### Estimation of income elasticities and their use in a CGE model for Palestine

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#### Abstract

A popular functional form for modeling the consumption block of a computable general equilibrium model (CGE) is the Linear Expenditure System (LES) for which the Engel curves are straight lines. The LES does not allow for the existence of inferior commodities, elastic demand and for gross substitution. To calibrate the parameters outside information on income elasticities and on the expenditure elasticity of the marginal utility of expenditure (Frisch parameter) is needed. In this paper we propose to use the Indirect Addilog System (IAS) that allows for non-straight Engel curves, inferior commodities, elastic demand and gross substitution, and for which the outside data requirement is the same as for LES. In the empirical part we estimate the income elasticities of the IAS from the 1998 Palestinian Expenditure and Consumption Survey (PECS). We replace the LES consumption block with a priori fixed income elasticities of the CGE model, that we previously constructed for Palestine based on the 1998 Social Accounting Matrix (SAM), by the IAS with estimated income elasticities and perform a sensitivity analysis for the choice of the Frisch parameter. A comparison with the results obtained by the LES-model with the same income elasticities makes it possible to further clarify the importance of using a IAS to represent consumption behaviors.

**Keywords**: CGE model, LES, indirect addilog system, estimation of income elasticities, Palestine

This paper is dedicated to Professor W.H. Somermeyer, Director of the Econometric Institute from 1966 until his untimely death on 31 May 1982. Wim Somermeyer was friend and guide to author Paul de Boer, as well as supervisor of his Ph.D. Thesis.

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1. Introduction

In a recent article (Missaglia and de Boer, 2004) we analyzed the provision of emergency assistance (food assistance, cash transfers, employment programs, etc.) to a country whose economy has been decimated since the start of the second intifada. We tried to simulate the different potential effects brought about by these different policies and, especially, to draw some policy implications concerning the Food-for-Work versus Cash-for-Work debate. To that end we constructed a general equilibrium model of the Palestinian economy that we calibrated on the (pre-intifada) Social Accounting Matrix (SAM) of 1998. We gave a so-called "intifada-shock" to construct a counterfactual "post-intifada" SAM which served as basis for our policy simulations. We showed that monetary aid from abroad is to be preferred to food aid from abroad and argued that a labor-oriented approach (subsidizing the most labor-intensive sectors) is to be preferred to a welfare-oriented approach where the subsidized sectors produce those goods that dominate the consumption basket.

Following common practice, we used the Linear Expenditure System (LES), augmented for leisure, for modeling the consumption block and assigned values to the income elasticities, to the Frisch parameter and the elasticity of labor supply in order to calibrate its parameters. The theoretical disadvantage of LES is the assumption that the marginal budget shares are constant so that the Engel curves, describing the relationship between expenditure on a particular commodity and total expenditure on commodities, are straight lines. From a theoretical point of view there are limitations: the LES does not allow for the existence of inferior commodities, elastic demand and gross substitution.

An alternative model for the consumption block is the Indirect Addilog System (IAS), augmented for leisure as well, which has the same outside requirements for calibration of its parameters as the LES, but allows for non-linear Engel curves, the existence of inferior commodities, elastic demand and commodities to be gross substitutes. Consequently, with the same data requirement, it is possible to describe more general patterns for the Engel curve, income and price responses so that, from an economical point of view, the use of IAS is to be preferred to LES.

In the empirical part of the paper we estimate the income elasticities of the IAS from the 1998 Palestinian Expenditure and Consumption Survey (PECS). We replace the LES consumption block with a priori fixed income elasticities of the CGE model, which we previously constructed for Palestine, by the IAS with estimated income elasticities. For the calibration of the parameters of the consumption block (including leisure) we need to assign a numerical value to the Frisch parameter (or, equivalently, to the own price elasticity of the reference commodity) and to the elasticity of labor supply.

The organization of this paper is as follows. In section 2 we review the well-known properties of the LES, section 3 is devoted to a detailed presentation of the IAS, whereas in section 4 we show how to estimate the income elasticities implied by IAS and present the estimates. Section 5 is devoted to a description of the model that we have constructed for Palestine, in section 6 we present the results of the policy shock and a sensitivity analysis with respect to the choice of the Frisch parameter. Section 7, finally, contains our conclusions.

# 2. The linear expenditure system (LES)

# 2.1 Survey of the standard results

The utility function is defined as:

$$U(x) = \prod_{i=1}^{n} (x_i - \mu_i)^{\alpha_i} \qquad x_i > \mu_i$$

$$= 0 \qquad x_i \le \mu_i$$
(1)

where:  $x_i$ : demand for commodity i (=1,..., n),

U : utility associated with the consumption bundle  $x' = (x_1, ..., x_n)$ ,

 $\alpha_i > 0$  : marginal budget share, and

 $\mu_i \ge 0$  : minimum quantity demanded from commodity i

Since utility is ordinal, any monotonous transformation of the utility function is a utility function as well. Therefore, without any loss of generality we impose the restriction:

$$\sum_{i=1}^{n} \alpha_i = 1 \tag{2}$$

Let m denote the consumer expenditure (income minus savings) and  $p_i$  the price of commodity i (i=1, ..., n).

The consumer is faced with the budget restriction:

$$\sum_{i=1}^{n} p_i x_i = m \tag{3}$$

and is assumed to maximize its utility function (1) under the budget restriction (3). The demand relations easily follow and read:

$$x_{i} = \mu_{i} + \alpha_{i} p_{i}^{-1} (m - \sum_{j} p_{j} \mu_{j})$$
 (4)

Since  $\mu_i$  is interpreted to be the minimum quantity demanded of commodity i,  $\sum_i p_j \mu_j$ 

represents the subsistence expenditure of the consumer and, as a consequence,  $(m-\sum_i p_j \mu_j)$  is called the *supernumerary* or *discretionary* income.

It easily follows from (4) that the *Engel curve*, the relationship between expenditure on commodity i and its budget, is equal to:

$$p_i x_i = p_i \mu_i + \alpha_i (m - \sum_j p_j \mu_j)$$
 (5)

which shows that the Engel curve is a *straight line*, originating from the point  $(p_i\mu_i-\alpha_i\sum_j p_j\mu_j,0)$ .

Next, we define the budget share:

$$W_{i} = \frac{p_{i}X_{i}}{m} \tag{6}$$

First, we derive from (4) the income elasticity:

$$E(x_i, m) = \frac{\alpha_i}{w_i} > 0 \tag{7}$$

which rules out the existence of inferior commodities.

Secondly, we derive from (4) the own price elasticities:

$$E(x_{i}, p_{i}) = -\frac{\alpha_{i} [1 - (\sum_{j \neq i} p_{j} \mu_{j}) / m)]}{w_{i}} = -\frac{\alpha_{i} [m - (\sum_{j \neq i} p_{j} \mu_{j})]}{(1 - \alpha_{i}) p_{i} \mu_{i} + \alpha_{i} [m - (\sum_{j \neq i} p_{j} \mu_{j})]}$$
(8)

It follows from (8) that:

$$-1 < E(x_i, p_i) < 0$$

so that the LES only allows for inelastic demand.

Finally, we derive from (4) the cross price elasticities:

$$E(x_{i}, p_{j}) = -\frac{\alpha_{i}(p_{j}\mu_{j}/m)}{w_{i}} < 0 \text{ for } i \neq j$$
(9)

so that all commodities are gross complements.

# 2.2 A special case: Cobb-Douglas

If all subsistence levels  $\mu_i=0$  then equation (1) boils down to the Cobb-Douglas utility function:

$$U(x) = \prod_{i=1}^{n} x_i^{\alpha_i}$$

Then the budget share is equal to the marginal budget share and from (7)-(9) the well-known disadvantages of Cobb-Douglas follow: the expenditure elasticity is equal to 1 (homothetic preferences), the own price elasticity is equal to -1 and the cross price elasticities are equal to zero.

### 2.3 Calibration of the parameters of the LES

First, we need a Social Accounting Matrix (SAM) (in our empirical example we dispose of the SAM 1998 of Palestine; for more information on this SAM we refer to Missaglia and de Boer, 2004). We put all prices equal to 1, we denote the household expenditure in the SAM by  $\,m^0\,$  and the budget shares by  $w_i^0\,$ . Secondly, we need to have an estimate of the income elasticities and thirdly, an estimate of the so-called Frisch parameter  $\phi$  which is the expenditure elasticity of the marginal utility of expenditure. In case of the LES this parameter is equal to:

$$\varphi = \frac{d\lambda}{dm} \cdot \frac{m}{\lambda} = -\frac{m}{(m - \sum_{i} p_{i} \mu_{i})}$$
 (10)

(see Blonigen, et al., 1997).

From (7) we derive the calibration of the marginal budget shares:

$$\alpha_i = W_i^0.E(x_i, m) \tag{11}$$

The calibration of the minimum subsistence levels follows from rewriting (4), making use of (10):

$$\mu_{i} = x_{i}^{0} + \alpha_{i} m^{0} \phi^{-1}$$
 (12)

# 2.4 Summary of the results for the LES

The Engel curves of the LES are *straight lines* originating from the point  $(p_i\mu_i-\alpha_i\sum_j p_j\mu_j,0)$  .

It rules out the existence of inferior commodities,

It does not allow for elastic demand.

It does not allow for gross substitutes.

For calibration we need to dispose of a SAM and of an estimate of the income elasticities and of the Frisch parameter (in case we introduce leisure, we need to have a value for the elasticity of labor demand, as well).

## 3. The indirect addilog system (IAS) of consumer demand

## 3.1 Introduction

In 1951 Statistics Netherlands conducted the first post-war budget survey and in the early fifties the question rose to which extent the income elasticities had changed after the major conflict (the Second World War). The last post-war budget survey was held in 1935/'36 and it was decided to estimate the income elasticities for both surveys. At that time (1956) only the Cobb-Douglas function and the LES were known. Somermeyer and Wit, for the theoretical reasons that we summarized in sections 2.2 for Cobb-Douglas and 2.4 for LES, were dissatisfied with both models and decided to

develop a new model. (They were, at that time, unaware of the fact that Leser, 1941, had already proposed the very same model). Their reasoning (Somermeyer and Wit, 1956) was as follows:

the demand for commodity i depends on its price and on expenditure. From the theory of utility maximization we know that demand equations are homogeneous of degree zero in prices and expenditure, so that we specify the function:  $f_i(p_i/m)$ . The sum of the

budget shares is equal to 1, so that we specify: 
$$w_{_i} = \frac{f_{_i}(p_{_i} \, / \, m)}{\sum_{_i} f_{_j}(p_{_j} \, / \, m)} \, .$$

They chose as functional form:  $f_i(p_i/m) = c_i(p_i/m)^{\alpha_i}$  so that the model reads:

$$W_{i} = \frac{c_{i}(p_{i}/m)^{\alpha_{i}}}{\sum_{k=1}^{n} c_{k}(p_{k}/m)^{\alpha_{k}}}$$
(13)

It was discovered later by Houthakker (1960) that there was an underlying indirect utility function. Houthakker's formulation is not very handy, so that we transform it to the Box-Cox formulation (see Heij et al, 2004, 297-298):

$$V(p,m) = \sum_{j} c_{j} \left[ \frac{(p_{j}/m)^{\alpha_{i}}}{\alpha_{i}} - 1 \right]$$
(14)

The demand relations (13) are obtained from the indirect utility function by applying Roy's identity:

$$x_{i} = -\frac{\partial V(p, m) / \partial p_{i}}{\partial V(p, m) / \partial m}$$
(15)

It is far from trivial to show (see de Boer et al., 2006), that the necessary and sufficient conditions for the indirect utility function to be well behaved are:

$$c_i \ge 0$$
 and  $\alpha_i \le 1$  for all i, (16)

the equality holding true for at most (n-1) commodities in the first case and at most for one commodity in the second case.

For the interpretation of the parameters we quote from Somermeyer and Langhout (1972): "The  $c_{\rm k}$ - with indeterminate level – may be interpreted as "preference coefficients" and the  $\alpha_{\rm k}$  as "reaction parameters"; the higher the value of  $\alpha_{\rm k}$  (i.e. the closer it is to 1), the more "urgent" the consumption of k may be considered to be, at least at lower income levels".

The preference coefficients  $c_i$  are indeterminate, that is to say: if we multiply each of them by the same factor, the equations (13) do not change. Therefore we impose the identifying restriction that the preference coefficients sum up to one:

$$\sum_{i=1}^{n} c_{i} = 1 \tag{17}$$

# 3.2 The Engel curve

From (13) we derive that the Engel curve is given by:

$$p_i x_i = \left[ \frac{c_i (p_i / m)^{\alpha_i}}{\sum_{k=1}^n c_k (p_k / m)^{\alpha_k}} \right] \times m$$
(18)

Since the first term in (18) is a non-linear function in m, the Engel curve is non-linear.

Next, we order the commodities such that:

$$\alpha_1 = \min_j \alpha_j \quad \text{and} \quad \alpha_n = \max_j \alpha_j$$
 (19)

It is shown in Somermeyer and Langhout (1972) that the Engel curve arises from the origin and that there are three main types of Engel curves, viz.:

- (1) unlimited monotonic increase if  $\alpha_i < 1 + \alpha_1$
- (2) monotonic increase to a maximum (saturation) level if  $\alpha_i = 1 + \alpha_1$ , and
- (3) decrease towards zero after having reached a maximum level if  $\alpha_1 > 1 + \alpha_1$ .

For more details, as well as an application to the Netherlands, we refer to Somermeyer and Langhout (1972).

It should be noted that types (2) and (3) cannot occur if  $\alpha_{_{\rm i}}>0$  .

## 3.3 Expenditure and price elasticities

It can easily be shown (Somermeyer and Langhout (1972) that the income (expenditure) elasticities are:

$$E(x_i, m) = \frac{\partial \ln x_i}{\partial \ln m} = 1 - \alpha_i + \overline{\alpha} \quad \text{where} \quad \overline{\alpha} = \sum_j w_j \alpha_j$$
 (20)

It should be noted that since the expenditure elasticities are not equal to one, preferences are non-homothetic.

It follows from (20) that commodity i is necessary when  $\alpha_i>\overline{\alpha}$  and luxury when  $\alpha_i<\overline{\alpha}$ .

The expenditure elasticities are lower-bounded as well as upper-bounded:

$$\alpha_1 < 1 + \alpha_1 - \alpha_i \le E(x_i, m) \le 1 + \alpha_n - \alpha_i$$

If the expenditure elasticity is positive, i.e. when

$$\alpha_{1} < 1 + \alpha_{1} \tag{21}$$

we have the first type of Engel curve.

The lower bound is zero if

$$\alpha_{i} = 1 + \alpha_{1} \tag{22}$$

in which case we have the second type of Engel curve,

and is negative, i.e. an inferior commodity if:

$$\alpha_{i} > 1 + \alpha_{1} \tag{23}$$

in which case we have the third type of Engel curve.

In Somermeyer and Langhout (1972) the own and cross price elasticities are given and read:

$$E(x_i, p_i) = (1 - w_i)\alpha_i - 1 < 0$$
(24)

so that Giffen goods are excluded.

It follows from (24) that when

$$\alpha_{i} < 0 \tag{25}$$

that  $E(x_i, p_i) < -1$  so that *elastic* demand is allowed for by IAS.

The cross price elasticities are easily be shown to be equal to:

$$E(x_i, p_j) = -w_j \alpha_j$$
 (26)

which means that all cross elasticities of a particular price are the same.

It follows from (21) that in case (20) holds true that:

$$E(x_i, p_i) > 0 \tag{27}$$

which means that IAS allows for gross substitutes, as well.

The price response implied by (27) is the following. If the price increase of  $p_j$  refers to a necessary commodity, i.e.  $\alpha_j$  positive, then the expenditure on all other commodities will *decrease* with a given percentage –  $w_j\alpha_j$ . If the price increase of  $p_j$  refers to a highly luxury commodity, i.e.  $\alpha_j$  negative, and of considerable magnitude, then all other expenditures will *increase* with –  $w_j\alpha_j$  (>0!). It follows that luxuries are *price elastic* and necessities *inelastic*. Thus both positive and negative cross price effects are distributed neutrally over all other commodities. In many circumstances such proportional effects do not seem to be an unreasonable price response in the framework of a CGE model.

3.4 A special case: the constant elasticities of substitution (CES) utility function

If we impose the restriction:

$$\alpha_i = 1 - \sigma \tag{28}$$

where  $\sigma$  denotes the elasticity of substitution, we obtain from (13):

$$W_{i} = \frac{c_{i}(p_{i}/m)^{1-\sigma}}{\sum_{k=1}^{n} c_{k}(p_{k}/m)^{1-\sigma}}$$
(29)

i.e., the budget shares following from the CES utility function, of which the Cobb-Douglas utility function ( $\sigma = 1$ ) and Leontief ( $\sigma = 0$ ) are special cases (see de Boer, 1997, who dealt with the indirect addilog counterpart in the theory of production, i.e. with Hanoch's HCDES production function).

### 3.5 Calibration of the parameters of IAS

It is derived by De Boer and Missaglia (2005) that for the addilog system the Frisch parameter is equal to:

$$\varphi = -(1 + \overline{\alpha}) \tag{30}$$

Consequently, it follows from (20) and (30) that the calibrated values of  $\alpha_i$  are:

$$\alpha_i = -[\varphi + E(x_i, m)] \tag{31}$$

Taking the identifying restriction (17) into account, and using (13) it is tedious to show that from (13) it follows that the calibrated values of  $c_i$  are equal to:

$$c_{i} = W_{i}^{0} [m^{0}]^{\alpha_{i}} / \sum_{i=1}^{n} W_{j}^{0} [m^{0}]^{\alpha_{j}}$$
(32)

Just as for the LES, we need for the calibration of the parameters to dispose of a SAM and of an estimate of the expenditure elasticities and the Frisch parameter which means that LES and IAS have the same data requirement (if we include leisure, then we need, like in the case of LES, a value for the elasticity of labor demand).

3.6 Summary of the results for IAS and a comparison with LES

The Engel curves of the LES are *non-straight lines* originating from the origin.

The existence of *inferior commodities* is allowed for.

Elastic demand is allowed for.

Gross substitutes are allowed for.

IAS has the same data requirement as LES for the calibration of the parameters.

Since LES and IAS have the same data requirement and since the IAS is able to describe a richer economic behavior than LES, the use of IAS is to be preferred to LES.

# 4. Estimating income elasticities implied by IAS from a budget survey

#### 4.1 Econometrics

In a budget survey, pertaining to a certain year, prices are not recorded, so that it is assumed that all consumers face the same price. Without loss of generality all prices are put equal to one. Introducing the index t, to denote the respondent, t=1,...T, t=1,..

$$w_{ti} = \frac{c_{i} m_{t}^{-\alpha_{i}}}{\sum_{k=1}^{n} c_{k} m_{t}^{-\alpha_{k}}}$$
 (33)

Somermeyer and Wit proposed to select a reference commodity that, without loss of generality, is commodity 1. It easily follows, after introducing an additive disturbance, that:

$$\log(\frac{W_{ti}}{W_{t1}}) = \log W_{ti} - \log W_{t1} = \gamma_i + (\alpha_1 - \alpha_i) \log(m_t) + \varepsilon_{ti} \quad i = 2,...,n$$
(34)

with  $\gamma_i = \log c_i - \log c_1$ 

Defining:

$$y_{i} = \begin{bmatrix} (\log w_{1i} - \log w_{11}) \\ \vdots \\ (\log w_{Ti} - \log w_{Ti}) \end{bmatrix}; X = \begin{bmatrix} 1 & \log m_{1} \\ \vdots & \vdots \\ \vdots & \vdots \\ 1 & \log m_{T} \end{bmatrix}; \beta_{i} = \begin{bmatrix} \gamma_{i} \\ \alpha_{1} - \alpha_{i} \end{bmatrix} \text{ and } \epsilon_{i} = \begin{bmatrix} \epsilon_{1i} \\ \vdots \\ \epsilon_{Ti} \end{bmatrix}$$

$$(35)$$

we can rewrite (34) to:

$$y_i = X\beta_i + \varepsilon_i \qquad i = 2,...,n \tag{36}$$

i.c. to a seemingly unrelated regression model (SUR) with identical explanatory variables for which it is known that ordinary least squares applied to each equation separately is efficient (see Heij et al, 2004, 687). It goes without saying that at the time of writing their article, Somermeyer and Wit were unaware of this fact, since SUR has been developed by Zellner (1962).

Having obtained the *differences* of the parameters of interest, we are now faced with the problem of obtaining the estimates of the expenditure elasticities. Fortunately, we only need these differences in order to estimate the expenditure elasticities. We rewrite (20) as follows:

$$E(\mathbf{x}_{i}, \mathbf{m}) = 1 - \alpha_{i} - \overline{\alpha} = 1 - \alpha_{i} + \sum_{j} \mathbf{w}_{j} \alpha_{j} = 1 + (\alpha_{1} - \alpha_{i}) - \sum_{j} \mathbf{w}_{j} (\alpha_{1} - \alpha_{j})$$
(37)

## 4.2 Calibration of the reaction parameters

The Frisch parameter, defined in (30), can be written in differences of parameters of interest:

$$\phi = -1 - \overline{\alpha} = \sum_{j} w_{j} (\alpha_{1} - \alpha_{j}) - (1 + \alpha_{1}) = d - (1 + \alpha_{1})$$
(38)

with  $\,d=\sum_{_j}w_{_j}(\alpha_{_1}-\alpha_{_j})$  , a constant following from the econometric estimation.

From (38) it follows that there is a direct relation between reaction parameter of the reference commodity and the Frisch parameter:

$$\alpha_1 = \mathbf{d} - (1 + \varphi) \tag{39}$$

On the other hand, it follows from (24) that the relation between the reaction parameter of the reference commodity and its own price elasticity is:

$$\alpha_1 = [1 + E(x_1, p_1)]/(1 - w_1)$$
(40)

This means that for calibration of the reaction parameter of the reference commodity we can either use the Frisch parameter or the own price elasticity.

Then, the calibrated reaction parameters of the other commodities follow from the estimated differences of the parameters of interest.

A word of caution: the values of the reaction parameters must, for theoretical reasons, be smaller than one. It should be verified that the calibrated values satisfy this restriction.

# 4.3 Application of IAS to the PECS 1998

The Palestinian Central Bureau of Statistics (1998) conducted the Palestinian Expenditure and Consumption Survey. For ease of exposition we have aggregated the 29 groups of expenditure into 3 groups of expenditure: the "Agrifood" sector, which contains agriculture and food processing industry (including beverages and tobacco), "Manufacturing" and "Services". We have chosen as reference commodity "Agrifood", which is non-zero for all 2,851 households that participated in the survey. Eight respondents reported zero expenditure for "Manufacturing" and six (other) respondents reported zero expenditure for "Services". We have replaced the zero expenditures by 1 Jordanian dinar. The results are given in table 1.

Table 1. Expenditure elasticities for Palestine, 1998

Expenditure group i	Budget Share	$\alpha_1 - \alpha_i$	Standard error	Income elasticity
Agrifood	0.3809			0.859
Manufacturing	0.2527	0.214	0.020	1.073
Services	0.3664	0.238	0.023	1.096

It follows that the income elasticity of "Agrifood" is lower than one, which means that it is a necessary commodity, confirming the famous law of Engel (1857). The other two groups turn out to be luxury commodities, since their income elasticity is larger than one. The estimated values are positive, so that none of the commodities is inferior, which makes sense in a situation where one distinguishes but three main commodity groups. It means that the Engel curves of these three commodities are of type (1), i.e. unlimited monotonic increase from the origin.

# 5. Description of the model

#### 5.1 Introduction

In the model we have five economic agents: three producers, one household, a bank that allocates savings over investments, the Palestinian Authority (PA) and the rest of the world (RoW). In their appendix 2 Missaglia and de Boer (2004) present the glossary of symbols and in their appendix 3 the equations of the model.

## 5.2 The producers

Intermediate inputs are combined into the intermediates by means of a Leontief technology, whereas capital and labor are combined into value added by means of constant elasticities of substitution (CES) technology. Both aggregates are, using the Leontief assumption, combined into the supply of the domestically produced commodity. This commodity is transformed via constant elasticities of transformation (CET) function into an export commodity and into a domestic commodity supplied to the domestic market. This commodity is combined with imports to produce the composite commodity.

To that end we adopt the Armington assumption by using a CES functional form. This commodity is either used in the production process (intermediate demand) or for final purposes: consumption, consumption of the PA and investment.

### 5.3 The household

The household owns the capital, receives transfers from the PA and from the RoW, and it disposes of a time endowment. The household is assumed to have preferences according to the IAS (or LES) with leisure.

In the model we use the unemployment theory delineated in the migration literature by Harris and Todaro (1970) to describe the wage gap between rural and urban jobs. In our framework, the wage rate paid by Palestinian firms to Palestinian workers must be equal, in equilibrium, to the expected wage rate of the Palestinian workers employed in Israel or in the settlements (for a more detailed description we refer to Missaglia and de Boer, 2004).

All sources of income (capital, transfers and wages earned in Israel and Palestine) together yield the household income.

The household pays income taxes and saves a fixed fraction out of its income after taxes. Subtracting taxes and savings from income yields the budget that it devotes to the purchase of commodities.

## 5.4 The Palestinian Authority (PA)

The PA derives its revenues from two sources: taxes (on imports, capital, labor, consumption commodities and on household's income) and foreign aid. These revenues are spent on transfers, savings and on other expenditures. With respect to the latter we assume that the PA maximizes a Cobb-Douglas utility function.

# 5.5 The bank

The household savings, the PA savings and the foreign savings are allocated over the investment demand for the commodities. To that end the bank is assumed to maximize a Cobb-Douglas utility function subject to the constraint that savings are equal to total investments.

# 5.6 The rest of the world (RoW)

For the Palestinian economy, the RoW basically coincides with Israel. Palestine earns revenues from the RoW via exports and other sources: foreign aid accruing to the PA, remittances from the workers employed in Israel or in the settlements, foreign transfers directly accruing to the households and foreign savings, i.e. the deficit in the current account balance. These revenues are spent on imports of goods. Imports and exports are treated in a rather standard way, through, respectively, an Armington-CES and a CET assumption.

# 6. Empirical results

It is useful, for the sake of our argument, to compare the outcomes of the same shock given to an IAS-version of the model (described in section 5) and a LES-version of the same model (Missaglia and de Boer, 2004). Since the focus of this paper is on methodological issues, giving the model a "realistic" shock is not strictly required. Yet, it makes sense trying to understand what happens when a sort of "intifada" shock hits the economy. Such a shock is rather complex and we can consider here a simplified version compared to that fully described in Missaglia and de Boer (2004). In particular, let us study the effects prompted by a 25% reduction in the capital stock (destruction of assets - just think about olive trees, or, as far as human capital is concerned, the difficulties for the young Palestinians to attend schools – was one of the most important feature of the second intifada) and a 50% reduction in the Palestinian labor force employed in Israel and the settlements because of the closure policy implemented by the Israeli authorities. Some of the combined effects of these changes - those more strictly related to the choice of the demand system - are illustrated in tables 2 and 3 (directly reported from the GAMS listing files) for the LES and the IAS model respectively. The rows report the prices (net of indirect taxes) of the three relevant commodities (PrAgrFood, etc.), their real consumption levels (ConAgr, etc.), gross expenditure (inclusive of indirect taxes) in each of the three items (ExpAg, etc.), the equivalent variation (EVar) associated to the "intifada" shock and, finally, GDP at constant (1998) prices (GDPCON). The second column of tables 2 and 3 refers to the benchmark, i.e. to the data included in the 1998 SAM mentioned above. Columns from "scenario0" to scenario3" report the effects of the "intifada" shock under different values of the Frisch parameter (respectively: -1.2 in scenario0, as in the benchmark; -1.32, -1.44 and -1.56 in scenarios from 1 to 3). The reason why sensitivity analysis was conducted by letting the Frisch parameter take different values lies in the important role this parameter plays both in a LES and in a IAS model. In a LES framework the Frisch parameter (see equation (10)) corresponds to minus the inverse of the ratio between discretionary income and total income and therefore its value directly affects the calibrated values of the minimum (subsistence) quantity of each commodity. In a IAS framework the value of the Frisch parameter affects both the "preference coefficients" and the "reaction parameters", as they were originally called by Somermeyer and Langhout (1972).

Table 2: the effects of the "intifada" shock, LES model (million US\$)

	benchmark	scenario0	scenariol	scenario2	scenario3
PrAgrFood	1.000	1.130	1.132	1.134	1.135
PrManuf	1.000	1.122	1.124	1.125	1.127
PrServ	1.000	1.119	1.121	1.122	1.123
ConAgr	1519.000	1326.067	1326.865	1327.568	1328.190
ConManuf	942.000	798.499	798.554	798.617	798.683
ConSer	1441.000	1219.977	1220.153	1220.260	1220.319
EXPAG	1551.000	1529.688	1533.442	1536.690	1539.525
EXPMA	1029.000	978.394	980.180	981.734	983.098
EXPSE	1493.000	1414.354	1416.521	1418.328	1419.858
EVar		-550.458	-552.428	-554.120	-555.588
GDPCON	4229.000	3493.980	3495.883	3497.509	3498.915

Table 3: the effects of the "intifada" shock, IAS model (million US\$)

	benchmark	scenario0	scenario1	scenario2	scenario3
PrAgrFood	1.000	1.144	1.155	1.167	1.181
PrManuf	1.000	1.135	1.145	1.157	1.170
PrServ	1.000	1.130	1.137	1.146	1.157
ConAgr	1519.000	1334.323	1337.972	1342.270	1347.325
ConManuf	942.000	797.290	797.732	798.344	799.153
ConSer	1441.000	1219.483	1220.688	1221.604	1222.160
EXPAG	1551.000	1558.756	1577.670	1599.343	1624.237
EXPMA	1029.000	988.684	998.025	1008.725	1020.997
EXPSE	1493.000	1427.165	1438.618	1451.018	1464.463
EVar		-614.379	-625.210	-637.369	-651.040
GDPCON	4229.000	3506.114	3516.019	3527.117	3539.584

Most of these numbers are very easy to interpret, the directions of changes are those one would expect. In any case, here we are not so interested in the economics of the shock (it is worth repeating that the intifada shock was much more complex than the simple exercise just presented); rather, we mainly focus on methodological aspects, in particular on the differences between a LES and a IAS framework. In this respect, one can see from the above numbers that even in this very simple three good-world (it goes without saying that in a larger framework the differences would be exacerbated) the two demand systems produce some remarkable differences. First, price increases due to the negative supply shock are higher when an IAS framework is considered. Second, in a LES framework the negative supply shock produces a reduction in the expenditure level for each commodity, whilst this does not apply to the IAS case, where expenditure on "Agrifood" goes up. It may be noted that this is not only due to the higher increase in the Agrifood price, but also to the lower reduction in real consumption of Agrifood. Third, the equivalent variation, a monetary measure of the cost to the households produced by the shock, is rather different under the two systems: around 17-18% of GDP in a IAS framework, 15-16% of GDP in a LES framework<sup>1</sup>. Two per cent of GDP – it would be even more if a more realistic, i.e. tougher "intifada" shock were considered - is a lot of money, especially when this money, at least in principle, should be used to compensate the losers.

<sup>&</sup>lt;sup>1</sup> The tables also show that the IAS model is less robust to variations of the Frisch parameter. This is not surprising: in a LES, the Frisch parameter only affects the subsistence quantity of each commodity, without affecting agents' behaviour once these minimum requirements are satisfied. On the contrary, in a IAS the Frisch parameter affects *all* the relevant parameters describing the consumption behaviour.

## 7. Concluding remarks

It is a common practice in CGE literature to model the consumption block using the well known Linear Expenditure System (LES). However easy to implement, the LES suffers from serious theoretical weaknesses; the associated Engel curves are straight lines, the LES does not allow for the existence of inferior commodities, for elastic demand and gross substitution. In this paper we have proposed to use a much more general demand system, the Indirect Addilog System (IAS), which allows for non-straight Engel curves, inferior commodities, elastic demand and gross substitution. We have shown that for the IAS to be estimated and calibrated, the outside data requirement is the same as for LES, so a priori there is no reason to use the LES in empirical applications. In the empirical part of the paper we have estimated the income elasticities from the 1998 Palestinian Expenditure and Consumption Survey (PECS). Then, we have replaced the LES consumption block with a priori fixed income elasticities of the CGE model, that we previously constructed for Palestine based on the 1998 Social Accounting Matrix (SAM). by the IAS with estimated income elasticities and we have performed a sensitivity analysis for the choice of the Frisch parameter. The results coming out of the comparison between a CGE cum LES and a CGE cum IAS are interesting, especially if we look at the equivalent variation, i.e. the monetary measure of how deeply households are disfavored by the negative shocks produced by the conflict. The relatively large difference in this measure – the IAS-equivalent variation is significantly larger than the LES-equivalent variation - together with the others illustrated in section 6, stimulate us to further investigate the implications of the use of the IAS in more refined CGE models. CGE models designed for poverty analysis, where consumption behavior of the different socio-economic groups plays of course a crucial role, constitute a natural candidate.

## References

Blonigen, B.A., J.E. Flynn and K.A. Reinert (1997), Sector-focused general equilibrium modelling, in: *Applied methods of trade policy analysis: a handbook*, J.F. Francois and K.A. Reinert (eds.), Cambridge University Press, Cambridge, UK, 189-230.

De Boer, P.M.C. (1997), On the relationship between input-output coefficients and Hanoch's linear homogeneous constant differences of elasticities of substitution production function, *Economic Systems Research* **9**, 259-264.

De Boer, P.M.C. and M. Missaglia (2005), Introducing the indirect addilog in a computable general equilibrium model: a case study for Palestine, Report 2005-25 of the Econometric Institute, Erasmus University Rotterdam, a revised version is submitted to *Applied Economics*.

De Boer, P.M.C., J. Bröcker, B.S. Jensen and J. van Daal (2006), The expenditure system of the CDES indirect utility function, Report El 2006-11 of the Econometric Institute, Erasmus University Rotterdam, available at: http://people.few.eur.nl/pmdeboer/research

Engel, E. (1857), Die Productions- und Consumptionsverhältnisse des Königreichs Sachsen, Zeitschrift des Statistisches Bureaus des Königlich Sächsischen Ministerium des Innern, reprinted in: Bulletin of the International Statistical Institute (1895) **9,** 1-54.

Engel, E. (1895), Die Lebenshaltung belgischer Arbeiterfamilien früher und jetzt, *Bulletin of the International Statistical Institute* (1895) **9**, 1-124.

Harris, J. and M. Todaro (1970), Migration, unemployment and development: a two-sector analysis, *The American Economic Review* **60**, 126-142.

Heij, C., P.M.C. de Boer, P.H. Franses, T. Kloek and H.K. van Dijk (2004), *Econometric methods with applications to business and economics*, Oxford University Press.

Houthakker, H. (1960), Additive preferences, Econometrica 28, 244-257.

Leser, C.E.V. (1941), Family budget data and price elasticity of demand, *Review of Economic Studies* **9**, 40-57.

Missaglia, M. and P.M.C. de Boer (2004), Food-for-work versus cash-for-work: emergency assistance in Palestine, *Economic Systems Research* **16**, 367-390.

Palestinian Central Bureau of Statistics (1998), Household expenditure and consumption survey.

Somermeyer, W.H. and A. Langhout (1972), Shapes of Engel curves and demand curves: implications of the expenditure allocation model, applied to Dutch data, *European Economic Review* **3**, 351-386.

Somermeyer, W.H. and J.W.W.A. Wit (1956), Een verdeelmodel (an allocation model), Report M14, Statistics Netherlands, Voorburg.

Zellner, A. (1962), An efficient method of estimating seemingly unrelated regressions and tests of aggregation bias, *Journal of the American Statistical Association* **57**, 348-368.