Modeling household behavior in a CGE model: linear expenditure system or indirect addilog?

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Abstract

We try to argue that in a computable general equilibrium model, household preferences should be modeled by the indirect addilog system (IAS) rather than by the frequently used linear expenditure system (LES). Both systems have the same data requirement and are as easy to implement, but IAS provides for a richer description of preferences. Contrarily to LES, its Engel curves are non-linear and it allows for inferior commodities, elastic demand and gross substitution. LES assigns zero utility to households with expenditure below a positive minimum value, whereas IAS assigns a positive utility, provided zero expenditure is replaced by a small positive number. In micro simulation models where the results of a macro CGE model (with one representative household) are used at micro level, this constitutes a clear advantage of IAS. In the framework of an expenditure survey, we find overwhelming statistical evidence that the IAS indirect utility function is likely to be (much) closer to the true indirect utility function than LES. Consequently, expenditure elasticities and welfare changes are likely to be (much) better estimated by IAS. Simulations with a CGE model for Palestine show that price responses and equivalent variation are considerably higher for IAS than for LES.

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This paper is dedicated to Professor W.H. Somermeyer, Director of the Econometric Institute from 1966 until his untimely death on 31 May 1982. Wim Somermeyer was friend and guide to Paul de Boer, as well as supervisor of his Ph.D. Thesis.

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I. INTRODUCTION

In applied economic research, computable general equilibrium (CGE) models are widely used. In these models, the behavior of several economic actors (e.g. firms, household, government, and rest of the world) is modeled in blocks; the links between these blocks are modeled, as well. In this paper we focus on one of these blocks, the model of household behavior.

The CPB Netherlands Bureau for Economic Policy Analysis, for instance, uses a recursively dynamic CGE model for the world economy, named WorldScan (Lejour et al*.,* 2006; Don and Verbruggen, 2006).The model is used both as a tool to construct long-term scenarios and as an instrument for policy assessments. In WorldScan the household is modeled by means of the Linear Expenditure System (LES).

Besides WorldScan, other well-known CGE models use LES specifications, as well. MIRAGE, the CGE model of CEPII (Centre d'Études Prospectives et d'Informations Internationales) uses a combination of the constant elasticities of substitution (CES) functional form (Arrow et al., 1961) combined with LES (Bchir et al., 2002, p. 47). Linkage, a CGE model of the World Bank, uses as default the LES augmented with savings (Van der Mensbrugghe, 2005, p. 21). GTAP, the Global Trade Analysis Project of the Purdue University, (Cranfield et al., 2000; Reimer and Hertel, 2004) allows, besides LES, for a generalization, the so-called AIDADS (An Implicitly Directly Additive Demand System due to Rimmer and Powell,1996). Van der Mensbrugghe (2005) discusses this model in his appendix G. AIDADS allows for a richer description of Engel curves (the relationship between expenditure on a certain commodity and total expenditure) than LES, but 'comes at the expense of an additional ($n-1$) parameters' (Rimmer and Powell, 1996, p. 1615). In many practical applications, this prevents AIDADS from being used.

The disadvantage of LES is that the Engel curves are straight lines. Moreover, LES does not allow for the existence of inferior commodities (expenditure elasticities smaller than zero), elastic demand (absolute value of the own price elasticity larger than one) and gross substitution (positive cross price elasticity); see Chung (1994, chap. 2). An alternative for modeling household behavior is the indirect addilog system (IAS), which is as simple to implement as LES, but which exhibits non-linear Engel curves and allows for the existence of inferior commodities, elastic demand and gross substitution. Consequently, IAS provides a theoretically richer description of household behavior than LES, while it is also easy to implement.

The purpose of this paper is to investigate whether in a CGE model household behavior should be modeled according to LES or to IAS. As an example we take the CGE model constructed by Missaglia and de Boer (2004) for the analysis of emergency assistance to the Palestinian economy. Following common practice, we used LES, augmented for leisure (De Melo and Tarr, 1992; Blonigen et al., 1997), for modeling the consumption block, and assigned values to the expenditure elasticities, to the Frisch parameter and the elasticity of labor supply in order to calibrate its parameters.

The outline of this paper is as follows. In section II we summarize the well-known properties of LES, give the theoretical properties of IAS and pay attention to the calibration of the parameters of both systems in the framework of a CGE model. Section III deals with the estimation of parameters of LES and IAS on expenditure survey data, with the likelihood ratio test for non-nested hypotheses of Vuong (1989), and with the distribution-free test due to Clarke (2007) for a formal statistical comparison of LES and IAS (De Boer and Paap, 2009). Sections IV and V are devoted to our empirical application. On the one hand, we dispose of the 1998 Social Accounting Matrix (SAM) constructed by the World Bank, used by Missaglia and de Boer (2004). On the other hand, we dispose of the Palestinian Expenditure and Consumption Survey (PECS, 1998) conducted by the Palestinian Central Bureau of Statistics (PCBS). Both datasets are only available at a rather high level of aggregation. The only level at which both datasets are compatible is the three-commodity-level adopted in this paper: Agrifood (agriculture and food processing industry); Manufacturing; and Services. In section IV we estimate the parameters of LES and IAS on the data of PECS (1998) and test LES versus IAS. It turns out that there is overwhelming evidence for the IAS specification over the LES specification. As a first consequence, the equivalent and/or compensating variation (Varian, 1992, p. 161), used in practice for the evaluation of policy changes, is likely to be (much) better estimated by IAS than by LES. As a second consequence, the estimated expenditure elasticities are likely to be (much) closer to the "true" expenditure elasticities than those estimated for by LES. We present the implied expenditure elasticities according to IAS and use them in the sequel. It turns out that Agrifood is a necessary commodity (expenditure elasticity smaller than one) and that Manufacturing and Services are luxury commodities (expenditure elasticities larger than one). We calibrate the parameters of both systems by fixing the so-called Frisch parameter at -1.20, corresponding to a subsistence expenditure of 16.7% of total expenditure in the framework of LES. This value has been used in the model of Missaglia and de Boer (2004) which showed a very good ex-post evaluation of macro-economic indicators for the Palestinian economy in 2002 (De Boer, 2009). The calibrated values of the minimum quantities demanded for the LES specification turn out to be such that 80.6% of the households that participated in the expenditure survey (PECS, 1998) are assigned a utility level of zero, whereas each household has a positive utility level according to the IAS specification. Consequently, in a micro simulation model where the results of a macro CGE model are used at micro level, the IAS specification has a clear advantage over LES. It turns out that the price elasticities of both specifications are close to each other and that the own price elasticity of Agrifood is about -0.80. Next, we calibrate the parameters of both systems by means of fixing this own price elasticity first at -0.72 (10% above -0.80) and second at -0.88 (10% below -0.80). In the first case, the Frisch parameter according to IAS, decreases by about 10% to -1.31, whereas it decreases by about 20% to -1.46 for LES, corresponding to a subsistence expenditure of about 30% of total expenditure, which does not seem to be unrealistic. The percentage of households to which LES assigns a utility level of zero, however, increases to 93.8, which is a quite unrealistic result. In the second case the minimum quantities demanded of Manufacturing and of Services become negative so that LES cannot handle an own price elasticity of Agrifood of -0.88. However, IAS can handle this value. It turns out that Manufacturing and Services are price elastic (absolute value of the own price elasticity larger than one) and that they are gross substitutes for Agrifood (cross price elasticities positive).

In section V we show a concrete Computable General Equilibrium (CGE) application of the addilog demand system. The application is related to previous work on the Palestinian economy (Missaglia and de Boer, 2004; De Boer and Missaglia, 2006, De Boer, 2009) and refers to the building of a counterfactual Social Accounting Matrix (SAM) for the Palestinian economy in 2002. The last available official SAM for the Palestinian economy dates back to 1998 and the need to build a new, counterfactual SAM comes from the tremendous shock suffered by the Palestinian economy after the outbreak (29 September 2000) of the second intifada (“uprising”), which is likely to have dramatically changed its size and composition. A short description of both the CGE model and the intifada-shock is given. First, we use the calibrated values of LES and IAS derived on basis of the value of -1.20 of the Frisch parameter. As expected, LES being a linear approximation of IAS around the equilibrium point, the real part of the CGE model does not exhibit much differences between both specifications. The price responses are stronger in the IAS version of the model: the Laspeyres consumer price index (CPI) turns out to be 114.0 for IAS and 112.4 for LES. The real equivalent variation (in absolute terms) is 10% higher for IAS than for LES (in nominal terms 11.6% higher). Consequently, in a LES framework the welfare losses are likely to be grossly underestimated since there was overwhelming evidence that the indirect utility function of IAS is (much) closer to the true indirect utility function than the one by LES. Second, we use the calibrated values derived on the basis of the own price elasticity of Agrifood of -0.72. The CPI turns out to be 114.6 for IAS and 112.7 for LES, a difference of almost 2 percentage points. The real equivalent compensation is now 11.8% higher for IAS (the nominal one 12.8% higher).

Section VI, finally, concludes. Since in the CGE model under consideration we take account of the demand of leisure in order to model the labor market, we devote appendix B to a treatment of the extended indirect addilog system that introduces leisure into the indirect utility function. Moreover we show how to calculate the equivalent and compensating variation. Appendix A is devoted to the derivation of one of the equations.

II. THEORY

Linear expenditure system (LES)

In production and consumer theory the mostly used production function and utility function is presumably the one due to Cobb and Douglas (1928). A major shortcoming of the Cobb-Douglas utility function is that preferences are homothetic implying unitary expenditure elasticities so that Engel curves are straight lines through the origin. Moreover, the expenditure shares are constant. Tinbergen (1942) proposed to generalize the Cobb-Douglas production function by introducing positive minimum amounts of capital and labor. Shortly after the war this idea was introduced in the theory of consumption in a series of articles: Klein and Rubin (1948-1949), Samuelson (1948), Geary (1949-1950) and Stone (1954). The function is known as the Stone-Geary utility function and reads:

$$U(x) = \prod_{i=1}^{n} (x_i - \mu_i)^{\alpha_i} \qquad x_i > \mu_i \tag{1}$$
$$= 0 \qquad x_i \leq \mu_i$$

where: x_i : demand for commodity i $(= 1,..., n \geq 2)$,
U : utility associated with the consumption bundle $x' = (x_1,..., x_n)$,
$0 < \alpha_i < 1$: marginal expenditure share, with $\sum_{i=1}^{n} \alpha_i = 1$, and
$\mu_i \geq 0$: minimum quantity demanded from commodity i.

Maximization of (1) subject to the expenditure restriction:

$$\sum_{i=1}^{n} p_i x_i = m \tag{2}$$

where: p_i : price of commodity i, and
m : total expenditure (income minus savings).

leads to the Linear Expenditure System (LES) (see a.o. Chung, 1994):

$$p_i x_i = p_i \mu_i + \alpha_i (m - \sum_j p_j \mu_j) \tag{3}$$

Since μ_i is interpreted to be the minimum quantity demanded of commodity i, $\sum_j p_j \mu_j$ represents the *subsistence expenditure* of the household and, as a consequence, $(m - \sum_j p_j \mu_j)$ is its *supernumerary* or *discretionary* expenditure. According to LES the household allocates its supernumerary expenditure in fixed fractions over the commodities. Consequently, the Engel curve, the relationship between expenditure on commodity i and total expenditure, is a *straight line*, originating from the point

$$\left(\sum_j p_j \mu_j, p_i \mu_i\right) \tag{4}$$

with slope equal to the marginal expenditure share α_i.

Expenditure and price elasticities

The expenditure share of commodity i is defined as:

$$w_i = \frac{p_i x_i}{m} \tag{5}$$

Then, the *expenditure elasticity* (Chung, 1994) reads:

$$E(x_i, m) = \frac{\alpha_i}{w_i} > 0 \tag{6}$$

which rules out the existence of *inferior commodities.*

The *own price elasticities*[1] can be shown to be equal to:

$$E(x_i, p_i) = -\frac{\alpha_i[1-(\sum_{j\neq i} p_j\mu_j)/m)]}{w_i} = -\frac{\alpha_i p_i\mu_i + \alpha_i[m-\sum_j p_j\mu_j]}{p_i\mu_i + \alpha_i[m-\sum_j p_j\mu_j)]} \tag{7}$$

It follows from (7) that $-1 < E(x_i, p_i) < 0$ so that the LES only allows for *inelastic demand*.

Finally, the *cross price elasticities* (Chung, 1994) are:

$$E(x_i, p_j) = -\frac{\alpha_i(p_j\mu_j / m)}{w_i} < 0 \quad \text{for } i \neq j \tag{8}$$

so that all commodities are *gross complements*.

Indirect addilog system (IAS)

The indirect addilog system has been introduced by Leser (1941) and, independently, by Somermeyer and Wit (1956) by directly specifying the functional form. Houthakker (1960) derived the system by applying Roy's theorem to the indirect addilog utility function[2]:

$$v(p, m) = \sum_{i=1}^{n} c_i \frac{(m/p_i)^{\beta_i} - 1}{\beta_i} \tag{9}$$

The expenditure share equations[3] of IAS are:

$$w_i = \frac{c_i(m/p_i)^{\beta_i}}{\sum_{k=1}^{n} c_k(m/p_k)^{\beta_k}} \tag{10}$$

In literature there is confusion about the restrictions to be imposed on the parameters[4]. Murty (1982), without proof, gives the correct restrictions:

$$c_i \geq 0 \text{ and } \beta_i \geq -1 \tag{11}$$

for all i, the equality holding for at most n-1 items in the first case and at most for one commodity in the second case. The proof has been supplied by de Boer et al. (2006).
The preference coefficients c_i are indeterminate, that is to say: if we multiply each of them by the same factor, the equations (10) do not change. Therefore we impose the identifying restriction that the preference coefficients sum up to one:

$$\sum_{i=1}^{n} c_i = 1 \tag{12}$$

The Engel curve

From (10) we derive:

$$p_i x_i = \left[\frac{c_i (m/p_i)^{\beta_i}}{\sum_{k=1}^{n} c_k (m/p_k)^{\beta_k}} \right] \times m \tag{13}$$

Since the first term in (13) is a non-linear function in m, the Engel curve is *non-linear*.

It is shown in Somermeyer and Langhout (1972) that the Engel curve arises from the origin and that there are three main types:

(i) unlimited monotonic increase if $\beta_i > \beta_{max} - 1$, where $\beta_{max} = \max_j(\beta_j)$

(ii) monotonic increase to a maximum (saturation) level if $\beta_i = \beta_{max} - 1$, and

(iii) decrease towards zero after having reached a maximum level if $\beta_i < \beta_{max} - 1$

It should be noted that types (ii) and (iii) can, in view of the parameter restriction (11) on β_i, not occur if $\beta_{max} < 0$.

Expenditure and price elasticities

It can easily be shown (Somermeyer and Langhout, 1972) that the expenditure elasticities are:

$$E(x_i, m) = \frac{\partial \ln x_i}{\partial \ln m} = 1 + \beta_i - \overline{\beta} \quad \text{where} \quad \overline{\beta} = \sum_j w_j \beta_j \tag{14}$$

It follows from (14) that commodity i is *necessary* when $\beta_i < \overline{\beta}$ and *luxury* when $\beta_i > \overline{\beta}$. If $\beta_i < \overline{\beta} - 1$, then the commodity is *inferior*[5].

They called the parameters β_i "reaction parameters": the lower the value of β_i (i.e. the closer it is to -1), the lower the expenditure elasticity and the more "urgent" the consumption of i may be considered to be.

Somermeyer and Langhout (1972) give the *own price elasticities*:

$$E(x_i, p_i) = -(1 - w_i)\beta_i - 1 < 0 \tag{15}$$

so that Giffen goods are excluded.

It follows from (15) that when

$$\beta_i > 0 \tag{16}$$

$E(x_i, p_i) < -1$ so that *elastic* demand is allowed for by IAS.

The *cross price elasticities* are shown to be equal to:

$$E(x_i, p_j) = w_j \beta_j \qquad (i \neq j) \tag{17}$$

which means that all cross elasticities of a particular price p_j are the same. In many low-income countries there is hardly any information on price responses so that such proportional effects seem to be the price to be paid for the scarcity of data.

It follows from (17) that in case (15) holds true:

$$E(x_i, p_j) > 0 \tag{18}$$

which means that IAS allows for *gross substitutes*, as well.

Calibration of parameters

In the framework of a computable general equilibrium model (CGE), we first need to dispose of a Social Accounting matrix (SAM) pertaining to a particular year (for Palestine we dispose of the SAM of 1998). We put all prices equal to 1 and denote the household expenditure in the SAM by m^0, the demand for commodity i by x_i^0, and the expenditure share by w_i^0. Second, we need to have an estimate of the expenditure elasticities $E(x_i, m)$ (for Palestine we dispose of the expenditure survey of 1998 enabling us to estimate the expenditure elasticities, as we will see in section III below). For the identification of the parameters one additional value must be fixed. In the literature on CGE modeling one usually assigns a value to the so-called Frisch parameter φ_m which is the expenditure elasticity of the marginal utility of expenditure.

Fixing the Frisch parameter

LES

From (6) we derive the calibrated values of the marginal expenditure shares α_i:

$$\alpha_i = w_i^0 . E(x_i, m) \tag{19}$$

where the superscript 0 denotes that it is the value of the expenditure share in the SAM. Note that for the calibration of α_i we do not need to have a value of the Frisch parameter. However, it is needed for the calibration of the minimum quantities μ_i. In case of the LES the Frisch parameter is shown to be equal to:

$$\varphi_m = \frac{\partial \lambda}{\partial m} . \frac{m}{\lambda} = - \frac{m}{(m - \sum_j p_j \mu_j)} \tag{20}$$

(see Blonigen, et al., 1997).

Since the subsistence expenditure $\sum_j p_j \mu_j$ is at least equal to zero, it follows from (20) that the value of the Frisch parameter is restricted to: $\varphi_m < -1$.
Putting prices equal to one, it follows from (3) and (20) that the calibrated values of μ_i are given by:

$$\mu_i = x_i^0 + \alpha_i m^0 \varphi_m^{-1} \tag{21}$$

In case of LES the Frisch parameter (20) has a clear economic interpretation. It is minus the inverse of the fraction of supernumerary expenditure in total expenditure, so that a Frisch parameter of -1.20 means that 16.7% of the expenditure consists of subsistence expenditure and 83.3% of supernumerary expenditure.

IAS

In order to calibrate the reaction parameters β_i we need to have exactly the same information as for the LES: a SAM, estimates of all expenditure elasticities and an estimate of the Frisch parameter.

The relationship between the indirect utility function and the marginal utility λ (see for instance equation (7.10) in Varian, 1992, page 108) is: $\lambda = \frac{\partial v(p,m)}{\partial m}$, so that it follows from (9) that for the indirect addilog system we have:

$$\lambda = \sum_j c_j m^{\beta_j - 1} p_j^{-\beta_j} \tag{22}$$

From (22) we derive: $\frac{\partial \lambda}{\partial m} = \sum_{j=1}^{n} (\beta_j - 1) c_j p_j^{-\beta_j} m^{\beta_j - 2}$. Consequently, it holds true that:

$$\varphi_m = \frac{m}{\lambda}\frac{\partial \lambda}{\partial m} = \frac{1}{\lambda}\sum_{j=1}^{n} (\beta_j - 1) c_j p_j^{-\beta_j} m^{\beta_j - 1} \tag{23}$$

Changing the summation in (22) from j to k and substitution in (23), using the expressions for w_i, i.e. (10), and for $\overline{\beta}$, i.e. (14), results in:

$$\varphi_m = \sum_{j=1}^{n} (\beta_j - 1) \left[\frac{c_j p_j^{-\beta_j} m^{\beta_j - 1}}{\sum_{k=1}^{n} c_k p_k^{-\beta_k} m^{\beta_k - 1}} \right] = \sum_{j=1}^{n} w_j (\beta_j - 1) = \overline{\beta} - 1 \tag{24}$$

In view of (11) the value of the Frisch parameter[6] is restricted to: $\varphi_m > -2$. When using calibration by means of the Frisch parameter, one should take account of this restriction.

It follows from (14) and (24) that the calibrated values of the reaction parameters β_i are:

$$\beta_i = E(x_i, m) + \varphi_m \tag{25}$$

In appendix A we prove:

$$c_i = x_i^0 [m^0]^{-\beta_i} / \sum_{j=1}^{n} x_j^0 [m^0]^{-\beta_j} , \tag{26}$$

the superscript 0 denoting again the value of the pertinent variable in the SAM.
After substitution of (25) in (26) the calibrated values of the preference parameters c_i read:

$$c_i = x_i^0 [m^0]^{-E(x_i, m)} / \sum_{j=1}^{n} x_j^0 [m^0]^{-E(x_j, m)} \tag{27}$$

Note that for the calibration of c_i we do not need to have a value of the Frisch parameter.

Fixing one own price elasticity

In case of IAS the Frisch parameter φ_m (cf. (24)) does not have a clear economic interpretation. We feel that a value of the own price elasticity of food of, say, -0.8 has a clearer economic interpretation than a value of, say, -1.20 for the Frisch parameter $\varphi_m = \overline{\beta} - 1$. Therefore, we propose to fix the own price elasticity of the reference commodity 1 as alternative.

LES

Putting prices equal to one, adding a superscript 0 to denote the value in the SAM, we derive from (3) that for commodity 1 it holds true that:

$$x_1^0 = \alpha_1(m - \sum_{j=2}^{n} \mu_j) + (1 - \alpha_1)\mu_1 \tag{28}$$

Putting prices equal to one, we derive from (7) that for commodity 1 it holds true that:

$$\alpha_1(m - \sum_{j=2}^{n} \mu_j) = -x_1^0 E(x_1, p_1) \tag{29}$$

Substitution of (29) into (28) and solving for μ_1 yields:

$$\mu_1 = \frac{[1 + E(x_1, p_1)]x_i^0}{(1 - \alpha_1)} \tag{30}$$

From (21) we derive for commodity 1:

$$m^0 \varphi_m^{-1} = -\frac{(x_1^0 - \mu_1)}{\alpha_1} \tag{31}$$

Substitution of (31) in (21) yields for $i = 2,...,n$:

$$\mu_i = x_i^0 - \frac{\alpha_i}{\alpha_1}(x_1^0 - \mu_1) \qquad \text{for } i = 2,...,n \tag{32}$$

IAS

From (15) we derive for β_1:

$$\beta_1 = -[1 + E(x_1, p_1)]/(1 - w_1^0) \tag{33}$$

Equation (25) yields for $i = 2,...,n$:

$$\beta_i = \beta_1 + [E(x_i, m) - E(x_1, m)] \qquad \text{for } i = 2,...,n \tag{34}$$

III. EXPENDITURE SURVEY DATA: ESTIMATION AND TESTING

Estimating LES

In an expenditure survey, pertaining to a certain year, prices are not recorded, so that it is assumed that all households face the same price. Without loss of generality all prices are put equal to one. Introducing the index t, to denote the respondent, (t=1,…T, T being the number of respondents), and an additive disturbance (ε_{ti}), the LES (3) boils down to:

$$x_{ti} = \mu_i + \alpha_i (m_t - \sum_j \mu_j) + \varepsilon_{ti} = \gamma_i + \alpha_i m_t + \varepsilon_{ti} \tag{35}$$

where $\gamma_i = \mu_i - \alpha_i \sum_j \mu_j$

Defining:

$$y_i = \begin{bmatrix} x_{1i} \\ . \\ . \\ . \\ x_{Ti} \end{bmatrix}; X = \begin{bmatrix} 1 & m_1 \\ . & . \\ . & . \\ . & . \\ 1 & m_T \end{bmatrix}; \ \pi_i = \begin{bmatrix} \gamma_i \\ \alpha_i \end{bmatrix} \text{ and } \ \varepsilon_i = \begin{bmatrix} \varepsilon_{1i} \\ . \\ . \\ . \\ \varepsilon_{Ti} \end{bmatrix} \tag{36}$$

we can rewrite (35) to:

$$y_i = X\pi_i + \varepsilon_i \qquad i = 1,...,n \tag{37}$$

i.c. to a s*eemingly unrelated regression model (SUR)* with *identical explanatory variables* for which it is known that ordinary least squares applied to each equation separately is efficient (Heij et al., 2004, p. 687). If the disturbances are normally distributed, the maximum likelihood estimator is equivalent to this procedure. We need the assumption of normality when testing LES against IAS (see below).

Estimating IAS

Somermeyer and Wit (1956) proposed to select a reference commodity that, without loss of generality, is commodity 1. Putting prices equal to one, using the index t to denote the respondent and introducing an additive disturbance, it easily follows from (10), that:

$$\log(\frac{x_{ti}}{x_{t1}}) = \log x_{ti} - \log x_{t1} = \delta_i + (\beta_i - \beta_1)\log m_t + \varepsilon_{ti} \quad i = 2,...,n \tag{38}$$

with $\delta_i = \log c_i - \log c_1$. (39)

Defining:

$$\tilde{y}_i = \begin{bmatrix} (\log x_{1i} - \log x_{11}) \\ . \\ . \\ . \\ (\log x_{Ti} - \log x_{T1}) \end{bmatrix}; \tilde{X} = \begin{bmatrix} 1 & \log m_1 \\ . & . \\ . & . \\ . & . \\ 1 & \log m_T \end{bmatrix}; \; \tilde{\pi}_i = \begin{bmatrix} \delta_i \\ \beta_i - \beta_1 \end{bmatrix} \text{ and } \; \varepsilon_i = \begin{bmatrix} \varepsilon_{1i} \\ . \\ . \\ . \\ \varepsilon_{Ti} \end{bmatrix} \tag{40}$$

we can rewrite (38) to:

$$\tilde{y}_i = \tilde{X}\tilde{\pi}_i + \varepsilon_i \qquad i = 2,...,n \tag{41}$$

which is a s*eemingly unrelated regression model (SUR)* with *identical explanatory variables* again, so that ordinary least squares applied to each equation separately is efficient, and equivalent to ML estimation in case of normally distributed disturbances, as well.
Having obtained the *differences* of the parameters of interest, we obtain the estimates of the expenditure elasticities by rewriting (14) to:

$$E(x_i, m) = 1 + \beta_i - \overline{\beta} = 1 + \beta_i - \sum_j w_j \beta_j = 1 + (\beta_i - \beta_1) - \sum_j w_j (\beta_i - \beta_1) \tag{42}$$

Testing LES versus IAS

For two reasons it is not possible to test LES against IAS by the usual likelihood ratio test. First, the variable to be explained in the LES model is the expenditure on commodity $i = 1,...,n$ (y_i , see (36)), whereas in the IAS model it is the log-change in expenditure (i.e. the logarithm of the expenditure on commodity $i = 2,...n$ in deviation from the logarithm of the demand of the reference commodity 1, $\tilde{y}_i$, see (40)). De Boer and Paap (2009) solve this problem by transforming the density function of IAS in terms of the log-change in expenditure to the density in terms of expenditure. The second problem is that the models are non-nested. De Boer and Paap (2009) apply the likelihood ratio test of Vuong (1989) for non-nested models and the distribution-free non-nested model test of Clarke (2007) to the problem at hand. For both tests the null hypothesis is that IAS and LES are equally close to the true model, the (one-sided) alternative being that IAS is closer to the true model than LES. Besides the test statistics and their distribution (for Vuong standard normal and Clarke binomial with number of trials equal to the number of respondents T and a probability of success of $p = 0.5$), they give the Gauss 6.0 code for the estimation of LES and IAS, and for the testing of LES versus IAS by means of Vuong (1989) and Clarke (2007).

IV. APLICATION TO PECS 1998

The Palestinian Central Bureau of Statistics (1998) conducted the Palestinian Expenditure and Consumption Survey (PECS, 1998). We aggregate the 29 groups of expenditure into 3 groups: the “Agrifood” sector, which contains agriculture and food processing industry (including beverages and tobacco), “Manufacturing” and “Services”. For IAS we choose as reference commodity “Agrifood”, which is non-zero for all 2,851 households that participated in the survey. Eight respondents reported zero expenditure for “Manufacturing” and six (other) respondents reported zero expenditure for “Services”; we replace them by 1 Jordanian dinar.
We estimate the parameters of LES and IAS according to (37) and (41). The average log-likelihood contribution of LES turns out to be −12.128, while for IAS we obtain −10.948 (in levels). The values of the log-likelihood functions of both models suggest that IAS is better than LES. To analyze whether this difference is statistically significant, we consider the Vuong and Clarke tests discussed in section III. The value of the Vuong test statistic equals 6.6079 and hence we reject the null that both specifications are equally close to the true specification versus the alternative that IAS is closer (using EViews we find a P-value of 1.95e-011). When we apply the Clarke test, we

find that in 82.1% of the cases the log-likelihood contribution of the IAS specification is larger than the log-likelihood contribution of the LES specification (using EViews we find that the P-value based on a binomial distribution with number of trials T =2,851 and p=0.5 is 0). Hence, the Clarke test also indicates that IAS is significantly closer to the true specification than LES.
According to both tests there is overwhelming evidence that the IAS indirect utility function is likely to be (much) closer to the "true" indirect utility function than LES. The first consequence is that the money metric indirect utility function (Varian, 1992, p. 110) used for the evaluation of policy changes in the form of equivalent and/or compensating variation (Varian, 1992, p.161), is likely to be (much) better estimated for by IAS than by LES. The second consequence is that the estimated expenditure elasticities by means of IAS are likely to be (much) closer to the "true" expenditure elasticities than the ones estimated by LES. Therefore, in the sequel, we shall use the IAS elasticities.
In Table 1 we provide the parameter estimates and the estimated expenditure elasticities of IAS. The latter are based on the total expenditure shares of the commodities in the SAM 1998 (the representative household in the CGE model of section V below).

Table 1 Expenditure elasticities* for Palestine, 1998

Expenditure group i	Expenditure (million US$)	Expenditure share w_i	$\beta_i - \beta_1$	Standard error	Expenditure elasticity	Standard Error
Agrifood	1,551	0.381			0.861	0.008
Manufacturing	1,029	0.253	0.214	0.020	1.075	0.020
Services	1,493	0.366	0.238	0.023	1.098	0.023
Total expenditure	4,073					

* The elasticities are computed using Equation (42), where w_i is set equal to the total expenditure share of commodity i in the SAM 1998.

It follows that the expenditure elasticity of "Agrifood" is lower than one, which means that it is a necessary commodity, confirming the famous law of Engel (1857). The other two groups turn out to be luxury commodities, since their expenditure elasticity is larger than one. The estimated values are positive, so that none of the commodities is inferior, which makes sense in a situation where one distinguishes but three main commodity groups.

Calibration of parameters: fixing the Frisch parameter

Following Missaglia and de Boer (2004) we calibrate the parameters using the approach of fixing the Frisch parameter. In the CGE model for Palestine (see section V below) the selected value was -1.20, corresponding to a subsistence expenditure of 16.7% of total expenditure. It turned out that the model had a (very) good ex-post evaluation of macro-economic indicators (De Boer, 2009) so that we adopt that value as well. In Table 2 we report on the results[7].

Table 2. Calibrated values of the parameters*

	IAS		LES	
	β_i	c_i	α_i	μ_i
1 Agrifood	-0.3393	0.8031	0.3258	445.11
2 Manufacturing	-0.1250	0.0897	0.2716	107.19
3 Services	-0.1017	0.1072	0.4026	126.53
Sum		1.0000	1.0000	678.83

* Based on the expenditure elasticities reported in Table 1 and a Frisch parameter of -1.20

The sum of the preference coefficients of IAS (c_i) is equal to one, as it should be. For the LES function the sum of the marginal expenditure shares (α_i) is equal to one and the minimum quantities demanded are all positive, as it should be. A household with expenditure below this value has, according to LES, a utility level of zero. The sum of the minimum quantities demanded (μ_i) represents the subsistence expenditure which, in the framework of the LES, is used as the poverty line (Lluch, et al., 1977; De Vos, 1991). In Table 3 we give the percentage of the (2,851) households that had an expenditure below the minimum quantities demanded and below the subsistence expenditure.

Table 3 Percentage of households in PECS 1998 below minimum quantities and below poverty line

	Minimum quantities demanded (μ_i) (US$) and poverty line $\left(\sum_i \mu_i\right)$	Percentage of households below minimum quantity or below poverty line
Agrifood	445.11	80.6
Manufacturing	107.18	30.5
Services	126.53	22.9
Subsistence Expenditure	678.82	47.9

It follows that 80.6% of the households have a utility level of zero and that 47.9% of the households are below the poverty line[8] based on LES. According to IAS each of these households has a positive utility level. Consequently, in micro simulation models where the results of a macro CGE model (with one representative household) are used at micro level, IAS has a clear advantage over LES because it assigns positive utility levels to all households (provided that zero expenditure on a commodity is replaced by a small positive number).

Engel curves

The maximum of the reaction parameters of IAS (β_i) is smaller than zero (cf. Table 1) which means that the Engel curves of all three commodities are of type (i): arising from the origin with an unlimited monotonic increase. The Engel curves of LES are straight lines originating from ($\sum_j \mu_j ; \mu_i$), i.c. (4) with prices put equal to one, and with slope equal to the marginal expenditure share α_i. To give an example: the Engel curve for Agrifood starts in the point (678.82; 445.11) (see Table 3) with slope 0.3258 (see Table 2). This Engel curve intersects the Engel curve of IAS in the equilibrium point (4,073; 1,551) (see Table 1). In Table 5 we give the estimated values of expenditure on the commodities according to both Engel curves for a number of values of expenditure (m).

Table 4 Total expenditure and expenditure on commodities (US$)

Total Expenditure	Expenditure on Agrifood		Expenditure on Manufacturing		Expenditure on Services	
m	LES	IAS	LES	IAS	LES	IAS
3,000	1,201	1,192	738	741	1,061	1,067
3,500	1,364	1,361	873	874	1,262	1,264
4,073	1,551	1,551	1,029	1,029	1,493	1,493
4,500	1,690	1,690	1,145	1,145	1,665	1,665
5,000	1,853	1,849	1,281	1,282	1,866	1,869

As expected, LES being a linear approximation of IAS around the equilibrium point, the values are very close to each other. As a consequence, it is to be expected that in simulations the real part of the CGE model will not produce a lot of differences between both specifications.
In table 5, finally, we give the implied price elasticities.

Table 5 Price elasticities

	Price elasticities LES			Price elasticities IAS		
Commodity	1	2	3	1	2	3
1	-0.807	-0.023	-0.027	-0.790	-0.032	-0.037
2	-0.117	-0.924	-0.033	-0.129	-0.907	-0.037
3	-0.120	-0.029	-0.949	-0.129	-0.032	-0.935

It turns out that the implied own and cross price elasticities which are based on the expenditure elasticities according to IAS and and a Frisch parameter of -1.20, are close to each other. The implied own price elasticity of the reference commodity (Agrifood) is equal to about -0.8.

Calibration of parameters: fixing one own price elasticity

First, we increase value of the own price elasticity of Agrifood, $E(x_1, p_1)$, from about -0.80 to -0.72 (10%). Then, for IAS, according to (33) the value of β_1 decreases and by virtue of (34), the values of β_i $(i = 2,..., n)$ decrease as well. As a consequence, the value of the Frisch parameter (24) decreases (recall that the preference coefficients c_i are identified). For LES the minimum quantity of commodity 1, μ_1, increases by virtue of (30) and by (32) so do μ_i $(i = 2,..., n)$. As a consequence, the subsistence expenditure increases, so that the Frisch parameter decreases, see (20) (recall that the marginal expenditure shares α_i are identified). In the columns 2-4 of Table 6 we summarize our findings.

Table 6 Calibrated values*

	Own price elasticity $E(x_1, p_1) = -0.72$			Own price elasticity $E(x_1, p_1) = -0.88$	
	IAS β_i	LES μ_i	Percentage below μ_i or $\sum_i \mu_i$	IAS β_i	LES μ_i
Agrifood	-0.4522	644.16	93.8	-0.1938	276.07
Manufacturing	-0.2378	273.10	75.8	0.0206	-33.72
Services	-0.2145	372.48	74.5	0.0439	-92.35
Subsistence expenditure		1,289.74	85.4		
Frisch parameter	-1.31	-1.46		-1.05	-1.04

* Based on the expenditure elasticities reported in Table 1.

It turns out that the Frisch parameter according to IAS decreases by about 10% from -1.20 to -1.31, whereas the Frisch parameter according to LES decreases by about 20% to -1.46 (corresponding to a subsistence expenditure of about 30% of total expenditure). The percentage of respondents with utility zero according to LES increases to 93.8%, whereas the percentage of households below the subsistence level increases to 85.4%, which is quite unrealistic.
Second, we decrease the value of the own price elasticity of Agrifood by 10% to -0.88. The results are reported in the columns 5 and 6 of Table 6. For IAS the reaction parameters for Manufacturing and Services become positive, whereas for LES the minimum quantities demanded become negative. Consequently, in this example LES cannot handle an own price elasticity of Agrifood of -0.88, whereas IAS can.
In Table 7 we give the price elasticities for the two scenarios.

Table 7 Price elasticities

	LES $E(x_1, p_1) = -0.72$			IAS $E(x_1, p_1) = -0.72$			IAS $E(x_1, p_1) = -0.88$		
Commodity	1	2	3	1	2	3	1	2	3
1 Agrifood	-0.720	-0.057	-0.078	-0.720	-0.047	-0.062	-0.880	0.004	0.013
2 Manufact.	-0.170	-0.807	-0.098	-0.135	-0.822	-0.062	-0.579	-1.015	0.013
3 Services	-0.174	-0.074	-0.851	-0.135	-0.047	-0.864	-0.579	0.004	-1.028

As before, the price elasticities between LES and IAS do not differ much in case of an own price elasticity of Agrifood of -0.72. Since LES is not applicable for an own price elasticity of -0.88, we only give them for IAS. The reaction parameters of Manufacturing and of Services are positive so that the demand for these commodities is elastic (absolute value of own price elasticity larger than one, see (16)) and commodities 2 and 3 are gross substitutes for Agrifood (cross price elasticities positive, cf. (18)).

V. APPLICATION TO THE CGE MODEL FOR PALESTINE

In this section we show a concrete Computable General Equilibrium (CGE) application of the addilog demand system. The application is related to previous work on the Palestinian economy (Missaglia and de Boer, 2004; De Boer and Missaglia, 2006, De Boer, 2009) and refers to the building of a counterfactual Social Accounting Matrix (SAM) for the Palestinian economy in 2002. The last available official SAM for the Palestinian economy dates back to 1998 and the need to build a new, counterfactual SAM comes from the tremendous shock suffered by the Palestinian economy after the outbreak (29 September 2000) of the second intifada ("uprising"), which is likely to have dramatically changed its size and composition. Before illustrating our results we give a short description of both the CGE model and the intifada-shock.

Model

We have seven economic agents: three firms, one household, a bank that allocates savings over investments, the Palestinian Authority (PA) and the rest of the world (RoW). In their appendix 2 Missaglia and de Boer (2004) present the glossary of symbols and in their appendix 3 the equations of the model.

The three firms combine intermediate inputs into the intermediates by means of a Leontief technology, whereas capital and labor are combined into value added by means of constant elasticities of substitution (CES) technology. Using the Leontief assumption, both aggregates are combined into the supply of the domestically produced commodity. This commodity is transformed via constant elasticities of transformation (CET) function into an export commodity and into a domestic commodity supplied to the domestic market. This commodity is combined with imports to produce the composite commodity. To that end we adopt the Armington assumption by using a CES functional form. This commodity is either used in the production process (intermediate demand) or for final purposes: consumption, consumption of the Palestinian authority (PA), and investment.

The household owns the capital, receives transfers from the PA and from the RoW, and it disposes of a time endowment. The household is assumed to have preferences according to IAS (or LES) augmented for leisure[9]. We use the unemployment theory delineated in the migration literature by Harris and Todaro (1970) to describe the wage gap between rural and urban jobs. In our framework, the wage rate paid by Palestinian firms to Palestinian workers must be equal, in equilibrium, to the expected wage rate of the Palestinian workers employed in Israel or in the settlements (for a more detailed description we refer to Missaglia and de Boer, 2004). All sources of income (capital, transfers and wages earned in Israel and Palestine) together yield the household expenditure. The household pays income taxes and saves a fixed fraction out of its income after taxes. Subtracting taxes and savings from income yields the expenditure that it devotes to the purchase of commodities.

The PA derives its revenues from two sources: taxes (on imports, capital, labor, consumption commodities and on the income of the household) and foreign aid. These revenues are spent on transfers, savings and on other expenditures. With respect to the latter we assume that the PA maximizes a Cobb-Douglas utility function.

The bank allocates the household savings, the PA savings and the foreign savings over the investment demand for the commodities. To that end the bank is assumed to maximize a Cobb-Douglas utility function subject to the constraint that savings are equal to total investments.

For the Palestinian economy, the RoW basically coincides with Israel. Palestine earns revenues from the RoW via exports and other sources: foreign aid accruing to the PA, remittances from the workers employed in Israel or in the settlements, foreign transfers directly accruing to the households and foreign savings, i.e. the deficit in the current account balance. These revenues are spent on imports of goods. Imports and exports are treated in a rather standard way, through, respectively, an Armington-CES and a CET assumption.

Shock

It is useful, for the sake of our argument, to compare the outcomes of the same shock given to an IAS-version of the model and a LES-version of the same model (Missaglia and de Boer, 2004). Since the focus of this paper is on methodological issues, giving the model a "realistic" shock is not strictly required. Yet, it makes sense trying to understand what happens when a sort of shock hits the economy. Such a shock is rather complex and we consider here a simplified version[10] compared to that fully described in Missaglia and de Boer (2004). In particular, we study the effects prompted by a 25% reduction in the capital stock and a 50% reduction in the Palestinian labor force employed in Israel and its settlements.

Results (fixing the Frisch parameter at -1.20)

In Table 8 we give the results for real consumption, exclusive of indirect taxes, and of real GDP.

Table 8 Real consumption* and real GDP before and after the shock

	Benchmark SAM 1998 (millions US$)	LES 2002 (millions US$, prices 1998)	Change (%)	IAS 2002 (millions US$, prices 1998)	Change (%)
Agrifood	1,519	1,326	-12.7	1,334	-12.2
Manufacturing	942	799	-15.2	797	-15.4
Services	1,441	1,220	-15.3	1,219	-15.4
Consumption	3,902	3,345	-14.3	3,350	-14.1
GDP	4,229	3,494	-17.4	3,506	-17.1

* Exclusive of indirect taxes.

As expected, the results of the real part of the model do not differ much between the LES and the IAS-version.

In table 9 we give the results for the price indexes and the equivalent variation (in prices 1998).

Table 9 Price indexes* (1998 = 100)
and equivalent variation (prices 1998)

	LES	IAS
Agrifood	113.0	114.4
Manufacturing	112.2	113.5
Services	111.9	113.0
Laspeyres price index (1998 =100)	112.4	114.0
Equivalent variation (prices 1998)	-489.3	-538.5

*Net of indirect taxes

The price index for the IAS version of the model is 1.4 percentage points higher for Agrifood, 1.3 percentage points for Manufacturing and 2.1 percentage points for Services than in the LES version. The Laspeyres consumer price index (CPI) is 1.6 percentage points higher according to the IAS version than for LES. Consequently, the price responses are substantially higher for IAS than for LES. The real equivalent variation (in absolute terms) is 10% higher for IAS than for LES. In nominal terms they amount to – 614 for IAS and – 550 for LES, or 11.6% higher for IAS than for LES. Consequently, in a LES framework the welfare losses are likely to be grossly underestimated since there was overwhelming evidence that the indirect utility function of IAS is (much) closer to the true indirect utility function than the one by LES.

Results (fixing the own price elasticity of Agrifood at -0.72)

Table 10 Real consumption*, real GDP, price indexes and
real equivalent variation before and after the shock

	LES 2002 (millions US$, prices 1998)	IAS 2002 (millions US$, prices 1998)	LES Price indexes (1998=100) and EV (prices 1998)	IAS Price Indexes (1998=100) and EV (prices 1998)
Agrifood	1,328	1,338	113.4	115.5
Manufacturing	799	797	112.5	114.5
Services	1,220	1,221	112.2	113.7
Consumption	3,347	3,356	112.7	114.6
GDP	3,498	3,516		
Equivalent Variation			-491.4	-545.6

* Exclusive of indirect taxes.

Compared to Table 9, the Laspeyres CPI according to the LES version of the model increases by 0.3 percentage points, whereas it increases by 0.6 percentage points according to IAS. When the own price elasticity is increased to -0.72, the price response, measured by the Laspeyres CPI, is 2 percentage points higher for IAS as compared to LES. The real equivalent variation (in absolute terms) is 11% higher for IAS, in nominal amounts they amount to -625 for IAS and -554 for LES, or 12.8% higher for IAS than for LES.

VI. CONCLUDING REMARKS

In this paper we try to give an answer to the question whether in a Computable General Equilibrium model (CGE) household preferences should be modeled according to the Linear Expenditure System (LES) or to the Indirect Addilog System (IAS). Both models describe household preferences by means of 2n parameters, where n denotes the number of commodities distinguished in the household's expenditure. The parameters of both models are uniquely identified ("calibrated") when one disposes of a Social Accounting Matrix (SAM), a value for the expenditure elasticities and a value for the Frisch parameter (or, alternatively, one price elasticity). In case one disposes of an expenditure survey, as well, the expenditure elasticities of both models can easily be estimated by means of application of ordinary least squares to linear equations separately. Consequently, both models are as easy to implement but IAS has the following advantages over LES.

1. Its Engel curves arise from the origin and are non-linear, whereas in LES the Engel curves are straight lines departing from a point of which the coordinates are positive. IAS allows for the existence of inferior commodities, elastic demand and gross substitution, whereas LES does not (recall the example where the own price elasticity of Agrifood is fixed at -0.88: the two other commodities exhibit elastic demand and are gross substitutes for Agrifood; whereas LES cannot be used). Consequently, IAS provides for a theoretically richer description of household behavior than LES.

2. In case one disposes of an expenditure survey, LES can be tested against IAS. In the case of the three commodities example (PECS, 1998 with 2,851 households) we already found overwhelming statistical evidence that IAS is (much) closer to the true system than LES. Vuong's parametric test statistic yields a value of 6.608, whereas the (one-sided) critical value at a level of significance of 5% is the well-known 1.645; applying Clarke's non-parametric test, we find that in 82% of the cases the log-likelihood contribution of the IAS specification is larger than the log-likelihood contribution of the LES specification. When one distinguishes more commodities, the statistical evidence of IAS being superior to LES becomes even stronger. De Boer and Paap (2009) who applied both tests to a ten commodities example (PECS, 2005 with 2,152 households) report a value of 31.033 for the Vuong statistic and a percentage of 91% of a higher log-likelihood contribution by IAS in case of Clarke's test.

Consequently, we find overwhelming evidence that the IAS indirect utility function is likely to be (much) closer to the "true" indirect utility function than LES. The expenditure elasticities estimated for by IAS are likely to be (much) closer to the true elasticities than the ones estimated using LES.

3. The money metric indirect utility function (Varian, 1992, p. 110) used for the evaluation of policy changes in the form of equivalent and/or compensating variation is likely to be (much) better estimated for by IAS than by LES. In the example of our CGE model for Palestine in case we use a Frisch parameter of -1.20 for the calibration of the parameters (implying an own price elasticity for Agrifood of -0.80) we find that the real equivalent variation (in absolute terms) is 10% higher for IAS than for LES; the nominal equivalent variation being 11.6% higher for IAS than for LES. In case we use an own elasticity of -0.72 for calibration, these percentages increase to 11% for the real equivalent variation and to 12.8% for the nominal one.

Consequently, in a LES framework the welfare changes are likely to be grossly underestimated.

4. In the framework of a LES, households with expenditure below one (or more) of the minimum quantities demanded are assigned a utility level of zero, whereas in IAS they are assigned a positive utility (provided that zero expenditure on a commodity is replaced by a small positive number). In our example we find that in the case of a Frisch parameter of -1.20, the percentage of households with utility zero equals 80.6%; in the case of an own price elasticity of Agrifood of -0.72 this percentage increases to 93.8%.

Consequently, in micro simulation models where the results of a macro CGE model (with one representative household) are used at micro level, IAS has a clear advantage over LES of assigning a positive utility to all households, whereas LES does not.

Last, but not least, we hope to have convinced practitioners of CGE models that the answer to the question in the title is: use IAS!

APPENDIX A DERIVATION OF EQUATION (26)

Putting prices equal to one we derive from (10):

$$\frac{x_i}{x_1} = \frac{c_i m^{\beta_i}}{c_1 m^{\beta_1}} \rightarrow c_i = c_1 \frac{x_i m^{-\beta_i}}{x_1 m^{-\beta_1}} \qquad i = 2,...,n \tag{A1}$$

Summation over $k = 2,...,n$ and using (12) leads to:

$$(1-c_1) = c_1 \frac{\sum_{k=2}^{n} x_k m^{-\beta_k}}{x_1 m^{-\beta_1}} \rightarrow c_1 \left[1 + \frac{\sum_{k=2}^{n} x_k m^{-\beta_k}}{x_1 m^{-\beta_1}} \right] = c_1 \left[\frac{\sum_{k=1}^{n} x_k m^{-\beta_k}}{x_1 m^{-\beta_1}} \right] = 1 \text{ leading to:}$$

$$c_1 = \frac{x_1 m^{-\beta_1}}{\sum_{k=1}^{n} x_k m^{-\beta_k}} \tag{A2}$$

Substitution in (A1) gives:

$$c_i = \frac{x_i m^{-\beta_i}}{\sum_{k=1}^{n} x_k m^{-\beta_k}} \qquad i = 2,...,n \tag{A3}$$

Combination of (A2) and (A3) yields:

$$c_i = \frac{x_i m^{-\beta_i}}{\sum_{k=1}^{n} x_k m^{-\beta_k}} \qquad i = 1,...,n \tag{A4}$$

Adding the superscript 0 (denoting the value in the SAM) to x_i and m we obtain equation (26).

APPENDIX B THE INDIRECT ADDILOG SYSTEM WITH LEISURE

Model and calibration

We assume that the household has an exogenously given time endowment, denoted by TS, that it allocates over labor supply, denoted by LS, and *leisure*, denoted by x_{n+1}, i.e.

$$TS = LS + x_{n+1} \tag{B1}$$

We take account of the consumption of leisure, valued at the wage rate p_{n+1}, define the *extended* household expenditure:

$$em = m + p_{n+1} x_{n+1} \tag{B2}$$

and assume that the household maximizes the *extended* indirect addilog utility function:

$$v(p, em) = -\sum_{j=1}^{n+1} c_j \frac{(em/p_j)^{\beta_j} - 1}{\beta_j} \tag{B3}$$

subject to the *extended* household expenditure (B2).
It follows straightforwardly that the optimal shares in *extended* expenditure are:

$$w_i = \frac{c_i (em/p_i)^{\beta_i}}{\sum_{k=1}^{n+1} c_k (em/p_k)^{\beta_k}} \qquad i = 1,...,n+1 \tag{B4}$$

As before, we need to have outside information on the expenditure elasticities and on the Frisch parameter (or on one own price elasticity) in order to calibrate the parameters β_i and c_i (i running from 1 to n for the commodities, while i is equal to $n+1$ for *leisure*). In this framework we need to have values of these elasticities (and of the Frisch parameter) with respect to *extended* household expenditure, em (including leisure), but in practice they are usually supplied with respect to expenditure, m (excluding leisure). Moreover, in practice a value of the elasticity of labor supply with respect to expenditure m is specified, rather than the expenditure elasticity of *leisure* with respect to *extended* expenditure.

First, we consider the case that i runs from 1 to n. Because (B2) implies that:

$$\frac{d(em)}{d(m)} = 1 \tag{B5}$$

it follows that:

$$E(x_i, em) = E(x_i, m).\frac{em}{m} \tag{B6}$$

Second, we consider leisure (x_{n+1}). Using (B1) we derive:

$$E(LS, m) = -\frac{\partial x_{n+1}}{\partial m}.\frac{m}{x_{n+1}}.\frac{x_{n+1}}{LS}.$$

Using the same reasoning as with (B6) and replacing x_{n+1} by ($TS - LS$) according to (B1), we arrive at:

$$E(x_{n+1}, em) = -E(LS, m).\frac{LS}{(TS - LS)}.\frac{em}{m} \tag{B7}$$

For the Frisch parameter with respect to extended expenditure, using (B5), we derive:

$$\varphi_{em} = \frac{em}{\lambda}.\frac{\partial \lambda}{\partial em} = \varphi_m.\frac{em}{m} \tag{B8}$$

Similarly to (25) we calibrate the parameters β_i:

$$\beta_i = E(x_i, em) + \varphi_{em} \qquad i = 1,...,n+1 \tag{B9}$$

The parameters c_i are calibrated in the same way as in (27):

$$c_i = x_i^0[em^0]^{-E(x_i,em)} / \sum_{j=1}^{n+1} x_j^0[em^0]^{-E(x_j,em)} \qquad i = 1,...,n+1 \tag{B10}$$

Equivalent and compensating variation

Suppose that we have two different policy regimes: the "benchmark equilibrium", and the "proposed change". Under the "benchmark equilibrium" the household faces prices and (extended) expenditure (p^0, em^0) , and under the "proposed change" it faces (p^1, em^1) .
The equivalent variation (EV) measures the expenditure change at current prices (p^0) that would be equivalent to the proposed change in terms of its impact on utility. Let em^{ev} denote the expenditure that at current prices (p^0) would yield utility level $V(p^1, em^1)$, i.e.:

$$V(p^0, em^{ev}) = V(p^1, em^1) \tag{B11}$$

Consequently, the equivalent variation is defined as:

$$EV = em^{ev} - em^0 \tag{B12}$$

It follows from the indirect utility function (B3) that em^v has to be solved numerically from:

$$\sum_{j=1}^{n+1} \frac{c_j[(em^1/p_j^1)^{\beta_j} - (em^{ev}/p_j^0)^{\beta_j}]}{\beta_j} = 0 \tag{B13}$$

The *compensating variation* (CV) measures the expenditure change that would be necessary to compensate the household for the price change induced by the "proposed change".
Let em^{cv} denote the expenditure that at prices p^1 would yield the utility level $V(p^0, em^0)$, i.e.:

$$V(p^1, em^{cv}) = V(p^0, em^0) \tag{B14}$$

Consequently, the compensating variation is equal to:

$$CV = em^1 - em^{cv} \tag{B15}$$

where em^{cv} has to be solved numerically from:

$$\sum_{j=1}^{n+1} \frac{c_j[(em^{cv}/p_j^1)^{\beta_j} - (em^0/p_j^0)^{\beta_j}]}{\beta_j} = 0 \tag{B16}$$

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[1] The own price elasticity given by Chung (1994) is not correct; the correct one is given in (7).

[2] The specification of Houthakker reads: $\sum c_i^*(m/p_i)^{\beta_i}$. Using the reparametrization $c_i = c_i^*\beta_i$ and subtracting the constant $\sum c_i/\beta_i$ we arrive at (9). Both specifications represent the same preferences, but the advantage of (9) is that the parameter restrictions can readily be derived, see Murty (1982) and de Boer et al. (2006), and that the special case $\beta_i = 0$ is defined to be equal to $\ln(m/p_i)$.

[3] In Leser (1941), Somermeyer and Wit (1956) and Somermeyer and Langhout (1972) the reaction coefficients are denoted by α_i (= $-\beta_i$). If all $\beta_j = \beta$ IAS reduces to the constant elasticity of substitution (CES) function (Arrow et al., 1961); the elasticity of substitution being $\sigma = (1-\beta)$. If, moreover, $\beta = 0$, we obtain the Cobb-Douglas function (Cobb and Douglas, 1928).

[4] Hanoch (1975), Deaton and Muelbauer (1980) and Chung (1994) give as restriction $\beta_i > 0$, excluding the region $-1 \le \beta_i < 0$. As a consequence, the existence of inelastic demand and of gross complementarity is, erroneously, excluded.

[5] Consider for instance the case of two commodities which have a budget share of 0.5 each, while $\beta_1 = -0.2$ and $\beta_2 = 2$. Then, commodity 1 is inferior.

[6] For the Cobb-Douglas utility function we have $\varphi_m = -1$ and for the CES utility function (see footnote 3): $\varphi_m = -(1-\beta) = -\sigma$.

[7] We calibrate the marginal budget shares α_i of LES by means of (19), because we use the income elasticities estimated for by IAS. If only LES is used, the marginal budget shares are estimated by applying OLS to (36). We calibrate the preference coefficients c_i by means of (27). Alternatively, we might have used the estimated values of γ_i obtained applying OLS to (41), denoted by $\hat{\gamma}_i$ ($i = 2,...,n$). From (39), using the identifying restriction (12), it can easily be derived that $\hat{c}_1 = 1/[1+\sum_{i=2}^{n}\exp(\hat{\gamma}_1)]$ and $\hat{c}_i = \hat{c}_1 \exp(\hat{\gamma}_i)$ for $i = 2,...n$.

[8] Using a poverty level of 2.1 US$ per day, the World Bank (2003) estimates the percentage of poor in 1998 at 23.2%.

[9] In their Appendix 7.1 Blonigen et al.(1997) give the relevant formulae for LES. In Appendix B we derive the relevant formulae for IAS. For the calibration of the parameters we used the income elasticities given in Table 1, a value of -1.20 for the Frisch parameter, and a value of -0.50 for the income elasticity of labor supply.

[10] The decline in real GDP according to the simplified version is 17.1%, see Table 6. The decline according to the full shock is 20.5% (De Boer and Missaglia, 2006).