Series-elastic properties of strips of smooth muscle from pig urinary bladder

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Abstract—The series elasticity of strips of smooth muscle from pig urinary bladder was investigated by means of a series of computerised quick-release and quick-stretch measurements with and without stimulation of the muscle, and at different shortenings and force levels. The results cannot be interpreted in terms of a discrete passive series-elastic element. They can, however, be interpreted in terms of the sliding-filaments model for contracting muscle.

Keywords—Quick-release technique, Series elasticity, Sliding filaments model, Smooth muscle, Urinary bladder

List of symbols

a = constant factor representing geometry of muscle

A = coefficient of exponential crossbridge elasticity

β = exponent of exponential crossbridge elasticity

 β/l_o = exponent in elastic characteristic of discrete elastic element

 ΔF_{s-} = corrected force decrease during stimulation and quick release (in the rising phase of the contraction)

 ΔF_{s+} = corrected force increase during stimulation and quick stretch (in the rising phase of the contraction)

 ΔF_{o-} = corrected force decrease after stimulation and during quick release (in the falling phase of the contraction)

 ΔF_{o+} = corrected force increase after stimulation and during quick stretch (in the falling phase of the contraction)

 Δl = extent of quick release or stretch

 Δx = elongation of series elastic element

 $E_o/l_o = \text{coefficient}$ in elastic characteristic of discrete elastic element

F = force

 $F_{\Delta l}$ = force measured during a quick release or quick stretch

 F_{iso} = maximum active isometric force during contraction

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 F_{trig} = preset force-trigger level

g(X) = force as a function of length of a crossbridge

h = upper limit of crossbridge lengths

l = length of muscle strip

 $n_o(t)$ = number of active crossbridges during an isometric contraction

n(X) = spectrum of lengths of crossbridges

 V_t = tissue volume of strip

X =length of crossbridge

x = macroscopic equivalent of X obtained by addition of lengths of crossbridges which are in series

 $\sigma_{\Delta i}$ = force measured during a quick release or quick stretch, normalised by dividing by the cross-sectional area of the strip

1 Introduction

THE series-elastic properties of muscle represent the elastic behaviour that can be measured by stretching or releasing a muscle during a contraction. This concept of series elasticity was first introduced by HILL (1938), for striated muscle. However, the model was very often found to be applicable to smooth muscle too; see e.g. GORDON and SIEGMAN (1971), HELLSTRAND (1979) and the review by MURPHY (1976). In this model for muscular behaviour, the development of active force in a muscle is described in terms of a contractile element (CE) in series with an elastic element (SE). During isometric contraction, the contractile element stretches the series-elastic element. thus developing force. It has been shown (VAN MASTRIGT et al., 1978b) that this model can be used to calculate fundamental properties of the contractile element (in particular the force-velocity relation

describing this element) from an isometric contraction.

To do this we need a mathematical description of the series-elastic element. ÅBERG (1967), ALEXANDER (1976), PARMLEY and SONNENBLICK (1967), MEISS (1978) and HALPERN et al. (1978) describe the series elasticity of smooth muscle as approximately exponential. If we interpret this in terms of an elastic element exerting a stress which depends exponentially on strain, the calculation becomes very simple (VAN DUYL et al., 1978), but results in an infinitely high maximum contraction velocity of the muscle, which is unrealistic (VAN MASTRIGT and GRIFFITHS, 1979a). An interpretation in terms of an element having an exponential elastic modulus yields more realistic results (VAN MASTRIGT and GRIFFITHS, 1979a) and is also in accordance with measurements of the passive properties of urinary bladder strips (VAN MASTRIGT et al., 1978a). Other authors like BLANGÉ et al. (1972), CHAPMAN and HARROWER (1977) and JULIAN et al. (1978), however, present measurements indicating that the series elasticity of muscle cannot be described in terms of a discrete passive series element, but depends on the activation of the muscle, or on the force the muscle exerts (Bressler and Clinch, 1974; Ford et al., 1977; HALPERN and MULVANY, 1976). This is in agreement with the sliding-filaments model for contracting muscle, introduced by HUXLEY (1957) for modelling striated muscle, if the series elasticity is assumed to be located in the crossbridges.

Although the muscle structure on which the slidingfilaments model is based cannot be seen in smooth muscle, the model is very often also applied to, or found to be applicable to, this type of muscle (BUTLER et al. 1976; GORDON and SIEGMAN, 1971; HUXLEY, 1957). However, many authors consider that the series elasticity in smooth muscle cannot be ascribed solely to the crossbridges but must be found at least partially external to them (GROOD and MATES, 1975; SIEGMAN et al., 1976; HALPERN et al., 1978; MEISS, 1978; HELLSTRAND and JOHANSSON, 1979). In view of the possible clinical value of a method of determining contractile properties from isometric contractions of the urinary bladder (VAN MASTRIGT et al., 1979b) the present study was undertaken to investigate the possibility of quantitative modelling of the series elasticity of urinary-bladder smooth muscle in terms of a discrete, passive elastic element.

Measurements were performed by means of the quick-release technique, because this method tests the series elasticity directly.

2 Methods

Experiments were performed on strips of pig urinary bladder wall measuring approximately 10×27 mm. The bladders were obtained from the local slaughterhouse. The strips were submerged in a physiological solution at a temperature of 37° C. Contractions were evoked by electrical stimulation.

Four insulated silver wires were inserted into the strips, parallel to the short side, and a mass electrode was placed into the immersion fluid. Rectangular pulses of 30 V, lasting 7 ms and at a repetition rate of 20 Hz (GRIFFITHS et al., 1979), were applied. One of

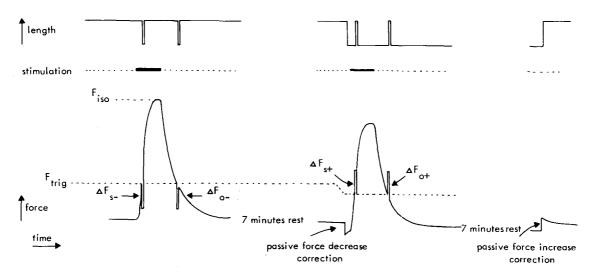


Fig. 1 Sequence of events during one measurement, comprising two contractions. Electrical stimulation of the muscle is indicated by a solid line, and stops as soon as force starts to decrease. During each contraction length changes are imposed during the rising and falling phases of the contraction, i.e. decreases in length during the first contraction and increases in length during the second. The resulting force changes are corrected for passive force changes by subtracting the response to an identical length change performed after the contractions. The symbol ΔF will be used for the force changes corrected in this way.

the short sides of the strip was connected to a Grass FT 03C force transducer via a clamp with penetrating pins. The force signal obtained was fed to a minicomputer. An identical clamp on the other short side of the strip could be moved by a pneumatic cylinder, thus releasing or stretching the muscle stepwise by up to 50 mm. Both the initial length and the amplitude of the length change of the muscle could be adjusted with micrometers. The electrical stimulation and the movement of the pneumatic cylinder were controlled by the minicomputer.

The strips were adjusted to an initial length where they were just taut. Prior to each stimulation, the force value was considered to be due to passive, parallel elastic properties of the strip, and was subtracted from all forces measured during contraction. One measurement consisted of the following sequence (see Fig. 1): Stimulation is turned on; when a preset trigger level (denoted by F_{trig}) is reached, the muscle strip is released for 250 ms and then reset to the original length. Next, stimulation is turned off as soon as the force starts to decrease, indicated by F_{iso} in Fig. 1. When the decreasing force again reaches the trigger level, a second quick release is performed. After a resting period of 7 min, a control measurement is carried out, starting with a release. After the stimulation has been turned on, two quick stretches are measured in the same way as during the first stimulation, and, finally, the muscle is reset to the original length.

All releases and stretches during one measurement sequence imply the same preset amount of shortening or elongation. A typical force/time recording for one release pulse prolonged to 500 ms is shown in Fig. 2. During each release or stretch pulse, 10 force samples are read by the computer at intervals of 25 ms. The minimum or maximum in this sequence was taken to be representative of the force during the release or stretch.

The force change thus measured is assumed to be composed of a change in active force and a change in passive force. The latter force is due to the stretching of passive viscoelastic structures (VAN MASTRIGT et al., 1978a). Because active and passive force are approximately additive (GRIFFITHS et al., 1979), we can correct for the passive force changes by subtracting from the measured force change the response to an

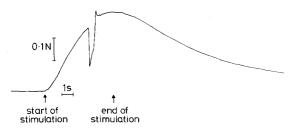


Fig. 2 Example of recorded force signal with one prolonged quick release and reset during contraction

identical length change performed without stimulation of the muscle. These passive force changes are measured at the beginning and the end of the control measurement, as shown in Fig. 1. Thus, each measurement yielded four corrected values in the rising and falling phase of a contraction, and during release and stretch, denoted ΔF_{s-} , ΔF_{o-} , ΔF_{s+} , ΔF_{o+} , respectively. The maximum isometric force F_{iso} which the strip yielded usually decreased during the course of the measurements. The first few contractions measured on each strip yielded an F_{iso} between 0.8 and 1.6 N. This yields an isometric active stress between 1×10^4 and 2×10^4 N m⁻².

This is low compared with values found for other smooth muscles $[2.5-35\times10^4\,\mathrm{N\,m^{-2}}$ (HALPERN et al., 1978; HELLSTRAND, 1979; MURPHY, 1976)], but it correlates well with the maximum pressures measured in the normal urinary bladder (VAN MASTRIGT and GRIFFITHS, 1979a).

Measurements were continued until F_{iso} was lower than F_{trig} . Generally 10 to 20 contractions could be measured on one strip. This rapid deterioration is at least partly due to the fact that quick stretches were also applied, which probably damaged the strip (FORD et al., 1977). However, a maximum possible number of about 20 contractions has been found before for strips prepared in this way, even without quick stretches (GRIFFITHS et al., 1979). The release and stretch speeds of the pneumatic device are in the order of $200 \,\mathrm{mm}\,\mathrm{s}^{-1}$. This is well above the maximum speed of contraction, which was estimated at 5 mm s⁻¹ for these strips (GRIFFITHS et al., 1979; VAN MASTRIGT and GRIFFITHS, 1979a). This v_{max} (normalised 0.2 strip lengths per second which agrees very well with values found for other smooth muscles (MURPHY, 1976)) is a factor of 20-100 lower than values found for striated muscle (MURPHY, 1976). This justifies the use of a relatively slow force transducer and low sampling rate.

3 Results of measurements with steps of equal length and constant trigger level

We consider the four measured ΔF s as the dependent variables in our experiment. There are three

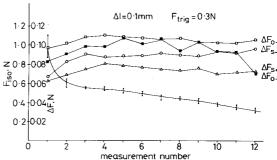


Fig. 3 Corrected force decrease and increase steps measured in experiment during which only F_{iso} varied

independent variables; F_{iso} , Δl and F_{trig} . In this Section we describe the relation between the ΔF s and F_{iso} . The latter varies considerably during each experiment. Measurements were performed on five strips. F_{trig} was fixed at 0.3 N and Δl was 0.1 mm for two strips and 0.3 mm for the other three. The results of one measurement are shown in Fig. 3. Two values of F_{iso} were obtained for each measurement. The higher plotted value was always measured in the first of the two contractions.

Although some kind of common pattern can be seen in the ΔF and F_{iso} curves, the dependence is so slight that, to a first approximation, we can regard the ΔF s as independent of F_{iso} . The averages and standard

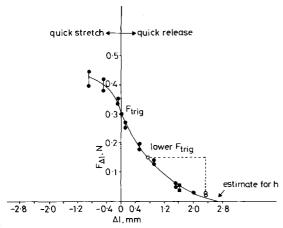


Fig. 4 Active force as a function of quick shortening or stretch. Dotted line shows how quick releases measured at a lower F_{trig} value should be plotted. Starting from a point on the curve corresponding to the lower F_{trig} (open circle), the length change during the quick release is plotted horizontally and the resulting force change is plotted vertically, yielding the two open circles (which do not fit the curve). The solid squares represent the same data corrected for the lower F_{trig} value by a factor $F_{trig} \ 2/F_{trig} \ 1$, as explained in Section 5

deviations for all measurements are shown in Table 1. A significant (student's t test, 95%), though rather small, difference between the four ΔFs is always seen. The ΔFs measured without stimulation (i.e. in the falling phase of the contraction) are 18% greater on the average than those measured with stimulation (in the rising phase of contraction). This holds good for quick-release as well as for quick-stretch measurements. Furthermore, when we omit the signs, the responses to quick stretches are 14% smaller on the average than those to quick releases. This will be discussed in the following Section. Apart from these second-order effects, we conclude that, to a first approximation, the measured decrease or increase in force is independent of F_{iso} .

4 Results of measurements with steps of varying length

Next, we investigated the dependence of the ΔF s on Δl at a constant F_{trig} of 0·3 N. Eleven experiments were performed. A typical plot of the active force $F_{\Delta l}$ exerted by the strip during shortening or elongation, as a function of Δl , is shown in Fig. 4. $F_{\Delta l}$ is calculated from the ΔF s by adding to F_{trig} with the appropriate sense, as follows:

$$F_{\Delta l} = F_{trig} - \Delta F_{s-}$$
 and $F_{\Delta l} = F_{trig} - \Delta F_{\sigma-}$ for $\Delta l > 0$ (shortening)

$$F_{\Delta l} = F_{trig} + \Delta F_{s+}$$
 and $F_{\Delta l} = F_{trig} + \Delta F_{o+}$ for $\Delta l < 0$ (elongation)

Thus, for each value of Δl , two force values are plotted, the upper one representing the force change measured in the falling phase of a contraction and the lower one the force change measured in the rising phase of the contraction. As can be seen, the results obtained with quick releases and quick stretches are consistent in that they yield a continuous curve. The decrease in the slope of the curve for high Δl can be understood from the fact that the force during the quick stretch then exceeds the maximum force F_{iso} , which causes the

Table 1. Averages and relative standard deviations of measurements made on six strips with constant Δl and constant F_{trig} (0.3 N), so that only F_{iso} varied

Δi	Variable	Average (N)	Relative standard deviation $\left(=\frac{\text{standard deviation}}{\text{average}} \times 100\%\right)$	Number of measurements
		0.005	%	20
D·1	$egin{array}{l} \Delta F_{s-} \ \Delta F_{o-} \ \Delta F_{s+} \end{array}$	-0.085 -0.108	14 15	30 30
	$\frac{\Delta F_o}{\Lambda E}$	0.086	9	30 29
	ΔF_{o+}	0.096	12	29
0.3	ΔF_{s-}	-0.131	18	18
	$\Delta \mathring{F_{a}}_{-}$	-0.143	13	18
	ΔF_{s+}	0.096	21	13
	$\Delta ar{F_{o}}_{-} \ \Delta F_{s}_{+} \ \Delta F_{o}_{-}$	0.127	21	13

contractile 'machinery' to 'slip' (HILL, 1938; LEVIN and WYMAN, 1927; JULIAN et al., 1978).

The curve shown in Fig. 4 can be understood as the elastic characteristic of a discrete passive series-elastic element. We plotted the 'stiffness' of this hypothetical elastic element (the derivative $dF_{\Delta l}/d\Delta l$ normalised with respect to the cross-sectional area of the strip to yield $d\sigma_{\Delta l}/d\Delta l$) as a function of σ in Fig. 5. The results of

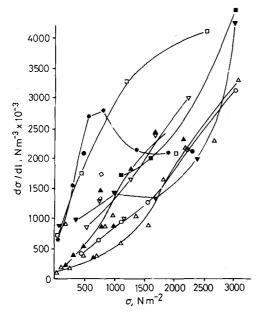


Fig. 5 Stiffness of hypothetical series-elastic element as a function of active stress exerted. The stress was calculated by dividing the force during a quick release by the cross-sectional area of a strip. The stiffness is the derivative of this stress with respect to the amplitude of the quick release

★ curve 2004	∇ curve 2010
▼ curve 2005	curve 2011
 curve 2006 	□ curve 2012
△ curve 2009	▲ curve 2013

the best eight out of the 11 measured curves are displayed. As the plotted curves are not straight lines, the measured quick-release curves (Fig. 4) are not directly exponential (see Introduction). It was concluded from measurements performed on whole bladders (VAN MASTRIGT and GRIFFITHS, 1979a) and bladder-wall strips (VAN MASTRIGT et al., 1978a) that the series elasticity of urinary bladder-wall muscle can be adequately described by the function

where

$$\frac{\beta}{l_o}$$
 and $\frac{E_o}{l_o}$ are parameters

 V_t is the tissue volume of the strip

l is the length of the strip

and Δx is the elongation of the series-elastic element.

This formula can be fitted to the measured $F_{\Delta l}$ curves by putting $\Delta x = h - \Delta l$, where h is the value of Δl when $F_{\Delta l} = 0$. The fitting was performed by estimating h, at $F_{\Delta l} = 0$, by estimating h from the $F_{\Delta l}$ against Δl plot (see Fig. 4), dividing $F_{\Delta l}$ by $(h - \Delta l)$ and plotting the result semilogarithmically. Nine out of the 11 curves measured could be described in this way.

Table 2. Averages and standard deviations of parameters of the hypothetical, discrete, passive series-elastic element, derived from the data of Fig. 5

Parameter	Average	Relative standard deviation	Number of measurements
		%	*
$rac{eta/l_o}{E_o/l_o}$	$674 (m^{-1})$ $475 \times 10^3 \text{Nm}^{-3}$	31 70	9

Averages and standard deviations of the resulting parameters are shown in Table 2. The average values can be compared with those measured on a whole bladder (VAN MASTRIGT and GRIFFITHS, 1979a) by applying a correction factor for the different geometry (VAN MASTRIGT, unpublished data). The high standard deviations found in Table 2 show that this modelling of the series elasticity of the urinary bladder wall is not very successful.

At the end of an experiment, when F_{iso} became lower than F_{trig} , one or more measurements were sometimes performed at a lower value of F_{trig} . In the case of Fig. 4, such a measurement was performed with $F_{trig} = 0.15$ N. On the assumption of a discrete, passive series-elastic element, such a measured point should be represented by the open circles shown in Fig. 4. Starting from the point along the curve corresponding with the lower value of F_{trig} (open circle), the amplitude of the length change during the quick release is plotted horizontally and the resulting force change vertically, yielding the two open circles seen near the foot of the curve. This was tried in five experiments but the extra measurements never fitted the original curve.

A better way of interpreting these quick-release data will be presented in Section 6.

5 Results of measurements with varying trigger forces

Finally, the dependence of the ΔFs on F_{trig} was investigated. Measurements were performed on five strips. For two strips, Δl was fixed at 0.2 mm; the other three were measured with $\Delta l = 1$ mm. The resulting

 ΔF s, which were divided by Δl to yield stiffnesses, were found to be directly proportional to F_{trig} . This is illustrated in Fig. 6. The quick-stretch measurements here were all performed at the same F_{trig} , to see whether this increased the reproducibility. This, however, was not the case, as can be seen in Table 3. The proportionality found between force change and F_{trig} necessitates a number of remarks.

First, we now have a method for correcting

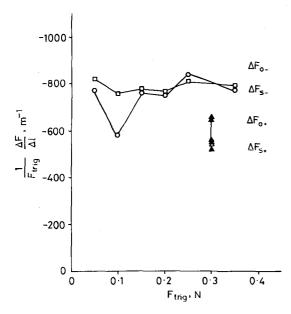


Fig. 6 Results of measurements on one strip made at constant Δl . F_{trig} is kept constant for the quick-stretch measurements, and is varied for the quick-release measurements. The results show that the quotient $1/F_{trig} \frac{\Delta F}{\Delta l}$ does not depend on F_{trig}

measurements made at different F_{trig} levels, as presented in Section 4. For instance, the measurement represented in Fig. 4 by open circles does fit (solid squares) when corrected for the lower F_{trig} value by the simple factor $F_{trig} \frac{1}{2} / F_{trig} \frac{1}{1}$.

In four out of the five experiments from Section 4 in which measurements at lower F_{trig} values were available, these could be fitted to the measured curves by applying this correction factor. Secondly the observation that the measured force change is proportional to the force level at which the length change is imposed has been made by many authors (Bressler and Clinch, 1974; Ford et al., 1977; JULIAN et al., 1978; MEISS, 1978; HELLSTRAND, 1979). Most of them conclude that this means that the force/length characteristic is exponential. We found this not to be the case (see Section 4). Furthermore, the proportionality was also found for large length changes ($\Delta l = 1 \,\mathrm{mm}$, which is 38% of h, the length change for which $F_{\Delta l} = 0$), which, in terms of a passive discrete series elasticity, can only be understood from a linear force/length characteristic parallel to the force

Finally, these findings have to be compared with those of Section 4. If we model the series elasticity of this type of smooth muscle by a discrete, passive series element, this comparison can be made in terms of stiffness, as explained in Fig. 7.

The stiffness data of Fig. 5 are replotted in Fig. 8. Two straight lines have been inserted representing the measurements presented in this Section for $\Delta l = 0.2 \, \mathrm{mm}$. It can be seen that especially at high forces the stiffnesses measured in the two different ways do not agree. We must thus abandon the idea of one discrete passive series element representing the elasticity of urinary bladder smooth muscle, as has also been concluded by other authors for other muscle types (Blangé et al., 1972; Chapman and Harrower, 1977).

Table 3. Results of measurements made with constant Δl ($\Delta l=1$ mm for three strips and $\Delta l=0.2$ mm for two strips) and varying or constant F_{tria}

Δl	F_{trig}	Parameter	Average value	Relative standard deviation	Number of measurements
			m ⁻¹	%	
mm	variable	$1/F_{trig} \times \Delta F_{s} - /\Delta l$	649	18	21
		$1/F_{trig} \times \Delta F_{o} - /\Delta l$	714	7	20
1	constant	$\frac{1/F_{trig} \times \Delta F_{s+}/\Delta l}{(F_{trig} = 0.3 \text{ N})}$	490	18	17
		$ \frac{1/F_{trig} \times \Delta F_{o+}/\Delta l}{(F_{trig} = 0.3 \text{ N})} $	611	23	17
0.2	variable	$1/F_{trig} \times \Delta F_{s} - /\Delta l$	529	44	12
		$1/F_{trig} \times \Delta F_{o} / \Delta l$	876	6	12
	constant	$1/F_{trig} \times \Delta F_{s+}/\Delta l$	789	19	14
		$1/F_{trig} \times \Delta F_{o+}/\Delta l$	795	6	14

6 Interpretation of results in terms of sliding filaments model

In the preceding Sections we described three findings which cannot be interpreted in terms of the model of HILL (1938) using a discrete passive series elastic element, namely

- (i) the proportionality of stiffness and force for large length changes (Section 5, Table 3)
- (ii) the differences in stiffness found by varying Δl and F_{trig} (Fig. 8)
- (iii) the fact that measurements made at a lower F_{trig} (Section 4) must be corrected by a factor F_{trig} 2/ F_{trig} 1 (Section 5)

We will now consider these findings in terms of the sliding-filaments model proposed by HUXLEY (1957, 1974). If we assume that there is no elasticity apart from that in the crossbridges (BLANGÉ et al., 1972; HUXLEY, 1957) the force the muscle exerts at any moment can be expressed as the integral of the forces exerted by the crossbridges over a spectrum of crossbridge lengths (HUXLEY, 1957):

where

F = force exerted by muscle

X = length of crossbridge

n(X) = spectrum of lengths of crossbridges

q(X) = force as function of length of crossbridge

a =constant factor representing geometry etc.

A real muscle, or muscle strip consists of a lot of

contractile units (sarcomeres) in series, taken into account by replacing the parameter X by x, its macroscopic equivalent. It is assumed that in the isometric case the crossbridge length is always distributed uniformly within a certain range (HUXLEY, 1957):

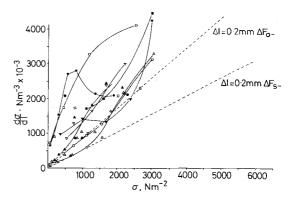
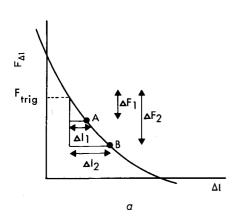


Fig. 8 The stiffness data of Fig. 5 (measured at constant trigger level and varying stretch or release length) compared with the two straight lines representing the stiffness measured as a function of F_{trig} at a constant value of Δl (Section 5). It will be seen that the agreement between the results obtained by these two methods is poor, especially at high forces





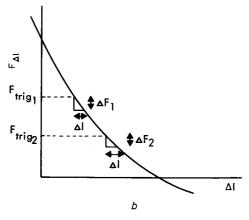


Fig. 7 Force changes measured as a function of Δl (a) can be compared with force changes measured as a function of F_{trig} (b) by comparing the stiffness of the hypothetical, discrete, passive series-elastic element at a given force in the two cases. In case (a), the stiffness (= the slope of the quick-release curve) can be approximated to by $(\Delta F_2 - \Delta F_1)/(\Delta l_2 - \Delta l_1)$. In case (b), we can approximate to the stiffness directly by calculating $\Delta F/\Delta l$, provided Δl is relatively small

During an isometric contraction the build-up of force is due to the successive formation of crossbridges:

What happens during a quick release depends on whether the crossbridges can exert a negative force or not. Some authors conclude that they cannot (BLANGÉ et al., 1972), though HUXLEY (1957) states that they can, because a quick-release curve approaches the Δl -axis sharply. This claim is not falsified by our data (it means that $d\sigma_{\Delta}/d\Delta l \neq 0$ for $\sigma_{\Delta l} = 0$ in Fig. 5) but we believe that this can be understood by assuming crossbridges which do not exert negative force but do exert force at zero extension (cf. eqn. 10). We will thus assume that

During a quick release, the length distribution of the crossbridges shifts to the origin:

$$F_{\Delta l} = an_o(t) \int_0^{h-\Delta l} g(x)dx . \qquad (6)$$

At this stage of our argument, we can understand why the results of measurements at different forces (presented in Section 5) yielded a constant quotient of ΔF and F_{trig} . Since F_{trig} is simply equal to F(t) at a given (t), we find:

$$\frac{1}{F_{trig}} \frac{\Delta F}{\Delta l} = \frac{1}{F_{trig}} \frac{F_{trig} - F_{\Delta l}}{\Delta l}$$

$$= \frac{\int_{0}^{h} g(x) dx - \int_{0}^{h - \Delta l} g(x) dx}{\Delta l \int_{0}^{h} g(x) dx} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

which does *not* depend on $n_o(t)$ and is constant at a constant value of Δl . For a quantitative description of our results we have to determine g(x). Eqn. 6 yields

Fig. 9 shows a semi-logarithmic plot of this derivative against Δl from seven out of the 11 quick-release curves measured, showing that they all fit the same straight line (apart from some marked outliers near $\Delta l = 0$).

The other measured curves showed large variations in $dF_{\Lambda l}/d\Delta l$. Thus

$$\frac{dF_{\Delta l}}{d\Delta l} = -Ae^{-\beta\Delta l} \qquad (9)$$

Combination of eqns. 8 and 9 yields

$$g(x) = \frac{A}{an_o(t)} e^{\beta(x-h)} . (10)$$

$$F_{\Delta l} = \frac{A}{\beta} \left[e^{-\beta \Delta l} - e^{-\beta h} \right] \qquad (11)$$

Here $\frac{A}{\beta}$ is not a constant but will probably depend on $n_o(t)$ in some way. The validity of eqn. 11 can be checked by estimating h from a quick-release curve as the point where $F_{\Delta l}=0$ (see Fig. 4, h=2.6 mm), next estimating $\frac{A}{\beta}$ from F_{trig} (where $\Delta l=0$) by inserting the

estimated h, and then plotting $\frac{F_{\Delta l}}{A/\beta} + e^{-\beta h}$ semi-logarithmically as a function of Δl . This should (and does) yield a straight line.

As a result of these calculations based on the sliding-filaments model, and some rather restrictive assumptions, we see that the findings presented in the beginning of this Section can easily be explained. The proportionality of stiffness and force follows from eqn. 7. The differences in stiffness obtained by varying Δl and F_{trig} are explained by the fact that in the first type of

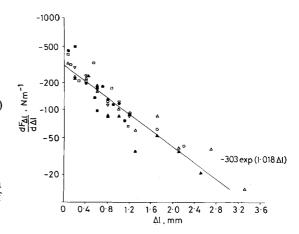


Fig. 9 Semilogarithmic plot of $dF_{\Lambda l}/d\Delta l$ versus Δl , calculated from quick-release curves such as that of Fig. 4



measurements eqn. 11 holds, and in the second case eqn. 7.

Finally, measurements made at lower F_{trig} values should be corrected according to eqn. 7, and not according to eqn. 11.

7 Conclusions and discussion

The quick-release and quick-stretch measurements we performed cannot be described in terms of one discrete passive series elastic element. They can however be described in terms of the sliding-filaments theory, assuming that

- (i) the series elasticity resides in the crossbridges
- (ii) the crossbridges cannot push, only pull
- (iii) the lengths of the crossbridges during an isometric contraction are distributed uniformly within a given range.

Especially for the first assumption, the parameter h is of crucial importance. This is the value of Δl which makes the force during a quick release zero, see eqn. 11. In the first place, this value is independent of the force level from which release starts. However large the contractile force is, it can always be made exactly zero by the same amount of release. This deduction is confirmed by the measurements of Section 5, and is in complete contradiction to the predictions made on the basis of a discrete passive series elastic element. Secondly, the parameter h varied from 1.2 to 5.5 mm in our measurements, with an average of 2.6 mm. Since our average strip length amounted to 27 mm, we see that a quick shortening by about 10% of the muscle length is necessary to remove all tension. This is a not uncommon value for smooth muscle (values found in literature range from 5 to 20%, MURPHY (1976)). It is however very large in comparison with values found for striated muscle, which range from 0.4 to 2% (FORD et al., 1977; Bressler and Clinch, 1974; Blangé et al., 1972). Most authors therefore conclude that a large part of the series elasticity of smooth muscle must be found outside the corssbridges (HELLSTRAND, 1979; HALPERN et al., 1978; SIEGMAN et al., 1976; MURPHY, 1976). Perhaps a large part of the series elasticity does not reside in the crossbridges, but in a structure in series with a number of crossbridges. Series elasticity might for instance be found in the filaments. In this case the dependence of stiffness on force and the other observations mentioned in Section 6 can still be understood in much the same way. The orientation of cells within the tissue is also relevant in this connection. It is not certain whether these are in series or in parallel (MURPHY, 1976) although there is some evidence that cell length is proportional to total tissue length (HELLSTRAND, 1979), which implies that no gross reorientation of cells within the tissue takes place as a result of stress. It will be clear that more anatomical data is necessary.

Finally, we should mention a pertinent observation

not covered by the theory presented. We noted that the force response both to quick releases and to quick stretches was always larger during the declining part of a contraction than during the rising part. The same effect was found by MEISS (1978) in rabbit mesotubarium smooth muscle. The reverse of this effect, i.e. a greater stiffness in the rising phase of contraction, was predicted by GROOD and MATES (1975) on the basis of a model with the series elasticity partially in the crossbridges and partially external to these. HELLSTRAND and JOHNSTON (1979) found this reverse effect in the rabbit urinary bladder.

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