \frac{x^2v^2}{2(1-x)^2p^2} - \frac{x^2v}{2(1-x)^2p}dp = \\ &= -\frac{x^2v^2}{2(1-x)^2p} - \frac{x^2v\ln p}{2(1-x)^2}\Bigg]_{\underline{p}}^{\lambda v},\end{aligned}$$

and we obtain:

$$\frac{x^2v}{2(1-x)^2}\left[\frac{-1}{\lambda}+\frac{2}{x}-1-\ln\left[\frac{(2-x)\lambda}{x}\right]\right].$$

The conditional expected minimum price is thus:

$$E(\min[p] \mid \min[p] \leq \lambda v) = \frac{\frac{x^2v}{2(1-x)^2}\left[\frac{-1}{\lambda}+\frac{2}{x}-1-\ln\left[\frac{(2-x)\lambda}{x}\right]\right]}{F_{\min}(\lambda v)} \text{ for } x < \overline{x}.$$

■

Proposition 2.5: Equilibrium for the case of partial channel substitution.

Proof. The result is established in the text. ■

Theorem 2.6: Characterization of equilibria.

Proof. It is clear that x can range in the interval $[0, 1-\mu]$. Moreover, when $\overline{x} < x \leq 1-\mu$ it has to be that $\alpha = 1$. From Fact 4 we know that there are either two, one or no solutions with x in the required interval to the equation $\Gamma(x; \lambda, \mu) = 0$. When there are two solutions, there is one stable equilibrium with $\alpha \in (0, 1)$ and one with $\alpha = 1$. When there is one solution, it follows (from Fact 1) that there is a unique stable equilibrium with $\alpha \in (0, 1)$. Finally, when there is no solution, it has to be that there is a unique equilibrium with $\alpha = 1$. We will now characterize which equilibrium situation arises when. To this end, we will fix μ at different levels and then see how the equilibrium configuration changes when λ takes on different values.

First, consider μ close to 0. From Facts 1 and 2 we know that at $x = 0$ and $x = \overline{x}$, $\Gamma(x; \lambda, \mu) < 0$ for all $\lambda < 1$. From Facts 6 to 8 we can then infer that there exist a $\widetilde{\lambda}(\mu)$ such that for all $\lambda < \widetilde{\lambda}(\mu)$, $\Gamma(x; \lambda, \mu) < 0$ and a unique equilibrium with $\alpha = 1$ exists. When $\lambda > \widetilde{\lambda}(\mu)$, $\Gamma(x; \lambda, \mu) = 0$ has two solutions in the interval $[0, \overline{x}]$. When λ gets close to 1, however, $\overline{x}$ approaches 1 and eventually becomes larger than $(1-\mu)$. Combining this with Fact 8 implies that there must be a $\widetilde{\widetilde{\lambda}}(\mu)$ such that for $\widetilde{\lambda}(\mu) < \lambda < \widetilde{\widetilde{\lambda}}(\mu)$, there exist two equilibria and for all $\lambda > \widetilde{\widetilde{\lambda}}(\mu)$ there is one stable equilibrium with $\alpha < 1$.

Fix then μ close to 1. As x lies in the interval $[0, 1-\mu]$, all permissable values of x lie close to 0. From Fact 3 and the continuity of $\Gamma(\cdot)$ and $\partial\Gamma(\cdot)/\partial x$ in a neighborhood of 0 (0 itself excluded) it follows that we can choose μ close enough to 1 such that $\partial\Gamma(\cdot)/\partial x > 0$ on the whole relevant interval. From Facts 6 to 8 it follows that there exist a $\widetilde{\lambda}(\mu)$ as defined in (A) of the Theorem.

We next show that for any given μ, $0 < \mu < 1$, the equilibrium configurations that arise when λ increases from 0 to 1, are exhausted by the two possibilities considered above. From Fact 7 it follows that for λ small enough there is only one equilibrium, namely where $\alpha = 1$. From Fact 8 it follows that for λ close to 1, there exist a unique stable equilibrium with $\alpha < 1$. Finally, from Fact 6 it follows that if for $\left(\widehat{\lambda}, \mu\right)$ there exists a unique stable equilibrium with $\alpha < 1$, then this is also the case for any (λ, μ) with $\lambda > \widehat{\lambda}$. Hence, no other possibilities exist than the two considered above.

We finally show that the existence of $\overline{\mu}$ as defined in the Theorem. To this end we show that if for a certain $\widehat{\mu}$ the equilibrium configuration for increasing λ shifts from a unique equilibrium with $\alpha = 1$, via two stable equilibria to a unique equilibrium with $\alpha < 1$, then this pattern has to exist for all $\mu < \widehat{\mu}$. In the proof of Fact 4 we show that $\partial\Gamma(\cdot)/\partial\mu < 0$. Moreover, the smaller μ, the larger the range of values for which $\Gamma(x; \lambda, \mu)$ is defined. This implies, together with Facts 1 and 2, that if $(\lambda, \widehat{\mu})$ is in the region where there are two equilibria, then (λ, μ) is also in this region for any $\mu < \widehat{\mu}$. Moreover, Facts 7 and 8 are true for any value of μ so that the existence of $\widetilde{\lambda}(\mu)$ and $\widetilde{\widetilde{\lambda}}(\mu)$ is guaranteed.

The observation that $\lim_{\lambda\to 1}\underline{\mu}(\lambda) = 0$ and $\lim_{\lambda\to 1}\overline{\mu}(\lambda) = 1$ in Proposition 2.2 follows from Fact 8. ■

Chapter 3

Internet Retailing as a Marketing Strategy

3.1 Introduction

The importance of the Internet as a marketplace has substantially grown over the past decade, even though expectations have been dramatically tempered since early 2000. A distinctive feature of doing business electronically is that transactions no longer require the physical coordination of buyers and sellers: market participants find each other at their screens. There are many aspects of market interaction which are affected by this online nature of trade. On the supply side, we may think of all kinds of cost reductions, resulting from new ways of organizing production and sales processes. On the demand side, the major impact of the Internet is on consumers' ability to acquire information about firms and their prices.

While electronic commerce may have considerable potential, it is still of minor importance compared to other retail channels even for books and CDs.[1] To understand the role of the Internet as a retail channel, we have to explain how its features influence market behaviour. This paper provides a theoretical framework that analyzes firms' incentives to sell online and the extent to which consumers will substitute the Internet for the con-

[0] This chapter is a version of an article co-authored by Maarten Janssen.

[1] According to a study by the Boston Consulting Group (2000) in the United States and Europe, e-commerce accounts for less than 5 percent of total sales in the books and CDs categories.

ventional retail channel. The interaction between incentives on both market sides sheds light on the different factors that determine the channel structure of the industry and the market share of electronic sales. Moreover, the analysis sheds a new light on the empirical evidence concerning prices on the electronic channel relative to prices in the conventional retail channel.

For consumers, online shopping makes it easy to find and 'visit' firms and compare their prices, but the online nature of the transaction leads to uncertainties. For example, in some product categories such as clothing and furniture, it is more difficult to assess how well a particular product fits a consumer's needs. Other uncertainties and inconveniences are related to the payment method, poor product declarations, slow delivery of goods, and unclear redemption policies.[2] A consumer weighs the conveniences of online shopping with the inconveniences and uncertainties. We explicitly take this trade-off into account in the consumer's choice of retail channel.

For firms, the Internet may be used as an alternative retail channel to gain market share. Moreover, if the online shopping convenience is large enough, firms may try to exploit the fact that consumers are willing to pay for this convenience by charging higher prices online. These positive aspects are offset by the fact that online competition is stronger as it is easier for consumers to compare prices. Moreover, the online infrastructure and the reorganization of sales processes requires that firms make an investment upfront. The paper models these different market forces in a consistent way and attempts to answer which factor is dominant and under what circumstances.

We present a two-stage game in which firms first decide whether to build an online retail channel before engaging in price competition, using the retail channels at their disposal. This game is analyzed in a variety of settings where initially there are always two incumbent bricks-and-mortar firms that sell horizontally differentiated products. Differentiation between firms is modelled a la Hotelling (1929) where firms are located at opposite

[2] A survey by Hummerston (2001) indicates that the most important reason for not purchasing online is related to payment security and the second most important reason is given by 'You don't know what you get.' and 'Easier/more fun to buy goods/services in a store.' In Andersen (2000) it is added that many "E-commerce web sites ... are not good at making the shopping experience easy and trustworthy for the online users."

sides of a line segment.[3] That is, incumbent firms have built a reputation that appeals more to some consumers than to others. Traditionally, differentiation between firms in Hotelling type of analyses is either with respect to physical location and transportation cost or with respect to heterogeneous products and tastes. In our model, differentiation between the two *bricks-and-mortar* firms is comprised of *both* factors. On the Internet, physical location is not important so that only the product differentation dimension remains in that retail channel. Accordingly, there is less differentiation online than in the conventional stores.

We analyse two settings of strategic interaction. The first setting is one where there is no threat of entry and consumers may or may not experience any additional convenience from buying online. The second setting we analyze is one where we allow for a pure online player to threaten the incumbent firms to enter the market and we investigate the possibilities for entry deterrence. In each setting equilibrium channel structures and prices are characterized.

Consumers know which of the two incumbent bricks-and-mortar firms they prefer the most. If these firms start an online retail channel, they bring the reputation that is built up in the conventional stores with them. A pure online player does not have any reputation among consumers so that consumers have to form expectations about their position on the product line.

We arrive at the following insights. First, in the absence of a threat of entry and no additional online shopping convenience, bricks-and-mortar firms do not become clicks-and-mortar firms. The reason is that a firm with a lower online price (than on its conventional channel) attracts two types of consumers: those who would otherwise buy from the rival and those who would otherwise buy form the conventional store of the same firm. The first effect can be called the business-stealing effect, while the second effect may be termed the cannibalisation effect. We find that the cannibalization effect dominates the business-stealing effect, and hence, firms find it optimal not to build an online presence as alternative retail channel.

[3] When we model the consumer's utility of buying from the entrant, it turns out that it is more convenient to interpret the line as a circle a la Salop (1979). The difference between the circle and the line segment is of no further importance, however, and therefore, we prefer to speak about differentiation along the line.

Second, if we allow for online shopping conveniences, some consumers will buy online even if the online prices are not lower. The cannibalization effect is in this case weaker as even at identical prices across retail channels, some consumers prefer to buy online. Moreover, we find that the business-stealing effect is strengthened in case the other incumbent firm did not open an alternative retail channel. This is because consumers who are fairly indifferent about the two firms, are more inclined to buy online than those who have a strong preference for a certain firm. Consequently, if the convenience of online shopping is high enough, individual firms have an incentive to start-up an online retail channel. However, if both firms become clicks-and-mortar, profits are lower than if both had stayed bricks-and-mortar firms. The reason is that even though some consumers are willing to pay more for a product online than in the conventional store, online competition for costumers is stronger as there is less differentiation online. Therefore, online prices will be lower. Depending on the cost of setting up an online retail channel, different equilibrium configurations are possible. If the cost is relatively low, firms face a Prisoner's Dilemma: individually they have an incentive to start an alternative retail channel, but they are both worse off if they both do so. For intermediate values of this cost, there is scope for only one click-and-mortar firm and there is a first-mover advantage of starting an online retail channel. When the cost is relatively high, none of the two firms will start an online channel. Interestingly, the only case where online equilibrium prices are higher than in the conventional store is when there is only one firm selling online and when the inconvenience associated with shopping online is not too large.

We next look into the possibilities for a pure online player to enter the market. As an entrant does not eat up its installed costumer base, the incentives for an entrant to go online are stronger than those of the bricks-and-mortar firms. Indeed, we find that both in the case where the incumbent firms do not have an online retail channel and in case they do, an entrant may make positive operating profits. Again, online prices can only be higher than in the conventional retail channel if firms enjoy some "monopoly power" online. We then analyze the incentives of both incumbents and entrants to make a strategic decision to be the first to start an online retail channel. For intermediate values of the cost of starting up an online retail channel, we show that an incumbent firm may set up an online infrastructure to deter entry and that if these costs are even lower, the incumbent firms face a situation of strategic uncertainty in the sense that they want to take identical strategies: either they want to jointly deter entry (as this is the way to deter entry) or they both want to remain a bricks-and-mortar firm.

In the case where the entrant considers making a strategic decision to be the first online player, many possible equilibrium reactions by incumbent firms may follow. This by itself shows the difficult situation online entrants face. Details of the interaction matter a lot; details that may be difficult to foresee. One important difficulty is that when online shopping convenience becomes larger or when the cost of building an online presence becomes smaller, it may be more profitable for incumbents to follow an entrant's decision to build an online retail channel thereby destroying the profitability of the entrant's decision. Another difficulty that may arise is that incumbents may face a coordination problem such that the entrant's decision to enter is only profitable if the incumbents solve the problem by not going online themselves. However, it may well be that the incumbents work out the other solution to the coordination problem in which case the entrant would have been better off not to enter in the first place. We see these theoretical possibilities as ways to explain the difficulties online players have been confronted with in the real world and as possible reasons why online shopping has not been as successful as it promised to be at the end of the previous millennium.

The empirical literature on the implications of the Internet for the competitiveness of markets has compared average prices in electronic marketplaces against those in conventional markets. The empirical evidence is mixed. Some studies find that prices in electronic markets are lower than in conventional markets (see, e.g., Brynjolfsson and Smith (1999)), while others report that prices in electronic markets are approximately equal to (Clay *et al.* (2000)) or even higher than in conventional markets (see, e.g., Lee (1997) and Bailey (1998)). Finally, Friberg et al. (2001) find that pure online retailers charge lower prices than the online channel of clicks-and-mortar firms. Our theoretical model provides an explanation for the latter result in terms of the fact that the online player does not compete with its own conventional store and therefore has less to loose from charging lower prices online. Our paper attributes the other mixed empirical evidence concerning conventional and online prices to different degrees of online market power and differences in online shopping convenience.

On the theoretical side, there are papers by Bakos (1997) and Janssen, Moraga and Wildenbeest (2005), among others, studying the implications of a reduction in search cost due to the emergence of online shopping on the competitiveness of markets. Bakos presents a model of circular product differentiation where consumers search for prices and product features. In his model, consumers can get to know all product characteristics if they engage in costly search. In our model, in contrast, consumers cannot get around some of the uncertainties associated with buying online. Commenting on Bakos' paper, Harrington, Jr. (2001) questions the validity of some of Bakos' results.

There are a few papers addressing the issue of channel substitution. Zettelmeyer (2000) focuses on the incentives firms have to increase the ease with which consumers can evaluate their products. In his paper, consumer search cost is a strategic variable of firms. He shows that firms may provide selected groups of consumers with different information (level of search cost), thereby providing monopoly power to the firms and reducing market competition. The focus of our model is different. The source of market power in our model lies in the fact that firms sell differentiated products. More recently, Liu et al. (2003) analyze an issue that is more closely related to the present paper, namely to what extent can a bricks-and-mortar retailer's online expansion affect a pure online player's entry decision? They find that for some parameter values, a bricks-and-mortar's decision to go online may tricker an entrant to go online as well, making it more profitable for a brick-and-mortar firm not to start an online retail channel in the first place. To get this result, Liu et al. (2003) assume that firms are restricted to set identical prices across different retail channels, that there is perfect competition online between homogeneous goods producing firms and that the pricing game is sequential. As we study a setting with heterogeneous goods and allow firms to set different prices across channels, this result cannot occur in our setting.

Lal and Sarvary (1999) pose a different question: When is the Internet likely to *decrease* price competition? In their model, the specific inconvenience of online shopping is that products cannot be evaluated physically to learn about non-digital attributes. Firms are present on two retail channels. Since digital attributes can be communicated over the web, the Internet changes the effective search cost structure: consumers do not have to visit the store to buy their familiar brand, and thus, the cost of trying another brand is higher than the cost of buying the familiar brand. In this way, the Internet may increase the effective cost of search, which decreases price competition.

Finally, Mazón and Pereira (2000) also analyze whether firms have incentives to open electronic retail channels and the different price equilibria these incentives generate. However, they focus on different issues such as retail cost reductions and their results depend crucially on the assumption that some ("old") consumers do not have Internet access.

The paper is organized as follows. Section 2 presents the model. Section 3 analyzes equilibrium properties in the absence of a threat of entry. Section 4 discusses the case of a pure online player threatening to enter. The issue of which market structures can obtain in equilibrium is discussed in Section 5. Conclusions and managerial implications are contained in Section 6 and proofs and calculations can be found in the Appendix.

3.2 The Model

There are two types of firms. Two incumbent bricks-and-mortar firms have established a certain brand reputation among consumers, while the possible pure online entrant does not have such a reputation. In the first stage of the game, firms decide whether or not to invest in setting up an online retail channel at fixed cost f. For the pure online player this decision is equivalent to the decision whether or not to enter the market. This first stage can be analyzed as a simultaneous move or as a sequential move game where one or two firms may try to pre-empt the other(s).

In the second stage, there are two (or three) vertically integrated firms on the supply side of the market. Firms produce the good at constant returns to scale and production cost is normalized w.l.o.g. to zero. Firms are horizontally differentiated, as in Hotelling (1929)'s linear city model. On the demand side of the market, there is a mass of consumers, normalised to 1. Incumbent firms 1 and 2 are located at $x = 0$ and $x = 1$, respectively.

Every consumer has a location x on the line segment and the preference for a firm is represented by the disutility of travelling the distance between the consumer's location and the firm. This travel cost consists of two components: the physical travel cost related to travelling to a firm's physical shop and the "utility cost" of buying a good that differs from her most preferred taste. For each unit travelled, consumers incur a linear cost $t > 0$.

The utility a consumer x gets from buying the product from firm 1 in its physical shop is given by

$$v - tx - p_{1c},$$

where v is the consumer's maximal willingness-to-pay[4] and p_{1c} is the price firm 1 sets for its conventional retail channel (i.e., its physical shop). Similarly, the utility consumer x gets from buying the product at firm 2's physical shop is given by

$$v - t(1 - x) - p_{2c}.$$

Firm i charges price p_{ic} in its store and p_{iE} on its electronic retail channel (if it has one). Consumers purchase at most one unit and to do so they can either go online, or they can visit one store.[5] The advantages and disadvantages of the different retail channels are modelled as follows. A first advantage of buying online is that consumers can easily compare the firms' prices. Moreover, the conveniences and inconveniences/uncertainties of buying on the internet such as poor product declarations and slow delivery of goods are modelled in two steps. First, when buying in a firm's online shop, the willingness-to-pay is multiplied by a factor λ. If $\lambda < 1$ ($\lambda > 1$), then the consumer's willing-to-pay is lower (larger) for an online purchase compared to a purchase in a conventional store. Second, a consumer does not have to travel to a firm's conventional store in order to buy, i.e., he only "pays" the mismatch between a firm's product and his most preferred commodity. This is modelled by multiplying the cost of "travelling" to a firm by a factor β, where $0 \leq \beta < 1$. The parameter β measures how much of the disutility of buying from firm i is attributed to product heterogeneity: for example $\beta = 0$ implies that products are homogeneous and differentiation between incumbent firms is entirely due to differences in their locations, whereas $\beta = 1$ implies that differentiation is entirely due to the heterogeneity of products. Hence, consumer x derives the following utility from buying from one of the incumbent

[4]It is assumed throughout that v is large enough so that the market is covered.

[5]Here we recognize that, although often suggested otherwise, searching on the Internet does take time. The equilibria we characterize are such that given the price and product offerings available, no consumer regrets the choice it has made and, therefore, allowing consumers to visit the Internet *and* the conventional channel would not alter the results.

firms' online retail channels:

$$\max[\lambda v - \beta tx - p_{1E}, \lambda v - \beta t(1-x) - p_{2E}]$$

It turns out to be useful to define a parameter α as $\alpha = (1-\lambda)v/(1-\beta)t$. The interpretation of α is straightforward whenever it is positive ($\lambda < 1$). If firm 1, resp. 2, sets identical prices in its conventional channel and its online channel, consumer α, resp. $(1-\alpha)$, is indifferent between buying in the conventional store and buying online. Another interpretation of α is that it measures the size of the online shopping inconvenience relative to the inconvenience of having to travel to the conventional shop. We will assume that $-1/2 < \alpha < 1/2$.

Since the potential third firm in the market is a newcomer and did not build up any reputation in its conventional channel, we assume that consumers do not know the x location of this firm. Each consumer therefore has to "travel" an expected distance of 1/2.[6] The expected utility of buying from firm 3, a pure Internet player, is then given by

$$\lambda v - \beta t/2 - p_{3E}.$$

An equilibrium of the second stage of the game is a set of prices, one for each retail channel on which a firm is active, such that each individual firm i maximizes its profits given the prices set by the other firms. Consumers buy at the firm where utility is maximized. We will focus on symmetric subgame perfect equilibria of the two-stage game.

3.3 No Threat of Entry

In this section we study incentives to open online channels and the subsequent price competition between two incumbent bricks-and-mortar firms. We study a two stage game in which firms first decide whether or not to sell online, and then compete in prices. The two incumbent players have both the possibility to open an additional retail channel,

[6] Strictly speaking, this is only true along a circle. Along the line, a consumer's expected travel to the Internet player depends on his location and is smaller the closer a consumer is located towards the middle of the line segment. Working with the proper line segment interpretation only complicates the analysis without bringing additional insights.

yielding four possible market structures in the second stage. The case in which neither firm opens an online presence is standard as it confirms the logic of Hotelling's (1929) linear city model. It is easy to see that in this case both firms set their prices equal to t and the two firms share the market equally so that each firm's profit is equal to $t/2$. The case in which only firm 2 sells online is equivalent to the case in which only firm 1 sells online. This leaves us with two cases to investigate further.

We first concentrate on the case where each duopolist is a multi-channel retailer.

3.3.1 Two Clicks-and-Mortar Players

Whenever $\lambda < 1$[7] and the prices $(p_{1c}, p_{1E}, p_{2c}, p_{2E})$ are in the relevant range, the picture of channel substitution is as in Figure 3-1,[8] where x_{1Ec} and x_{2Ec} represent the consumer that is indifferent between buying from a respective firm's conventional and online channel and x_{1E2E} represent the consumer that is indifferent between buying from the two online channels. One can easily calculate that

$$x_{1Ec} = \frac{(1-\lambda)v - (p_{1c} - p_{1E})}{(1-\beta)t}; x_{2Ec} = 1 - \frac{(1-\lambda)v - (p_{2c} - p_{2E})}{(1-\beta)t}; x_{1E2E} = \frac{\beta t - (p_{1E} - p_{2E})}{2\beta t}.$$

In this picture, the area to the left of x_{1Ec} forms the demand for firm 1's conventional channel, while the area between x_{1Ec} and x_{1E2E} constitutes the consumers who buy via firm 1's online channel. Mirror areas represent Firm 2's demand on its conventional and online channel, respectively. From Figure 3-1 it is clear that the profit function for firm 1 is given by:

$$\begin{aligned}
\pi_1 &= p_{1c}x_{1Ec} + p_{1E}(x_{EE} - x_{1Ec}) \\
&= \frac{(1-\lambda)v - (p_{1c} - p_{1E})}{(1-\beta)t}(p_{1c} - p_{1E}) + p_{1E}\frac{\beta t - (p_{1E} - p_{2E})}{2\beta t}.
\end{aligned}$$

[7] The main reason why we focus in the text mostly on this case is that the expressions that hold true when $\lambda > 1$ are less complicated and easily follow once the case $\lambda < 1$ is clearly understood. Also, one may argue that this is the most natural case to look at as the main advantage of shopping online, the fact that one does not incur geographical travelling cost, is already captured by the parameter β.

[8] Here, it is implicitly assumed that equilibrium prices are such that all channels by all firms are visited by at least some consumers. Below, we will specify the parameters for which this is the case.

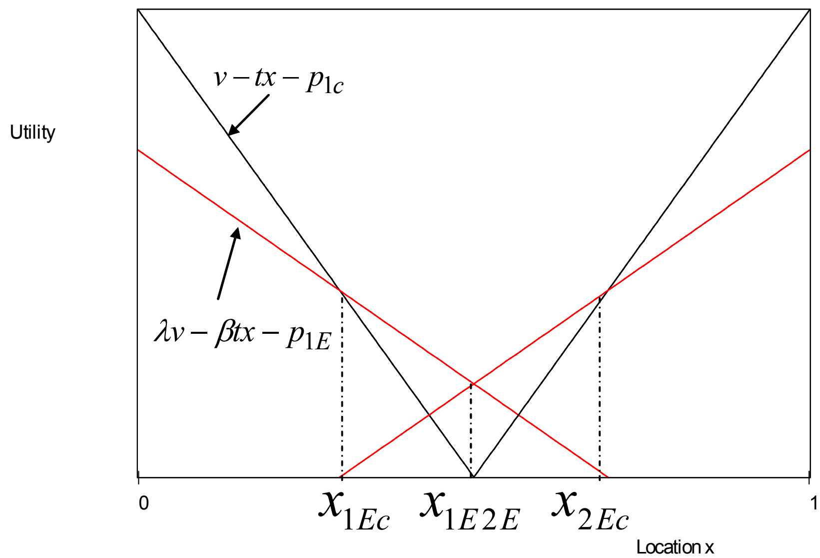


Figure 3-1: Two clicks-and-mortar players

A similar equation holds for firm 2. The equation has a simple interpretation. By means of its online channel, a firm competes with the other firm to get a larger market share. The division of a firm's market share between the conventional and the electronic channel is entirely determined by a firm's internal pricing policy: a firm has some monopoly power over the consumers who buy from them and can set the price difference $p_{1c} - p_{1E}$ so as to maximize profits. It is easy to see then that whenever all channels are visited by some consumers, equilibrium prices are given by:

$$
\begin{aligned}
p_{1E} &= p_{2E} = \beta t \\
p_{1c} &= p_{2c} = \beta t + \frac{(1-\lambda)v}{2}
\end{aligned}
$$

and the equilibrium indifferent consumers are given by

$$x^*_{1Ec} = \frac{(1-\lambda)v}{2(1-\beta)t}; x^*_{2Ec} = 1 - \frac{(1-\lambda)v}{2(1-\beta)t}; x^*_{EE} = 1/2.$$

As it has to be the case that $0 \leq x_{1Ec} \leq 1/2$, it follows that this equilibrium holds whenever $0 < (1-\lambda)v \leq (1-\beta)t$, or given the restrictions we have imposed on α, whenever

α is positive.[9] The Proposition below summarizes the above and also considers the case where $\alpha < 0$.

Proposition 3.1 *When both firms have built the infrastructure to sell online, there are three cases to consider: (a) if $\alpha \leq 0$, all consumers will buy online, $p_{1E} = p_{2E} = \beta t$, and equilibrium profits will be $\beta t/2$; (b) if $\alpha > 0$, all retail channels will be used and $p_{1E} = p_{2E} = \beta t$, $p_{1c} = p_{2c} = \beta t + \frac{(1-\lambda)v}{2}$ and equilibrium profits will be $\beta t/2 + \frac{[(1-\lambda)v]^2}{4(1-\beta)t}$.*

Proposition 3.1 is easily understood by taking the Hotelling result as a reference point and therefore, a more formal proof is omitted. If Internet purchases are considered to be relatively inconvenient overall ($\alpha > 0$), firms use both channels. Competition between the firms mostly takes the form of competition online as the consumer that is indifferent between buying from the two firms is a consumer that buys online. The online prices that result are exactly equal to the equilibrium prices in a Hotelling model where transportation costs are equal to βt (the online "transportation costs"). When both firms are clicks-and-mortar firms, online prices are always lower than prices in the conventional channel. More precisely, conventional prices are set as a monopoly mark-up on the online prices: there is no effective competition for these inframarginal consumers and firms will use their monopoly power over these consumers. This also helps to explain the result in case online purchases are considered to be more convenient ($\alpha < 0$). In this case, firms have to set lower conventional prices than online prices to motivate consumers to buy in their conventional stores. From the discussion provided above for the case $\alpha > 0$ it follows that this cannot be profit maximizing. Therefore, we get a situation where the online retail channel dominates all sales. As online "transportation costs" are lower, competition is more severe and prices are lower than when firms compete with their conventional stores.

As competition online is more severe, we can therefore arrive at a preliminary conclusion that it is not in the interest of incumbent bricks-and-mortar firms to make the online shopping experience very convenient.

[9] One may observe that many relevant expressions in this paper, like the ones here for the indifferent consumers (in equilibrium), include a term like $(1-\lambda)v/(1-\beta)t$. One of the advantages of defining α in the way we did is that one does not have to impose each time additional and different restrictions on β (to have it bounded away from 1). One should note, however, that given the values of the other parameters, α negatively depends on β and in particular that α is close to 0 whenever β is close to 1.

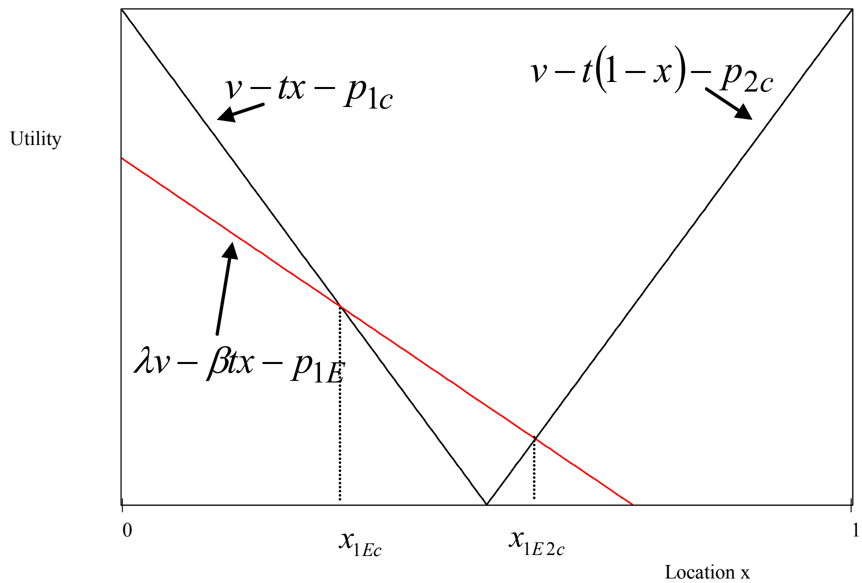


Figure 3-2: Clicks-and-mortar vs. bricks-and-mortar

3.3.2 Clicks-and-Mortar vs. Bricks-and-Mortar

When one firm (say firm 1) has opened an electronic retail channel and the other has decided not to do so, the picture of channel substitution for the relevant range of prices and parameter values is as in Figure 3-2, where

$$x_{1Ec} = \frac{(1-\lambda)v - (p_{1c} - p_{1E})}{(1-\beta)t}; x_{1E2c} = \frac{t - (1-\lambda)v + (p_{2c} - p_{1E})}{(1+\beta)t}.^{10}$$

In this picture, the segment to the left of x_{1Ec} forms the demand for firm 1's conventional channel, while the segment between x_{1Ec} and x_{1E2c} constitutes the consumers who buy via firm 1's Internet channel.

The remaining segment represents firm 2's demand on its conventional channel. Using Figure 3-2 the profit function for firms 1 and 2 are given by:

$$\begin{aligned}
\pi_1 &= p_{1c}x_{1Ec} + p_{1E}(x_{1E2c} - x_{1Ec}) \\
&= \frac{(1-\lambda)v - (p_{1c} - p_{1E})}{(1-\beta)t}(p_{1c} - p_{1E}) + p_{1E}\frac{t - (1-\lambda)v + (p_{2c} - p_{1E})}{(1+\beta)t}; \\
\pi_2 &= p_{2c}x_{1E2c} = p_{2c}\frac{\beta t + (1-\lambda)v - (p_{2c} - p_{1E})}{(1+\beta)t}.
\end{aligned}$$

The first-order conditions that yield the subgame perfect equilibrium prices are given in the Appendix. Solving these equations and substituting them back into the profit function given above gives the Proposition below.

Proposition 3.2 *Suppose one firm has decided to build the infrastructure to sell online, say firm* 1*, and firm* 2 *has not. Then, if* $\lambda < 1$,[11] *the unique symmetric subgame equilibrium prices are given by*

$$\begin{aligned} p_{1c} &= \frac{(4+2\beta)t + (1-\lambda)v}{6}; \\ p_{2c} &= \frac{(1+2\beta)t + (1-\lambda)v}{3}; \\ p_{1E} &= \frac{(2+\beta)t - (1-\lambda)v}{3}, \end{aligned}$$

and equilibrium profits equal

$$\begin{aligned} \pi_1 &= \frac{[(1-\lambda)v]^2}{4(1-\beta)t} + \frac{[(2+\beta)t - (1-\lambda)v]^2}{9(1+\beta)t}; \\ \pi_2 &= \frac{[(1+2\beta)t + (1-\lambda)v]^2}{9(1+\beta)t}. \end{aligned}$$

There are a few interesting observations to make. First, if only one firm opens an electronic retail channel, it will set higher prices online than the other firm with its conventional and it may even set higher prices online than the average conventional price. The first point follows as $(1-\beta)t > 2(1-\lambda)v$ whenever $\alpha < 1/2$ and is explained by the fact that the clicks-and-mortar firm exploits its monopoly position online. The second point holds true whenever $\alpha < 1/5$ and is explained by the fact that in this case the online inconveniences are considered to be fairly small (or non-existent in case $\alpha < 0$) so that the clicks-and-mortar firm can further exploit its online monopoly position. A second observation is that the operating profits of the clicks-and-mortar firm are always higher than those of the bricks-and-mortar firm. Finally, both the online and the conventional prices of the clicks-and-mortar firm are lower than in case both firms had stayed out of online retailing altogether. The main reason for this is that the bricks-and-mortar

[11] *In case* $\lambda > 1$, *firm 1 will not use its conventional channel, the remaining two equilibrium prices will not be affected and equilibrium profits are simply given by* $\pi_1 = \frac{[(2+\beta)t-(1-\lambda)v]^2}{9(1+\beta)t}$ *and* $\pi_2 = \frac{[(1+2\beta)t+(1-\lambda)v]^2}{9(1+\beta)t}$.

firm considers the online channel of its competitor more aggressive than the competitor's conventional retail channel. In reaction, it will price lower, which forces the clicks-and-mortar firm also to lower its price.

3.3.3 Clicks or just Bricks?

In the previous two subsections we have characterized the price equilibria of the second stage of the game. We now go one step back and analyze the first stage decision whether or not to build an online retail infrastructure. The above analysis can be summarized in the payoff matrix below (for the case where $\lambda < 1$),[12] where because of symmetry only the pay-offs of firm 1 are mentioned.

		2	
	y/n	n	y
1	n	$\frac{t}{2}$	$\frac{[(1-\lambda)v+(1+2\beta)t]^2}{9(1+\beta)t}$
	y	$\frac{[(1-\lambda)v]^2}{4(1-\beta)t} + \frac{[(2+\beta)t-(1-\lambda)v]^2}{9(1+\beta)t} - f$	$\frac{\beta t}{2} + \frac{[(1-\lambda)v]^2}{4(1-\beta)t} - f$

Table 3-1. Pay-offs for firm 1 depending on whether or not itself and its rival incumbent start an online channel and given that a pure online retailer stays out of the market.

A firm contemplating to attract additional consumers via its online retail channel has to trade-off two effects. First, it will steal some consumers away from its competitor, but at the expense of lower overall prices. Moreover, the online channel also steals consumers from its own conventional store. The calculations in the Appendix and Figure 3-3 below show that unless α is a relatively large negative number the first, business-stealing, effect is weaker than the second, cannibalization, effect. In fact, the Appendix and Figure 3-3 show that for many parameter constellations the firms have a dominant strategy not to open an online channel. This implies that without a threat of entry, incumbent firms will not find it profitable to start an online retail channel for most parameter values. It is

[12] It follows from the analysis above that the pay-off matrix in case $\lambda > 1$ differs only to the extent that the term $\frac{[(1-\lambda)v]^2}{4(1-\beta)t}$ is missing in both cells in the bottom row of the matrix.

interesting to observe that this result holds true even if f, the cost of starting such a retail channel, is very small.

If α is a relatively large negative number and f is small enough,[13] a firm may be better off having an online channel and the situation where both firms did not build an online retail channel cannot arise in equilibrium. Two cases may result. First, for a range of parameters where α is not close to $-1/2$ when β takes on middle range values (see the picture below for an example), there exist asymmetric equilibria where only one firm opens an online channel. In this case one firm effectively uses only its online retail channel, while the other firm uses the only channel it has, the conventional one. Online prices in this case will be higher than offline prices. If α is quite close to $-1/2$ and β is not close 0 or to 1, firms face a Prisoner's Dilemma: both firms have a dominant strategy to build on online channel and to use it as their only effective retailing channel, but as competition online is more severe than off-line competition, both firms would be better off if they had not opened an online channel!

The possible equilibrium configurations are summarized in Figure 3-3 below for the case when $f/t = 0.01$. The Figure shows for which parameter constellations, which equilibrium structure arises. The Figure shows that the Prisoner's Dilemma situation described above cannot arise when β is close to 0. In this case, locational differences are the main source of product differentiation, hence online competition is very severe. In this case, firms want to avoid both having an online presence. When β is close to 1, differences between firms are due to differences in the products they sell and in this case firms cannot benefit from an online presence, no matter how large the online shopping conveniences are! In this case without a threat of entry, geographical differences between shops and the consumer cost of travelling associated with it are thus important features explaining whether or not firms will start an online retail channel. Note also that on average firms are worse off, or not better off, if online shopping conveniences increase (α decreases). This is easily seen in the two regions where a symmetric equilibrium exists. In the asymmetric equilibrium case, one can easily show that the average pay-off, relevant if

[13] Of course, when f is relatively large, it is never optimal to build an online channel.

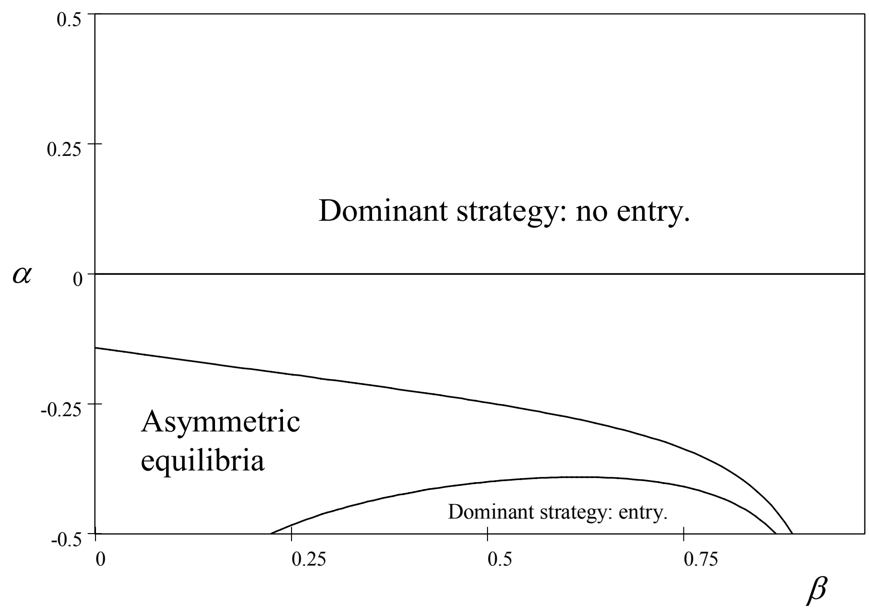


Figure 3-3: Equilibrium structures when no threat of entry, $f/t = 0.01$.

firms do not know which one of them will be the one with an online presence, is declining in α.

3.4 Threat of Entry by a pure Internet player

We now turn to the role the Internet can play as an alternative marketing and distribution channel in case a pure online retailer threatens to enter the market. We are interested in the incentives to enter and in the question whether the two conventional firms have an incentive to start operating an Internet channel in view of this threat of entry. Assuming the entrant enters, there are three relevant situations to consider depending on how many incumbent firms open an online channel: no incumbent firm sells online, one incumbent sells online or both incumbents sell online. Each of these three cases is analyzed in turn in the next Subsections. The case where the entrant, firm 3, decides not to enter is analyzed in the previous section.

3.4.1 Bricks-and-Mortar vs. Pure Internet Retailer

We first consider the subgame where the two incumbent firms have not built the infrastructure to sell online, while the entrant has done so and uses the online channel as the sole retail channel. In this case, it is easy to see that for the set of relevant prices, the indifferent consumers are given by the following expressions:

$$x_{1c3E} = \frac{(1-\lambda)v - (p_{1c} - p_{1E}) + \frac{\beta t}{2}}{t}; x_{2c3E} = 1 - \frac{(1-\lambda)v - (p_{2c} - p_{2E}) + \frac{\beta t}{2}}{t},$$

where for example x_{1c3E} denotes the consumer who is indifferent between buying from firm 1's conventional store and the entrant's (firm 3's) online store. Consumers located to the left of x_{1c3E} buy from firm 1's conventional store, those located to the right of x_{2c3E} buy from firm 2's conventional store and those located between x_{1c3E} and x_{2c3E} buy from the entrant's online store. Given these indifferent consumers, one can easily derive the firms' profit functions.

Proposition 3.3 *When the two incumbent firms have committed not to use the Internet channel and a pure internet retailer has entered, the subgame equilibrium prices are given by*[14]

$$p_{1c} = p_{2c} = \frac{(1+\beta)t + 2(1-\lambda)v}{6}$$

$$p_{3E} = \frac{(1-\frac{1}{2}\beta)t - (1-\lambda)v}{3}.$$

Operating profits are given by

$$\begin{aligned} \pi_1 &= \pi_2 = \frac{[(1-\lambda)v + (\frac{1}{2}+\frac{1}{2}\beta)t]^2}{9t} \\ \pi_3 &= \frac{2[(1-\frac{1}{2}\beta)t - (1-\lambda)v]^2}{9t} \end{aligned}$$

The Proposition shows that entry by a pure online retailer forces incumbent bricks-and-mortar firms to lower their prices considerably: straightforward calculations show that the subgame perfect equilibrium prices are smaller than $\frac{t}{2}$ implying that they are more than two times smaller than without the presence of the pure online retailer. In this

[14] *Note that equilibrium prices are nonnegative for* $-1/2 \leq \alpha \leq 1/2$.

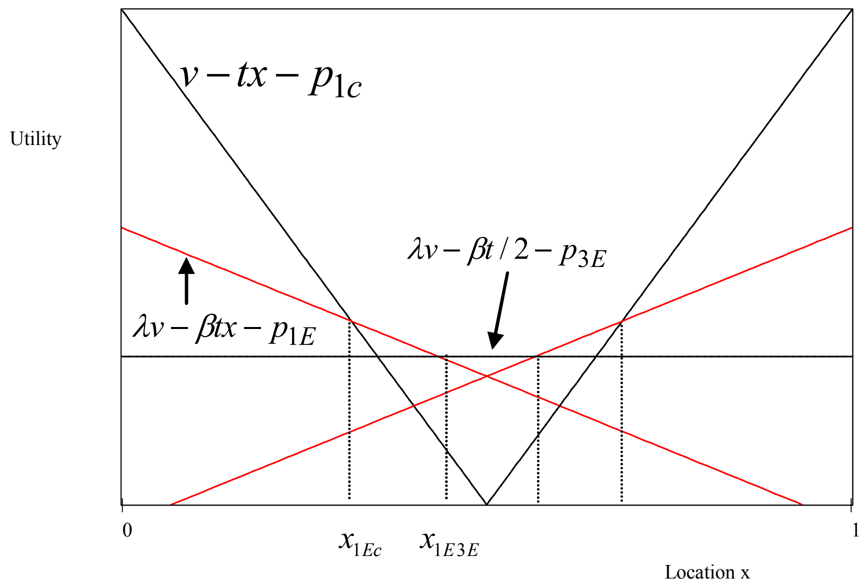


Figure 3-4: Clicks-and-mortar vs. pure Internet retailer.

case, the entrant has a monopoly position online and because of this may even enter with higher prices than the incumbent firms who have an established reputation. Online prices are larger whenever $(2-\beta)t - 2(1-\lambda)v > (1+\beta)t + 2(1-\lambda)v$. This can be rewritten as: $\alpha < \frac{1-2\beta}{4(1-\beta)}$. This inequality is satisfied whenever α and β are both relatively small, i.e., in sectors where the online inconvenience is perceived to be fairly small (or online shopping is considered to be more convenient) and products are close substitutes. This has a clear economic significance. In case β is small, consumers do not have a clear preference for one of the incumbents' brands; they mainly dislike travelling to the conventional shop. The online retailer benefits in case the online shopping convenience is not too bad. Note that it is in the interest of the entrant to increase online shopping convenience if possible.

3.4.2 Clicks-and-Mortar vs. Pure Internet Retailer

Let us now consider the case where all firms opened the possibility of selling through the online channel. In case $\lambda < 1$, Figure 3-4 applies for relevant values of the parameters and prices, where (in addition to the notation used earlier) x_{1E3E} denotes the consumer who is indifferent between buying from the online channel of firm 1 and the online channel of firm 3. Consumers located relatively close to $1/2$ buy from firm 3.

As firms 1 and 2 are symmetric with respect to each other, we only concentrate on firm 1. Using this notation and the figure one can derive that the indifferent consumers are given by

$$x_{1Ec} = \frac{(1-\lambda)v - (p_{1c} - p_{1E})}{(1-\beta)t} \text{ and } x_{1E3E} = \frac{1}{2} - \frac{p_{1E} - p_{3E}}{\beta t}.$$

Using these equations and the derivations in the Appendix, we can state the following Proposition.

Proposition 3.4 *If* $\lambda < 1$,[15] *the unique symmetric subgame equilibrium prices are as follows in case all three firms have opened an online retail channel :*

$$p_{1E} = p_{2E} = \frac{\beta t}{3} \tag{3.1}$$

$$p_{1c} = p_{2c} = \frac{\beta t}{3} + \frac{(1-\lambda)v}{2}, \tag{3.2}$$

and

$$p_{3E} = \frac{\beta t}{6}. \tag{3.3}$$

Some consumers buy via the incumbents Internet channel. Operating profits are given by

$$\begin{aligned} \pi_1 &= \pi_2 = \frac{\beta t}{9} + \frac{[(1-\lambda)v]^2}{4(1-\beta)t} \\ \pi_3 &= \frac{\beta t}{18}. \end{aligned}$$

With three retailers with online presence, online prices are lower than prices in the conventional retail channel. Moreover, the fact that pure online retailers do not have an established reputation, forces them to gain market share by setting quite low prices: their equilibrium prices are just half of the online prices of incumbent firms and these are again lower than their conventional prices. Also, when β is close to 0, online prices are close to

[15] *In case $\lambda > 1$, it easily follows that the conventional channel will not be used by the incumbent firms and the equilibrium profits of these firms will be simply $\beta t/9$. All other expressions remain the same.*

0 as well: in this case, products are almost homogeneous and due to a lack of locational difference, online competition is very severe.[16]

3.4.3 Asymmetric Incumbents

The last case to consider in the presence of entry is when one incumbent has set up an online retail channel, say firm 1, and the other has not. The profit functions in this case are given by the following equations:

$$\begin{aligned}
\pi_1 &= \frac{(1-\lambda)v-(p_{1c}-p_{1E})}{(1-\beta)t}(p_{1c}-p_{1E})+p_{1E}(\frac{1}{2}-\frac{p_{1E}-p_{3E}}{\beta t});\\
\pi_2 &= \frac{(1-\lambda)v-(p_{2c}-p_{3E})+\frac{\beta t}{2}}{t}p_{2c};\\
\pi_3 &= [\frac{1}{2}-\frac{(1-\lambda)v-(p_{2c}-p_{3E})+\frac{\beta t}{2}}{t}+\frac{p_{1E}-p_{3E}}{\beta t}]p_{3E};
\end{aligned}$$

Using the derivations in the Appendix, we can state the following Proposition.

Proposition 3.5 *In the asymmetric case where only incumbent firm 1 and the entrant have an online retail channel and firm 2 not, the unique symmetric subgame equilibrium prices are, in case $\lambda < 1$,[17] given by:*

$$p_{1E} = \frac{\beta}{1+\beta}[\frac{(3+\beta)t}{6}-\frac{(1-\lambda)v}{6}]; \tag{3.4}$$

$$p_{1c} = p_{2c} = \frac{\beta}{1+\beta}\frac{(3+\beta)t}{6}+\frac{3+2\beta}{1+\beta}\frac{(1-\lambda)v}{6}; \tag{3.5}$$

and

$$p_{3E} = \frac{\beta}{1+\beta}[\frac{(3-\beta)t}{6}-\frac{(1-\lambda)v}{3}]. \tag{3.6}$$

[16] It is easy to see that $x_{1Ec} \leq 1/3$ whenever $\alpha \leq 2/3$, which is always satisfied given the assumption on α we imposed.

[17] *When $\lambda > 1$, there are two subcases two consider. In both cases, as before, the incumbent firm with an online channel decides not to use its conventional retail channel implying equation (5) stops being relevant. The first subcase arises when the expression for p_{2c} remains positive. This is the case as long as $\alpha(3+2\beta)(1-\beta)+\beta(3+\beta) > 0$. Apart from the fact that we have to delete the expression $\frac{[(1-\lambda)v]^2}{4(1-\beta)t}$ in the equilibrium profits of the incumbent, all other expressions remain unchanged. When $\alpha(3+2\beta)(1-\beta)+\beta(3+\beta) < 0$, firm 2 cannot make positive profits and drops out of the market altogether. One can easily check that in this case, the equilibrium prices are $p_{1E} = p_{3E} = \beta t/2$ and equilibrium pofits are given by $\pi_1 = \pi_3 = \beta t/4$. One may easily verify that given the condition on α and β, even the consumer located at 1 prefers buying from the entrant at this price (yielding a pay-off of $\lambda v - \beta t$) to getting the good for free from firm 2 (yielding a pay-off of v).*

Operating profits in this case are given by

$$\begin{aligned}
\pi_1 &= \frac{[(1-\lambda)v]^2}{4(1-\beta)t} + \frac{\beta}{36(1+\beta)^2 t}[(3+\beta)t - (1-\lambda)v]^2;\\
\pi_2 &= \frac{1}{36(1+\beta)^2 t}[(3\beta+\beta^2)t + (3+2\beta)(1-\lambda)v]^2;\\
\pi_3 &= \frac{\beta}{36(1+\beta)t}[(3-\beta)t - 2(1-\lambda)v]^2.
\end{aligned}$$

With asymmetric incumbent firms there are again some interesting results that deserve some further elaboration. First, in case online shopping is considered to be less convenient ($\alpha > 0$) both incumbents charge the same equilibrium offline price. This price is simply equal to the online price plus the mark-up $\frac{(1-\lambda)v}{2}$ we have seen in previous cases. Second, the entrant's online price can never be larger than the incumbent's online price. From equations (3.4) and (3.6) one can conclude that the entrant's online price is larger in case $2\beta + \alpha(1-\beta) < 0$. However, in this case the condition that $p_{2c} \geq 0$ is violated, implying that the equilibrium online prices of both firms equal $\beta t/2$ (see footnote 18). We can therefore conclude already that the entrant's (online) prices can only be larger than the incumbents' prices in case the entrant has a monopoly position online, the case examined in subsection 3.4.1.

3.5 Equilibrium Market Structures: Only Bricks or also Clicks?

We now analyze the first stage of the game and ask the question what are the incentives of firms to develop an infrastructure to sell online at a fixed cost f. In other words, under what conditions on the exogeneous parameters can which of the above market structures arise? We consider two different ways the first stage may be played. First, we analyze the case where the incumbents decide whether or not to start an online channel before the entrant does. Here, we ask the question whether starting an online channel may be a way to deter a pure online player to enter. Second, we analyze the case where the entrant decides whether or not to start an online channel before the incumbents do.

Before we delve into the implications of these two different decision sequences, we provide some analysis that is used in both cases. We first summarize the pay-offs in the different cases *where the entrant always enters* in the following two matrices, where due to the symmetry between firm 1 and 2, we have only one matrix for both of them.[18]

		2	
	y/n	n	y
1	n	$\frac{[(1-\lambda)v+(\frac{1}{2}+\frac{1}{2}\beta)t]^2}{9t}$	$\frac{[(3\beta+\beta^2)t+(3+2\beta)(1-\lambda)v]^2}{36(1+\beta)^2t}$
	y	$\frac{[(1-\lambda)v]^2}{4(1-\beta)t}+\frac{\beta[(3+\beta)t-(1-\lambda)v]^2}{36(1+\beta)^2t}-f$	$\frac{\beta t}{9}+\frac{[(1-\lambda)v]^2}{4(1-\beta)t}-f$

,

Table 3-2. Pay-offs for firm 1 in case $\lambda < 1$ depending on whether or not itself and its rival incumbent start an online channel and given that a pure online retailer enters.[19]

		2	
	y/n	n	y
1	n	$\frac{2[(1-\frac{1}{2}\beta)t-(1-\lambda)v]^2}{9t}-f$	$\frac{\beta[(3-\beta)t-2(1-\lambda)v]^2}{36(1+\beta)t}-f$
	y	$\frac{\beta[(3-\beta)t-2(1-\lambda)v]^2}{36(1+\beta)t}-f$	$\frac{\beta t}{18}-f$

,

Table 3-3. Pay-offs for firm 3 in case $\lambda < 1$ depending on which of the incumbent firms starts an online channel and given that itself enters.[20]

On the basis of Table 3-3, we can easily argue that firm 3's incentive to enter are monotone in the number of firms that have an online channel: the more incumbents have an online retail channel, the less operating profits a pure online retailer can get.[21]

Looking at the matrices, one can see that the parameters λ and v only enter together in the combination $(1-\lambda)v$. Using the definition of α we can therefore substitute this

[18] Note that the cells are defined in terms of the actions taken by firms 1 and 2. Hence, this is not a proper pay-off matrix.

[19] In case $\lambda > 1$ the term $\frac{[(1-\lambda)v]^2}{4(1-\beta)t}$ has to be deleted from the two cells in the bottom row. In case $\lambda > 1$ and, in addition, $\alpha(3+2\beta)(1-\beta)+\beta(3+\beta) < 0$, the bottom left cell will have to be replaced by $\beta t/4 - f$ (see footnote 17) and the upper right cell becomes zero.

[20] In case $\lambda > 1$ and $\alpha(3+2\beta)(1-\beta)+\beta(3+\beta) > 0$, the expressions remain identical. In case $\lambda > 1$ and $\alpha(3+2\beta)(1-\beta)+\beta(3+\beta) < 0$, the bottom left cell will have to be replaced by $\beta t/4 - f$ (see footnote 17).

[21] See the Appendix for the calculations.

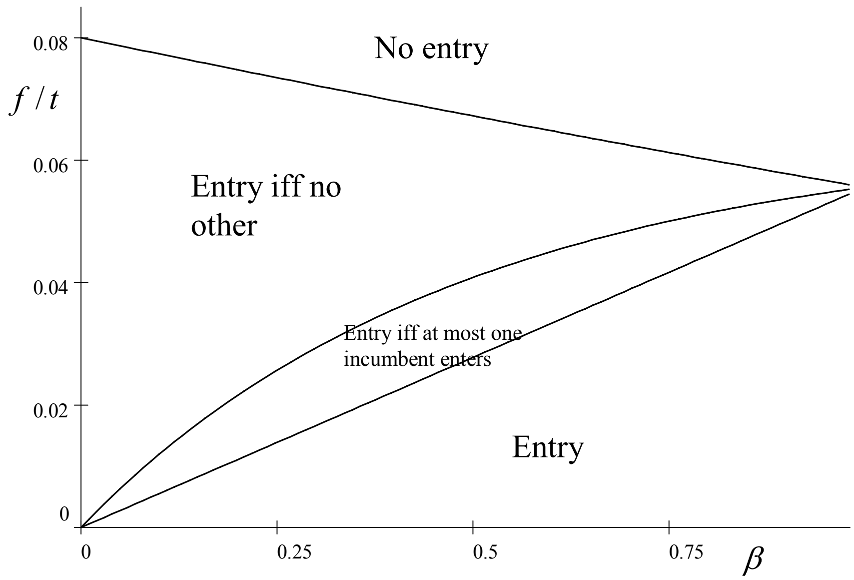


Figure 3-5: Incentives for firm 3 to enter, $\alpha = 0.4$.

expression without loss of generality for $\alpha(1-\beta)t$, where α is relatively small (large) when the shopping inconvenience or the willingness-to-pay is relatively small. Once the substitution is made, one can also see that all operating profits are linear in t so that in fact we are left with three relevant parameters: α, β and f/t. Figure 3-5 shows for $\alpha = 0.4$, the monotonicity of the incentives of firm 3 to enter: for any β, if f/t is relatively large firm 3 will never enter even, not even if no incumbent has built the facilities to sell online. If f/t becomes smaller we first enter an area of parameter values in which firm 3 can only profitably enter when no incumbent firm has an online retail channel. If f/t becomes even smaller we next enter an area of parameter values in which firm 3 can only profitably enter if at most one incumbent firm has an online retail channel. Finally, if f/t is very small firm 3 will always want to enter.

3.5.1 Online Retailing as an Entry Deterrence Strategy

We first consider whether or not incumbent firms have an incentive to make a strategic move by opening an online retail channel first. We have seen that in the absence of an entry threat the incentives to open an online channel are limited to the case where the online shopping conveniences are relatively large. By looking at the case where incumbents decide first whether or not to start an online retail channel before a pure online player decides to enter the market, we therefore inquire whether the possibility to deter entry gives incumbent firms more incentives to open online retail channels. The analysis of this section is facilitated by the fact presented above that the entrant's incentives to enter are monotonic in the number of incumbent firms having an online retail channel at their disposal. It turns out that in line with Figure 3-5, there are four possible cases for the entrant's reaction to the incumbents' online presence. To discuss these cases in an easy way, we denote by $\pi_i(a, b, c)$ the total pay-off (including the cost f of opening an online channel) of player i when player 1, 2 and 3 take decisions a, b and c respectively, where a, b and c are either equal to n (not to start an online retail channel) or y (start an online channel).

First, if the cost of setting up an electronic retail channel are relatively high, $\pi_3(n, n, y) < 0$, the entrant will never enter. In the appendix we show that if the costs are really that high that an entrant will never enter, then incumbents will also not start an online channel. In other words, the incentives to start an online retail channel are larger for a pure online entrant than for an incumbent. This is easily explained by the fact that an incumbent partially competes with its own conventional store when it sets up its own online channel.

Second, if the cost of setting up an electronic retail channel is somewhat smaller, $\pi_3(n, n, y) > 0 > \pi_3(y, n, y)$, the entrant will enter if, and only if no incumbent firm has started an electronic channel. In the appendix it is shown that in this case the only subgame perfect equilibrium has one incumbent incurring the cost of starting an online retail channel. Depending on the parameters, the incumbent firms then may face a free-rider problem whenever both of them prefer the other to incur the cost of deterring entry. Free riding may occur for example when β is close to 1 and α is close to 0 so that product differentiation is mostly due to product heterogeneity and if there is an online shopping inconvenience, it is fairly small. As we have seen in Section 3.3, online prices may be higher in this market structure (when α is small) than average conventional prices due to the fact that one of the incumbents has some online monopoly power. Unlike in Section 3.3, this market structure may now also arise when $\alpha > 0$ as incumbent firms have an incentive to deter entry.

Third, if the cost of setting up an electronic retail channel is even smaller, $\pi_3(y, n, y) > 0 > \pi_3(y, y, y)$, the entrant will enter if and only if, at most one incumbent firm has started an electronic channel. The appendix shows that in this case two things may happen. The more interesting possibility occurs when $\alpha > 0$ and it may also arise when α is not too small. In this case, the incumbent firms face a situation of severe strategic uncertainty in the sense that they only would like to start an online retail channel if the other firm will do likewise so that they together deter entry. This strategic uncertainty is reflected by the fact that the incumbent firms face a coordination game and there are two subgame perfect equilibria: either they both start an online retail channel and together deter entry, or none of them builds an online channel and entry occurs. When entry occurs, as explained in subsection 3.4.1., online prices may be higher than the prices in the conventional stores due to the "monopoly power" of the entrant. When there are large online shopping conveniences, only one online shop owned by the entrant is a very big threat to the incumbents and it becomes a dominant strategy for the incumbent firms to start an online channel irrespective of the behavior of the other incumbent. The net effect in this case is that entry is deterred.

Fourth, if the cost of setting up an electronic retail channel is very small, $f < \frac{\beta t}{18}$, the entrant will always enter. If the cost of setting up the online retail channel is really very low, it may be (as a subcase) that both incumbent firms nevertheless decide to go online, sometimes even when $\alpha > 0$. Hence, even if entry does occur it may facilitate incumbents also to start an online retail channel as without entry they would not do so.

An overview of the different cases presented in this subsection is given in (α, β) space in Figure 3-6 below. The vertical line in the Figure represents the boundary between a region where the entrant always enters (to the right) and a region where the entrant's decision is conditional on what incumbents do. In the left-hand part of the Figure we see the different possibilities to deter entry (either by one firm, or jointly as part of a coordination game or due to dominant strategies) as discussed above. The right-hand part shows that when α decreases it becomes more attractive for incumbents to start an online retail channel as well even though this will not affect the entrant's choice.

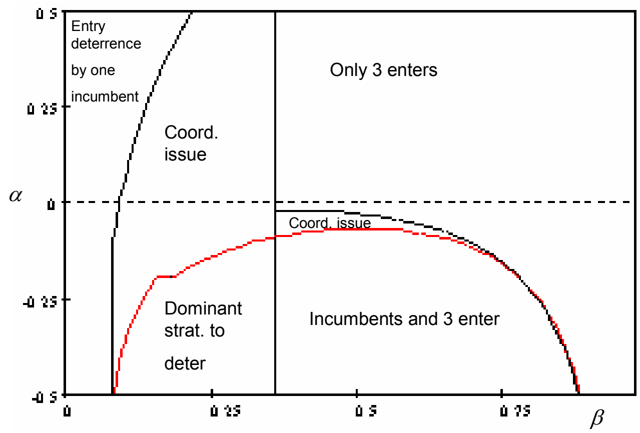


Figure 3-6: Equilibrium structures, $f/t = 0.02$.

3.5.2 Following the entrant's lead

Next, we analyze the case where the entrant is considering whether or not to make a strategic move and invest already early on (i.e., before the incumbents consider doing so) in building an online retail channel. More formally, the entrant now decides first whether or not to enter before the incumbents decide whether or not they follow and start their own retail channel. Of the cases considered in the previous subsection, the first and the last case ($\pi_3(n, n, y) < 0$ and $\pi_3(y, y, y) > 0$) are very similar to the previous analysis where the incumbents decide first and are therefore omitted. So, we concentrate on the intermediate cases where $\pi_3(n, n, y) > 0 > \pi_3(y, y, y)$.

Unlike in the previous subsection, now many possibilities arise and we don't attempt to give a complete characterization. The most obvious cases arise when the incumbents have a dominant strategy either to start or not to start an online channel after the entrant has decided to enter. For quite a few parameter values when $\alpha > 0$, the incumbents have a dominant strategy not too follow the entrant and the market outcome is one where the entrant is the only one having online presence. On the other hand, when α is relatively small, there are many parameters values (especially when β is not too close to 0) for which the incumbents have a dominant strategy also to start an online retail channel whenever the entrant has done so. In this case, the entrant is better off not to start an online channel when $\pi_3(y, y, y) < 0$. Whether or not the incumbents will then set up an online channel depends on the specific parameter values and this case is already analyzed in section 3.3.3.

There are, however, also quite a few other more interesting situations that may occur. First, it may be that the incumbents face a situation of strategic uncertainty, formally represented by a coordination game, following the entrant's decision to go online. This is for example the case in an area around the following parameter constellation: $\alpha = 0, \beta = 0.1$ and $f/t = 0.007$. The incumbent firms would either like to enter both or not at all as $\pi_1(y, y, y) > \pi_1(n, y, y)$ and $\pi_1(n, n, y) > \pi_1(y, n, y)$. Turning back to the decision of the entrant makes clear that when the incumbents face a coordination problem, the entrant also faces a very risky decision: if incumbents will not follow, it can make a profit starting an online channel as $\pi_3(n, n, y) > 0$. However, if incumbents follow suit, the entrant cannot recover its start-up costs and goes bankrupt as $\pi_3(y, y, y) < 0$. As the entrant cannot logically predict what the incumbents will do, as both possibilities may occur as equilibrium responses, the entrant cannot really figure out what to do. We may interpret this theoretical possibility as one of the possible rational explanations for the shake-out we have seen in recent years in the online business world: Entrants may have expected that incumbent brick-and-mortar firms will not follow and they had good reasons to expect so as not going online is an equilibrium response by incumbent firms. However, there exists another equilibrium path where incumbents do go online and where entrants make losses. The model tells that this possibility arises when both α and β are close to 0, i.e., when the online shopping inconvenience is fairly small and goods are relatively homogeneous. Interestingly, these are exactly the sectors where one may expect online retailing to be quite successful!

A second interesting observation is that the number of firms deciding to start an online retail channel may very well be non-monotonic in the cost of building the infrastructure to sell online. The following parameter constellation serves as an example. For $\alpha = 0, \beta = 0.3$ and $f/t = 0.10$, it is optimal for the entrant to enter and start a pure online shop as

$\pi_3(n,n,y) > 0$, knowing that the incumbents will not follow suit as $\pi_1(y,n,y) < \pi_1(n,n,y)$ and $\pi_1(y,y,y) < \pi_1(n,y,y)$. However, when f/t drops to $f/t = 0.05$, ceteris paribus, one incumbent will find it optimal also to start an online retail channel following the entrant's decision to do so as $\pi_1(y,n,y) > \pi_1(n,n,y)$. This, however, makes it unprofitable for the entrant to start an online retail channel in the first place as $\pi_3(y,n,y) < 0$. When the entrant does in fact abstain from entering (as it is able to accurately predict the incumbent's response), the incumbents will find it optimal not to start an online channel themselves. Thus, when $f/t = 0.05$, no online retail channel is built: the threat to enter in this case is, in a sense, incredible in case the incumbents can respond by going online themselves! When the cost of building an online retail channel falls further to, for example, $f/t = 0.03$ the entrant will find it optimal to enter again ($\pi_3(n,n,y) > 0$) even if it knows that an incumbent will do likewise.[22] Thus, this numerical example shows that it is not generally true that by subsidizing the building of online retail channels, more online retail channels will be built.

A similar situation may arise with respect to the online shopping convenience perceived by the consumers. For $\alpha = 0, \beta = 0.4$ and $f/t = 0.07$, it is optimal for the entrant to enter and start a pure online shop, knowing that the incumbents will not follow suit. However, when the online shopping inconvenience drops to $\alpha = -0.3$, keeping the other parameter values equal, one incumbent will find it optimal to go online as well, making the entrant's decision to do so unprofitable. As above, when the entrant does not enter in the first place, the incumbents will not go online at all. This makes the point that it is not generally true that increasing the online shopping convenience as perceived by consumers, will increase the possibility that firms build an online presence. Hence, increasing online shopping convenience may actually not be in the interest of consumers!

[22] The implicit point in the above discussion is that for some parameter configurations the incumbent's decision to go online is positively affected by the entrant's presence online. This can be explained by the fact that by doing so an incumbent will want to recover some of the business it has lost to the entrant.

3.6 Conclusions and Managerial Implications

In this paper we have explored the implications of the possibility for firms of building a retail channel online. The setting we analyze is one where two incumbent firms compete for costumers in a market with horizontally differentiated products. Firms are differentiated because they sell differentiated products and because their physical locations are different. In the online market, only the first type of differentiation plays a role. Consumers know which of the two incumbent firms sell products closest to their tastes. They can decide to go either to the conventional store of a firm or they can go online. Consumers find one aspect of online shopping certainly better than conventional shopping, which is the fact that they do not have to travel a physical distance. Apart from that, consumers may find online shopping more or less convenient from going to a conventional shop depending on whether or not they feel confident about things like payment method, delivery time and the thrustworthiness of redemption policies.

Firms play a two-stage game. In the first stage they decide whether or not to build the infrastructure needed to sell online at a fixed cost. The interesting cases arise when this fixed cost is not prohibitively high so as to make it unprofitable to start an online retail channel. The main question at this stage is what are the incentives for a bricks-and-mortar firm to go online and what are the incentives to enter for a pure online player. In the second stage, firms decide which prices to charge on different retail channels.

The more interesting managerial implications are as follows. First, bricks-and-mortar firms are worse off if they open an online retail channel as the cannibalization effect (stealing consumers from its own conventional store at lower prices) is likely to be more important than the business stealing effect (attracting demand from rival firms). When online shopping is considered to carry some conveniences, incumbent firms also will not have an incentive to start an online retail channel. When there are large online shopping conveniences, firms may individually have an incentive to go online, but total profits decline as online competition is more severe due to the absence of differentiation in location. In some cases, firms may in fact face a Prisoner's Dilemma.

A second managerial implication relates to the question who benefits from online shopping conveniences. By increasing the online shopping conveniences (or reducing the inconveniences), firms are able to direct more of their costumers through the online channel. If there are no important cost advantages related to this shift and there is no (threat of) entry, incumbent firms will be worse off as more bricks-and-mortar firms will open an online retail channel and competitive pressure online is larger. When there is a threat of entry, however, increasing online shopping convenience may be a way for incumbent firms to credibly commit to start an online channel if pure only players do so. In such situations, online players may abstain from entry as entry will be followed by an incumbent going online, making the entry unprofitable. These are situations where incumbent firms may benefit from online shopping conveniences at the expense of entrants and consumers.

A third implication relates to the viability of pure online retailers. Our analysis shows that in most circumstances these retailers have to charge fairly low prices in order to get a fair market share. The operating profits of online retailers are therefore relatively low as they do not benefit from a bricks-and-mortar reputation. Moreover, they may find it difficult to predict the response of incumbent firms to their entering the market. There are situations when incumbent firms face a coordination problem after entry. This makes the entry decision a risky decision as the consequences in terms of profitability are difficult to predict. Moreover, it may be that incumbent firms will face lower cost of building up an online retail channel as they may learn from the entrant's experience. We have seen how sensitive the entrant's decision problem is to the non-monotonicity in the cost of building an online retail channel. All these factors question the viability of pure online retailers, especially in markets where product differentiation is largely due to the geographical location of bricks-and-mortar firms.

A last managerial implication relates to a firm's pricing policy. Online prices depend on the type of firm: a clicks-and-mortar firm charges higher online prices than a pure online player. Also, online prices depend on the market structure and on the size of the online shopping convenience. Online prices will only be higher than prices in the conventional stores if firms have some market power online, which is partly founded by the size of the online shopping convenience. In future research it would be interesting to see whether this theoretical prediction can be empirically verified.

The results may explain the fact that online shopping has not become as important a retail channel as many believed at the end of the previous millennium. The online shop may cannibalize a firm's own costumer base and when it does not, it may increase competition between firms if everyone builds an online retail channel. Managers should be aware of these facts and should weigh the importance of them against the possibility that new costumers are attracted to the market by using an alternative retail channel. In future theoretical work, it would be interesting to see to what extent the conclusion drawn here for a more or less saturated market continue to hold when the online retail channel is able to attract new costumers.

3.7 Appendix

First-order conditions Section 3.3.2.

There are three first-order conditions, one for firm 2 and two for firm 1:

$$\begin{aligned} p_{1c} &= p_{1E} + \frac{(1-\lambda)v}{2}; \\ 0 &= t - (1-\lambda)v + p_{2c} - 2p_{1E}; \\ 0 &= \beta t - (1-\lambda)v - 2p_{2c} + p_{1E}; \end{aligned}$$

Solving these three equations for the three unknown prices, yield the subgame equilibrium prices given in the main text.

Calculations establishing firms have a dominant strategy in Section 3.3.3 when $\lambda < 1$

When firm 2 does not start an online channel, it is better for firm 1 to do likewise iff

$$\frac{1}{2} \geq \frac{\alpha^2(1-\beta)}{4} + \frac{[2-\alpha+(1+\alpha)\beta]^2}{9(1+\beta)} - f/t.$$

This inequality certainly holds if it holds for $f = 0$. In this case, the inequality can be rewritten as

$$18(1+\beta) \geq 9\alpha^2(1-\beta^2) + 4[2-\alpha+(1+\alpha)\beta]^2.$$

It is easy to verify that this inequality holds true for both $\beta = 1$ (with equality) and $\beta = 0$. As the coefficient in front of the β^2 term of the RHS is positive for $\alpha > 0$, the inequality has therefore to hold for all $0 \leq \beta \leq 1$.

When firm 2 starts an online channel, it is better for firm 1 not to do likewise iff

$$\frac{[1+\alpha+(2-\alpha)\beta]^2}{9(1+\beta)} \geq \frac{\beta}{2} + \frac{\alpha^2(1-\beta)}{4} - f/t.$$

For $f = 0$, this reduces to

$$4[1+\alpha+(2-\alpha)\beta]^2 \geq 18\beta(1+\beta) + 9\alpha^2(1-\beta^2).$$

It is easy to verify that this inequality holds true for both $\beta = 1$ (with equality) and $\beta = 0$. If we bring all terms to the LHS the coefficient in front of β^2 is negative, implying that the inequality has to hold for all $0 \leq \beta \leq 1$.

Calculations establishing the equilibrium structure in Section 3.3.3 when $\lambda > 1$

When firm 2 does not start an online channel, it is better for firm 1 to do likewise iff

$$\frac{1}{2} \geq \frac{[2-\alpha+(1+\alpha)\beta]^2}{9(1+\beta)} - f/t.$$

In this case, the inequality can be rewritten as

$$\left(1+2\frac{f}{t}\right)9(1+\beta) \geq 2[2-\alpha+(1+\alpha)\beta]^2.$$

The upper curve in Figure 3-2 depicts this relation in case we replace the inequality by an equality sign and $f/t = 0.01$.

When firm 2 starts an online channel, it is better for firm 1 not to do likewise iff

$$\frac{[1+\alpha+(2-\alpha)\beta]^2}{9(1+\beta)} \geq \frac{\beta}{2} - f/t.$$

The lower curve in Figure 3-2 depicts this relation when $f/t = 0.01$ and when replacing the inequality by an equality sign.

First-order conditions Section 3.4.1.

There are three first-order conditions, one for each firm. As firms 1 and 2 are in a symmetric position and we are looking for symmetric equilibria, we only give the first-order conditions for firm 1 and 3:

$$\begin{aligned}(1-\lambda)v - 2p_{1c} + p_{3E} + \frac{\beta t}{2} &= 0;\\ t - 2(1-\lambda)v + p_{1c} + p_{2c} - 4p_{3E} - \beta t &= 0;\end{aligned}$$

Imposing symmetry ($p_{1c} = p_{2c}$) we can solve these two equations and obtain the subgame equilibrium prices given in Proposition 3.3.

First-order conditions Section 3.4.2.

In case $\lambda < 1$, there are five first-order conditions, two for firms 1 and 2 and one for firm 3. As firms 1 and 2 are in a symmetric position and we are looking for symmetric equilibria, we only give the first-order conditions for firm 1 and 3, taking p_{1E} and $p_{1c} - p_{1E}$ as choice variables:

$$\begin{aligned}p_{1c} - p_{1E} &= \frac{(1-\lambda)v}{2};\\ \frac{1}{2} - \frac{2p_{1E} - p_{3E}}{\beta t} &= 0;\\ p_{1E} + p_{2E} - 4p_{3E} &= 0;\end{aligned}$$

Imposing symmetry ($p_{1c} = p_{2c}$ and $p_{1E} = p_{2E}$) we can solve these three equations and obtain the subgame equilibrium prices given in the main text.

First-order conditions Section 3.4.3.

In case $\lambda < 1$, there are four first-order conditions, two for firm 1, one for firm 2 and one for firm 3:

$$
\begin{aligned}
p_{1c} - p_{1E} - \frac{(1-\lambda)v}{2} &= 0; \\
\frac{1}{2} - \frac{2p_{1E} - p_{3E}}{\beta t} &= 0; \\
(1-\lambda)v - 2p_{2c} + p_{3E} + \frac{\beta t}{2} &= 0; \\
\frac{1}{2} - \frac{(1-\lambda)v - p_{2c} + 2p_{3E} + \frac{\beta t}{2}}{t} + \frac{p_{1E} - 2p_{3E}}{\beta t} &= 0;
\end{aligned}
$$

The second and third conditions together imply that $p_{2c} = p_{1E} + \frac{(1-\lambda)v}{2}$. It is important to note that this is independent of whether or not the first-order condition holds, i.e., in case $\lambda > 1$ and the first condition does not hold the second firm's offline price is still the usual mark-up over the online price. Substituting this equation for p_{2c} into the fourth first-order condition and using the second yields

$$
\frac{1}{2} - \frac{\frac{(1-\lambda)v}{2} + \frac{3}{2}p_{3E} + \frac{\beta t}{4}}{t} + \frac{\frac{1}{2}p_{3E} + \frac{\beta t}{4} - 2p_{3E}}{\beta t} = 0.
$$

This can be solved for the subgame perfect equilibrium value of p_{3E} given in the Proposition. Substituting this value into the second and third condition yields the subgame perfect equilibrium values of p_{1E} and p_{2c}. The expressions for equilibrium profits, can be obtained by substituting the equilibrium prices in the expression for profits given in the text.

Section 3.5: The entrant's incentive to enter are monotone in the number of incumbents with an online channel

On the basis of Table 3-3 we first show for the case when $\lambda < 1$ or the case where $\lambda > 1$ and $\alpha(3+2\beta)(1-\beta) + \beta(3+\beta) > 0$ that the operating profits of the entrant are larger when no incumbent has an online channel than when one of them has one. This is the case when[23]

$$
8(1+\beta)[1-\alpha+(\alpha-\frac{1}{2})\beta]^2 > \beta[3-2\alpha+(2\alpha-1)\beta]^2.
$$

[23] In case $\lambda > 1$ and $\alpha(3+2\beta)(1-\beta) + \beta(3+\beta) < 0$, the inequality takes the form $8(1+\beta)[1-\alpha+(\alpha-\frac{1}{2})\beta]^2 > 9\beta$. Straightforward calculations show that whenever this inequality is relevant, it holds.

As $\beta/(1+\beta) < 1/2$, it follows that this is certainly the case when

$$1-\alpha+(\alpha-\frac{1}{2})\beta > \frac{3-2\alpha+(2\alpha-1)\beta}{4}.$$

One can easily verify that this is the case if $\alpha < 1/2$.

Next, we show that the operating profits of the entrant are larger when one incumbent has an online channel than when both of them have one.[24] This is the case when

$$[3-2\alpha+(2\alpha-1)\beta]^2 > 2(1+\beta).$$

As the LHS is decreasing in α, this inequality certainly holds if it holds for $\alpha = 1/2$. Substituting this into the inequality yields $4 > 2(1+\beta)$, which holds for all $\beta < 1$.

Section 3.5.1: If f is to large for the entrant to enter in case the incumbents have no online retail channel, then incumbent will not start an online retail channel either

We have to show that $\pi_3(n,n,y) < 0$ implies $\pi_1(y,n,n) < \pi_1(n,n,n)$. This is the case when $\frac{2[(1-\frac{1}{2}\beta)t-\alpha(1-\beta)]^2}{9} < f/t$ implies $\frac{[(2+\beta)t-\alpha(1-\beta)]^2}{9(1+\beta)} - f/t < \frac{1}{2}$. This implication holds true if

$$\begin{aligned} & (1+\beta)\{2-2\beta+\frac{1}{2}\beta^2-4\alpha(1-\beta)(1-\frac{1}{2}\beta)+\alpha^2(1-\beta)^2\}+\frac{9}{2}(1+\beta) \\ > \ & 4+4\beta+\beta^2-4\alpha(1-\beta)(1+\frac{1}{2}\beta)+\alpha^2(1-\beta)^2. \end{aligned}$$

This inequality can be rewritten as $2\frac{1}{2}+\frac{1}{2}\beta-2\frac{1}{2}\beta^2+\frac{1}{2}\beta^3+2\alpha\beta^2+\alpha^2\beta(1-\beta)^2 > 0$. The terms involving α reach a minimum value of -β^3 at $\alpha = -\beta/(1-\beta)$. Given this, it is easily seen that the inequality always holds true.

Section 3.5.1: Entry deterrence when the entrant enters if, and only if, no incumbent sells online

We have to show that $\pi_3(n,n,y) > 0$ implies $\pi_1(y,n,n) > \pi_1(n,n,y) > 0$. In case $\lambda < 1$ (and when $\lambda > 1$) this implication certainly holds true if $\frac{2[(1-\frac{1}{2}\beta)t-(1-\lambda)v]^2}{9t} > f$

[24]In this case the relevant expressions for the entrant's pay-off are independent of whether or not $\lambda < 1$.

implies $\frac{[(2+\beta)t-(1-\lambda)v]^2}{9(1+\beta)t} - f > \frac{[(1-\lambda)v+(\frac{1}{2}+\frac{1}{2}\beta)t]^2}{9t}$. This can be rewritten as

$$\frac{[2-\alpha+(1+\alpha)\beta]^2}{9(1+\beta)} > \frac{[\frac{1}{2}+\alpha+(\frac{1}{2}-\alpha)\beta]^2}{9} + \frac{2[1-\alpha+(\alpha-\frac{1}{2})\beta]^2}{9}$$

The LHS of this inequality is certainly larger than $(2-\alpha)^2/9$, while the RHS is certainly smaller than $(1+2(1-\alpha)^2/9$. It is easily seen that $(2-\alpha)^2 > 1+2(1-\alpha)^2$ for all values of $\alpha < 1/2$.

Coordination issue between incumbents when the entrant enters if, and only if, at most one incumbent firm has started an electronic channel (Section 3.5.1).

We have to show two things: (i) $\pi_3(y,y,y) < 0$ and $\alpha > 0$ implies $\pi_1(n,n,y) > \pi_1(y,n,y)$ and (ii) $\pi_3(y,n,y) > 0$ implies $\pi_1(n,y,y) < \pi_1(y,y,n)$

(i) The first implication says $f > \frac{\beta t}{18}$ implies $\frac{[(1-\lambda)v+(\frac{1}{2}+\frac{1}{2}\beta)t]^2}{9t} > \frac{[(1-\lambda)v]^2}{4(1-\beta)t} + \frac{\beta[(3+\beta)t-(1-\lambda)v]^2}{36(1+\beta)^2 t} - f$. We think of the LHS and the RHS of this inequality as function of α. We prove three facts: (a) When $\alpha = 0$, the implication holds true if the following inequality holds: $\frac{(1+\beta)^2}{36} > \frac{\beta(3+\beta)^2}{36(1+\beta)^2} - \frac{\beta}{18}$, which can be rewritten as $1 - 3\beta + 4\beta^2 + 5\beta^3 + \beta^4 > 0$, which holds for all $0 \leq \beta \leq 1$; (b) When $\alpha = 1/2$, the implication holds true if the following inequality holds: $\frac{1}{9} > \frac{\beta(5+3\beta)^2}{144(1+\beta)^2} + \frac{1-\beta}{16} - \frac{\beta}{18}$, which can be rewritten as $16(1+\beta)^2 > \beta(5+3\beta)^2 + 9(1-\beta) - 8\beta$, or, $-\beta^3 - 14\beta^2 + 24\beta + 7 > 0$, which also holds for all $0 \leq \beta \leq 1$; (c) Further, it is easy to evaluate the derivative of both LHS and RHS with respect to α. The derivative of the LHS is positive on the relevant domain of α, whereas the derivative of the RHS is first negative and then positive. These three facts together imply that the implication always holds.

When $\lambda > 1$ and $\alpha(3+2\beta)(1-\beta) + \beta(3+\beta) > 0$, the implication does not need to hold. To see this note that the implication holds if, and only if, $\frac{[\alpha(1-\beta)+(\frac{1}{2}+\frac{1}{2}\beta)t]^2}{9} > \frac{\beta[(3+\beta)t-\alpha(1-\beta)]^2}{36(1+\beta)^2} - \frac{\beta}{18}$. One may check that for example when $\alpha = -1/2$ and $\beta = 0.4$ the inequality does not hold.

(ii) In case $\lambda < 1$ the second implication certainly holds true if $\frac{\beta[(3-\beta)t-2(1-\lambda)v]^2}{36(1+\beta)t} > f$ implies $\frac{[(3\beta+\beta^2)t+(3+2\beta)(1-\lambda)v]^2}{36(1+\beta)^2 t} < \frac{\beta t}{2} + \frac{[(1-\lambda)v]^2}{4(1-\beta)t} - f$. This implication holds true if

$$\frac{\beta}{2} + \frac{\alpha^2(1-\beta)}{4} > \frac{[(3\beta+\beta^2)+\alpha(3+2\beta)(1-\beta)]^2}{36(1+\beta)^2} + \frac{\beta[3-2\alpha+2(\alpha-1)\beta]^2}{36(1+\beta)}$$

As the last term of the RHS is decreasing in β, this term is smaller than $\beta/4$ when $\alpha > 0$, and thus, this inequality holds if

$$\frac{\beta}{4} + \frac{\alpha(1-\beta)}{4}\left(\alpha - \frac{\alpha(3+2\beta)^2(1-\beta)}{9(1+\beta)^2} - \frac{2\beta(3+2\beta)(3+\beta)}{9(1+\beta)^2}\right) > \frac{\beta^2(3+\beta)^2}{36(1+\beta)^2}.$$

As the term on the LHS in brackets is increasing in α, this is certainly the case when

$$\frac{\beta}{4} - \frac{\alpha(1-\beta)}{4}\frac{2\beta(3+2\beta)(3+\beta)}{9(1+\beta)^2} > \frac{\beta^2(3+\beta)^2}{36(1+\beta)^2}.$$

As $\alpha < 1/2$, it is a straightforward exercise to show this is the case.

Similar calculations hold when $\lambda > 1$ and $\alpha(3+2\beta)(1-\beta) + \beta(3+\beta) > 0$.[25] In this case the implication holds when

$$\frac{\beta}{2} > \frac{[(3\beta+\beta^2) + \alpha(3+2\beta)(1-\beta)]^2}{36(1+\beta)^2} + \frac{\beta[3-2\alpha+2(\alpha-1)\beta]^2}{36(1+\beta)}.$$

As the first term of the RHS is increasing in α and as $0 > \alpha > -1/2$, this inequality certainly holds if

$$\frac{1}{2} > \frac{\beta(3+\beta)^2}{36(1+\beta)^2} + \frac{16\beta}{36(1+\beta)}.$$

It is straightforward to show that this is always the case.

[25] When $\lambda > 1$ and $\alpha(3+2\beta)(1-\beta) + \beta(3+\beta) < 0$ the implication reads as $\beta t/4 - f > 0$ implies $\beta t/2 - f > 0$,which obviously holds true.

Chapter 4

Discrimination in Insurance

4.1 Introduction

When purchasing life insurance, insurees are often required to do a medical test, the result determining at what terms one can be insured, if at all. There is ungoing debate about insurers' information use, for example about genetic discrimination and risk selection (Lemmens 2003). Insurers usually motivate information use by stating that it is a necessary underwriting strategy, since they suffer from adverse selection: insurees have private information about their risk profile and base their decision to buy insurance on this information. Consequently, in a heterogeneous population only the high-risks will purchase insurance. Even though adverse selection is mitigated by this underwriting, the loss of insurance coverage for the high-risks forms a welfare loss. We develop a model of adverse selection in which the two opposing effects are incorporated. Insurers may obtain information about an applicant's risk profile by using some screening technology. We postulate that if the applicant possesses a certain trait (e.g. a gene mutation or virus), his or her risk profile exceeds some upperbound. The objective is to compare welfare in an economy where risk profiles above this upperbound are rejected with an economy in which this is not the case.

We address this question in a setting that has not been studied before. One of the main novelties of our model is that we use a continuum of risk types. The advantage of such a realistic setup is that asymmetric information remains after excluding some of the highest risks. Furthermore, it provides us with a continuous relation between the demand for insurance and important market variables, such as risk aversion. The equilibrium we find has more risk types than contracts. This is not the case in the existing contributions where individuals come in a discrete number of known risk profiles, usually two. In a two-type model, exluding one risk type would imply full information about the remaining pool and the welfare effect of discrimination would be trivial: it would implement the full-information outcome. Our setting permits the important mechanism that when the highest risks are excluded, the expected risk in the pool decreases but individual risks remain unknown. It follows that the actuarially fair premium decreases as well. This in turn gives rise to the following three effects on aggregate surplus: (i) the fall in premium paid due to the overall lower risk in the economy, (ii) some low risks will switch to purchasing insurance (softening adverse selection) and (iii) loss of insurance for the high risk types. Our analysis fully balances these three effects and thus shows when total welfare improves due to discrimination and when this is not the case. We thereby contribute to the debate on the desirability of information usage by insurers.

Since discrimination is a mechanism that potentially softens adverse selection, we want to look at adverse selection in isolation from other solutions, like offering an incentive compatible menu-of-contracts where different types choose different combinations of premium and deductible. Additionally, we want to portray a market where there are more risk types than contracts as clearly as possible, and therefore we assume that insurers compete only on the price dimension.

Our results are as follows. The loss of insurance effect (iii) depends on the treshold, which in turn is determined by the nature of the peculiarity tested for and the test technology. For a given treshold, the total effect on welfare depends therefore on the magnitude of effect (i) and (ii) mentioned above. How large will the premium decrease be and how many low risks will be persuaded to buy insurance as a result? The answer depends on the risk attitude in the population. It turns out that when risk aversion is low, the price elasticity of the demarcating low risk consumer is high and many low-risks are gained. As a result, total welfare increases in that case. When risk aversion is very low, a market does not emerge if insurers do not discriminate, while it always emerges in the discrimination regime. On the contrary, when risk aversion is high, even the low risks already have a relatively high willingness to purchase insurance and not many are gained by a premium decrease. Hence, the welfare effect of discrimination is negative in that case.

It is interesting to compare our result with a lemons market model, initiated by Akerlof (1970). In our model, the volume of transactions decreases as a result of discrimination, when risk aversion is high. However, in the lemons model an increase in the lowerbound of the quality distribution always increases the volume of transactions.[1]

The literature on adverse selection (in insurance) provides the background for the motivation of this paper.[2] There is quite some literature on discrimination and risk selection. Lemmens (2003) gives an overview of policy and legal issues of genetic discrimination in Canada. The relevance of risk selection in its many forms is discussed by Eggleston (2000), it is furthermore suggested that both demand-side cost sharing and risk selection are ways to discourage over-utilization in health insurance. The optimal combination of the two is being analyzed.

A concern raised by insurers is the discrepancy between premium paid and expected costs, while on the contrary others stress the role of solidarity in insurance: low risks should pay more than their expected costs to subsidize the high risks. According to the first concern, a better identification of risk type can be seen as a *reduction* of discrimination since the discrepancy between premium paid and expected costs decreases. Hoy and Lambert (2000) analyze this effect for genetic screening and assert that the possibility of misclassification is a counterforce.

In contrast to the above papers, we focus on the simple question whether risk selection increases welfare. Our contribution captures the effect that by excluding some risks, the price for the remaining pool decreases, softening adverse selection. We provide clear-cut conclusions about aggregate surplus.

[1] Suppose that quality is distributed uniformly on $\left[\underline{\theta}, \overline{\theta}\right]$ and that buyers are willing to pay $v.E(\theta)$ and that a seller is willing to trade only if the price exceeds his/her quality. Then it can be shown that in equilibrium the sellers in $\left[\underline{\theta}, \frac{v}{2-v}\underline{\theta}\right]$ trade and the volume of transactions equals $2\frac{v-1}{2-v}\underline{\theta}$ and is thus increasing in the lowerbound of quality $\underline{\theta}$.

[2] This literature was initiated by Rothschild and Stiglitz (1976) and is surveyed in Dionne *et al.* (2000). For empirical evidence on adverse selection, see Puelz and Snow (1994).

A related common underwriting strategy is risk classification. The difference with risk selection is that with classification, all risk categories are being offered a contract, whereas with selection the highest risks are simply rejected. We use simulations in which the rejected individuals are offered a risk adjusted alternative. This addition does not change our results.

Crocker and Snow developed a theory of risk classification and review it in Crocker and Snow (2000). No sophisticated tests are used but simple data such as age, gender and domicile. Instead, we allow for insurers to gather information by some costly technology, that is potentially more informative, e.g. genetics. They allow for screening of individuals by menus of contracts that have to be made incentive compatible. In contrast, by precluding screening we isolate the merits of risk classification from such other mechanisms that partly resolve adverse selection. Related to this, our simple modelling setup relates clear-cut welfare implications to the degree of risk aversion, as opposed to their conclusions that are more ambiguous. This is due to the complexity of their separating equilibrium and maximization problems, furthermore they need to implement complicated transfers to attain a welfare improvement.

Insurers may also employ some information technology *ex-post* to investigate the information that was provided by the insuree himself in the insurance application. Insurance law allows for contestability, *i.e.* when a claim is filed and falsehood is proven, the contract may become void and the payment of indemnity can be denied. This issue is analyzed in Dixit and Picard (2002) and in Chapter 5 of this volume.

Finally, an issue quite different from risk selection occurs when *symmetric* information is the starting point and an increase of information available to the insuree may *create* adverse selection. In Subramanian *et al.* (1999) consumers may learn more about their own risk by doing a genetic test. Using mortality rates for a known gene mutation, they discuss the consequence of this private information for the expected costs of the insurer.

Section 2 presents the model, followed by Sections 3 on equilibrium and 4 on welfare. Section 5 concludes. The proofs are in the Appendix.

4.2 The Model

Consider an insurance market, where the demand side consists of a continuum of individuals who seek to insure the amount of R. Individuals have private information about their probability of loss a_i and are strictly risk averse, with constant absolute risk aversion (CARA). We assume that $v(m) = -\exp(-cm)$ represents the utility of income, where $c > 0$ denotes the CARA coefficient. If no damage occurs, wealth equals y_1 and in case of damage it equals y_2, where $y_1 - y_2 \geq R$.

The supply side has many insurance firms competing by setting a price P. Insurance companies are risk-neutral, have no administration costs and know that the distribution of a_i is uniform with support $[0,1]$. In our main result we will assume full insurance: $y_1 - y_2 = R$.

Consider the contract P, for individual i the expected utility if this contract is purchased equals:

$$a_i v(y_2 + R - P) + (1 - a_i) v(y_1 - P), \tag{4.1}$$

and if no insurance is purchased, expected utility equals:

$$a_i v(y_2) + (1 - a_i) v(y). \tag{4.2}$$

Let:

$$\begin{aligned} D_1(P) &= v(y_1) - v(y_1 - P); \\ D_2(P) &= v(y_2 + R - P) - v(y_2). \end{aligned}$$

Expression D_1 denotes the ex-post utility cost of insurance if no accident occurred and D_2 denotes the ex-post utility gain of insurance in case an accident occurred. From (4.1) and (4.2), we can derive that, for type i, the utility of taking the contract in excess of the utility of not taking the contract equals:

$$a_i D_2(P) - (1 - a_i) D_1(P).$$

This expression is intuitive: if an accident occurs (with probability a_i), the utility gain D_2 is enjoyed, otherwise the cost D_1 is incurred. We can rewrite the expression as:

$$F(a_i, P) = (D_1(P) + D_2(P))\, a_i - D_1(P).$$

An individual purchases the contract iff $F > 0$. Since F is increasing in a_i, $F(0) < 0$ and $F(1) > 0$ it follows that there exists $\hat{a}$ such that an individual purchases insurance iff. $a_i > \hat{a}$, where

$$\hat{a} = \frac{D_1(P)}{D_1(P) + D_2(P, R)}.$$

We now introduce the information technology that allows insurers to exclude the highest risks. We assume that a technology is available that allows an insurer to determine whether an individual has a certain trait that is informative about the risk profile (*e.g.* have a disease or gene mutation). Individuals that are found not to possess this trait are known to have a risk profile that does not exceed a certain treshold, denoted by $a^h < 1$.[3] Yet, their precise risk profile remains unknown beyond that knowledge. We assert that a^h depends on the characteristic that is being tested for and on the accuracy of the test.

From the analysis above it follows that the selection of types that demands insurance is then $\left[\hat{a}, a^h\right]$ and the expected risk of an insuree equals $E(a_i) = \frac{\hat{a}+a^h}{2}$. The benchmark case of no discrimination corresponds to $a^h = 1$. In what follows, we first characterize equilibrium and consequently investigate the welfare effects of discrimination, *i.e.* a regime in which a^h is less than but close to 1. Total welfare is defined as the sum of all actors' utilities.

[3] Insurers may have learned this through past experience.

4.3 Equilibrium

Due to perfect competition on the side of insurers, we require that, in expectation, they break even on the contract P:

$$P - E(a_i).R = 0 \text{ or}$$
$$f(P) \equiv 2\frac{P}{R} - \frac{D_1(P)}{D_1(P) + D_2(P)} = a^h. \tag{4.3}$$

Observe that for a market to exist, we must have that in equilibrium, $\hat{a} < a^h$. The following defines an insurance market in equilibrium:

Definition *Let the discrimination treshold be* $a^h \in (0, 1]$. *The premium* P^* *characterizes an insurance market in equilibrium if (i)* $f(P^*) = a^h$ *and (ii)* $P^* < R$. *Consumers in* $\left[\hat{a}, a^h\right]$ *purchase insurance. If no such* P^* *exists, no market emerges.*

To verify that this is an equilibrium, note that any firm that deviates by setting a lower premium will make a loss because of (4.3) and that any firm that sets a higher premium attracts no customers. The optimality of the consumer decision is derived above.

We now provide a claim about $f(P)$, that is useful to characterize the equilibrium and to show it is unique. In the Appendix we show that:

Claim 4.1 $f(0) = 0,\ f(R) = 1;\ \frac{df}{dP}(0) > 0$ *and* $\frac{d^2 f}{dP^2} < 0$.

The above claim implies that equilibrium is unique: f is concave and if it has two intersections with a^h then one is in the point where $P = R$ which cannot be an equilibrium. Note that these claims hold for any concave utility function.

Lemma 4.2 *For any concave utility of income, the insurance market equilibrium is unique. If* $a^h < 1$, *a market always emerges.*

The claim indicates that the premium increases in the upperbound a^h. Now since the lowerbound of participating types $\hat{a}$ increases in P (which can easily be inferred from the proof of Claim 4.1), it follows that lowering a^h has two opposing effects on market volume. The first is simply that individuals on the upper end of the distribution will be excluded. The second is that more individuals on the lower end of the risk distribution will demand insurance, because the premium decreases. This effect increases market volume and thus softens adverse selection. This will show to be of importance when assessing the welfare effects.

4.4 Welfare Analysis

The following two results will show to be convenient:

Claim 4.3 *$f(P)$ increases in c.*

Claim 4.4 *We have $\lim_{c\to 0}\hat{a} = P/R$ and $\lim_{c\to\infty}\hat{a} = 0$.*

Claim 4.3 implies that P^* decreases in c. The more risk averse people are, the more they are willing to pay for insurance, yet the equilibrium premium decreases. This is because $\hat{a}$ decreases in c (more low risks will buy insurance), and the expected risk profile decreases in c and due to the zero profit condition, this decreases the premium.

We now continue by investigating the welfare that obtains in this market. Since firms make zero expected profits in equilibrium, we add the utility of all types. Formally:

$$\int_0^1 \left[a_i v\left(y_2\right) + (1-a_i)v\left(y_1\right)\right] da_i + \int_{\hat{a}}^{a^h} F\left(a_i, P\right) da_i.$$

Note that the density function of a_i equals 1 and is omitted. Also, the first integral is independent of endogeneous variables and since F is lineair in a_i it is easy to see that it equals $\frac{v(y_1)-v(y_2)}{2}$. This part of welfare obtains independent of the market. The second integral can be attributed to the operation of a market for insurance and we denote it by W. It can be written as:

$$W = 1/2((D_1+D_2)\left(a^h\right)^2 + \frac{\left(D_1\right)^2}{D_1+D_2} - 2D_1 a^h).$$

When $a^h = 1$ we have $W = \frac{1}{2}\frac{[D_2]^2}{D_1+D_2}$. Importantly, note that this part of welfare is only realized when a market emerges. The next proposition characterizes the effect of discrimination, i.e. decreasing a^h marginally, on welfare. The case where R is smaller than $y_1 - y_2$ as well as a wider variation of a^h will be provided by numerical simulations below.

Proposition 4.5 *Let $y_1 - y_2 = R$. There exists c_1 and c_2, $0 < c_1 \leq c_2$, such that the effect of a marginal decrease of a^h is as follows: (i) if $c < c_1$ then welfare (W) increases and (ii) if $c > c_2$, then welfare decreases.*

The result weighs the three effects that were mentioned in the introduction and can be explained by looking at market volume. When risk aversion is low ($c < c_1$), the demand for insurance by low risks is relatively low (high lowerbound $\hat{a}$) and their price elasticity is relatively high. The premium decrease due to a lower a^h then induces a large decrease in $\hat{a}$, and hence, a large increase in the market volume.

When risk aversion is high however ($c > c_2$), the low risks are already more inclined to buy insurance (low lowerbound $\hat{a}$) and a premium decrease does not induce a large decrease of $\hat{a}$ and thus, the market volume and welfare decrease due to lower a^h. The important mechanism at work is thus the price sensitivity of the pivot $\hat{a}$ which, in turn, is determined by the risk attitude c.

Above we assumed that the excluded risks are lost for this economy. It has been shown that this is plausible in real-world insurance, for example by Eggleston (2000). However, it is also observed in some markets that the high risks are offered an alternative insurance policy (known as risk classification). By using numerical simulation, we next analyze if the above welfare result continues to hold if the excluded risk types are being offered a competitive alternative, geared to their risk profile. The risk types that get this contract are in $\left[a^h, 1\right]$ and the premium that breaks even for this contract is $P = R\frac{a^h+1}{2}$.[4] The

[4]It can be shown that there is no adverse selection of the consumers $\left[a^h, 1\right]$: all types purchase the contract, formally it must be that $F\left(a^h, R\frac{a^h+1}{2}\right) \geq 0$.

c:	$a^h = 0{,}8$	$a^h = 0{,}85$	$a^h = 0{,}9$	$a^h = 0{,}95$	$a^h = 1$
0,001	0,00000001	0,00000001	0,00000000	0,00000000	0,00000000
0,5	0,00354953	0,00226201	0,00114772	0,00033047	0,00000000
1	0,02013119	0,01479152	0,00903650	0,00337247	0,00000000
1,5	0,05354369	0,04526396	0,03448210	0,01983445	0,00000000
2	0,09500343	0,08807487	0,07815561	0,06254627	0,03148265
2,5	0,13312970	0,12974048	0,12454256	0,11554422	0,09754395
3	0,16262105	0,16264971	0,16233828	0,16057401	0,15483261
3,5	0,18300086	0,18551296	0,18891018	0,19271931	0,19580298
4	0,19571150	0,19974196	0,20556658	0,21315842	0,22217796
4,5	0,20255250	0,20733971	0,21455982	0,22450944	0,23736602
5	0,20511071	0,21011745	0,21798604	0,22925902	0,24448385
5,5	0,20462931	0,20950261	0,21750835	0,22938325	0,24596011
6	0,20203231	0,20655539	0,21436285	0,22634866	0,24357739
6,5	0,19798910	0,20204082	0,20944004	0,22120994	0,23860989
7	0,19297856	0,19650271	0,20337115	0,21471377	0,23195704
7,5	0,18734128	0,19032454	0,19659956	0,20738414	0,22424970
8	0,18131897	0,18377544	0,18943484	0,19958666	0,21592868

Figure 4-1: $W + W^h$ for $y_1 = 1.1$, $y_2 = .1$, $R = 0.8$ and different values of c and a^h.

welfare generated by this contract is:

$$W^h = \int_{a^h}^{1} F\left(a_i, R\frac{a^h+1}{2}\right) da_i,$$

note that since $F\left(a^h, \cdot\right) > 0$, the area under F is a square plus a triangle, therefore:

$$W^h = \frac{1}{2}\left(1 - a^h\right)\left[\left(v\left(y_1\right) - v\left(y_2\right)\right) a^h + 2v\left(y_1 - P\right) - v\left(y_2\right) - v\left(y_1\right)\right],$$

where $P = R\frac{a^h+1}{2}$. We computed $W + W^h$ in Figure 4-1.[5] The Table shows that the result continues to hold when high-risks get a contract, there is 20% coinsurance (R is smaller than $y_1 - y_2$) and the decrease in a^h is more than marginal. It nicely illustrates Proposition 4.5: when the CARA coefficient c is 3.5 or higher, excluding some of the highest risks decreases welfare. However, when risk aversion is lower than 3.5, welfare is maximized for $a^h < 1$: the welfare loss for high risks that results from discrimination is in that case more than offset by the welfare gain of the intermediate risks. When $c < 2$

[5]The algorithms are available upon request.

a market doesn't emerge at all if insurers do not discriminate, and in that case welfare is maximized when the 80% lowest and 20% highest risks obtain separate contracts. Of course, the level of a^h that can be set in the market depends on screening technologies and the underlying risk determinants.

4.5 Concluding remarks

By setting up a model of adverse selection with a continuum of risk types, we assessed the welfare effects (in terms of aggregate surplus) of risk selection (or discrimination) in insurance. The use of the continuum made the problem non-trivial: after selecting some risks out, uncertainty about individuals' risk remains. It also highlights the role of risk aversion in explaining the demand for insurance. Insurers compete on the price dimension only, allowing us to obtain an equilibrium with more risk types than contracts.

Our results contribute to the debate on the desirability of information use by insurers. Only when risk aversion is low, discrimination can be justified on the basis of aggregate surplus. In fact, in this case it might prevent the market from not emerging at all.

4.6 Appendix

In the Appendix we denote by L the loss $y_1 - y_2$.

Claim 4.1 $f(0) = 0,\ f(R) = 1;\ \frac{df}{dP}(0) > 0$ and $\frac{d^2 f}{dP^2} < 0$.

Proof. The first two observations are trivially verified. The derivative equals:

$$\frac{df}{dP} = \frac{2}{R} - \frac{D_1'(P)\, D_2 - D_1 D_2'(P)}{\left[D_1 + D_2\right]^2},$$

and the derivative in 0 is

$$\frac{df}{dP}(0) = \frac{2}{R} - \frac{D_1'(P)}{D_2(P)} = \frac{2}{R} - \frac{v'(y_1)}{v(y_2 + R) - v(y_2)}.$$

For the latter to be positive it has to be required that:

$$2 > \frac{v'(y_1)}{\frac{v(y_2+R)-v(y_2)}{R}}.$$

Now note that the denominator is simply the slope of the secant line through y_2 and $y_2 + R$ and that $y_2 + R$ is at most y_1. From the concavity of $v(\cdot)$ it then follows that $v'(y_1) < \frac{v(y_2+R)-v(y_2)}{R}$ and this proves the third observation.

Next, note that:

$$\frac{d^2 f(P)}{dP^2} = -\left[\frac{D_1''(P)\, D_2 - D_1 D_2''(P)}{[D_1 + D_2]^2} - \frac{2\,[D_1'(P)\, D_2 - D_1 D_2'(P)]\,[D_1'(P) + D_2'(P)]}{[D_1 + D_2]^3}\right].$$

Now since $D_1 > 0$, $D_2 > 0$, $D_1' > 0$, $D_2' < 0$, $D_1'' > 0$ and $D_2'' < 0$ and the fact that $D_1'(P) + D_2'(P) < 0$, this derivative is negative. ■

Claim 4.3 *Let* $v(m) = -\exp(-cm)$. $f(P)$ *increases in* c.

Proof. We have to show that $\hat{a}$ decreases in c. Recall that

$$\hat{a} = \frac{D_1(P)}{D_1(P) + D_2(P)}$$

and let $D_{ic}' = \frac{\partial D_i}{\partial c}$ for $i = 1, 2$. Then we can write:

$$\frac{\partial \hat{a}}{\partial c} = \frac{D_{1c}' D_2 - D_{2c}' D_1}{[D_1 + D_2]^2},$$

and for the latter expression to be negative, we require that:

$$\frac{D_{1c}'}{D_1} < \frac{D_{2c}'}{D_2}.$$

For the specified utility function, this becomes:

$$\frac{-(y_1 - P)\exp(-c(y_1 - P)) + y_1 \exp(-cy_1)}{\exp(-c(y_1 - P)) - \exp(-cy_1)} < \frac{-y_2 \exp(-cy_2) + (y_2 + R - P)\exp(-c(y_2 + R - P))}{\exp(-cy_2) - \exp(-c(y_2 + R - P))},$$

which is equivalent to:

$$\frac{-(y_1-P)\exp(cP)+y_1}{\exp(cP)-1}<\frac{(y_2+R-P)\exp(-c[R-P])-y_2}{1-\exp(-c[R-P])}\quad\Leftrightarrow$$

$$-\frac{P\exp(cP)}{1-\exp(cP)}-\frac{(R-P)\exp(-c[R-P])}{1-\exp(-c[R-P])}<y_1-y_2.$$

Now let $x_1=-P$, $x_2=R-P$, and bring the exponential terms to the denominator, then we can rewrite the above expression as:

$$\frac{x_1}{\exp(cx_1)-1}-\frac{x_2}{\exp(cx_2)-1}<L. \tag{4.4}$$

Now we will show that (i): $\frac{x}{\exp(cx)-1}$ decreases in x and (ii): check inequality (4.4) for $x_1=-L$ and $x_2=R$:

(i): The derivative equals $\frac{\exp(cx)(1-cx)-1}{[\exp(cx)-1]^2}$ and we can demonstrate that $e^x(1-x)<1$ for $x>0:$ in $x=0$, LHS=RHS and $\frac{dLHS}{dx}=-xe^x<0$.

(ii) We have $\frac{-L}{\exp(-cL)-1}-\frac{R}{\exp(cR)-1}<L$, which is equivalent to $-\frac{R}{\exp(cR)-1}<L\frac{\exp(-cL)}{\exp(-cL)-1}$, which, in turn, is equivalent to $\frac{R}{\exp(cR)-1}>\frac{L}{\exp(cL)-1}$.Now since $R\leq L$, and $\frac{x}{\exp(cx)-1}$ decreases in x (shown in (i) above) the inequality holds for all $R<L$. When $R=L$, LHS equals RHS.

Due to step (i), it is enough to find for some $x_1<-P$ and $x_2>R-P$ a weak inequality of (4.4). This is done in step (ii). ■

Claim 4.4 *We have (a)* $\lim_{c\to 0}\hat{a}=P/R$ *and (b)* $\lim_{c\to\infty}\hat{a}=0$.

Proof. (a) Write $\hat{a}$ as $1/y(c)$, where $y(c)=1+D_2/D_1$. By applying the rule of l'Hopital once we obtain that $D_2/D_1\to(R-P)/P$ and hence $y(c)\to R/P$.

(b) From Claim 4.3 we know that $\hat{a}$ decreases in c, therefore it converges. Suppose for sake of contradiction that it converges to a strictly positive number, $\lim\hat{a}>0$. This implies that there exists small $\epsilon>0$ such that $\lim_{c\to\infty}F(\epsilon,P)<0$ (a low risk type does not buy insurance when $c\to\infty$). Now since $F(0,P)=-D_1(P)$ and $D_1(P)\to 0$ this is contradicted and the result follows. ■

Proposition 4.5 *Let $R = L$. There exists c_1 and c_2, $0 < c_1 \leq c_2$, such that the effect of a marginal decrease of a^h is as follows: (i) if $c < c_1$ then welfare (W) increases and (ii) if $c > c_2$, then welfare decreases.*

Proof. (i)
We know that when $a^h = 1$, a market emerges iff

$$2 < \frac{v'(y_2)}{\frac{v(y_1)-v(y_1-R)}{R}}, \tag{4.5}$$

while a market always emerges when $a^h < 1$. Hence, when the above inequality is violated, a marginal decrease of a^h means that in stead of foreclose, a market emerges, which is a welfare improvement. We will now show that a violation of (4.5) is equivalent to a low value of c. When $R = L$, (4.5) reads as $\frac{cL}{1-\exp(-cL)} > 2$ and LHS increases in cL. To complete the proof of the statement, note that $\lim_{c\to 0} \frac{cL}{1-\exp(-cL)} = 1$.
(ii)
Recall that P is a function of a^h via (4.3). When $R = L$ we have that $D_1 + D_2$ is independent of endogeneous variables. We have that $W = \frac{1}{2}\left\{(D_1 + D_2)\left(a^h\right)^2 + \frac{(D_1)^2}{D_1+D_2} - 2D_1 a^h\right\}$ and taking the derivative w.r.t. a^h we obtain:

$$\frac{\partial 2W}{\partial a^h} = (D_1 + D_2)\, 2a^h + \frac{1}{(D_1 + D_2)} 2D_1 \frac{\partial D_1}{\partial P}\frac{\partial P}{\partial a^h} - 2\left(\frac{\partial D_1}{\partial P}\frac{\partial P}{\partial a^h} + D_1\right),$$

now substitute $a^h = 1$, the expressions for D_1 and D_2 and rearrange:

$$[v(y-L) - v(y-P)]\left\{\frac{v'(y-P)}{v(y) - v(y-L)}\frac{\partial P}{\partial a^h} - 1\right\},$$

with the specific utility function this becomes:

$$[\exp(-c(y-P)) - \exp(-c(y-L))]\left(\frac{c}{\exp(c(L-P)) - \exp(-cP)}\frac{\partial P}{\partial a^h} - 1\right)$$

We will now show that the latter equality is positive for c large enough. Observe that the first multiplicand is negative. Now recall that when $c \to \infty$, $\widehat{a} \to 0$ and hence $\frac{\partial P}{\partial a^h} \to \frac{R}{2}$ in that case. We will next investigate the fraction between brackets $\frac{c}{\exp(c(L-P))-\exp(-cP)}$.

We have that the derivative has the sign of $[1 - c(L-P)]\exp(c(L-P)) - (1+cP)\exp(-cP)$ and is therefore negative when:

$$1 - \frac{cL}{1+cP} < \exp(-cL).$$

Now note that the fraction on the LHS increases in c, implying that LHS becomes eventually negative. This demonstrates that the derivative is negative for c large enough. We will now show that

$\lim_{c\to\infty} \frac{c}{\exp(c(L-P))-\exp(-cP)} = 0$. Applying the rule of l'Hopital once we obtain that the limit equals $\lim_{c\to\infty} \frac{1}{(L-P)\exp(c(L-P))+P\exp(-cP)} = 0$. Therefore, the fraction between brackets decreases in c for c large enough and converges to zero and since $\frac{\partial P}{\partial a^h} \to \frac{R}{2}$ their product becomes less than 1 for c large enough. The above demonstrates that $\partial W/\partial a^h > 0$ for such c and completes the proof. ■

Chapter 5

Contestability in Insurance

5.1 Introduction

When purchasing life insurance, insurees are required to provide detailed information about their health and life style on their application. Insurance companies use that information to determine whether to accept an insuree and at what price. The accuracy and thruthfullness of this information remains important even after the purchase of the contract: the insurance contract may became void if irregularities are discovered. The following quote from *www.insurance.com* (a resource for consumers) illustrates the point:

> The point is, if you lie about your smoking, insurance companies have ways of uncovering your falsehood and will almost certainly reject your application if they find out you've lied. In addition, if a company sells you a life insurance policy and then finds out that you lied about your smoking, they may be able to terminate your life insurance coverage immediately (leaving your beneficiaries without protection). The reason: most life insurance policies are subject to a contestability period (generally, the first two years that the policy is in force) during which the company has the right to cancel the contract based on any false statements you made on your application. For example, if you died from emphysema due to smoking a year after you bought life insurance and

told the insurance company you don't smoke, your beneficiaries might not be entitled to the policy death benefit.[1]

It is not only smoking behaviour that is the subject of exclusion clauses. Other grounds for dispute are for example pre-existing medical conditions and the cause of death (whether it was self-inflicted or an accident). The legal clause in the policy, the disputes in courts and consumer advice such as the one above all demonstrate that contestability is an important phenomenon. Since the matters are often ambiguous, this not only holds for those deliberately lying. Contestability reduces the insuring effect of the insurance policy, especially with life insurance since one is not able to file the claim oneself. Insurers usually motivate information use by stating that it is a necessary underwriting strategy to alleviate adverse selection.

Our objective is to analyze the welfare effect of contestability, weighing opposing effects. We take smoking behaviour as the leading example. Insurers offer different rates for non-smokers and smokers, where the first type of contract is contestable and the latter is not.[2] We will therefore compare the aggregate surplus obtained in an economy in which two contracts are offered with one in which only one, non-contestable contract is for sale.

This issue has so far only been addressed in the two-types, two-contracts setting, as initiated by Rothschild and Stiglitz (1976). In such a model, it is quite easy to see that contestability increases welfare, since the possible investigation of a claim discourages a high-risk to purchase the contract designed for the low type. Dixit (2000) shows that this implies that the incentive compatibility constraint can be relaxed, compared to the benchmark model, and hence the low risks enjoy a better contract and a Pareto improvement is obtained.

However, since in reality there are far more many risk types than the number of contracts in the menu, this setting can be questioned. To adhere to this observation, we construct a model with infinitely many risk types and one contract only in the benchmark regime of no contestability. Importantly, the conclusion found by Dixit (2000) is being contradicted completely.

[1] Source: http://www.insurance.com/FAQs/lifeFAQDetail.aspx/index/17.

[2] That is, not on smoking behaviour.

Our setting thus features a continuum of privately informed individuals that demand insurance on a market with a competitive supply side. They evince constant absolute risk aversion (CARA). The feature that we use a continuum of risk types is rather unusual for the asymmetric information literature. The advantage of such a realistic setup is that asymmetric information remains even if individuals select into two contracts. Furthermore, it provides us with a continuous relation between the demand for insurance and important market variables, such as risk aversion. This is not the case in e.g. the seminal paper by Rothschild and Stiglitz (1976).

To obtain a market with with more risk types than contracts, we assume that insurers only compete on the price dimension. Moreover, since we study a mechanism that potentially softens adverse selection, an additional motivation for this assumption is that we want to isolate contestability from other solutions, like offering incentive compatible menu-of-contracts.[3]

Finally, the risk of contestation is modeled by ascribing a probability of contest to every risk profile. We next postulate that this probability of contest is increasing in risk profile.

The model provides the following insights. When there is only one contract on the market, only the individuals above some demarcation level will insure (hence, the market suffers from adverse selection). When two contracts are offered, the highest risks will still prefer the more expensive non-contestable contract, since it provides more insurance. Some intermediate risks will switch to the contestable contract, however. Additionally, some consumers who did not purchase insurance before will now buy the contestable contract. As a result, the volume of transactions will increase and thus adverse selection is softened.

The important countereffect on welfare is that the premium for the non-contestable contract increases substantially, due to the fact that the risk in that pool increased. We thus quantify the two opposing effects of contestability on welfare: (i) some relatively low risks are being insured, softening adverse selection, while (ii) the more certain non-contestable contract is purchased by less individuals and at a higher price.

[3] Also known as screening or self-selection.

We computed the welfare effect for risk attitudes that can be considered plausible[4] and for a wide range of risk of contestation. A market for the contestable contract does not always emerge. When it does, welfare in the two contracts world is lower than welfare in the one contract world. Hence, contestability decreases welfare.

The literature on adverse selection (in insurance) provides the background for the motivation of this paper.[5] Our topic is highly related to risk selection and classification. With these strategies, insurers use an information technology on all applicants *before* the contract is signed. See for an overview Crocker and Snow (2000) and Chapter 4 in this volume for a recent contribution. With contestability, the clause in the contract has a discouraging and risky effect because insurers might investigate matters *ex-post* and no information technology needs to be actually employed.

As far as I know, there are only three papers that discuss clauses in the insurance contract. I already mentioned Dixit (2000) and this model is being extended in Dixit and Picard (2002) by relaxing the assumption that the insured has full knowledge of his own risk. In sharp contrast to our findings, their equilibrium Pareto improves on the benchmark model of no contestability. The important difference with our setting is set out above: because of the continuum, the interaction between the two contracts causes the high-risks to be worse off. Finally, Garatt and Marshall (2001) analyze exclusions and the demand for property insurance. They model interaction between exclusions in one contract and the demand for another.

Section 2 presents the model and shows how the two contracts operate in isolation from one another. Section 3 provides the equilibrium, Section 4 computes welfare effects and finally, Section 5 concludes.

[4] Using a television game show as natural experiment, Beetsma and Schotman (2001) find robust evidence of substantial risk aversion and show that CARA and CRRA perform equally well.

[5] This literature was initiated by Rothschild and Stiglitz (1976) and is surveyed in Dionne *et al.* (2000). For empirical evidence on adverse selection, see Puelz and Snow (1994).

5.2 The Model

Consider an insurance market, where the demand side consists of a continuum of individuals who seek to insure the amount of R. Individuals have private information about their probability of loss a_i and are strictly risk averse, with constant absolute risk aversion (CARA). We assume that $v(m) = -\exp(-cm)$ represents the utility of income, where $c > 0$ denotes the CARA coefficient. If no damage occurs, wealth equals y_1 and in case of damage it equals y_2, where $y_1 - y_2 \geq R$.

The supply side has many insurance firms competing. Insurance companies are risk-neutral, have no administration costs and know that the distribution of a_i is uniform with support $[0, 1]$. They choose a price P^k, accept all applicants and pay R in case of damage (the coverage), where $k \in \{ns, s\}$ denotes a non-smokers contract and a smokers contract, respectively. The first type of contract may be disputed when a claim is filed. In this section, we study the two contracts in isolation of one another.

The Non-Contestable (Smokers) Contract

This contract coincides with the benchmark model in Chapter 4 of this volume. We set out by determining an insuree i's expected utility, when purchasing the contract P^s :

$$a_i v(y_2 + R - P^s) + (1 - a_i) v(y_1 - P^s). \tag{5.1}$$

When no insurance is purchased, expected utility equals:

$$a_i v(y_2) + (1 - a_i) v(y_1). \tag{5.2}$$

Let:

$$\begin{aligned} D_1(P^s) &= v(y_1) - v(y_1 - P^s) \\ D_2(P^s) &= v(y_2 + R - P^s) - v(y_2). \end{aligned}$$

When convenient, we use D_1^k and D_2^k, for $k \in \{ns, s\}$. Expression D_1 denotes the ex-post utility cost of insurance if no accident occurred and D_2 denotes the ex-post utility gain of insurance in case an accident occurred. From (5.1) and (5.2), we can derive that, for type i, the utility of taking the contract in excess of the utility of not doing so equals:

$$a_i D_2 (P^s) - (1 - a_i) D_1 (P^s) .$$

This expression is intuitive: if an accident occurs (with probability a_i), the utility gain D_2 is enjoyed, otherwise the cost D_1 is incurred. We can rewrite the expression as:

$$F^s (a_i, P^s) = (D_1 (P^s) + D_2 (P^s)) a_i - D_1 (P^s) .$$

An individual purchases the contract iff $F > 0$. Since F is increasing in a_i, $F(0) < 0$ and $F(1) > 0$ it follows that there exists $\underline{a}^s$ such that an individual purchases insurance iff. $a_i > \underline{a}^s$:

$$\underline{a}^s = \frac{D_1 (P^s)}{D_1 (P^s) + D_2 (P^s)}.$$

Perfect competition on the side of insurers requires that, in expectation, they break even on the contract P^s. Observe that the expected risk profile equals $(\underline{a}^s + 1)/2$, therefore:

$$\frac{P^S}{R} - \frac{1}{2}\left(\frac{D_1 (P^S)}{D_1 (P^S) + D_2 (P^S, R)} + 1\right) = 0. \tag{5.3}$$

Proposition 5.1 *A unique equilibrium where insurers set $P^s < R$ and consumers purchase insurance if and only if $a_i \geq \underline{a}^s$ is defined by (5.3) if such P^s exists. A market does not emerge otherwise.*

See Chapter 4 for the proof.

The Contestable (Non-Smokers) Contract

Next, consider a contract that is intended for non-smokers and specifies that the claim can be denied in case the applicant provided false information about his smoking habits. Suppose that for an individual with risk profile a_i, the perceived probability of a successful dispute by the insurance company in case of death is $g(a_i) \in [0, 1]$. The probability g depends on many factors, including factual smoking behaviour, other health hazards and the legal institutions. Below we show how g depends on a_i.

We will first show how such a clause affects the demand for insurance. As before, we have that utility in case of no insurance equals $(1-a_i)\, v(y_1) + a_i v(y_2)$. When insurance is purchased, utility equals:[6]

$$(1-a_i)\, v(y_1 - P^{ns}) + a_i (1 - g(a_i))\, v(y_2 + R - P^{ns}) + a_i g(a_i)\, v(y_2 - P^{ns}).$$

The first term is the usual utility of income if no accident happens and the premium is paid. The second term represents the contingency that the damage is undisputed and the reimbursement of R is being made. Finally, the third term shows the execution of the exclusion clause: with probability g the damage is successfully disputed and in that case, the reimbursement is denied. Hence, an individual buys insurance iff

$$F^{ns}(a_i, P^{ns}) = -D_3(P^{ns})\, a_i g(a_i) + (D_1(P^{ns}) + D_2(P^{ns}))\, a_i - D_1(P^{ns}) > 0,$$

where $D_3(P^{ns}) = v(y_2 + R - P^{ns}) - v(y_2 - P^{ns})$ denotes the difference in the gain of insurance when the damage occurs due to the covered risk, instead of the excluded risk. As before, it is easily verified that adverse selection is present: $F^{ns}(0, P^{ns}) = v(y - P^{ns}) - v(y) < 0$.

It is likely that $g' > 0$: the higher the risk profile, the more smokers there are amongst this risk profile.[7] Many increasing specifications of g are conceivable but for tractability, we assume that $g(a_i) = \alpha a_i$, for some $0 < \alpha \leq 1$.

High risks are deterred by the clause when $F^{ns}(1, P^{ns}) < 0$ or $\alpha > D_2/D_3$: α is large enough or the individual is risk averse enough. A high-risk individual is quite sure of incurring damage, but by purchasing a policy that pays only with probability $1-g$, he or she buys in fact a lottery. The insurance contract is therefore too risky for a high-risk individual. Henceforth, we assume that either risk aversion or α is sufficiently high so as to ensure $F^{ns}(1, P^{ns}) < 0$.

[6] Here we abstract from the question how the deceased enjoys the money paid to beneficiaries.

[7] We incorporate the generally believed positive correlation between mortality risk and smoking. Indeed, smokers pay a higher premium.

Since F^{ns} is a polynomial in a_i it is useful to provide its two roots explicitly (suppressing arguments when this causes no confusion):

$$
\begin{aligned}
\underline{b}\left(P^{ns}\right) &= \frac{D_1+D_2}{2\alpha D_3} - \frac{\sqrt{\left(D_1+D_2\right)^2 - 4\alpha D_1 D_3}}{2\alpha D_3}; \\
\overline{b}\left(P^{ns}\right) &= \frac{D_1+D_2}{2\alpha D_3} + \frac{\sqrt{\left(D_1+D_2\right)^2 - 4\alpha D_1 D_3}}{2\alpha D_3}.
\end{aligned}
$$

Thus, the expected risk profile equals $\frac{D_1^{ns}+D_2^{ns}}{2\alpha D_3^{ns}}$ and zero-profitability implies:

$$
\frac{P^{ns}}{R} - \frac{D_1^{ns}+D_2^{ns}}{2\alpha D_3^{ns}} = 0. \tag{5.4}
$$

In summary:

Proposition 5.2 *Equilibrium for a contestable contract has firms setting $P^{ns} < R$ according to (5.4) and consumers with a risk profile a_i in $\left[\underline{b}\left(P^{ns}\right), \overline{b}\left(P^{ns}\right)\right]$ purchasing insurance.*
This equilibrium exists only if the condition $\left(D_1^{ns}+D_2^{ns}\right)^2 - 4\alpha D_1^{ns} D_3^{ns} > 0$ is met. If this equilibrium does not exist, a market does not emerge.

5.3 Two-Contracts Equilibrium

Our objective is to compare an economy in which only the smokers contract is for sale with one in which two contracts are for sale. In the first case, equilibrium is given by Proposition 5.1.

In the two-contract economy, we distinguish two situations. Firstly, the parameters of the market are such that the contracts as defined above do not overlap. The two-contracts equilibrium is then given by Propositions 5.1 and 5.2. Secondly, when the contracts do overlap ($\underline{a}^s$ that follows from Proposition 5.1 is lower than the $\overline{b}$ that follows from Proposition 5.2) the market is not in equilibrium. Figure 5-1 illustrates; consumers in a right neighborhood from $\underline{a}^s$ will switch from the smokers contract to the non-smokers contract, increasing the risk profile in the smokers pool.

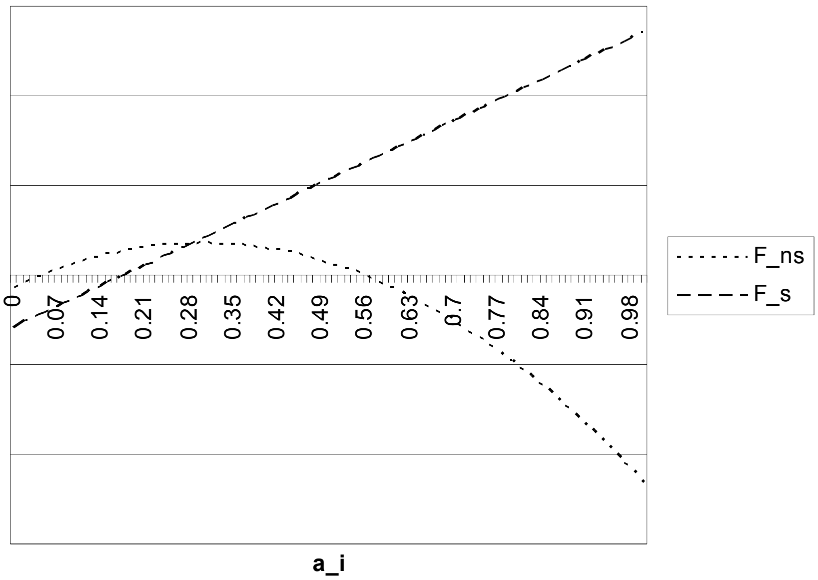


Figure 5-1: Overlapping contracts. Some will switch from the s contract to the ns contract.

In that case, a two contracts equilibrium can be found by a demarcation level d and prices (P^{ns}, P^{s}) such that:

$$\begin{aligned} P^{ns} &= \frac{\underline{b}(P^{ns}) + d}{2} R; \qquad P^{s} = \frac{d+1}{2} R \quad \text{ and} \\ F^{s}(d, P^{s}) &= F^{ns}(d, P^{ns}) > 0. \end{aligned} \tag{5.5}$$

The slopes of F^{ns} and F^{s} are such that when these conditions hold, consumers left of d prefer the ns contract and consumers to the right of d prefer the s contract. Figure 5-2 shows such an equilibrium, in which d is found at the intersection of F^{ns} and 'F^{s} two contracts'.

5.4 Welfare Analysis

Note that when there is no overlap between the two contracts, it must be that welfare with two contracts is higher than welfare with only one contract. We now illustrate the other case by using Figure 5-2. The one contract case is depicted by the line 'F^{s} only s'

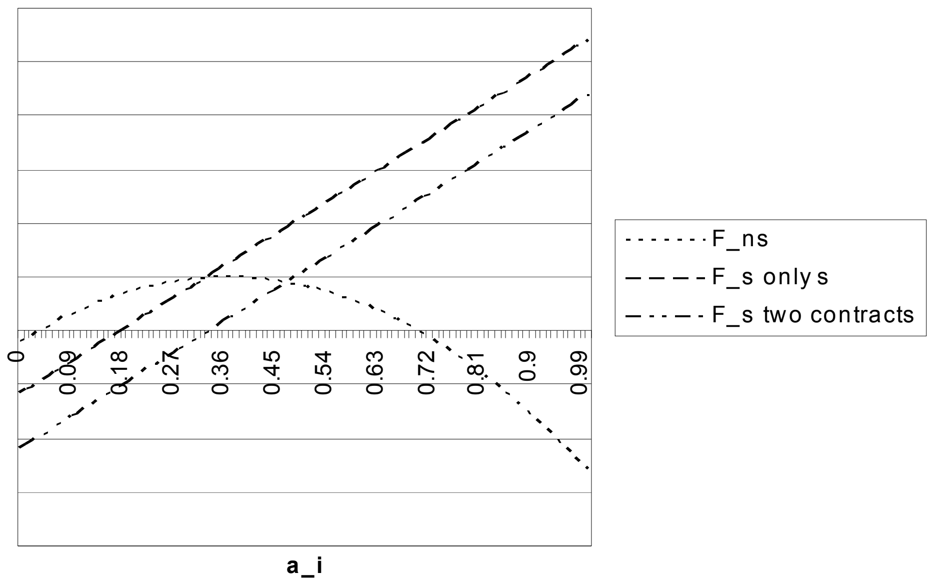


Figure 5-2: Two contracts equilibrium for $y_1 = 1.1$, $y_2 = .1$, $R = 1$, $c = 4$ and $\alpha = .5$.

and hence, aggregate utility generated by the insurance market equals the area under this line. When the ns contract is introduced, P^s increases, as reflected in the downward shift of 'F^s only s' to 'F^s two contracts'. The welfare generated by that contract is decreased by area between the two lines. The positive impact of course is the welfare generated by the ns contract, the area under F^{ns} between d and $\underline{b}$. In the example of the figure, it can be seen that the latter effect is smaller. Formally welfare in the two regimes equals:

$$\begin{aligned} W^s &= \int_{\underline{a}}^{1} F^s\left(a_i, P^s\right) da_i; \\ W^{ns,s} &= \int_{\underline{b}(P^{ns})}^{d} F^{ns}\left(a_i, P^{ns}\right) da_i + \int_{d}^{1} F^s\left(a_i, P^s\right) da_i, \end{aligned}$$

where the density functions equal 1 and are omitted. Unfortunately, the determination of d satisfying (5.5) cannot be done analytically. Therefore we computed it for a wide range of parameters: the following Table reports $W^s - W^{ns,s}$ for different values of risk aversion c and contest probability parameter α.[8]

[8]The algorithms are available upon request.

c:	α =0.1	α =0.15	α =0.2	α =0.25	α =0.3	α =0.35
1	#	#	#	#	#	#
1.5	#	#	#	#	#	#
2	#	#	#	#	#	#
2.5	#	#	#	0.0586	0.0363	0.0289
3	#	#	0.0908	0.0772	0.0661	0.0570
3.5	#	0.1246	0.1046	0.0895	0.0778	0.0685
4	#	0.1250	0.1039	0.0887	0.0773	0.0684
4.5	0.1474	0.1148	0.0941	0.0798	0.0694	0.0615
5	0.1306	0.0985	0.0795	0.0670	0.0581	0.0515
5.5	0.1089	0.0798	0.0636	0.0534	0.0462	0.0409
6	0.0860	0.0617	0.0489	0.0409	0.0354	0.0313
6.5	0.0650	0.0461	0.0363	0.0303	0.0263	0.0233
7	0.0475	0.0334	0.0263	0.0220	0.0191	0.0169

Figure 5-3: $W^s - W^{ns,s}$ for $y_1 = 1.1$, $y_2 = .1$, $R = 1$ and different values of CARA coefficient c and α. The sign # indicates that a ns market does not emerge.

Figure 5-3 indicates how much in aggregate utility can be gained when the market offers only one, instead of two, contract. Note that in all cases, there was overlap. The sign # indicates that a ns market does not emerge and in that case $W^s - W^{ns,s} = 0$. This happens when risk aversion is relatively low; it reduces the demand for insurance and shifts F^{ns} downwards.

The table shows that for all cases that we computed, whenever a two contracts equilibrium is viable, it reduces welfare. The negative impact on high-risks' welfare is larger than the positive effect of insuring some intermediate risks that where uninsured before, hence contestability decreases welfare.

5.5 Concluding remarks

We analyzed the welfare effect of having two contracts, instead of one. The two contracts are intended for different types and insurance law provides that a claim can be disputed if it is shown that an insuree provided false information about his or her type. Hence, we analyze the welfare effect of this legal possibility. Our results show that for a wide range of parameters, the two contracts economy generates less welfare than the one contract economy. This is striking in view of the fact that many actual markets are organized as the two contracts regime of our model.

Smoking in life insurance is our leading example but the insights are not limited to that type of insurance. The model can also be applied to insurance markets where no explicit disclosure of information has to be made, but some perils are simply excluded. Examples are certain kinds of accident and injury, such as self-inflicted injury and not taking care (e.g. not locking the car), drunkenness and so on and so forth. Of course, these perils are not type related but depend on behaviour. Yet we think whenever a successful dispute by the insurer cannot be excluded with full certainty, the insurance policy looses some of its insuring effect and the mechanisms we studied play a role.

Chapter 6

Competition for a Prize

6.1 Introduction

Consider a scientific committee that has the task to organize a contest for a research prize. Having no knowledge of the qualities of potential applicants, the committee faces the problem of advertising the prize and designing the application procedure in order to award the best candidate. With only a limited (time) capacity to read and extract information from the applications, the committee runs the potential risk of being overloaded with piles of applications, when too many individuals apply. At the same time, the committee wishes not to discourage applying too much, since in that case *nobody* will apply. The problem is thus to design the contest in such a way that attention will be devoted only to the better candidates in the population, while ensuring that some people do apply. The scholars know their own quality but are uncertain about the number of other applicants and their qualities. To decide whether to apply or not, individuals weigh their chances of winning and the size of the prize against the trouble of writing the application. To what extent should entry be troublesome?

It is this interaction that is the subject of this paper: the competition by a number of privately informed candidates for a single prize, awarded by a committee that has imperfect ability to rank applicants. This interaction characterizes a number of markets: examples include recruitment in the labour market, architectural competition, competition for licences, conference presentations and publications. I will answer the above question by relating the (optimal) cost of applying to the size of the market and the available selection technology.

Since the pioneering work of Spence (1973), asymmetric information in the above markets has been studied quite extensively, especially in the literature on signaling and mechanism design. There are two pillars that virtually all models in the traditional literature rest on: *(i)* the uninformed side of the market relies on incentives solely in order to mitigate informational asymmetry and *(ii)* these incentive mechanisms, like signaling, require the informed agents to be heterogeneous in some observable way, being for example their output or education level. Moreover, in the latter case the individual's cost of sending a certain signal must be correlated to the hidden information (*e.g.* ability), to enable separation.

In some markets, like prize competition, these two assumptions do not seem adequate. Turning to the first, it is widely observed that effort is spent on acquiring information about trading partners simply by investigating the candidates. For example, the assessment of job candidates is a widespread phenomenon. Firms try to discover ability of applicants by using interviews or assessment centers in their hiring decisions. In trade, buyers may audit and compare the features of different sellers. A program committee of a conference tries to select the best papers by *reading* the papers. [1]

Hence, it is plausible that for many types of information asymmetries, the uninformed uses some kind of information technology to obtain imperfect estimates of the other side of the market. Using these estimates, different candidates can then be compared. It is likely that the uninformed side of the market can hereby improve its decisions and this practice should be allowed for in a formal model of asymmetric information.

I now turn to the secondly mentioned pillar, taking education as an example of a signaling device, as in Spence (1973). I will argue that such a device may not always work and may leave the problem of asymmetric information unsolved. In the first place, when asserted that the cost of education decreases in ability, it is often overlooked that more

[1] In an overview article on signaling and screening, Riley (2001) asserts on this point that "*there is a strong incentive for the market to seek alternative means of information transmission* (for signaling or screening devices). *It is likely that in environments where this is the case, there will be evidence of direct testing, early monitoring, etc. -all provided to greatly reduce, if not eliminate, asymmetric information.*" (p. 474).

able people incur higher opportunity costs of education, due to unfulfilled other profitable activities. When contented with this assertion, two additional objections can be made. Firstly, the contest may be specialized in such a way that the pool of potential applicants is *de facto* homogeneous. Think for example about a research grant for Ph.D. students: they all have a university degree and some working paper ready to send. Thus, sorting by education level may be too crude and not go far enough. Secondly, the timeframe of a contest does not always allow for applicants to adjust their (educational) choices to the menu of contracts offered by a firm. When the scientific committee announces the research prize, contestants will typically send existing papers, rather than write new ones. In conclusion, in real world markets costs may not always serve as a sorting device. [2]

I present a strategic model in which both aforementioned pillars are relaxed. In my framework, incentives and technology are both means of dealing with informational imperfections and I do not require applicants to have different costs of applying. The ability of each agent is modelled as an independent draw from a common pool of abilities. Each individual applicant knows his own ability, but does not know the abilities of the other applicants, nor the distribution of these rivals' abilities.

The committee strives for awarding the best candidate in the population, that is, the committee cares about the *rank* of the winner in the original population and is *ex ante* uninformed about individuals' abilities. It is thus implicit in my model that, although qualities are unknown in the selection stage, they matter later on. The assumption is natural in situations where the merit of the hired candidate is realized when he or she 'beats' the other candidates. The following example clarifies: a university seeks to hire a PhD for a PostDoc position. Since publishing takes time, the quality of recent graduates is relatively hard to observe. After some years, however, it will become more clear which candidates have research potential, since time allowed them to publish. At that stage, quality is easier to observe and it is likely that the largest research funds and international standing can be gained only if the best candidate was hired.

[2] Supporting this assertion, Riley (2001) emphasizes "*the need for further discussion of equilibrium in which screening/signaling costs are not perfectly negatively correlated with quality.*" (p. 475).

In a sequential game, the committee announces a contest to a total of T agents. Participation in this contest is made troublesome by the committee and I denote the utility cost of applying by c. To control c, the committee can, for example, require all kinds of formalities to be fulfilled and numerous documents be sent. Then individuals decide whether to participate or not and finally, the committee processes N applications with a selection technology and allocates the award to one candidate.[3]

The endogenous choice of how many contestants to process must take into account the imperfect and costly nature of any information processing technology. I capture this fact by implicitly assuming a constraint on time, budget or attention available for processing information. More specifically, I model the allocation of scarce limited attention by letting the reliability of the ranking of candidates decrease in the quantity of applications processed. As a result, a trade-off emerges between including the best candidate in the investigation and identifying the best candidate. I will call the decreasing reliability of the ranking technology the *overload effect*.

My results about equilibrium behaviour in this game are the following. Firstly, to avoid overload, the contest organizer sometimes chooses to disregard some pieces of information and thus randomly chooses N applications out of the total number. The overload effect thus causes the contest organizer to leave unattended some potential trading alternatives.

Secondly, I find an equilibrium of 'favorable selection', in the sense that each candidate participates if and only if the ability is above some demarcation level. Separation emerges not due to some cost heterogeneity but due to the fact that $N > 1$: the winner is determined by comparison of informative estimates of ability.[4] Individuals with higher abilities have higher probabilities of winning and are therefore easier inclined to incur the cost c, while lower abilities will not find it worthwhile to apply. I will label this the *selection effect*. I thus introduce a new mechanism for a separating equilibrium to emerge.

Thirdly, the number of participants is stochastic in equilibrium and there is a *risk of delay* or breakdown in the contest. It might occur that in equilibrium, no candidate finds

[3] We abstract from the question whether ignoring applicants is possible, from an institutional perspective.

[4] The accuracy may be quite low.

it worthwhile to participate as all candidates perceive their ability level as too low, that is, below the demarcation level.

I finally study two design problems. The first insight is that the selection effect of c is used to balance the risk of delay and the overload effect. I then analyze how the demarcation level depends on the market size T and find that it increases in T, for fixed c. The latter finding implies that the larger the potential market, the smaller the expected share of the active market. In fact, for some parameters, the expected total number of applications may actually decrease in the number of potential applicants!

Thus the sheer number effect of market size T is not the only effect to consider when overload is feared: the competitive effect, due to strategic interaction, works in the opposite direction: it discourages applying. The above suggests that the risk of delay becomes an issue for larger markets. Indeed, even though the committee incurs overload, I find that the optimal application costs c decrease in the size of the market T.

Since in some contests not the trouble of applying (c) can be controlled easily, but committees rather invest in their selection technology, the second design problem I investigate is whether for fixed c and market size T, it is optimal to invest in a perfect selection technology. The surprising result I find is that when applying is troublesome enough, it is not optimal to have a perfect selection technology, even when this technology has zero costs. The reason is that the risk of delay becomes an issue when c is high and lowering the accuracy of the selection technology is then a way to dampen this risk.

The issues in this study relate to several strands in the literature. Firstly, it is related to the traditional signaling literature. I have already discussed the features that differentiate my analysis from this field. The paper that comes closest is Janssen (2002), who studies competition for a job and the effect of the number of competitors on signaling activity and the wage the firm sets.

Secondly, another vast area is that of contests and tournaments. A distinction should be made between mechanisms that have the purpose to select one candidate out of a group (*e.g.* whom to hire) and mechanisms that serve to induce a desired behaviour of all group members (*e.g.* work hard). The term 'contest' is often understood to include the latter purpose. I should therefore emphasize that I only study the selection role of contests. For the incentive role I refer to Lazear and Rosen (1981) and subsequent papers.[5]

Most papers that study contests as selection devices share the feature that contestants provide effort (or make expenditures) that increases their chances of winning the prize.[6] I have argued that a selection procedure does not always allow for contestants to adjust their efforts. In my setup, contestants merely decide whether to apply or not and it excludes effort considerations.

Thirdly, there is some literature that discusses imperfect information acquisition. A study that comes close to my model is Ficco (2004). His emphasis is on the conditions for information overload to occur in equilibrium, defined as a situation in which the number of candidates that applies is larger than the number that will be processed. An important feature that differentiates it from my work, is that the distribution of the rivals' abilities is known. Hence, there is no risk that nobody applies, and consequently the effect of costs to invoke a selection effect is unambiguous and is not an issue in his model. Malueg and Yongsheng Xu (1997) investigate the optimal acquisition of information in order to assign a worker to a job. Information acquisition also plays a role for prospective home buyers, for whom the beliefs of quality also depend on other consumers' decisions and the time the house has been on the market. This interaction is analyzed by Taylor (1999).

Finally, there are two ingredients of prize competition that dissociate it from a typical auction: *(i)* the (common) value is deterministic and *(ii)* the committee's main objective is not to raise revenue, but to allocate the prize (contract, licence etc.) to the best candidate.[7] It is also worth noting that auctions may have undesirable self-selection effects in procurement, as shown by Manelli and Vincent (1995).[8]

[5] A more recent contribution is Clark and Riis (1998) who study competition over more than one prize.

[6] Typically some contest success function is assumed that increases in effort or rent-seeking expenditures. Recent contributions are Morgan (2003), who studies sequential expenditures by contestants, and Hvide and Kristiansen (2003), who study the degree of risk taking as a strategic variable.

[7] Moreover, auction mechanisms are not always available as prices are sometimes fixed due to regulation or other reasons, for example house rents in the Netherlands.

[8] In spite of these two important differences, a connection with the auction literature can be made, as there are some papers that study the role of participation cost and the potential number of bidders, see Levin and Smith (1994) and Menezes and Monteiro (2000). Both studies also find that participation should be costly.

The paper is organized as follows. The next section discusses the ranking of candidates by using some technology. Section 3 analyses the game and is followed by Section 4 on contest design. Section 5 relates the market size to the competitiveness of the market and Section 6 concludes and discusses further work. The lengthier proofs are in the Appendix.

6.2 Selection Technology

In this section, I introduce a selection technology, the committee's corresponding preference relation and its use of the selection technology. Consider a population S with k members. Each member i of this population has a certain ability a_i, determined by an independent random draw from a common pool of abilities. Ability a_i is private information held by agent i and the realization of abilities is unknown.

In this section, I abstract from agents' incentives and I assume all k agents apply. The committee can make N random draws from this set of applying candidates, denote this sample by X_N, with $X_N \subseteq S$. The committee investigates the ability of all $i \in X_N$ and its decision is to choose a single element from X_N.[9]

One can think of various objectives for a committee to strive for. For example, to make sure that only candidates that exceed a treshold apply, or to maximize the expected ability of the winner. Instead, I assume that the objective of the committee is to maximize the probability that the best candidate will win the prize. As mentioned in the Introduction, good arguments can be given for this assumption. The committee strives for awarding the best candidate and obtains utility u_h in that event and u_l if another was awarded, where $u_h > u_l$, w.l.o.g. I normalize $u_l = 0$.[10]

As mentioned above, an important aspect I want to capture, is that the precision of the investigation depends on N. I introduce an assessment method that suggests one candidate from X_N as winner. With a certain probability, this candidate is indeed the best candidate and with the remaining probability this candidate is a random draw from X_N. The following definition is used throughout.

[9] We assume it is not possible to award a candidate $i \notin X_N$ that was not processed.

[10] In the model, both the preference relation and the technology treat all candidates but the best in the same way. It can be seen that in a setting where all ranks are selected and evaluated differently, all the insights of the paper would obtain. Such a setting would only complicate matters needlessly.

Definition *An imperfect selection technology* $\pi : \mathbb{N} \rightarrow [0,1]$ *suggests a winner. The probability that the winner is* $\arg\max_{i \in X_N} \{a_i\}$, *that is, the best candidate, is* $\pi(N)$ *and the probability that the winner is a random draw from* X_N *is* $1 - \pi(N)$.

To incorporate the scarce nature of the capacity to process information we assume that technology π satisfies one of the two following conditions:

(A) $\pi(N+1) < \pi(N)$ *for all* N *and* $\lim_{N \rightarrow \infty} \pi(N) = 0$.

(B) For some capacity constraint $\widehat{N} \in \mathbb{N}$ *and constant* $\overline{\pi} \in [0,1]$,

$$\pi(N) = \begin{cases} \overline{\pi} & if\ N \leq \widehat{N} \\ 0 & if\ N > \widehat{N} \end{cases}.$$

Technology (A) exhibits the trade-off in quantity and quality of information processing most straightforwardly, whereas technology (B) is a step-wise approach and can be the result of outsourcing the test to an assessment center, with whom a contract $\left\{\overline{\pi}, \widehat{N}\right\}$ is agreed that specifies a reliability and a capacity of the test.

The above definition implies that, when k candidates apply and N will be processed, the probability of awarding the best in S equals:

$$\phi(N; k, \pi) \equiv \pi(N) \frac{N}{k} + [1 - \pi(N)] \frac{1}{k}. \tag{6.1}$$

This can be understood as follows. With probability $\pi(N)$, the best in X_N will be chosen and since the best in S is included in X_N with probability $\frac{N}{k}$ the first term follows. With probability $1 - \pi(N)$, the winner was a random draw from X_N and in this case the probability that the best is drawn is $\frac{1}{k}$.

The trade-off in information processing can now be seen in the first term: (a) as more candidates are being assessed the more likely it is that a really good candidate is amongst them (the fraction $\frac{N}{k}$) but as (b) the limited attention is divided over more applications, the more superficial the assessment will be, obscuring the informativeness of every single candidate's application ($\pi(N)$ decreases). Hence, this trade-off is one of processing many alternatives and the informativeness of the investigation. The problem of choosing how many alternatives to process can then be written as follows:

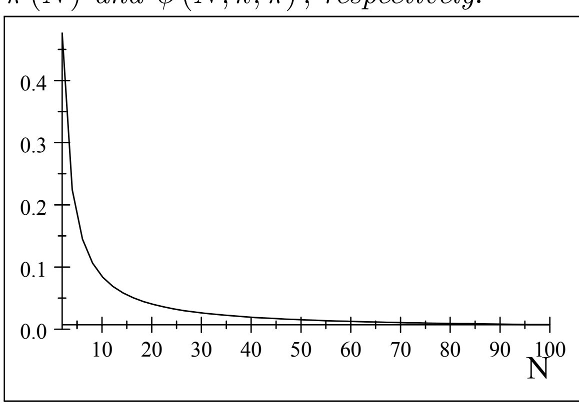
$$\max_{1 \le N \le k} \quad u_h \phi(N; k, \pi) \tag{6.2}$$

A few observations about the solution can be made. Firstly, since ϕ is a probability, it exists. Secondly, the solution to the unconstrained problem, say N^*, is independent of k and fully determined by π. Therefore I can write the solution to (6.2) as $N = \min\{k, N^*\}$: N^* tells the committee how many applications to process *at most*. Thirdly, the variable u_h has no impact of interest and will therefore be normalized to $u_h = 1$. Finally, since $\phi(2; k, \pi) > \phi(1; k, \pi)$, I have that $N^* > 1$. These results hold for any specification of π. In summary:

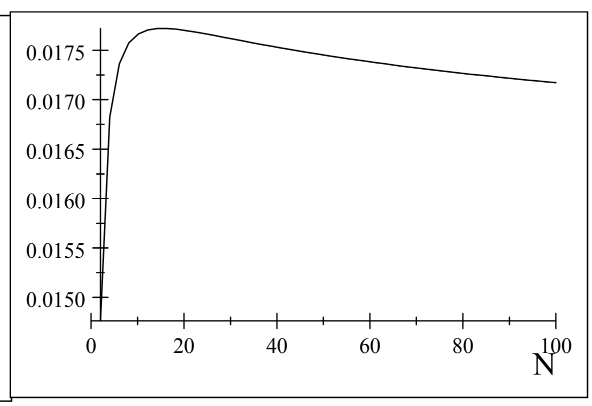
Lemma 6.1 *The committee processes at most N^* applications, ignoring applications randomly if necessary, where N^* is the solution to (6.2). N^* is larger than 1, independent of the number of applications actually received and fully determined by the selection technology π.*

The following example illustrates:

Example *Suppose $\pi(N) = N^g$ for some $g < 0$. Then the following are graphs of $\pi(N)$ and $\phi(N; k, \pi)$, respectively.*



$\pi(N) = N^{-1.07}$.



$\phi(N; 100, \pi(N))$ for $\pi(N) = N^{-1.07}$.

In the above example, $N^* = 15$. I thus conclude that imperfect decision making may require to process a finite amount of information, implying that some sources are left unattended.

6.3 The Game

Above we studied the problem of the committee in isolation from applicants' incentives. In this section I introduce the entire game and its equilibrium. There is a population of T agents. The committee announces to all T agents a contest $C\left\langle \frac{c}{V}, \pi; T\right\rangle$ for prize V that costs c to participate in. The timing of events is as follows:

0. The committee determines and announces the contest C.

   Nature draws abilities $\{a_1, a_2, ..., a_T\}$ from a uniform distribution with support $[0, 1]$.

1. Each agent i privately learns a_i and decides whether to participate (*i.e.* to apply) or not. By participating in the contest he incurs cost c.

2. The committee observes the number of applications and decides how many to investigate, N.

3. The investigation of the committee yields a winner and the prize is awarded.

We will postpone the determination of c/V and π in stage 0 to the next section. Before analyzing stages 1 and 2 in detail, I introduce a first result:

Lemma 6.2 *Let $\pi(N) > 0$. If in an equilibrium of the game a candidate with ability a' applies then all candidates with ability $a_i > a'$ apply as well.*

Proof. First note that all candidates have the same costs and benefits of applying and that they only differ in their chances to win. Hence, I prove the claim by arguing that the probability to be chosen by the committee increases in true ability a_i on $[a', 1]$. The probability to be chosen is equal to the probability to be suggested as winner by π. The winner, in turn, is either determined correctly or by chance. Thus, the higher the ability, the higher the probability to win. Furthermore, note that $\pi(N)$ is independent of a_i and then the result follows. ■

This result implies that there exists a demarcation level α such that a candidate i applies if and only if $a_i \geq \alpha$. I will now analyze Stages 2 and 1 subsequently. Let us denote by $S \subseteq T$ the subset of candidates that send an application to the contest organizer in Stage 1, with $\mid S \mid = k$. The contest organizer has to decide in Stage 2 on N, the number of applications to consider. S consists of the k best candidates in T and Lemma 6.1 applies.

I now turn to stage 1, the incentive to apply. Due to Lemma 6.2 we can restrict the quest for equilibrium strategies to those of the form "apply if and only if $a_i \geq \alpha$" and I will now investigate how the demarcation level α is determined.[11] We have that k denotes the unknown total number of applications. In determining the expected payoff of applying, the expectation about the number of rivals applying $(k-1)$ will turn out to play a major role. A potential contestant i with ability $a_i \in [0, \alpha]$ has to conjecture two uncertain events to decide whether or not to apply: $i \in X_N$ (being processed) and the event of being selected (being labeled as winner). I now proceed by investigating how the probability to win for such a type is determined.

First, given that $i \in S$, $\Pr\left(i \in X_N \mid i \in S\right) = \frac{N}{k}$. Second, given that i applies $(i \in S)$ and that i will be investigated $(i \in X_N)$, i wins if the test with technology π suggests him as best candidate.

Consider first the case $1 < k \leq N^*$. All k candidates will be processed and thus agent i will be considered surely. If the winner is determined correctly, he looses for sure since at least one other candidate applied and this other candidate has a higher ability. If the winner is determined by chance, he wins with probability $\frac{1}{k}$. Thus, the probability to win for given $k-1$ in case all will be processed equals

$$\left[1 - \pi\left(k\right)\right] \frac{1}{k} \text{ for } 1 < k \leq N^*,$$

and when $k > N^*$, then i will be considered with probability $\frac{N^*}{k}$ and only N^* candidates will be processed, hence the probability to win is then:

$$\frac{N^*}{k} \left[1 - \pi\left(N^*\right)\right] \frac{1}{N^*} \text{ for } k > N^*.$$

[11] Note that we have in fact two kinds of contestant-strategy that are part of an equilibrium: "apply iff $a_i > \underline{a}$" and "apply iff $a_i \geq \underline{a}$". The distinction is unimportant.

Now since, if $k=1$, agent i is the only one and wins for sure, we can write the probability to win for all types $a_i \in [0,\alpha]$ as:

$$\Gamma(\alpha,\pi,N^*,T) \equiv \sum_{k=1}^{T} P(k-1)\,F(k-1)\,, \text{ where } F(k-1) = \begin{cases} 1 & \text{if} \quad k=1 \\ \frac{1-\pi(N)}{k} & \text{if} \quad k>1 \end{cases}, \tag{6.3}$$

$N=\min\{k,N^*\}$ and $P(k-1) \equiv \binom{T-1}{k-1}\alpha^{T-k}(1-\alpha)^{k-1}$ denotes the density function of $k-1$.

To relate the demarcation α that arises in equilibrium to the other variables of the model, it is important how $\Gamma(\alpha,\cdot)$ depends on α. When α increases, the probability that a given rival candidate applies decreases and thus, probability mass shifts to lower k. How this effects $\Gamma(\alpha,\cdot)$ depends on the shape of $F(k)$, as can been seen in expression (6.3). For $k>1$ and $N^*>T$, we have that $F(k-1)=(1-\pi(k))/k$ and two counterforces are at work: when k decreases, the easier it is to be selected when the selection is random, but the lower the probability that the selection will be random since the technology is more accurate for lower k. Thus, both numerator and denominator of F increase in k. To characterize equilibrium I want Γ to be monotone, however. The following assumption resolves this:

Assumption 1 Let $\hat{k}=(T-1)(1-\alpha)+1$. Technology π is such that

$$\begin{aligned} F(k-1) &\geq F\left(\hat{k}-1\right) \quad \forall\, k \leq \hat{k}, \\ F(k-1) &< F\left(\hat{k}-1\right) \quad \forall\, k > \hat{k}. \end{aligned}$$

It is immediate that a constant technology (type B) always satisfies this assumption (since π does not vary). For type (A), we have that $\widehat{k}-1$ is the expected number of competitors so this condition requires that, for a non-applying agent, the chance to win if $k-1$ is below (above) average is higher (lower) than the chance to win if $k-1$ is average.[12] The assumption allows us to show:

[12] Technologies characterized by a very steep decrease of π (the ability to select the best deteriorates very fast in the number processed) might not satisfy this assumption. What $\Gamma(\alpha,\cdot)$ might look like in that case is left as an exercise.

Lemma 6.3 *The probability* $\Gamma(\alpha, \pi, N^*, T)$ *increases in* α.

The following Theorem characterizes the equilibrium for the game.

Theorem 6.4 *(i) Suppose* $(1-\pi(N^*))/T < c/V < 1$, *then in every subgame perfect equilibrium of the game, only agents with ability* $a_i \geq \alpha$ *apply, where* $\alpha > 0$ *is the unique solution to*

$$\Gamma(\alpha, \pi, N^*, T) = c/V. \tag{6.4}$$

(ii) Suppose that $c/V \leq (1-\pi(N^*))/T$. *Then an equilibrium in which all agents apply exists.*

In both cases the committee processes at most a finite number of applications, which is fully determined by π.

Proof. In Lemma 6.2 I have demonstrated the existence of a demarcation level α as stated. Furthermore, the uniqueness of the solution $\Gamma(\alpha, \pi, N^*, T) = c/V$ follows from Lemma 6.3.

(i) I proceed by showing that in every equilibrium: (a) $\alpha > 0$ and (b) the determination of demarcation level α as in (6.4).

(a) When every candidate applies ($\alpha = 0$), the probability distribution $P(m)$ collapses such that all probability mass shifts to $m = T-1$, hence $\Gamma(0, \pi, N^*, T) = (1-\pi(N^*))/T < c/V$, from which it follows that a sender with $a_i = \varepsilon$, for small $\varepsilon > 0$, is strictly better off by not applying, yielding a contradiction.

(b) Denote the demarcation level by $\widetilde{a}$. We will rule out the situations $\Gamma(\widetilde{a}, \pi, N^*, T) < c/V$ and $\Gamma(\widetilde{a}, \pi, N^*, T) > c/V$. The first implies that $\widetilde{a}$ is strictly better off by not applying and the second point implies that all in $[0, \widetilde{a}]$ apply. Both points contradict the fact that $\widetilde{a}$ is a demarcation level.

(ii) That an equilibrium with all T candidates applying exists easily follows. ■

I note that non-applying agents have the same incentive to apply as the candidate with ability level α. The reason for this is the following. Incentives to apply are constant in a_i for $a_i < \alpha$ as low-ability people can only be chosen if the winner is determined by chance or if $k = 1$, and hence, their winning is independent of their true ability. I therefore assign a strategy to low-ability people not to apply, although they do not have a strict incentive to do so, namely they are indifferent, as agent α would be. I have shown that this profile of strategies is the only subgame perfect equilibrium profile.

The equilibrium proposed above shows how information asymmetry can be overcome, when the market does not allow for signaling. Because the uninformed compares 2 or more candidates (with a precision that may be quite low), the higher types have a higher probability of winning. As a result, only the higher types are willing to incur the participation costs and separation is induced. I will label this the *selection effect* of participation costs c.

In what follows I characterize the contest design. Since variations in c and π will have no effect on the incentive to apply as long as $c/V \leq (1 - \pi(N^*))/T$, I will focus on the properties of the equilibrium described under (i) of the Theorem, rather than (ii).

6.4 Contest Design

In this section I consider two design problems: (i) for given market size and selection technology, how troublesome should applying be made and (ii) for given market size and cost of applying, is it optimal to have a perfect selection technology ?

Since what matters in equilibrium condition (6.4) is the ratio c/V, all my results on c can be applied to V as well.[13] For simplicity, we normalize $V = 1$. The committee's objective function is given by:

$$U(c, \pi; T) \equiv \sum_{k=1}^{N^*} P(k; \alpha) \phi(k; k, \pi) + \sum_{k=N^*+1}^{T} P(k; \alpha) \phi(N^*; k, \pi),$$

where, with a slight change in notation from (6.3), $P(k; \cdot)$ denotes the probability that k out-of T apply. There is a probability that *no one* applies ($k = 0$) and when that happens, the contest is delayed. I will label this the *risk of delay* and the payoff in that event is normalized to 0.[14]

[13] This is the case to the extent that V is not a transfer.

[14] The dynamic process of delay and the resulting updating of information is left out of the model.

For $k \geq 1$ the above expression can be understood by recalling that when k agents apply, the probability to award the winner is $\phi(\cdot)$, as was elucidated in (6.1). Note that when $k > N^*$, the test capacity is overloaded and then the best candidate might not be considered, and I will label this the *overload effect*.

In the remainder of the paper I will study the interaction between the selection effect, the overload effect and the risk of delay to characterize the optimal design of the contest. First observe that Lemma 6.3 and equilibrium equation (6.4) imply that $\frac{\partial \alpha}{\partial c} > 0$. This leaves us with investigating one other variable, market size T.

For an individual candidate, the number of potential competitors affects the chance of winning. First, recall that a type $a_i \in [0, \alpha]$ can only win if the test picks up a candidate randomly. Then, as T increases, for given demarcation α more candidates apply, and the discussion that proceeds assumption 1 is again relevant: the increase in k makes it more difficult to be selected if selection is random, but increases the probability that the selection is in fact random. The latter effect stimulates low-ability types to apply. By assumption 1 the first effect dominates and consequently I can show:

Lemma 6.5 *The demarcation level α increases in T.*

We now use this result to characterize the two design problems.

Designing c

For a given technology π, I relate the optimal cost c to the market size T. If the demarcation level α would be independent of T, one might conclude that, since for given α, k increases in T, optimal cost increase in T. The intuition would be that the larger the market, the more troublesome applying should be, in order to avoid overload. However, since Lemma 6.5 shows that α does depend on T, this reasoning lacks the strategic effect of the market size. To accurately incorporate the fact that $U(c, \pi; T)$ depends on T via α as well as directly via $P(k; \cdot)$, I resort to a type (B) technology.[15] Doing so, I find that:

Proposition 6.6 *Suppose that the selection technology is of type (B). There exist $\widetilde{\pi} \leq 1$ such that if $\overline{\pi} < \widetilde{\pi}$, then $c(T+1) < c(T)$ for all T.*

[15]The difficulty with a type (A) technology is that Γ contains a sum.

The above Proposition demonstrates that the strategic effect of T inverts the logic that in larger markets it should be more troublesome to apply: even though overload constitutes a problem, application cost should decrease in market size!

Designing π

It can easily be verified that when $N^* > T$ and $\overline{\pi} = 1$ (perfect information acquisition), $c(T) = 0$ for all T. In many prize competitions however, the utility cost of applying cannot be controlled to that extent. Therefore, I now look at the design of π : this variable can be controlled by investing time and budget in the selection technology. When c is given, I investigate whether it is optimal to have perfect information acquisition:

Proposition 6.7 *There exists $\widetilde{c} < 1$, such that if $c > \widetilde{c}$, the committee prefers an imperfect selection technology over a perfect one with $\overline{\pi} = 1$ and $N^* > T$.*

To give an idea about what the level of $\widetilde{c}$ might be, I computed it for a market with $T = 50$ potential applicants: when the utility cost of applying for the prize amounts to more than 20% of the value of winning it, the committee prefers having an imperfect selection technology to a perfect one! The reason is that a very accurate selection technology works discouraging and by lowering it, the risk of delay is dampened. Note that the result does not require the technology to be costly.

6.5 Market Size

The analysis above shows how a committee best balances the risk of receiving too many applications with the risk of receiving no applications. Surprisingly, when the market size increases and the information technology is poor (low π), the selection effect of c should not be used to discourage applications, but to encourage them. Indeed, when information acquisition is perfect, participation cost c should be zero. Conversely, when c cannot be that low, information acquisition should not be perfect. This points out that the risk of market break down might increase with market size and invalidates the commonly held belief that the more players there are, the more competitive the outcome will be.

Since additionally, perfect information acquisition is believed to enable lush competition, I will in this section use the model to investigate the hypothesis that having perfect information and a large number of competitors will lead to a competitive outcome. We denote by $E(k \mid c, T)$ the expected number of applications:

Proposition 6.8 *Let $\pi(\cdot)$ be of type (B) with $\overline{\pi} = 1$.*
(i) The risk of break down $P(0; \cdot)$ increases in T;
(ii) For all finite T, there exist $\widehat{c}(T) < 1$ such that $E(k \mid c, T) > \lim_{T \to \infty} E(k \mid c, T)$ for $c \in (\widehat{c}(T), 1]$.

The proposition shows that having a larger market does not imply having a more competitive outcome (understood as the expected number of participants), even when processing capacity is unbounded. This is a relevant insight for those designing markets. Applied to the labour market, the above implies that in times of high unemployment, people could actually apply less in the aggregate, due to the low expectations they have about their chances of being accepted!

My result corresponds to findings in the consumer search literature in which the expected price is related to the number of firms in the market. Janssen and Moraga-Gonzalez (2004) show for example that an increase in the number of firms may lead to an increase in the expected price. In their study, some consumers engage in costly search for prices, others are fully-informed and firms randomize their price. Now when the number of rival firms increases, it becomes less likely to be the lowest-priced firm (*i.e.* to win the competition for selling to the informed consumers) and in that case a firm can be better off by targeting the consumers that search for one price only, instead of targeting the fully-informed consumers. Hence, the expected price increases.

6.6 Concluding Remarks

A model of prize competition was presented. My model intends to describe markets where the contest is specialized in such a way that the applicants cannot be sorted on the basis of heterogenous variables. I show that in this case, the uninformed market side finds an alternative means of information acquisition: directly comparing candidates by some technology. As a result, better candidates are more likely to win and separation is induced. A novel feature for the traditional asymmetric information literature is that both technology and incentives reduce informational asymmetry. My model thus fills the gap in markets where the traditionally studied mechanisms are not applicable.

The technology is imperfect and as a result, resources will not be spend on *every* application received. This resembles the practice in selection procedures in which numerous applications are first ranked on rather trivial, uninformative attributes and consequently, costly interviews are given to a select few.

The nature of information asymmetry I use deserves some discussion. In my setup, besides not knowing which candidate has which ability (as in most models), it is also unknown whether there is an applicant out there with ability above some given level.[16] For an applicant, knowing her own ability, this implies that she does not know whether a *better* candidate considers applying. Especially in those situations where the population of potential applicants is not too small (in that case people may know each other too well) one cannot be sure that one of the applicants possesses any pre-specified ability.

The finding that there is a positive probability that *nobody* applies, is driven by the fact that realizations of ability are unknown. As argued above, for many markets, not requiring players to know the distribution is appropriate, and hence, the contingency that no one applies is an important one. A contest designer must simply take into account that it might happen that nobody finds it worthwhile to apply: all agents perceive their chance of winning too low compared to the prize. If no one applies, information about abilities in the population is revealed and lower quality candidates are encouraged to apply. This dynamic process, which is left out of the theoretical model, delays the contest.

The variables c and V of the model are not treated as transfers but instead incurred, resp. enjoyed, only by the agents. They lend themselves therefore for various, non-monetary interpretations. The cost parameter c for example, can be seen as the trouble of collecting the application documents and the value of the prize V can be seen as

[16] Again, as exception, I have this feature in common with Janssen (2002).

prestige. In case of journal submissions, the cost c can be seen as the waiting time for an editorial decision.

The above mentioned selection and overload effects and risk of delay were analyzed and showed that the actual level of competition may vary with market size in an unanticipated way. Insights about optimal contest design that we obtained are firstly, that the larger the market, the easier applying should be, in order to mitigate the discouraging effect of competition. This effect of market size dominates the potential overload of applications. Secondly, when the utility cost of applying is fixed and high enough, an accurate selection technology also discourages applicants. In that case, it is in the interest of the committee to have a less than perfect technology.

Besides the typical contests for prizes and contracts, these insights can be applied to the allocation of public assets, such as frequencies and licences for example, when they are procured via a beauty contest rather than via an auction. In this light, I should emphasize that the committee's sole objective in this model is to allocate the prize to the best possible candidate, rather than revenue maximization. A comparison of the two mechanisms constitutes an avenue for future research.

6.7 Appendix

We investigate the impact on $\Gamma(\cdot)$ of marginal changes in α and T :

Lemma 6.3 *The probability* $\Gamma(\alpha, \pi, N^*, T)$ *increases in* α.

Proof. For convenience, we denote $m \equiv k-1$. First note that:

$$\frac{\partial P(m,\alpha)}{\partial \alpha} \begin{cases} > 0 \text{ for } m < \widehat{m} \\ = 0 \text{ for } m = \widehat{m} \\ < 0 \text{ for } m > \widehat{m}, \quad \text{where } \widehat{m} = (T-1)(1-\alpha). \end{cases} \tag{6.5}$$

This easily follows from the fact that:

$$\frac{\partial P(m,\alpha)}{\partial \alpha} = \binom{T-1}{m} \alpha^{T-m-1} (1-\alpha)^m \left(\frac{(T-1)(1-\alpha) - m}{\alpha(1-\alpha)} \right).$$

Let $P' = \frac{\partial P(m,\alpha)}{\partial \alpha}$, the claim can then be stated as

$$\frac{\partial \Gamma(\cdot)}{\partial \alpha} = \sum_{m=1}^{\widehat{m}} P' F(m) + \sum_{m=\widehat{m}+1}^{T-1} P' F(m) + \frac{\partial P(0,\alpha)}{\partial \alpha} > 0.$$

By Assumption 1 and (6.5) this is surely the case if $F(\widehat{m}) \sum_{m=1}^{T-1} P' + \frac{\partial P(0,\alpha)}{\partial \alpha} > 0$ which implies

$$F(\widehat{m}) \sum_{m=0}^{T-1} P' + (1 - F(\widehat{m})) \frac{\partial P(0,\alpha)}{\partial \alpha} = (1 - F(\widehat{m})) \frac{\partial P(0,\alpha)}{\partial \alpha} > 0,$$

now since $F(\widehat{m}) < 1$ this inequality holds. This completes the proof. ■

Lemma 6.5 *The demarcation level α increases in T.*

Proof. A change in T changes only the probability distribution and the possible values m can take. Therefore, let us denote the distribution of m when there are $T+1$ senders in total by $\widetilde{P}(m)$, (omitting the argument α for convenience):

$$\begin{aligned} P(m) &= \binom{T-1}{m} \alpha^{T-1-m} (1-\alpha)^m, \\ \widetilde{P}(m) &= \binom{T}{m} \alpha^{T-m} (1-\alpha)^m = \frac{\alpha T}{T-m} P(m), \end{aligned}$$

hence

$$\begin{aligned} \Gamma(\alpha, \pi, N^*, T+1) &= \sum_{m=1}^{N^*-1} \widetilde{P}(m) \left\{ \frac{(1-\pi(m+1))}{m+1} \right\} + \\ &+ \sum_{m=N^*}^{T-1} \widetilde{P}(m) \left\{ \frac{(1-\pi(N^*))}{m+1} \right\} + \widetilde{P}(T) \frac{1-\pi(N^*)}{T+1} + \alpha P(0), \end{aligned}$$

and thus we need to show that

$$\Gamma(\alpha, \pi, N^*, T+1) - \Gamma(\alpha, \pi, N^*, T) = \sum_{m=1}^{N^*-1} \left[\widetilde{P}(m) - P(m) \right] \left\{ \frac{(1-\pi(m+1))}{m+1} \right\} +$$
$$\sum_{m=N^*}^{T-1} \left[\widetilde{P}(m) - P(m) \right] \left\{ \frac{(1-\pi(N^*))}{m+1} \right\} + \widetilde{P}(T) \frac{1-\pi(N^*)}{T+1} - (1-\alpha) P(0) < 0$$

Now note that $\widetilde{P}(m) - P(m) = \frac{m-(1-\alpha)T}{T-m} P(m)$ such that the coefficients on $F(m)$ are negative for small m and positive for large m. Indeed, we can use the same method as in Lemma 6.3, employing again (6.5) and Assumption 1.

A sufficient condition for the inequality above is then:

$$\sum_{m=1}^{\widehat{m}} \left[\widetilde{P}(m) - P(m)\right] F(\widehat{m}) + \sum_{m=\widehat{m}+1}^{T-1} \left[\widetilde{P}(m) - P(m)\right] F(\widehat{m}) + \\ + \widetilde{P}(T) \frac{1-\pi(N^*)}{T+1} - (1-\alpha) P(0) < 0$$

Repeatedly rewriting of LHS we get:

$$\begin{aligned} & F(\widehat{m}) \sum_{m=1}^{T-1} \left[\widetilde{P}(m) - P(m)\right] + \widetilde{P}(T) \frac{1-\pi(N^*)}{T+1} - (1-\alpha) P(0) \\ = \; & F(\widehat{m}) \left\{ \sum_{m=1}^{T-1} \widetilde{P}(m) - \sum_{m=0}^{T-1} P(m) \right\} + \widetilde{P}(T) \frac{1-\pi(N^*)}{T+1} - (1-\alpha - F(\widehat{m})) P(0) \\ = \; & \widetilde{P}(T) \left\{ \frac{1-\pi(N^*)}{T+1} - F(\widehat{m}) \right\} - (1-\alpha)(1-F(\widehat{m})) P(0) < 0, \end{aligned}$$

and since $\frac{1-\pi(N^*)}{T+1} = F(T) < F(\widehat{m})$ this inequality is satisfied. ■

The next two statements are on contest design. For easy reference, we state the objective function for a type (B) technology:

$$\begin{aligned} U(c,\pi;T) \;=\; & \pi\left[1-\alpha^T\right] + (1-\pi)\left\{ \sum_{k=1}^{T} \binom{T}{k} \frac{1}{k} (1-\alpha)^k \alpha^{T-k} \right\} + \\ & +\pi \left\{ \sum_{N^*+1}^{T} \binom{T}{k} (1-\alpha)^k \alpha^{T-k} \left[\frac{N^*}{k} - 1\right] \right\}. \end{aligned}$$

Proposition 6.6 *Suppose that the selection technology is of type (B). There exists $\widetilde{\pi} \leq 1$ such that if $\overline{\pi} < \widetilde{\pi}$, then $c(T+1) < c(T)$ for all T.*

Proof. Take the first-order condition w.r.t. α for a maximum of U :

$$-T\pi\alpha^{T-1}+(1-\pi)\frac{T}{\alpha}\left\{\sum_{k=1}^{T}\binom{T}{k}\frac{1}{k}(1-\alpha)^{k}\alpha^{T-k}-\frac{1-\alpha^{T}}{T(1-\alpha)}\right\}+$$
$$+\pi\sum_{N+1}^{T}\binom{T}{k}\alpha^{T-k}(1-\alpha)^{k}\frac{T(1-\alpha)-k}{\alpha(1-\alpha)}\left[\frac{N}{k}-1\right]=0, \tag{6.6}$$

The proof consists of three steps.

Step (1).

Denote by $G(T)=\sum_{k=1}^{T}\binom{T}{k}\frac{1}{k}(1-\alpha)^{k}\alpha^{T-k}$ and by $H(T)=\sum_{N^{*}+1}^{T}\binom{T}{k}(1-\alpha)^{k}\alpha^{T-k}\left[\frac{N^{*}}{k}-1\right]$. We then obtain that (6.6) implies:

$$c(T)=(1-\pi)G(T)+\pi\left(\alpha^{T-1}-\alpha^{T}\right)+\pi\frac{\alpha}{T}H(T).$$

Step (2).

Rewrite $G(T)$:

$\sum_{k=1}^{T}\binom{T}{k}\frac{1}{k}(1-\alpha)^{k}\alpha^{T-k}=\alpha^{T}\sum_{k=1}^{T}\binom{T}{k}\frac{1}{k}\left(\frac{1-\alpha}{\alpha}\right)^{k}$. Now let $z=\frac{1-\alpha}{\alpha}$, then we get: $G(T)=\alpha^{T}\sum\binom{T}{k}\frac{1}{k}z^{k}$. Now since $\frac{1}{k}z^{k}=\int_{0}^{z}x^{k-1}dx$, we get $\alpha^{T}\sum\binom{T}{k}\int_{0}^{z}x^{k-1}dx=\alpha^{T}\int_{0}^{z}\left(\sum_{0}^{T}\binom{T}{k}x^{k-1}-\frac{1}{x}\right)dx=$ $\alpha^{T}\int_{0}^{z}\left(\frac{1}{x}\sum_{0}^{T}\binom{T}{k}x^{k}-1\right)dx=$

$$\alpha^{T}\int_{0}^{z}\frac{1}{x}\left((1+x)^{T}-1\right)dx=\alpha^{T}\int_{0}^{\frac{1-\alpha}{\alpha}}\frac{(1+x)^{T}-1}{x}dx.$$

It is convenient to eliminate α from the bounds of integration. Let $y=\frac{\alpha}{1-\alpha}x$ and $dx=\frac{1-\alpha}{\alpha}dy$, then the above can be rewritten as $\alpha^{T}\int_{0}^{1}\frac{\left(1+\frac{1-\alpha}{\alpha}y\right)^{T}-1}{\frac{1-\alpha}{\alpha}y}\frac{1-\alpha}{\alpha}dy$ or:

$$G(T)=\int_{0}^{1}\left[(\alpha+(1-\alpha)y)^{T}-\alpha^{T}\right]\frac{dy}{y}.$$

Using this expression, take the first-order condition w.r.t. α :

$$\frac{\partial U}{\partial\alpha}=-T\pi\alpha^{T-1}+(1-\pi)\frac{dG}{d\alpha}+\pi H(T)=0 \text{ or}$$

$$-T\pi\alpha^{T-1}+(1-\pi)\left\{\int_0^1\frac{dy}{y}\left(T\left[\alpha+(1-\alpha)y\right]^{T-1}(1-y)-T\alpha^{T-1}\right)\right\}+\pi H(T)=0\Rightarrow$$

$$\begin{aligned}-\pi\alpha^{T-1}+\frac{\pi}{T}H(T)+(1-\pi)\int_0^1\frac{dy}{y}\left[\alpha+(1-\alpha)y\right]^{T-1}-\alpha^{T-1}=\\ (1-\pi)\int_0^1 dy\left[\alpha+(1-\alpha)y\right]^{T-1}\quad\Longrightarrow\end{aligned}$$

$$(1-\pi)G(T-1)-\pi\alpha^{T-1}+\frac{\pi}{T}H(T)=(1-\pi)\frac{1-\alpha^T}{T(1-\alpha)}$$

Hence, f.o.c. (6.6) when the market size equals T implies that:

$$c(T)=(1-\pi)G(T-1)+\frac{\pi}{T}H(T).$$

Step (3).

Now, when convenient, denote by $\alpha(T)$ the level of α that satisfies the f.o.c. when the market volume is T. From Steps (1) and (2) we know:

$$\begin{aligned}c(T) &= (1-\pi)G(T,\alpha(T))+\pi\left(\alpha^{T-1}-\alpha^T\right)+\pi\frac{\alpha}{T}H(T)\\ c(T+1) &= (1-\pi)G(T,\alpha(T+1))+\frac{\pi}{T+1}H(T+1).\end{aligned}$$

Thus, to prove the result we will show that:

$$\begin{aligned}(1-\pi)G(T,\alpha(T))+\pi\left(\alpha^{T-1}-\alpha^T\right)+\pi\frac{\alpha}{T}H(T)>\\ (1-\pi)G(T,\alpha(T+1))+\frac{\pi}{T+1}H(T+1)\ \text{ or}\end{aligned}$$

$$G(T,\alpha(T))-G(T,\alpha(T+1))>\frac{\pi}{1-\pi}\left(\frac{H(T+1)}{T+1}-\left(\alpha^{T-1}-\alpha^T\right)-\frac{\alpha}{T}H(T)\right).$$

Note that, for $\alpha(T)$ we have $\frac{\partial G}{\partial\alpha}=\frac{\pi}{1-\pi}\left(T\alpha^{T-1}-H\right)$ due to (6.6). Since $\frac{\pi}{1-\pi}$ increases, we have that $\frac{\partial G}{\partial\alpha}$ goes to zero when π goes to zero. This, in turn, implies that when π goes to zero, $G(T,\alpha(T))$ goes to the maximum of G w.r.t. α. Now since $\alpha(T+1)$ is different from $\alpha(T)$ in a discrete way, we can conclude that there exists a $\hat{\pi}$ such that LHS of the latter inequality is positive for all $0\leq\pi\leq\hat{\pi}$. Finally, since the RHS equals zero for $\pi=0$, we can conclude that there exists $\widetilde{\pi} > 0$ such that the inequality holds for all $\pi \in [0, \widetilde{\pi}]$. This completes the proof. ■

Proposition 6.7 *There exists $\widetilde{c} < 1$, such that if $c > \widetilde{c}$, the committee prefers an imperfect selection technology over a perfect one with $\overline{\pi} = 1$ and $N^* > T$.*

Proof. We will investigate $\partial U / \partial \pi$ in the point $\overline{\pi} = 1$ and $N^* > T$, and use the same notation as in the previous proof. We need to show that:

$$\frac{\partial U}{\partial \pi} = 1 - \alpha (\pi)^T - T\alpha (\pi)^{T-1} \frac{\partial \alpha}{\partial \pi} \pi - G + \frac{\partial G}{\partial \pi} (1 - \pi) + H + \frac{\partial H}{\partial \pi} \pi < 0$$

Since $N^* > T$, the last two terms vanish and then substituting $\pi = 1$ we get

$$1 - \alpha (\pi)^T - T\alpha (\pi)^{T-1} \frac{\partial \alpha}{\partial \pi} - G < 0. \tag{6.7}$$

First, we investigate $\frac{\partial \alpha}{\partial \pi}$. The demarcation α depends on π via, (6.4) and we thus use the Implicit Function Theorem to obtain that, in $\pi = 1$:

$$\frac{\partial \alpha}{\partial \pi} = -\frac{\alpha^{T-1} - \frac{1-\alpha^T}{T(1-\alpha)}}{(T-1)\, \alpha^{T-2}},$$

thus $T\alpha^{T-1} \frac{\partial \alpha}{\partial \pi} = \frac{1}{T-1} \left(\frac{\alpha\left(1-\alpha^T\right)}{1-\alpha} - T\alpha^T \right)$. Inequality (6.7) thus becomes

$$\begin{aligned} 1 - \alpha^T - G &< \frac{1}{T-1} \left(\frac{\alpha\left(1-\alpha^T\right)}{1-\alpha} - T\alpha^T \right) \quad \text{or} \\ 1 - G &< \frac{1}{T-1} \frac{\alpha - \alpha^T}{1-\alpha}. \end{aligned} \tag{6.8}$$

Now from the proof of Proposition 6.6 we know that
$G = \int_0^1 \left[(\alpha + (1-\alpha) y)^T - \alpha^T \right] \frac{dy}{y}$ and
$\frac{dG}{d\alpha} = \int_0^1 \frac{dy}{y} \left(T \left[\alpha + (1-\alpha) y \right]^{T-1} (1-y) - T\alpha^{T-1} \right)$. Label LHS and RHS of (6.8) by $LHS(\alpha)$ and $RHS(\alpha)$, respectively. We will now show that (i) $LHS(1) = RHS(1)$ and (ii) $\frac{dLHS}{d\alpha}(1) > \frac{dRHS}{d\alpha}(1)$.
(i) It is easily shown that $G(\alpha = 1) = 0$ and RHS can be written as $\frac{1}{T-1} \alpha \frac{1-\alpha^{T-1}}{1-\alpha}$. By applying l'Hopital's rule once we get that $\lim_{\alpha \to 1} \frac{1-\alpha^{T-1}}{1-\alpha} = T-1$, showing that $RHS(1) = 1$.

(ii) We have $\frac{dG}{d\alpha}(1) = \int_0^1 \frac{dy}{y}(T(1-y)-T) = -T$ and thus $dLHS/d\alpha = T$. Now, $\frac{dRHS}{d\alpha}(1) = \frac{1}{T-1}\lim_{\alpha\to 1}\frac{1-T\alpha^{T-1}}{1-\alpha}$ and by applying l'Hopital's rule twice we obtain that the limit equals $\frac{dRHS}{d\alpha}(1) = T/2$.

Due to points (i) and (ii) there exists $\alpha' < 1$ such that (6.8) holds for all $\alpha > \alpha'$ and the fact that α monotonically increases in c implies the existence of $\widetilde{c}$ as stated. ■

Proposition 6.8 *Let $\pi(\cdot)$ be of type (B) with $\overline{\pi} = 1$.*

(i) The risk of break down $P(0;\cdot)$ increases in T;

(ii) For all finite T, there exist $\widehat{c}(T) < 1$ such that $E(k \mid c, T) > \lim_{T\to\infty} E(k \mid c, T)$ for $c \in (\widehat{c}(T), 1]$.

Proof. (i) In equilibrium, $\alpha(T)^{T-1} = c$ and hence $P(0;\cdot) = \alpha(T)^T = \alpha(T)c$. Now since RHS increases in T, LHS must increase in T as well.

(ii) We first show that $E(k \mid c, T)$ converges to $-\ln(c)$. We have $E(k \mid c, T) = T\left[1-(c)^{1/(T-1)}\right]$. Write this as $\frac{T}{1/\left[1-(c)^{1/(T-1)}\right]}$ and note that both numerator and denumerator converge to infinity. The limit can then be obtained by repeatedly applying l'Hôpital's rule.

Next, we show

$$T\left(1-c^{1/(T-1)}\right) > -\ln(c).$$

First observe that both sides are decreasing in c. Then we will prove by showing that $LHS > RHS$ for c high enough. We have that $\frac{\partial LHS}{\partial c} = \frac{-T}{T-1}c^{-T/(T-1)}$ and $\frac{dRHS}{dc} = -\frac{1}{c}$ and $LHS(1) = RHS(1)$ and thus the inequality is satisfied in a left neighbourhood of $c = 1$.

■

Chapter 7

Summary and Conclusions

Several important economic consequences of information and communication technologies (ICTs) were discussed in this thesis. While incorporating the promising features of these ICTs, each essay also addressed the important drawbacks that are naturally associated to them. In this way, we attempted to provide a balanced analysis of an ICTs significance for society. Indeed, our analysis might shed some light on the question why these technologies have not invaded the economic sphere totally. This holds especially for Part I. The essays in Part II are to a lesser extent focused on the emergence of the Internet, but discuss information acquisition and the advances in technology that allow risk selection (Chapters 4 and 5) and how best to use the possibilities of information acquisition (Chapter 6). In this final Chapter I summarize each individual Chapter and attempt to provide some implications for further research and policy.

In Chapter 2 we analyze a market where firms compete in a conventional and an electronic retail channel. Consumers easily compare prices online, but some incur purchase uncertainties on the online channel. We investigate the market shares of the two retail channels and the prices that are charged. We find that the share of the electronic channel is decreasing in the size of the uncertainty. Furthermore, searching consumers do not always buy. They drop out when the uncertainty associated with buying online is not compensated by a low price. Finally, the model exhibits price dispersion and surprisingly, the expected price is increasing in the magnitude of the online purchase uncertainty and in the number of confident online shoppers. Most of the findings are in line with observations on e-commerce. Our model thus provides a sound explanation of the low share of Internet sales, emphasizing the need of trust in the online environment and the need for intangible product information. The importance of these issues has been confirmed by OECD (2004). Additionally, the equilibrium attributes price dispersion to the fact that firms' operate their different retail channels jointly.

Chapter 3 analyzes the incentives for incumbent bricks-and-mortar firms and new entrants to start an online retail channel in a differentiated goods market. To this end we set up a two-stage model where firms first decide whether or not to build the infrastructure necessary to start an online retail channel and then compete in prices using the channels they have opened up. Consumers trade-off the convenience of online shopping and the ease to compare prices, with online uncertainties. Without a threat of entry by a third pure online player we find that for most parameter constellations firms' dominant strategy is not to open an online retail channel as this cannibalizes too much on their conventional sales. As the cannibalization effect is not present for a pure Internet player, we show that these firms will start online retail channels under a much wider range of parameter constellations. The threat of entry may force incumbent bricks-and-mortar firms to deter entry by starting up an Internet retail channel themselves. We also show that a low cost of building up an online retail channel or online shopping conveniences may not be to the benefit of online shopping as the strategic interaction between firms may be such that no online retail channel is built when the circumstances seem to be more favourable. Finally, we provide arguments for the low viability of pure online retailers that was observed in the last decade. This essay thus explains the rather low share of e-commerce in retailing by investigating the profitability for firms to enter online. An interesting issue for future research is whether the theoretical predictions can be verified empirically. In future theoretical work, it would be interesting to see to what extent the conclusion drawn here for a more or less saturated market continues to hold when the online retail channel is able to attract new costumers.

Chapter 4 studies an insurance model characterized by a continuum of risk types, private information and a competitive supply side. We use the model to investigate the welfare effects of discrimination (also known as risk selection). We postulate that a test is available that determines whether an applicant's risk exceeds a treshold. Excluding the highest risks softens adverse selection, but constitutes a welfare loss for the high risks. In contrast to a *lemons market* intuition, we find that aggregate surplus decreases when risk aversion is high. When risk aversion is low however, discrimination increases aggregate surplus and may even prevent a market from being eradicated by adverse selection. The important mechanism that drives this finding is the price elasticity of the demand for insurance. The result contributes to the lively policy debate on the desirability of insurers' use of medical and genetic information: the effect of discrimination on aggregate welfare depends on the risk preferences of the insurees. Our theoretical work thus reemphasizes the need for understanding and measuring the empirical attitudes towards risk, and how they vary between markets.

A model that largely coincides with the one above is employed in Chapter 5 to investigate the contestability clause in the policy: when a claim is filed, the insurer may dispute it on grounds of the information provided by the insuree. Smoking in life insurance is our leading example: there are different rates for smokers and non-smokers. We compare the aggregate utility in a two contracts economy with a one, non-contestable contract economy. Having two contracts alleviates adverse selection, but increases the risk in the smokers pool. The negative effect dominates: contestability decreases welfare. This result strongly opposes earlier studies and is striking in view of the fact that the two-contracts situation is in fact the way the market is organized in the U.S. and many other countries. Whether the theoretical controversy will find its counterpart in the policy debate remains to be seen.

The final essay presents a model in which individuals compete for a prize by choosing to apply or not. Abilities are private information and in attempt to select the best candidate, the committee compares applicants with an imperfect technology. The choice of application cost, size of the prize and use of information technology are being characterized. In equilibrium, the number of applicants is stochastic and may overload the committee. I show that in spite of overload, the optimal cost (size of the prize) is decreasing (increasing) in market size. Furthermore I show that in case the control of application cost is limited and this cost therefore cannot be set low enough, the committee prefers an *imperfect* information technology over a perfect one.

From a conceptual perspective, it is surprising that a model built to study overload and spam (resulting from a limited cognitive capacity) finds that the more potential applicants there are, the less their applying should be discouraged by cost. The mechanism of discouragement behind this may even render perfect information processing inferior! This trade-off between preventing overload and avoiding desired information being missed is a policy challenge recognized by OECD (2004).

The applications of this model are versatile. Typical examples are the competitions for prizes, grants and awards that are common in the academic world. Interestingly, my result indicates that the increased reach of competitions due to Internet should go hand in hand with easy application, which is natural for an e-mail or Internet environment. The overload effect of the high reach is offset by the discouragement of this reach for each individual applicant, making it undesirable to discourage by costs. This is however not all to it; an increased reach due to Internet does not necessarily imply that the number of potential competitors is large since it is limited by the number of individuals that is eligible for application. The paper shows that the optimal design of the competition varies with this number.

Another example that comes very naturally is architectural competition. Since in this market the costs of preparing an application are always substantial, my results indicate that here it may pay for a committee to have a not-so-perfect selection technology in order to avoid the possibility that applicants feel too uncertain to apply.

To conclude, the essays in this book cover several topics in the large field of information economics. We showed how phenomena such as bounded human information processing capacities, the importance of non-digital information, the indirect nature of e-commerce and discrimination due to disclosure of personal information can be incorporated in theoretical models. The endeavours may contribute to understanding the impact of information and communication technologies.

Chapter 8

Samenvatting (Summary in Dutch)

In het laatste decennium heeft veel vooruitgang in informatie- en communicatietechnologieën (ICT) plaatsgevonden. Deze technologieën spelen een grote rol in de wereldeconomie (OECD 2004): de opkomst van het Internet heeft de verspreiding van informatie vergemakkelijkt en technologische ontwikkeling heeft de verwerving van informatie verbeterd, bijvoorbeeld in de gezondheidszorg. Desondanks is de verwerking van informatie nog steeds begrensd door de menselijke natuur en het gemak waarmee individuen en bedrijven elkaar informatie toezenden werkt overlading ('overload') in de hand. Andere grenzen aan de mogelijkheden van informatiestromen zijn dat niet alles van waarde in digitale vorm gegoten kan worden, dat transacties via Internet indirect zijn en dat informatie discriminatie in de hand kan werken. De gevolgen van ICTs op de maatschappij zijn daarom niet zondermeer duidelijk. Zo vraagt OECD (2004) aandacht voor de volgende beleidsonderwerpen: veiligheid en vertrouwen in de online omgeving, waarborgen van privacy en de bestrijding van spam.

Dit proefschrift bestaat uit vijf essays dat elk een van de bovenstaande thema's behandelt en de impact van een ICT op het economische verkeer onderzoekt. De eerste twee hoofdstukken analyseren op welke wijze en in welke mate het Internet gebruikt kan worden voor (detail)handel. De voor- en nadelen van Internet voor zowel consumenten als bedrijven worden gemodelleerd om het functioneren van de markt in kaart te brengen, waardoor zicht ontstaat op het belang van e-commerce. Hoofdstukken 4 en 5 gaan over de informatieachterstand die verzekeraars ondervinden ten opzichte van kandidaat-verzekerden en welke technologieën zij in de praktijk gebruiken om deze achterstand te overbruggen. Zoals gebleken is in beleidsdiscussies, over bijvoorbeeld het gebruik van genetische informatie, kan deze informatieverwerving leiden tot discriminatie en schending van privacy. De twee essays onderzoeken elk het effect van een vorm van informatiegebruik op de totale maatschappelijke welvaart en dragen zo bij aan het debat over de wenselijkheid ervan. Het laatste hoofdstuk behandelt het thema overlading, wanneer een selectiecommissie een kandidaat uit velen moet kiezen, bijvoorbeeld bij de werving van een werknemer. Een van de vragen die aan bod komen is: als de omvang van de kandidatenmarkt groeit, moeten zij dan ontmoedigd worden om te solliciteren? Ik zal nu elk hoofdstuk afzonderlijk samenvatten en enige aanbevelingen geven.

In Hoofdstuk 2 introduceren we een model waarin twee bedrijven elk de beschikking hebben over twee verkoopkanalen: het Internet (e-commerce) en een conventionele winkel. Consumenten dienen te kiezen of en waar ze een aankoop willen doen: bij een van de conventionele winkels of online. Daarbij ondervinden sommige 'onervaren' consumenten de volgende voor- en nadelen. Online kunnen ze gemakkelijk prijzen vergelijken, zodat ze zeker weten dat ze de aankoop tegen de laagste prijs doen. Het nadeel van de online omgeving bestaat uit twee factoren. Ten eerste kunnen niet alle producteigenschappen onderzocht worden omdat het product niet fysiek bekeken en betast kan worden. Ten tweede is de uitvoering van de transactie niet direct: betaling en levering vinden niet direct plaats. Dit betekent dat men niet zeker weet of er niet te veel via bijvoorbeeld credit card wordt afgeschreven en of het product wel geleverd wordt. Deze nadelen noemen we hieronder de 'online onzekerheid' en deze vormen de voordelen van de conventionele winkel: producten kunnen uitgebreid 'besnuffeld' worden en goede afloop van betaling en levering zijn direct vast te stellen.

Het marktaandeel van e-commerce draait daarom op twee grootheden: de zekerheid de laagste prijs te betalen (online) en de online onzekerheid. De resultaten zijn als volgt. Hoe groter de online onzekerheid, hoe kleiner het marktaandeel van e-commerce. Consumenten die voor het Internet kiezen, doen niet altijd een aankoop: de online onzekerheid wordt voor hen niet voldoende gecompenseerd door een lage prijs. Verder is het zo dat in sommige marktomstandigheden er geen enkele combinatie van prijzen bestaat die tot evenwicht leidt en de prijzen zijn in zo'n markt daarom zeer moeilijk te voorspellen.

Alhoewel beide bedrijven hetzelfde product verkopen en op Internet aanbieden, kan het dus voorkomen dat hun prijzen ver uit elkaar liggen! Het meest verrassende resultaat tenslotte is dat de verwachte prijs toeneemt in de omvang van de online onzekerheid en in het aantal consumenten dat deze onzekerheid niet ervaart.

De meeste bevindingen komen overeen met empirische waarnemingen over het marktaandeel van e-commerce. Ons model geeft dus een theoretische verklaring voor het lage marktaandeel van e-commerce, met nadruk op vertrouwen in de online omgeving en het belang van product eigenschappen die niet digitaal te communiceren zijn.

Hoofdstuk 3 kijkt daarnaast ook naar de winstgevendheid voor bedrijven om een online verkoopkanaal op te zetten, zowel voor bedrijven die al een conventionele winkel gevestigd hebben als voor bedrijven die alleen online zullen opereren. Wanneer de dreiging van toetreding door een zuivere Internet speler afwezig is, loont het onder de meeste marktomstandigheden niet om te investeren in een online verkoopkanaal, omdat de afzet in dat kanaal vooral ten koste zou gaan aan afzet die anders in de eigen winkel behaald zou worden, tegen een hogere prijs. Zuivere online spelers ondervinden dit kannibalisatie effect niet en zullen daarom eerder toetreden en die mogelijkheid beweegt de al aanwezige bedrijven dan toch actief te worden op het Internet, om zo de toetreding te voorkomen. Op die wijze verklaren wij de lage winsten die zuivere online bedrijven het laatste decennium hebben behaald. Dit essay schrijft het geringe aandeel van e-commerce dus toe aan de lage winstgevendheid voor bedrijven, die resulteert uit de strategische toetredingsbeslissingen en de kannibalisatie van de omzet in het conventionele kanaal.

De resterende hoofdstukken gaan over informatieverwerving: in veel markten heeft de ene partij een informatieachterstand op de andere. Een voorbeeld: de werkgever die potentiële werknemers vergelijkt weet minder van de kwaliteiten van die kandidaat dan de kandidaat zelf. Sinds Nobelprijswinnaar George Akerlof in 1970 hierover publiceerde hebben veel auteurs laten zien hoe een vaststaande informatieasymmetrie kan leiden tot drastische gevolgen, zoals het totaal uitblijven van handel. Mijn essays volgen een andere aanpak: hoe kan de achtergestelde partij zijn informatiegebrek overbruggen middels een informatietechnologie? Dit zien we overal: een werkgever interviewt een sollicitant, een koper van een tweedehands auto maakt een testrit en een verzekeraar onderzoekt een kandidaat-verzekerde. Ik onderscheid deze technologische aanpak nadrukkelijk van oplossingen die gebaseerd zijn op evenwichtsconstructies zoals signalling en screening.

In de hoofdstukken 4 en 5 is anti-selectie (adverse selection) bij verzekering het vertrekpunt. De belangrijke aanname hier is dat de kandidaat-verzekerde zijn eigen risico beter kan inschatten dan de verzekeraar. Die informatieachterstand van de verzekeraar op kandidaat-verzekerden zorgt ervoor dat de verzekeraar de premie moet baseren op het verwachte risico van zijn klantenbestand. Omdat bij deze premie de individuen met de allerlaagste risico's de verzekering te duur zullen vinden, zullen zij geen klant worden. Dit drijft het verwachte risico en daarmee de premie omhoog, waardoor weer meer lage risico's zullen afhaken. Dit mechanisme (anti-selectie) kan ertoe leiden dat er in het evenwicht geen enkele verzekering afgesloten zal worden.

Verzekeraars proberen deze informatieachterstand zo goed mogelijk te overbruggen en dit leidt tot maatschappelijke onrust omdat betere informatie automatisch leidt tot differentiatie en daarmee kan afdoen aan het solidariteitsbeginsel in verzekering en wellicht ook tot schending van privacy. Een voorbeeld van een debat is de toegang tot verzekering voor HIV/Aids patiënten. De onthulling van genetische persoonlijke informatie reikt nog verder en werpt allerlei ethische dilemma's op.

In hoofdstuk 4 is een verzekeringsmarkt met anti-selectie het vertrekpunt. Ik bestudeer de praktijk waarin verzekeraars middels een test bepalen of een kandidaat-verzekerde tot een hoge risico groep behoort. Te denken valt bijvoorbeeld aan medische (waaronder genetische) tests in geval van het ziektekosten- of arbeidsongeschiktheidsrisico. Met de uitslag kan de verzekeraar de hoge risico-groep een aangepast contract aanbieden of uitsluiten. Anti-selectie wordt hierdoor verzacht, maar het nadeel is dat de hoge risico's geen of een ongunstiger verzekering zullen hebben. Er zijn dus twee tegenstrijdige effecten op de totale maatschappelijke welvaart: een nadelig effect voor de hoge risico's en een voordelig effect voor de andere individuen. Het model stelt ons in staat de optelsom van deze effecten analytisch vast te stellen. Deze blijkt af te hangen van de gevoeligheid voor onzekerheid (ofwel risicoaversie) van de verzekerden. In geval verzekerden erg gevoelig zijn voor onzekerheid, zullen relatief veel lage risico's een verzekering afsluiten. Het probleem van adverse selection is dan gering. Als in die situatie de test wordt ingevoerd, zal de verzachting van adverse selection weinig extra verzekerden opleveren. Het totale welvaartseffect van discriminatie is dan negatief. Wanneer deze risicoaversie echter laag is, lijdt men weinig onder onzekerheid en is de prijs zeer belangrijk voor het bepalen van de omvang van de markt. Adverse selection is daarom een groot probleem en kan zelfs de markt geheel vernietigen. Als vanuit deze situatie de discriminerende test wordt ingevoerd, zal de totale maatschappelijke welvaart stijgen. Het verlies voor de hoge risico's wordt in dat geval gecompenseerd door de winst van de relatief lage risico's. Mijn analyse laat dus zien dat de wenselijkheid van discriminatie afhangt van de risicoaversie onder verzekerden. Dit resultaat noopt tot meer kennis over de empirische risicoaversie in verschillende markten.

Hoofdstuk 5 bestudeert het gebruik van informatie door verzekeraars nadat het contract is gesloten. Voorafgaand aan het tekenen van het contract vullen kandidaten een vragenlijst in over hun medische situatie. Het contract bevat een clausule die de verzekeraar de mogelijkheid geeft schadeuitkering te weigeren indien zij deze informatie kan bestrijden. In de Verenigde Staten gebeurt dit bijvoorbeeld met roken bij levensverzekering: verzekerden verklaren zelf of zij roken of niet. Niet-rokers betalen een lagere premie maar lopen het risico dat de verzekering aangevochten wordt wanneer er aanspraak op wordt gemaakt. Weer zijn er tegenovergestelde effecten: de clausule zorgt ervoor dat niet-rokers een beter contract aangeboden krijgen waar meer individuen gebruik van zullen maken, terwijl rokers erop achteruit gaan. Daarnaast neemt de zekerheid voor niet-rokers af omdat het contract na schade aangevochten kan worden. In dit hoofdstuk vergelijk ik de maatschappelijke welvaart in een wereld waarin dit is toegestaan met die in een wereld waar dat niet is toegestaan. Het negatieve effect domineert: de clausule verlaagt de maatschappelijke welvaart. Dit weerlegt resultaten uit eerdere studies en zet vraagtekens bij deze in de praktijk veelvoorkomende clausule.

Hoofdstuk 6 is het laatste essay en gaat over het probleem van een selectiecommissie wanneer zij een kandidaat uit velen moet kiezen en over de keuze van een kandidaat om te solliciteren of niet. De keuze van de commissie is enerzijds hoe de competitie georganiseerd moet worden en anderzijds hoe met een teveel aan sollicitaties omgegaan moet worden als tijd en middelen voor selectie beperkt zijn. Een sollicitant moet zijn kans om te winnen inschatten en daarmee afwegen of het loont zich de moeite van sollicitatie te getroosten. De interactie wordt speltheoretisch geanalyseerd en is niet alleen van toepassing op de arbeidsmarkt maar op al die situaties waar een partij moet kiezen uit velen om een contract mee te sluiten. Voorbeelden zijn competities voor architectuuropdrachten, de keuze van een huurder door een verhuurder en het toewijzen van subsidies aan kunstenaars en wetenschappers. De cruciale informatieachterstand is dat de kandidaten beter weten in welke mate ze aan de doelstelling van de commissie zullen voldoen dan de commissie zelf.

De volgende vragen komen aan bod. Hoe moet met een eventueel teveel aan sollicitaties worden omgegaan? Is het in het belang van de commissie om sollicitaties middels een drempel te ontmoedigen en hoe verhoudt de optimale drempel zich tot de omvang van de kandidatenmarkt? Onder welke omstandigheden loont het te investeren in een betere selectietechnologie of commissie? Analyse van de drijfveren van zowel kandidaat als commissie levert de volgende aanbevelingen voor een commissie op. Een commissie doet er goed aan willekeurig sollicitanten te negeren als het totale aantal boven een vaststaande bovengrens uitkomt. Wanneer tijd en middelen inderdaad beperkt zijn, is het altijd optimaal een drempel op te werpen, financieel of anderszins. Wanneer de accuratesse van de selectietechnologie laag genoeg is, moet deze drempel dalen in de omvang van de kandidatenmarkt: hoe meer (potentiële) kandidaten er zijn, hoe makkelijker het solliciteren moet zijn. Merk op dat juist wanneer deze accuratesse laag is, overlading eerder optreedt wat het resultaat extra verrassend maakt.

Tenslotte, in die markten waar de drempel relatief hoog is en niet gemakkelijk is te verlagen (bijvoorbeeld bij architectuur competities), is het optimaal voor de commissie om een imperfecte selectietechnologie te hebben, omdat kandidaten anders zozeer ontmoedigd worden dat het risico dat niemand reageert te groot zou zijn.

Ik heb een aantal thema's in informatie-economie besproken. Ik heb laten zien hoe fenomenen zoals begrensde informatieverwerking, het belang van niet-digitale informatie, de indirecte aard van uitwisseling op het Internet en discriminatie als gevolg van informatieverwerving verwerkt kunnen worden in theoretische modellen. Hopelijk dragen ze bij aan het inzicht in de gevolgen van informatie en communicatietechnologieën op economie en maatschappij.

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