STEP-WISE CYSTOMETRY OF URINARY BLADDER

New Dynamic Procedure to Investigate Viscoelastic Behavior

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ABSTRACT—Step-wise cystometry has been developed to analyze the pressure-relaxation phenomena of the bladder in mongrel dogs. The results show that a mathematical model consisting of two exponentials and a constant can be fitted to the measurements. This model is interpreted in a mechanical model of the bladder wall in terms of viscoelasticity.

Recent advances in urodynamics have indicated that to understand better the results of urodynamic investigations, it is necessary to have more information about the mechanical properties of the bladder wall. For example, the properties of the wall can affect the storage capacity and the expulsion-force of the bladder.

While the storage-(adaptation) properties formerly were ascribed exclusively to active mechanisms, we now cannot exclude that passive mechanisms are also involved. Qualitatively the same behavior was found in the bladder, in vitro, several hours after death and even in rubber balloons.[1]

In this report we describe the over-all behavior of the bladder in terms of viscoelasticity and geometry of the bladder.

Viscoelasticity

For purely elastic material, the relation between stress and strain is given by Hooke's law:

$$S = E\epsilon$$

where S is the stress, E is the elastic modulus, and $\epsilon = (l - l_0)/l_0$ is the strain.

For purely viscous material the relation between stress and the rate of strain is given by Newton's law:

$$S = \eta\dot{\epsilon}$$

where S is the stress, η is the viscosity modulus, and $\dot{\epsilon} = d\epsilon/dt$ is the rate of strain.

Viscoelasticity is a combination of the two. The stress in such viscoelastic material can be described in terms of exponential time functions.

Geometry of Bladder

The law of Laplace is commonly used to describe the relationship between pressure and bladder-wall tension:

$$T = \frac{PR}{2}$$

where T is force per unit length. A basic assumption in this model is that the bladder is a thin-walled sphere. Rather than the tension, we need force per unit wall thickness, per unit length as a parameter for bladder-wall stress. If wall thickness is included, one obtains:

$$S = \frac{3PV}{2V_t}$$

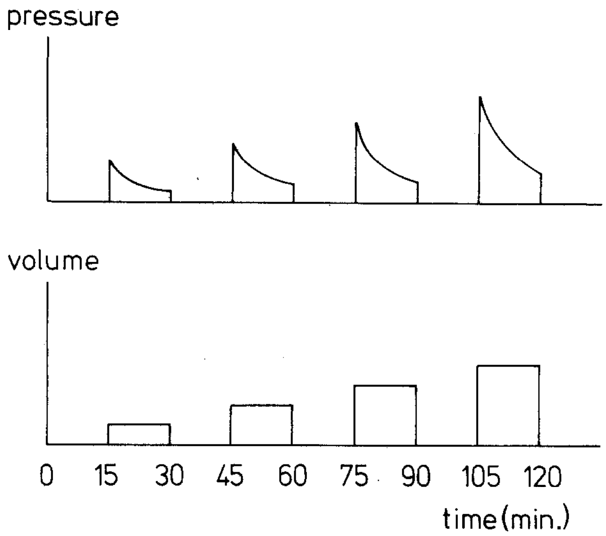

FIGURE 1. *Chart showing step-wise volume changes and corresponding pressure-decay curves.*

where S is stress, P is intraluminal pressure, V is intraluminal volume, and V_t is tissue volume. Accepting that V_t is a constant, it can be measured in an experimental situation. When S is an exponential function of time, P will also be an exponential function of time.

Method of Investigation[2]

Many phenomena may influence the intraluminal bladder pressure. To isolate the pressure-volume relationship, we have applied a systems approach, that is, we treat the bladder as a "black box" and investigate the response to step-changes in intraluminal volume. A step-wise cystometry was used (Fig. 1).

Experiments were done on mongrel dogs during continuous pentobarbital anesthesia, on dog-cadavers, bladders-in-vitro, and bladder strips. Using a systems approach, a step-volume change would be ideal. Since this cannot be realized, steep ramp volume changes with known rise-times were used and the results appropriately corrected. To obtain reproducible results, it was necessary to wait about fifteen minutes between each change in volume.

Mathematical Model

When a semilogarithmic plot of the pressure-decay curve was made, we found that the pressure-decay is not monoexponential.[3] Thus a mathematical model consisting of two exponentials and a constant:

$$P = Ae^{-\alpha t} + Be^{-\beta t} + K_2$$

and a model of three exponentials and a constant:

$$P = Ae^{-\alpha t} + Be^{-\beta t} + Ce^{-\gamma t} + K_3$$

were fitted to the measurements according to a "least square" fitting procedure by a digital computer. Digital computing is preferred to graphic hand-methods, because digital computing gives an idea about the deviations of the fitted parameters and is not subjected to individual interpretations.

As may be expected, a three-exponential model gives a better fit than a two-exponential one. For mathematical reasons, the three-exponential model is not reliable because of extraordinary sensitivity to measurement errors. This results in a lack of uniqueness of the fitted set of parameters.

The two-exponential model plus a constant gives an acceptable fit and may be practicable in clinical application. For quick analysis a device has been constructed to analyze the pressure-decay curves by electronic simulation. In this device a fit-criterium is used similar to the "least square" procedure in digital computing.

Interpretation of model

As a consequence of our model, at least 2 structures with different viscoelastic properties are distinguished. The coefficients are corrected for the fact that ramp inflow functions have been used instead of mathematical steps. The exponents are not affected. The corrected coefficients represent the elastic contributions of yet undefined functional structures.

The values of K are especially significant because, with increasing volume levels, the values of K give us the static cystometrogram. That is, only if the contribution of the time-dependent exponentials have been sufficiently diminished, the pressure will approach the value of K. The exponents represent the viscoelastic properties of the different functional structures and describe the relaxation-process.

The mathematical model which consists of two exponentials and a constant can be translated into a mechanical model consisting of two Maxwell elements in parallel with a Hooke element (Fig. 2).

Results

About 200 pressure-decay curves were analyzed according to the mathematical models described. In almost all cases a good fit was attained with the two-exponential model. The

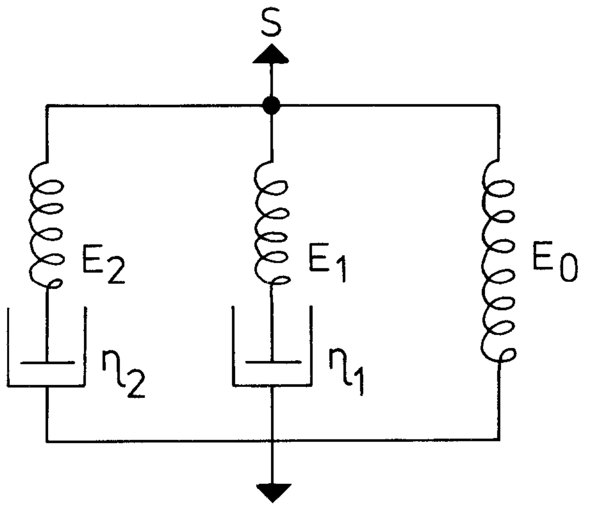

FIGURE 2. *Mechanical model of bladder wall:*

$$S = \epsilon E_1 e^{-\frac{E_1}{\eta_1}t} + \epsilon E_2 e^{-\frac{E_2}{\eta_2}t} + \epsilon E_0$$

parameters appeared to be reproducible when experiments were done at equal volume-levels in the same animal.

The influence of contractions by pelvic nerve stimulation during pressure-decay has also been studied. The results suggest that contractions are simply superimposed on the pressure-decay curve found without stimulated contractions. Only contractions given less than two minutes before the volume-step influence the pressure-decay curve.

When using increasing volume levels, a systematic trend was demonstrated for the coefficients A, B, and K. However, no systematic relationship could be found between volume levels and the exponents. Similar experiments performed on bladder-strips demonstrated constant values of the exponents at different strain levels.

From the slowest exponential we calculate the infusion rate which would be necessary to obtain a pseudostatic cystometry within an acceptable error, compared with a static cystometrogram based on the K values. The appropriate equation can be shown to be:

$$A = \frac{\beta f K V}{B}$$

where A is the infusion rate, β is the exponent of the slowest exponential, B is the coefficient of the slowest exponential, K is the static pressure, V is the intraluminal volume, and f is the relative error.

By substitution of experimental data we found that the infusion rate for an error of 5 per cent should be about 0.2 ml. per minute.

Since the parameters E_0, E_1, E_2, η_1, and η_2 are dependent on the volume levels, our model must be considered to be nonlinear. A step-wise analysis of the system, however, appears to be an appropriate method of investigation.

Conclusion

The proposed step-wise cystometry elicits viscoelastic parameters of the bladder which cannot be derived from commonly used cystometry. Step-wise cystometry, in addition, gives a more reliable static cystometrogram. A disadvantage of step-wise cystometry, when taken at several volume levels, is that this procedure requires too much time to be clinically applicable. However, if one is interested in the viscoelastic properties, the procedure may be restricted to one convenient volume level, because the exponents are rather independent of the volume levels. In this case, the procedure is clinically feasible.

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