

COMMENT

A SHORT NOTE ON THE PERFORMANCE OF TWO COMPUTER PROGRAMS FOR THE ESTIMATION OF THE PARAMETERS OF A MULTI-EXPONENTIAL MODEL

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During the fitting of a multi-exponential model to experimental visco-elastic stress-decrease curves determined after stepwise straining of urinary bladder-wall tissue, Coolsaet *et al.* [1] noted a discrepancy between the results obtained with the two computer programs used to estimate the parameters of the multi-exponential model (i.e. the coefficients and exponents of the various exponential terms). A sophisticated program called Exposum [2], based on a Marquardt iteration, was used initially [3]. It was later found [1] that a lower sum of squares, indicating a better fit, was obtained with the program Expostep described by van Mastrigt [4], which is based on a simple stepwise exploration of the parameter space. Kuik [5] made the following remarks concerning this discrepancy in results:

1. If Exposum is used in double precision for analysis of the urological data, it gives the same sum of squares as Expostep or even a slightly lower one (owing to the fact that the parameters are determined at a higher precision).

2. The minimum in the sum of squares found in fitting these urological stress-decrease curves is situated in a long straight trough parallel to one of the parameter axes, probably as a result of the large difference between the exponents. The straight form of this trough is the reason why the minimum can be easily located by simple stepwise approach as used in Expostep.

3. The minimum in the sum of squares of other data, e.g. the nuclear data analysed by Kirkegaard [2], is often situated in a curved trough known in the literature as a "banana-shaped valley". Expostep fails to converge with data of this type, but Exposum still determines the minimum without difficulty. Summing up, we may state that both programs are satisfactory for analysis of our stress-decrease data found in the investigation of the viscoelastic behaviour of bladder-wall tissue; however, Exposum has the advantage of being significantly faster, whereas Expostep requires appreciably less memory space. With other data, Expostep might fail to converge if the minimum in the sum of squares was not situated in a straight trough in parameter space. In general therefore, Exposum is preferable for analysis of data of the type indicated above, provided it is implemented in double precision.

REFERENCES

1. B. L. R. A. Coolsaet, R. van Mastrigt, W. A. van Duyl and R. E. F. Huygen, Viscoelastic properties of bladder wall strips at constant elongation, *Invest. Urol.* **13**, 435-440 (1976).
2. P. Kirkegaard, A FORTRAN IV version of the sum-of-exponential least-squares code Exposum, Danish Atomic Energy Commission, Research Establishment Risø, Report Risø-M-1279.
3. B. L. R. A. Coolsaet, W. A. van Duyl, R. van Mastrigt and J. W. Schouten, Viscoelastic properties of bladder wall strips, *Invest. Urol.* **12**, 351-356 (1975).
4. R. van Mastrigt, Constant step approximation of multi-exponential signals using a least squares criterion, *Comput. Biol. Med.* **7**, 231-247 (1977).
5. A. J. Kuik, Estimation of the parameters of a multi-exponential model, *Afstudeerverslag deel I*. Available on request from: werkgroep Signaalverwerking, Afdeling der Technische Natuurkunde, Technical University Delft, Delft, the Netherlands.