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PRICE FORMATION OF FISH*

An Application of an Inverse Demand System

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Inverse demand systems explain price variations as functions of quantity variations. They have properties analogous to those of regular demand systems. There are very few examples of their empirical application. In part this is due to lack of data for which price is the decision variable and the quantity given. The case of fish landed at Belgian sea ports appears to suit an inverse demand system well. A Rotterdam variant of such a system is estimated. Allais interaction intensities have been derived and show a reasonable pattern.

1. Introduction

Gorman's well-known but unpublished paper at the Amsterdam Meeting of the Econometric Society in 1959 has established 'fish' as a respectable, challenging, subject in demand analysis. The present paper shares with Gorman's study more than only the mention of 'fish' in its title. It also aims at explaining why people pay for various types of fish the recorded prices. Gorman started off from the proposition that the price of fish depends in part on a specific factor, a function of its quantity consumed and income, and in part on the shadow prices of basic characteristics shared by all types of fish – see also Boyle, Gorman and Pudney (1977). The present approach follows Gorman by relating the price of each type of fish to its quantity traded and to total real expenditure on fish. The interactions with other types of fish are represented here by the quantities available of these other

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types. This explanation is cast in the form of an inverse demand system. Such a system expresses the relative or normalized prices paid as a function of total real expenditure and the quantities available of all goods. It appears to be a very natural model for the price formation of quickly perishable goods for which the quantities cannot adjust in the short run, as is the case for fish.

The justification of the use of an inverse demand system for fish is the topic of the next section. To estimate such a system a particular parametrization has to be selected. This issue is taken up in section 3.

The coefficients of the quantities in the various inverse demand relations reflect interactions among the goods in their ability to satisfy wants. To describe these interactions the measure of complementarity and substitution proposed by Allais (1943) is used. Section 4 is addressed to this issue. After presenting the main characteristics of the data in section 5 estimation results are given in section 6. A last section contains concluding remarks.

2. Inverse demand systems

Gossen's second law describes a consumer equilibrium as the proportionality between the vector of prices and that of the consumer's marginal utilities. The latter are functions of the quantities of commodities. Under regularity conditions this equilibrium implies a relation between price variations and quantity variations. If one writes this relation with the quantities expressed as a function of the prices one has a (regular) consumer demand system. From a theoretical point of view one could just as well express the prices as a function of the quantities. One then has what is known as an inverse demand system – see e.g. Katzner (1970), Salvat-Bronsard et al. (1977), Laitinen and Theil (1979) and Anderson (1980).

From an empirical point of view, however, inverse and regular demand systems are not equivalent. To avoid statistical inconsistencies the right-hand side variables in such systems of random decision rules should be the ones which are not controlled by the decision maker. In most industrialized economies the consumer is a price taker and a quantity adjuster for most of the products and services usually purchased. A regular demand system is then indicated.

For certain goods, like fresh vegetables or fresh fish, supply is very inelastic in the short run and the producers are virtually price takers.

Price taking producers and price taking consumers are linked by traders who select a price which they expect clears the market. In practice this means that at the auction the wholesale traders offer prices for the fixed quantities which, after being augmented with a suitable margin, are sufficiently low to induce consumers to buy the available quantities. The traders set the prices as a function of the quantities. The causality goes from quantity to price.

In the present case of eight kinds of fresh sea fish we will assume weak separability of the total commodity bundle into these types of fish on the one hand and other groups on the other hand. We can then – see e.g. Barten and Böhm (1982) – treat the demand for these fish in isolation from the demand for other products. Only the quantities and prices for these fish and total expenditure for this group matter. We also assume that collective consumer behaviour for fresh sea fish can be adequately described as that of the rational representative consumer. We can then express market demand by a system of Marshallian demand functions

$$q = f(m, p), \quad (1)$$

where q is the n -vector of quantities of fish, p the corresponding price vector and $m = p'q$ total expenditure on fish. In view of the homogeneity of degree zero in m and p , we can also write (1) as

$$q = h(\pi), \quad (2)$$

where $\pi = (1/m)p$ is the normalized price vector – cfr. e.g. Samuelson (1947) and Anderson (1980). π_i is the fraction of total expenditure paid for one unit of good i . Note that π is the same for wholesale and retail prices if the traders' margin is proportional to the price.

The traders will select π such that the given quantities q are bought. The prices they offer to the producers (fishing industry) result from inverting (2) i.e. from the inverse demand system

$$\pi = h^{-1}(q) \quad (3)$$

which reflects all the properties of (1) and (2).

To estimate such a system we will have to be more specific about these properties and about an adequate parametrization. Recalling that the properties of (1) and (2) are derived from the first-order conditions for a conditional maximum of a (partial) utility function $u(q)$, we can deduce the properties of (3) directly from these conditions:

$$u_q = \mu p, \quad p'q = m, \quad (4)$$

where $u_q = \partial u(q)/\partial q$ is the vector of marginal utilities and μ is a Lagrange multiplier. In terms of the normalized prices (4) reads

$$u_q = \lambda \pi, \quad \pi'q = 1 \quad (5)$$

with $\lambda = \mu m$. This system can be solved for π :

$$\pi = (1/\lambda)u_q = (1/q'u_q)u_q \quad (6)$$

which is equivalent to (3).

To study this relation between π and q in more detail we will consider the shift in π for a small change in q . Note that $du_q = U dq$ where $U = [\partial^2 u / \partial q \partial q']$ is the Hessian matrix of the utility function. We have

$$\begin{aligned} d\pi &= (1/q'u_q) [-\pi u'_q dq + (I - \pi q') du_q] \\ &= -\pi \pi' dq + (I - \pi q') V dq, \end{aligned} \quad (7)$$

where $V = (1/q'u_q)U$ is a symmetric matrix. A minor rearrangement results in

$$\begin{aligned} d\pi &= -[\pi - (I - \pi q') V q] \pi' dq + (I - \pi q') V (I - q \pi') dq \\ &= g \pi' dq + G dq \end{aligned} \quad (8)$$

with $g = -[\pi - (I - \pi q') V q]$ and $G = (I - \pi q') V (I - q \pi')$.

Result (8) describes the change in π as the effect of two shifts. The first one, $g \pi' dq$, can be interpreted as a *scale effect* – see Anderson (1980). Consider a proportionate increase in q , i.e. $dq = \kappa q$, κ positive scalar. It follows from (5) that then $\pi' dq = \kappa \pi' q = \kappa$. Now $Gq = 0$. Consequently, the second effect in (8) $Gdq = \kappa Gq = 0$ for a proportionate increase. The change in scale only works by way of the first effect. The change in scale is monotonously related to a change in utility. Let du be such a change. One has, using (5), $du = u'_q dq = \lambda \pi' dq = \lambda \kappa$ with $\lambda > 0$. This means that Gdq is the (utility or real income) compensated or substitution effect of quantity changes. G is the counterpart of the Slutsky matrix for regular demand systems and known as the *Anonelli (substitution) matrix* – Antonelli (1886), Salvas-Bronsard et al. (1977), Laitinen and Theil (1979), Anderson (1980). Gdq represents the move along an indifference surface, $g \pi' dq$ the move from one indifference surface to another.

It is useful to point out that the scale measure

$$\pi' dq = \sum_i \pi_i dq_i = \sum_i \pi_i q_i d \ln q_i = \sum_i w_i d \ln q_i = d \ln Q, \quad (9)$$

where $w_i = \pi_i q_i = p_i q_i / m$ is the share of expenditure on i in total expenditure. One may thus consider $\pi' dq$ also as the change in the *Divisia quantity index*. A further property follows from the differential form of $\pi' q = 1$, namely $\pi' dq + q' d\pi = 0$, yielding $q' d\pi = -\pi' dq = -d \ln Q$.

From this property and from the definitions of g and G one has the *adding-up* conditions $q'g = -1$ and $q'G = 0$. The property $Gq = 0$ can be named *homogeneity* condition because it ensures that a proportionate increase in q is neutralized as far as this substitution effect is concerned. The

matrix G is obviously symmetric. It is moreover negative semidefinite of rank one less than its order. This last property follows from the strong quasi-concavity condition of the underlying utility function, which implies that $x'Ux < 0$ for all $x \neq 0$ such that $p'x = 0$ – see Barten and Böhm (1982). This condition is equivalent to $x'Vx < 0$ for all $x \neq 0$ such that $\pi'x = 0$. Then, for $y = (I - q\pi')z$

$$z'Gz = z'(I - \pi q')V(I - q\pi')z = y'Vy$$

is zero if and only if z is proportional to q , because then $y = 0$. Otherwise it is negative, since $\pi'y = \pi'(I - q\pi')z = 0$. One consequence of this property is the negativity of the diagonal elements of Antonelli matrix G .

The properties of (8) appear to be analogous to those of a regular demand system. This suggests a similar approach to the choice of parameters, the topic of the next section.

3. Parametrization

The adding-up and homogeneity conditions for the vector g and the Antonelli matrix G involve the vector of the variable quantities. Using the g and G as constants is then not very attractive, at least if one wants to use these conditions as constraints on the parameter estimation.

A similar situation occurs for a regular demand system in differentials. Theil (1965) proposed to multiply the i th demand equation through by π_i to arrive, after some rearrangements, at a choice of constants which satisfy the usual conditions in a natural way. The resulting system is known as the Rotterdam system. In the present context we will multiply the inverse demand equations

$$d\pi_i = g_i d \ln Q + \sum_j g_{ij} dq_j \quad (10)$$

through by q_i :

$$q_i d\pi_i = h_i d \ln Q + \sum_j h_{ij} dq_j/q_j \quad \text{with} \quad (11)$$

$$h_i = q_i g_i, \quad h_{ij} = q_i g_{ij} q_j \quad (12)$$

as constants. For the variable on the left-hand side one has

$$q_i d\pi_i = q_i \pi_i d \ln \pi_i = w_i d \ln \pi_i.$$

Eq. (11) can then be written as

$$w_i d \ln \pi_i = h_i d \ln Q + \sum_j h_{ij} d \ln q_j, \quad i, j = 1, \dots, n \quad (13)$$

with the following properties of h_i and h_{ij} :

$$\Sigma_i h_i = -1 \quad \Sigma_i h_{ij} = 0 \quad (\text{adding-up}) \quad (14)$$

$$\Sigma_j h_{ij} = 0 \quad (\text{homogeneity}) \quad (15)$$

$$h_{ij} = h_{ji} \quad (\text{Antonelli symmetry}) \quad (16)$$

$$\Sigma_i \Sigma_j x_i h_{ij} x_j < 0 \quad \forall x \neq \theta i, \theta \in \mathbb{R} \quad (\text{negativity}). \quad (17)$$

System (13) is the inverse analogue of the regular Rotterdam system. It will be named the *Rotterdam* inverse demand system. Actually, the inverse demand system of Laitinen and Theil (1979) is somewhat different. It can be obtained by adding to both sides of (13) $w_i d \ln Q$ and treating the $c_i = h_i + w_i$ as constants. The variable on the left-hand side is then

$$\begin{aligned} w_i(d \ln \pi_i + d \ln Q) &= w_i(d \ln p_i - d \ln m + d \ln Q) \\ &= w_i(d \ln p_i - d \ln P) = w_i d \ln (p_i/P) \quad \text{with} \\ d \ln m - d \ln Q &= d \ln m - \Sigma_i w_i d \ln q_i = \Sigma_i w_i d \ln p_i = d \ln P, \end{aligned} \quad (18)$$

which is the Divisia price index. One then has

$$w_i d \ln (p_i/P) = c_i d \ln Q + \Sigma_j h_{ij} d \ln q_j, \quad i, j = 1, \dots, n. \quad (19)$$

The dependent variable involves now the relative price of commodity i rather than the normalized price. System (19) relates to system (13) as the CBS regular demand system of Keller and Van Driel (1985) does to the regular Rotterdam system. We will name it the *CBS* inverse demand system. Note that in (19) the adding-up condition $\Sigma_i c_i = 0$ holds. Another variant is possible. Add to both sides of (19) $w_i(d \ln q_i - d \ln Q)$. On the left-hand side one then has, in view of (18),

$$w_i(d \ln p_i + d \ln q_i - d \ln P - d \ln Q) = w_i d \ln w_i = dw_i.$$

Consequently,

$$dw_i = c_i d \ln Q + \Sigma_j c_{ij} d \ln q_j \quad (20)$$

with the $c_{ij} = h_{ij} + w_i \delta_{ij} - w_i w_j$ (δ_{ij} is Kronecker delta) now treated as constants. This is the inverse analogue of the linear version of the regular differential Almost Ideal Demand System (AIDS) of Deaton and Mullbauer (1980a). Replacing in (14), (15) and (16) the h_{ij} by $c_{ij} - w_i \delta_{ij} + w_i w_j$ it is simple

to verify that also the c_{ij} are subject to adding-up, homogeneity and symmetry conditions. There is no parallel to negativity condition (17) in this case, however. It is obvious to designate (20) as the AI inverse demand system of *AIIDS*.

Clearly, CBS system (19) is a cross between the Rotterdam and the *AIIDS*. Which of the three versions should one use? The answer to this question will not be undertaken here. Our empirical application uses the Rotterdam inverse demand system. Before turning to that it is useful to first look into the possibility of further interpretation of the elements of the matrix H . The next section discusses an approach to this issue which is originally due to Allais (1943).

4. Allais coefficients

One aspect of the original Gorman paper is the analysis of the structure of preferences for the various types of fish. The matrix $H=[h_{ij}]$ or for that matter $C=[c_{ij}]$ reflects to a certain degree the interactions between the goods in their ability to satisfy wants. Restricting our attention to H , the Antonelli substitution matrix of the Rotterdam and CBS inverse demand systems, we have by definition

$$\begin{aligned} H &= \dot{q}G\dot{q} = (\dot{q} - wq')V(\dot{q} - qw') \\ &= (1/q'u_q)(\dot{w} - ww')\bar{\pi}^{-1}U\bar{\pi}^{-1}(\dot{w} - ww'), \end{aligned} \quad (21)$$

where $\bar{\pi}$ over a vector indicates that it is a diagonal matrix with the elements of the vector as diagonal elements. Moreover w is the vector of the share of expenditures for each good in total expenditure.

The negativity condition for H implies that the h_{ii} , the diagonal elements are negative. More of good i means that one is willing to pay a lower price for i . One may also say that a good is its own substitute. Extending the notion of substitution to all negative h_{ij} , it is natural to consider a positive h_{ij} as an indication of complementarity between i and j . Note that for $i \neq j$ complementarity will dominate in an inverse demand system, because the adding-up condition $\sum_i h_{ij} = 0$ together with $h_{jj} < 0$ means that $\sum_{i \neq j} h_{ij} > 0$. The dominance does not come from the structure of preferences but from the condition $\pi'q = 1$. It makes the h_{ij} imperfect measures of the interaction of goods in their satisfaction of wants. (Analogously, the dominance of substitution in the Slutsky matrix of a regular demand system pleads against the use of the elements to describe such interaction.)

The alternative to let the signs of the off-diagonal elements of the matrix U indicate the direction of interaction among goods is also not attractive because these are not invariant for monotone increasing nonlinear transfor-

mations of the utility function – see e.g. Barten and Böhm (1982). We would prefer an ordinal measure of the direction of interaction.

Barten (1971) gives such a measure of substitution and complementarity. As pointed out by Charette and Bronsard (1975) a similar and slightly superior indicator was already proposed by Allais (1943). It appears to have been lost out of sight by the profession. Neither a contemporary like Samuelson (1947 and most notably 1974) who treats these issues at length, nor the more recent extensive and in many respects excellent survey of Deaton and Muellbauer (1980b) mention the approach of Allais.

Allais essentially works with a transformation of the Hessian matrix U such that the result is invariant under any monotone transformation of the utility function and can be considered to reflect interactions within the preference order independently of how it is represented. Let $A=[a_{ij}]$ be the matrix of the Allais coefficients a_{ij} . Then, by definition

$$A=(1/q'u_q)\tilde{\pi}^{-1}U\tilde{\pi}^{-1}-a_{ii}' \quad (22)$$

Here i is the n -vector of all elements equal to one while a is a scalar defined as $(1/q'u_q)u_{rs}/\pi_r\pi_s$. In this definition of a the subscripts r and s refer to some standard pair of goods r and s . The scalar a makes $a_{rs}=0$. Thus $a_{ij}>0$ indicates that i and j are more complementary than r and s , while $a_{ij}<0$ reflects that i and j are stronger substitutes than r and s . Clearly, $a_{ij}=0$ then means that i and j have the same type of interaction as r and s .

Combining (21) and (22) yields

$$H=(\dot{w}-ww')A(\dot{w}-ww') \quad (23)$$

because $i'(\dot{w}-ww')=w'-w'=0$. Observe that the negative semi-definite nature of H requires A to be also negative definite or semi-definite. There is no reason why A should not have full rank.

Result (23) expresses the relation between Antonelli substitution effects and the Allais coefficients. Because of the pre- and postmultiplication of A by the nondiagonal and singular matrix $\dot{w}-ww'$ the signs of the elements of matrix A are not necessarily carried over to the corresponding elements of H . Can we unscramble the a_{ij} values from H ?

In answering this question it is first to be realized that also h , the vector of scale effects can be expressed in terms of w and A . From its definition we have

$$\begin{aligned} h &= \dot{q}g = -[w - (I - w i') \dot{q} V q] \\ &= -[w - (1/u'_q q)(\dot{w} - ww') \tilde{\pi}^{-1} U \tilde{\pi}^{-1} w] \\ &= -[I - (\dot{w} - ww') A] w. \end{aligned} \quad (24)$$

This expression can be used to write

$$H = \dot{w}A\dot{w} - h\dot{w}' - \dot{w}h' - \beta\dot{w}\dot{w}', \quad (25)$$

where $\beta = 2 + \dot{w}'A\dot{w}$, a scalar. Consequently,

$$A = \dot{w}^{-1}H\dot{w}^{-1} + \dot{w}^{-1}h\dot{w}' + \dot{w}'\dot{w}^{-1} + \beta\dot{w}\dot{w}'. \quad (26)$$

For estimated H and h and some vector w one can determine A if β were known. By selecting r and s as the standard pair, β can be determined from (26) for $a_{rs} = 0$. This means that

$$a_{ij} = h_{ij}/w_i w_j - h_{rs}/w_r w_s + (h_i/w_i - h_r/w_r) + (h_j/w_j - h_s/w_s). \quad (27)$$

This relation will be used in the empirical part to describe the interactions between the various types of fish.

It may be pointed out that Allais also proposed a measure of the *intensity of interaction*, namely

$$\alpha_{ij} = a_{ij} / \sqrt{(a_{ii}a_{jj})} \quad (28)$$

which for a negative definite matrix A varies between -1 (perfect substitution) and $+1$ (perfect complementarity).

Being able to ascertain the nature of the interaction is of course not the same as explaining why some goods are substitutes or complements. If common sense or prior knowledge about consumer technology does not yield the answer one may analyse the matrix A by a technique of diagonalization, somewhat along the lines of the preference independence transformation derived by Brooks (1970), quoted and further extended by Theil (1976) for a regular demand system. One is then very close to the original factor analysis approach of Gorman.

5. Data

Our data refer to the eight major types of fish landed at Belgian fishery ports. Table 1 lists their names and the average share in the expenditure on these fish in total over the sample period. For instance, in 1987 they covered 77 percent of the total landings. The data, taken from various issues of the *Statistisch Tijdschrift* published by the Nationaal Instituut voor de Statistiek, Brussels, consist of monthly figures on the amounts landed measured in tons of 1,000 kilograms and on the average monthly prices in (Belgian) francs per kilogram. The earliest set of observations used is that of December 1973, the latest that of December 1987. The sample covers 169 months.

Table 1
Fish types, shares in returns, variation in quantities.
December 1973–December 1987.

Type of fish	Sample average share in total sales (%)	q_{\min} (tons)	q_{\max} (tons)
1. Haddock	3	7	442
2. Cod	23	173	1,604
3. Whiting	4	55	468
4. Redfish	3	1	274
5. Plaice	13	104	1,315
6. Sole	47	81	1,098
7. Ray	4	50	236
8. Turbot	3	8	42

The types of fish in table 1 are all white fish, relatively expensive and lean. Haddock, cod, whiting and redfish are roundfish swimming close to the sea bottom. Plaice, sole, ray and turbot are flatfish or bottom fish laying on the sea bottom. There is a considerable degree of joint production of fishes with the same habitat because of the fishing technique used (beam or otter trawling, e.g.).

Table 1 shows per type of fish the extremes in the landed quantities. These display a wide range. Part of the variation is seasonal, part of it is trendlike. The roundfish catches have severely suffered from the extension of the territorial fishing waters by Iceland in the seventies. Plaice and sole have increased their role. Sole is the prime fish of Belgian sea fishing. The catches are substantial, it is well liked by the consumer who is willing to pay a good price for it. Its share in total returns is on average 47 percent and still increasing.

The prices of the various types of fish are monthly averages. The average is not only taken over the days of the months but also over the various qualities (size, degree of freshness). Prices may be influenced by some measures of intervention in the market. For instance, in 1985 (1984) about 4.6 (7) percent of the total landings of fishery products in Belgian sea ports were withdrawn from the market in order to maintain the minimum price – see Welvaert (1986). Minimum price regulations are degressive in order to avoid too much overproduction. No precise information was available about the extent in which the monthly average price was affected by price support measures.

A small fraction of Belgian fishermen land (part of) their catches at foreign ports. Only few foreign fishermen offer their catches at Belgian auctions. The strong seasonal variation in the landed amounts should in principle work out on the prices by way of the quantity effects. The remaining seasonal variation in the price formation appeared not to be essential.

6. Estimation

For the eight types of fish mentioned in the preceding section the following inverse demand equation has been estimated

$$\bar{w}_{it} \Delta \ln \pi_{it} = h_i \Delta \ln Q_t + \sum_j h_{ij} \Delta \ln q_{jt} + v_{it}, \quad (29)$$

where $\bar{w}_{it} = (w_{it} + w_{i,t-1})/2$ is the two months moving average in the share of good i in total sales, $\Delta \ln x_t = \ln x_t - \ln x_{t-1}$ for x_t being π_{it} and q_{jt} , respectively, and $\Delta \ln Q_t = \sum_j w_{jt} \Delta \ln q_{jt}$. The h_i and h_{ij} are constants. The v_{it} is a disturbance term, normally distributed with mean zero. The 8×8 contemporaneous covariance matrix of the v_{it} is Ω . Intertemporal covariances have been set at zero.

In (29) one recognizes the finite difference and dated version of (13), the typical equation of the Rotterdam inverse demand system. Evidently, the h_i and h_{ij} are subject to conditions (14) through (17) which are complemented by the adding-up condition

$$\sum_i v_{it} = 0. \quad (30)$$

This condition causes the contemporaneous covariance matrix Ω to be singular since it implies $\iota' \Omega = 0$.

In the present context the quantities are treated as exogenous variables. Consequently, their covariance with the current or lagged disturbance terms is taken to be zero.

The set of eight equations (6.1) has been estimated jointly by a maximum likelihood procedure – see e.g. Barten (1969) and Barten and Geyskens (1975). The DEMMOD computer program designed for the estimation of regular demand systems needed only few modifications to also estimate inverse demand systems. (A mainframe or PC version of DEMMOD is available from the authors on request.)

As it turned out the estimated h_{ij} satisfy the negativity condition spontaneously, i.e. without it having been imposed on the estimation. Table 2 gives the estimates for the h_i and h_{ij} together with their (asymptotic) standard errors in parentheses. For easier presentation the entries have been multiplied by 100.

As table 2 shows the *scale effects* h_i have all been estimated negatively. As the aggregated quantity increases the normalized price goes down. This is to be expected. Assume that the p_i , the *absolute* prices, stay constant. An increase in the aggregated quantity means an increase of total expenditure m , hence a decrease in $\pi_i = p_i/m$. If *relative* prices do not change as the consequence of a change in scale, (19) suggests that $c_i = h_i + w_i = 0$, implying $h_i = -w_i$, i.e. negative h_i . One may note that the estimated scale coefficients are rather close to minus the average w_i of table 1. The scale coefficients h_i

Table 2
Coefficients ($\times 100$) of scale and quantity variables, price formation of fish, January 1974–December 1987.

	Quantity effects								
	Scale effects	Haddock	Cod	Whiting	Redfish	Plaice	Sole	Ray	Turbot
1. Haddock	-2.38 (0.24)	-0.35 (0.05)	-0.32 (0.11)	-0.25 (0.05)	0.01 (0.03)	0.18 (0.09)	0.27 (0.11)	0.21 (0.05)	0.25 (0.05)
2. Cod	-22.71 (1.27)	-0.32 (0.11)	-2.62 (0.63)	0.14 (0.15)	-0.01 (0.11)	0.67 (0.34)	1.86 (0.58)	0.43 (0.12)	-0.14 (0.12)
3. Whiting	-4.73 (0.34)	-0.25 (0.05)	0.14 (0.15)	-0.53 (0.13)	-0.06 (0.05)	0.09 (0.13)	0.62 (0.17)	0.02 (0.09)	-0.03 (0.10)
4. Redfish	-2.14 (0.28)	0.01 (0.03)	-0.01 (0.11)	-0.06 (0.05)	-0.25 (0.04)	0.09 (0.09)	0.25 (0.12)	0.01 (0.04)	-0.04 (0.04)
5. Plaice	-13.15 (0.79)	0.18 (0.09)	0.67 (0.34)	0.09 (0.13)	0.09 (0.09)	-2.48 (0.34)	0.84 (0.34)	0.15 (0.11)	0.47 (0.11)
6. Sole	-46.17 (1.69)	0.27 (0.11)	1.86 (0.58)	0.62 (0.17)	0.25 (0.12)	0.84 (0.34)	-5.20 (0.78)	0.81 (0.14)	0.56 (0.14)
7. Ray	-5.22 (0.26)	0.21 (0.05)	0.43 (0.12)	0.02 (0.09)	0.01 (0.04)	0.15 (0.11)	0.81 (0.14)	-1.71 (0.13)	0.08 (0.11)
8. Turbot	-3.50 (0.48)	0.25 (0.05)	-0.14 (0.12)	-0.03 (0.10)	-0.04 (0.04)	0.47 (0.11)	0.56 (0.14)	0.08 (0.11)	-1.15 (0.72)

Table 3
Scale and own substitution elasticities, price formation of fish. January 1974–December 1987.

Type of fish	Scale elasticity	Own substitution elasticity
1. Haddock	-0.82 (0.08)	-0.12 (0.02)
2. Cod	-1.00 (0.06)	-0.12 (0.03)
3. Whiting	-1.15 (0.08)	-0.13 (0.03)
4. Redfish	-0.77 (0.10)	-0.09 (0.02)
5. Plaice	-1.02 (0.06)	-0.19 (0.03)
6. Sole	-0.99 (0.04)	-0.11 (0.02)
7. Ray	-1.14 (0.06)	-0.37 (0.03)
8. Turbot	-1.06 (0.15)	-0.35 (0.05)

can be converted into *scale elasticities* by dividing by w_i . It follows from (13), (19) and (20) that

$$\frac{h_i}{w_i} = \frac{\partial \ln \pi_i}{\partial \ln Q} = \frac{\partial \ln (p_i/P)}{\partial \ln Q} - 1 = \frac{\partial \ln w_i}{\partial \ln Q} - 1$$

using the relation $h_i = c_i - w_i$. A value of -1 for this elasticity means that the relative price and the sales share are constant. If the scale elasticities are all equal to -1 preferences are homothetic. The estimated values for the scale elasticities are given in table 3 together with their approximate standard errors (in parentheses). The elasticities are evaluated for the w_i 's given in table 1. It appears that the scale elasticities are rather close to unity, suggesting homotheticity. Still, one should be cautious in making inferences from these elasticity estimates. They are not estimated as constants. There is quite some variation in the w_i from month to month with a concomitant variability of the elasticities.

Next turn to the estimated *Antonelli substitution* or *Quantity effects* of table 2. The own substitution effects have all been estimated negatively with a high degree of precision. The relatively low standard errors are no doubt due to the rather large sample size of 168 months. By dividing them by the average w_i of table 1 they have been converted into elasticities which are given in table 3. One observes that these elasticities are low and in a rather narrow range. Their being small corresponds with high price elasticities, since we are

Table 4
Allais interaction intensities for eight types of fish.

	Haddock	Cod	Whiting	Redfish	Plaice	Sole	Ray	Turbot
1. Haddock	-1	-0.66	-0.75	-0.36	-0.40	-0.53	-0.13	0
2. Cod		-1	-0.61	-0.60	-0.68	-0.82	-0.41	-0.45
3. Whiting			-1	-0.55	-0.53	-0.60	-0.35	-0.35
4. Redfish				-1	-0.47	-0.57	-0.33	-0.35
5. Plaice					-1	-0.73	-0.38	-0.23
6. Sole						-1	-0.44	-0.39
7. Ray							-1	-0.20
8. Turbot								-1

dealing with an *inverted* demand system. The negative sign of the h_{ii} is in accordance with negativity condition (17). The estimated matrix H is indeed a negative semidefinite matrix.

As already stated the off-diagonal elements of the matrix H , representing cross substitution, are not the appropriate measures of non-trivial interactions among the various types of fish. Only 7 of the 28 different cross effects are negative. If one would consider a negative h_{ij} as an indication of substitution, the small number of negative h_{ij} does not agree with the notion that most types of fish are mutual substitutes.

In section 4 the Allais coefficients (22) were proposed as a more adequate measure of interaction between commodities in their ability to satisfy needs than the coefficients of the Antonelli matrix H . Expression (27) expresses the Allais coefficients as a function of the h_{ij} and the scale coefficients h_i . To apply this relation to the results of table 2 one has to identify a standard pair of goods. We have selected for this purpose the interaction between turbot and haddock for the simple reason that then all other Allais interactions are negative. This expresses the intuitive idea that all the types of fish considered here are substitutes in consumption. For the w_i the sales shares of table 1 have been used.

Although not strictly required, the Allais matrix calculated in this way is negative definite, a sufficient condition for H to be negative semidefinite. It appears that interaction intensities (28) are more easily interpretable than the a_{ij} themselves. The interaction intensities can also be more easily compared across the various pairs. Table 4 presents the results. By construction the diagonal entries are -1 , consistent with the notion that a good is its own perfect substitute. Also by construction the interaction intensity between turbot and haddock is zero. Of the other 27 intensities 14 are less than 0.46 in absolute value.

Haddock and whiting, plaice and sole and cod appear to be substitutes. These are all white fish. The substitutability of plaice and sole being both flatfish confirms intuition. In general, the interaction intensities for plaice and

Table 5
Coefficients of determination (R^2)
and of autocorrelation of residuals
($\hat{\rho}$), price formation of fish.
January 1974–December 1987.

Type of fish	R^2	$\hat{\rho}$
1. Haddock	0.535	-0.122
2. Cod	0.745	-0.116
3. Whiting	0.644	-0.266
4. Redfish	0.364	-0.260
5. Plaice	0.711	-0.127
6. Sole	0.924	-0.265
7. Ray	0.801	-0.134
8. Turbot	0.704	-0.224

sole with the other types of fish are very close. Together with cod, plaice and sole display the strongest interaction intensities. Ray and turbot appear to be very specific kinds of fish. They interact only weakly with other types of fish. Redfish takes an intermediate position.

To conclude this section some statistical performance measures are presented. Table 5 gives the coefficients of determination (R^2) as an indication of relative fit and the autocorrelation coefficients of the residuals ($\hat{\rho}$) as a measure of unexplained dynamics. Note that the R^2 's have not been maximized as such since the system is estimated jointly. In effect the determinant of the residual covariance matrix of the full system (minus one equation) has been minimized. Still, the R^2 's are rather high for a specification in first differences. This is in part due to the large variation in the data. The negative $\hat{\rho}$ values reflect the differencing. Perhaps one month is too short to complete adjustment to a new equilibrium position. The rather low values for these autocorrelations, however, do not seem to give too much reason for worry on that score.

7. Concluding remarks

Two issues were taken up in this paper: the formulation of an inverse demand system and the analysis of preference interaction among goods using the approach suggested by Allais in 1943. The price formation of sea fish at auctions in Belgian fishery ports provided the empirical context.

Frequently, demand systems are estimated for large aggregates with ill-defined and varying technical characteristics and average prices which are perhaps very partially representative. In the case of the inverse demand model for fish the various types of fish leave little doubt about their nature.

Hardly any processing takes place before the catches reach the market. The prices, although averages, pertain to the same commodities and are usually left free to clear the market. This type of empirical material is as close as possible to the ideal as one can hope to get with reason.

By and large, the data fitted nicely the Rotterdam parametrization of an inverse demand system. The Allais interaction intensities obtained from the estimated Antonelli substitution matrix make sense. A further factorization of these intensities may shed more light on the explanation of the interaction pattern.

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