Baryogenesis Through Mixing of Heavy Majorana Neutrinos

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Abstract

A mechanism is presented, in which the mixing of right handed heavy Majorana neutrinos creates a $CP-$asymmetric universe. When these Majorana neutrinos subsequently decay more leptons than antileptons are produced. The lepton asymmetry created by this new mechanism can exceed by a few orders of magnitude any lepton asymmetry originating from direct decays. The asymmetry is finally converted into a baryon asymmetry during the electroweak phase transition.

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The generation of the baryon asymmetry in the universe has been discussed in many articles [1, 2]. Two prominant scenarios are the Grand Unified Theories [3, 2] and the production of an asymmetry through extended solutions of field theories (sphalerons) [4, 5]. At this time both scenarios generate asymmetries which are small. This comes about because in the former case $CP-$violation is produced by higher order effects and in the latter the tunneling rate through potential barriers is small (in addition, if the higgs particles are heavier than 80 GeV, then the baryon asymmetry thus generated will be completely erased) [5].

A third scenario includes heavy Majorana neutrinos whose decays generate a lepton asymmetry, which later on is converted into the baryon asymmetry. By their very nature Majorana neutrinos posses $\Delta L = 2$ transitions and in addition they may have couplings which allow them to decay into the standard higgs and leptons, i.e., $N \to \phi^\dagger l^-$ [6]. In these models the $CP-$violation is introduced through the interference of tree-level with one-loop diagrams in the decays of heavy neutrinos, which we shall call $\epsilon'-$type effects (direct $CP-$violation) [6, 7]. An additional contribution from self energies was included in ref [8-10]. A new aspect was pointed out [9] when it was realised that the heavy physical neutrino states are not $CP-$ or lepton-number–eigenstates. Therefore as soon as the physical states are formed there is imprinted on them a $CP-$asymmetry and a lepton–asymmetry. This we shall call $\epsilon-$type effects (indirect $CP-$violation). It is a property which appears in the eigenstates of the Hamiltonian at an early epoch of thermodynamic equillibrium, even before the temperature of the universe falls down to the masses of the heavy Majorana particles.

We demonstrate the phenomenon with a Gedanken experiment. Consider a universe consisting of a large $p-\bar{p}$ collider which produces $s-$ and $\bar{s}-$pairs. The s and $\bar{s}$ quarks hadronize into K° and $\overline{K^\circ}$ mesons whose superpositions are the physical states

$$K_{L,S} \propto [(1+\epsilon)K^\circ \pm (1-\epsilon)\overline{K^\circ}].$$

The probability of finding a $|K^\circ>$ is proportional to $|1+\epsilon|^2$ and the probability for a $|\overline{K^\circ}>$ is proportional to $|1-\epsilon|^2$ which are not equal. When the particles decay

$$K_L \to \pi^\pm e^\mp \overset{(-)}{\nu} \quad \text{and} \quad K_S \to \pi^\pm e^\mp \overset{(-)}{\nu}$$

the above asymmetry survives as an asymmetry of the detected e^+'s and e^-'s. The electron-positron asymmetry generated is the same for the K_L and K_S [11].

In this article we point out that a similar situation arises in the formation and decays of Majorana neutrinos. We adopt the wave function formalism to calculate the eigenstates of the Hamiltonian and their decay rates. In this formalism we can extend the region of our calculation to the case of very small mass differences using degenerate perturbation theory. We find that for small mass differences between the two generations the indirect $CP-$violation ($\epsilon-$type) produces a lepton asymmetry much larger than that of the $\epsilon'-$type.

We work in an extension of the standard model where we include one heavy right handed Majorana field per generation of light leptons (N_i, $i = 1,2,3$). These new fields are singlets with respect to the standard model. The lagrangian now contains a Majorana mass term and the Yukawa interactions of these fields with the light leptons,

$$\begin{aligned}\mathcal{L}_{int} &= \sum_i M_i[\overline{(N_{Ri})^c}N_{Ri} + \overline{N_{Ri}}(N_{Ri})^c] \\ &+ \sum_{\alpha,i} h^*_{\alpha i}\,\overline{N_{Ri}}\,\phi^\dagger\,\ell_{L\,\alpha} + \sum_{\alpha,i} h_{\alpha i}\,\overline{\ell_{L\,\alpha}}\phi\,N_{Ri} \\ &+ \sum_{\alpha,i} h^*_{\alpha i}\,\overline{(\ell_{L\,\alpha})^c}\,\phi\,(N_{Ri})^c + \sum_{\alpha,i} h_{\alpha i}\overline{(N_{Ri})^c}\,\phi^\dagger\,(\ell_{L\,\alpha})^c\end{aligned} \tag{1}$$

where $\phi^T = (-\overline{\phi^\circ}, \phi^-)$ is the higgs doublet of the standard model, which breaks the electroweak symmetry and gives mass to the fermions; $l_L\alpha$ are the light leptons, $h_{\alpha i}$ are the complex Yukawa couplings and α is the generation index. We have adopted the usual convention for charge conjugation : $N^c = C\overline{N^T}$. Without loss of generality we work in a basis in which the Majorana mass matrix is real and diagonal with eigenvalues M_i.

The states $|N_i>$ decay only into leptons, while the states $|N_i^c>$ decay only into antileptons (figure 1). For this reason the states $|N_i>$ and

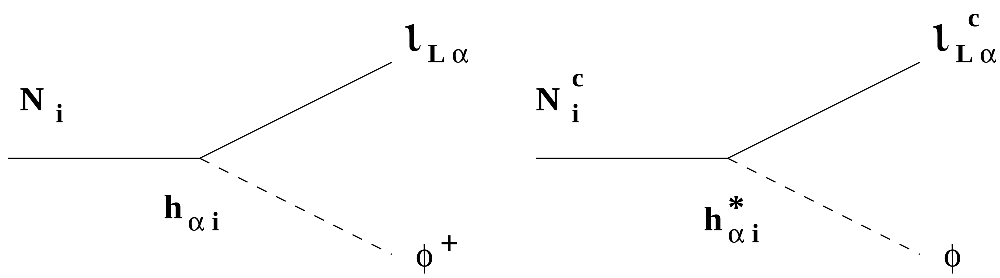

Figure 1: N_i and N_i^c decaying into $l_{L\alpha}$ and $(l_{L\alpha})^c$

$|N_i^c>$ have definite lepton numbers and are the appropriate states to describe $CP-$violation in the leptonic sector. They are analogous to the K° and $\overline{K^\circ}$ states. The idea is as follows: Through the presence of the Yukawa interactions we obtain one loop corrections to the mass matrix (figure 2), such that the corresponding mass eigenfunctions are no longer the $|N_i>$ and $|N_i^c>$ states, but a mixture of them. It is these physical eigenstates which evolve in time with a definite frequency. If they are shown to be asymmetric linear combinations of the $|N_i>$ and $|N_i^c>$'s then we have created a $CP-$asymmetric universe. By asymmetric linear combinations we mean that the $|N_i>$ and $|N_i^c>$'s enter with different complex phases into the decomposition of the eigenfunctions. The subsequent decay of these fields will produce the desired lepton asymmetry.

As a result even if we start with equal numbers of $|N_i>$ and $|N_i^c>$, they will evolve according to the asymmetric eigenstates of definite time developement. Since $|N_i>$ and $|N_i^c>$ carry different lepton numbers given by their interactions, this means that a lepton asymmetry is established through mixing before the fields actually decay. Herein lies the main difference between our model and the literature.

For the sake of simplicity we consider two generations of Majorana neutrinos, where the indices i and j take the values 1 and 2. We assume the hierarchy $M_2 > M_1$. In the basis $(|N_1^c > |N_2^c > |N_1 > |N_2 >)$ the effective Hamiltonian of this model can be written as,

$$\widehat{\mathcal{H}}^{(0)} = \begin{pmatrix} 0 & 0 & M_1 & 0 \\ 0 & 0 & 0 & M_2 \\ M_1 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 \end{pmatrix}. \tag{2}$$

Once we include the one loop diagram of figure 2, there is an additional contribution to the effective Hamiltonian, which introduces $CP-$violation in the mass matrix. We treat the one loop contributions as a small perturbation to the tree level Hamiltonian.

$$\widehat{\mathcal{H}}^{(1)} = \begin{pmatrix} 0 & 0 & H_{11}^{(1)} & H_{12}^{(1)} \\ 0 & 0 & H_{12}^{(1)} & H_{22}^{(1)} \\ H_{11}^{(1)} & \widetilde{H}_{12}^{(1)} & 0 & 0 \\ \widetilde{H}_{12}^{(1)} & H_{22}^{(1)} & 0 & 0 \end{pmatrix} \tag{3}$$

with

$$H_{ij}^{(1)} = H_{ji}^{(1)} = \left[M_i \sum_\alpha h_{\alpha i}^* h_{\alpha j} + M_j \sum_\alpha h_{\alpha i} h_{\alpha j}^* \right] (g_{\alpha ij}^{dis} - \frac{i}{2} g_{\alpha ij}^{ab}) \tag{4}$$

$$\widetilde{H}_{ij}^{(1)} = \widetilde{H}_{ji}^{(1)} = \left[M_i \sum_\alpha h_{\alpha i} h_{\alpha j}^* + M_j \sum_\alpha h_{\alpha i}^* h_{\alpha j} \right] (g_{\alpha ij}^{dis} - \frac{i}{2} g_{\alpha ij}^{ab}) \tag{5}$$

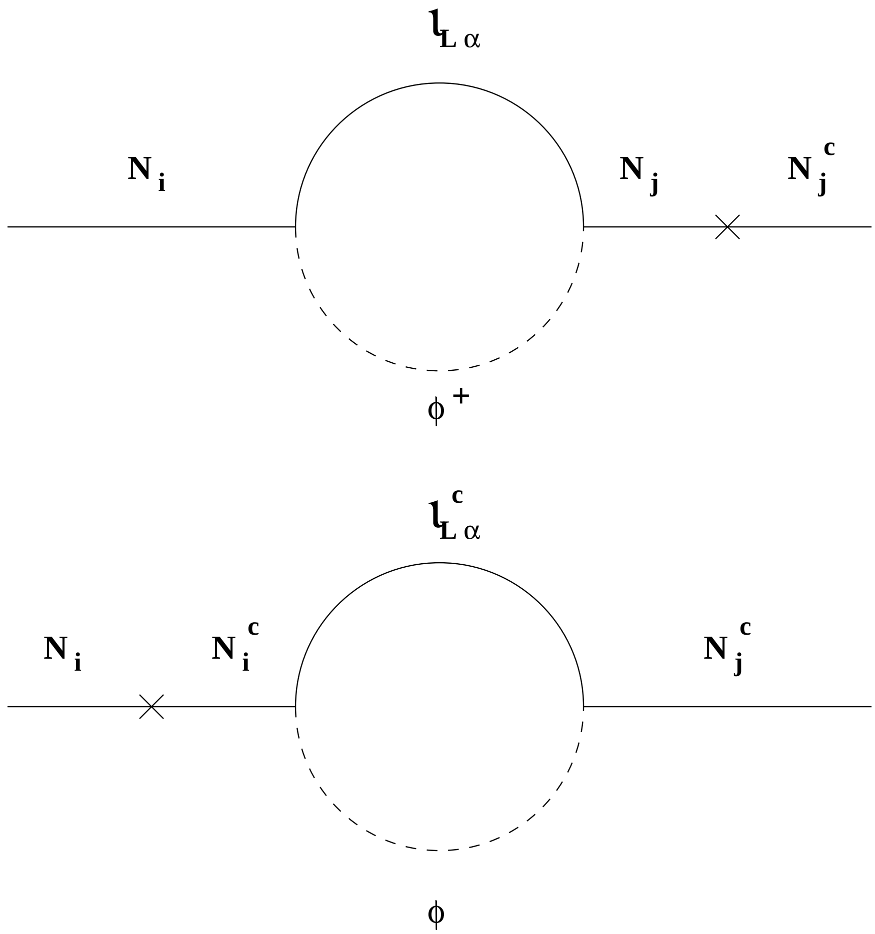

Figure 2: One loop contributions to the mass matrix

and

$$H_{ii}^{(1)} = \widetilde{H}_{ii}^{(1)} = \left[2M_i \sum_\alpha h_{\alpha i} h_{\alpha i}^*\right] (g_{\alpha ij}^{dis} - \frac{i}{2} g_{\alpha ij}^{ab}) \tag{6}$$

as can be easily read off from figure 2. The dispersive part $g_{\alpha ij}^{dis}$ can be absorbed in the wave function and mass renormalization. The absorbtive part $g_{\alpha ij}^{ab}$ of the loop integrals is given by,

$$g_{\alpha ij}^{ab} = \frac{1}{16\pi} \tag{7}$$

neglecting terms of order $O\left(m_\alpha^2/p^2\right)$, $O\left(m_\phi^2/p^2\right)$ with $p^2 \geq M_i^2$.

Employing ordinary first order perturbation theory the eigenfunctions of the effective Hamiltonian $\widehat{\mathcal{H}} = \widehat{\mathcal{H}}^{(0)} + \widehat{\mathcal{H}}^{(1)}$ are found to be

$$\begin{aligned}
|\Psi_1> &= \frac{1}{\sqrt{\mathcal{N}}}(|N_1> +\alpha_2|N_2> +|N_1^c> +\alpha_1|N_2^c>) \\
|\Psi_1'> &= \frac{1}{\sqrt{\mathcal{N}}}(|N_1> +\alpha_2|N_2> -|N_1^c> -\alpha_1|N_2^c>) \\
|\Psi_2> &= \frac{1}{\sqrt{\mathcal{N}}}(|N_2> -\alpha_1|N_1> +|N_2^c> -\alpha_2|N_1^c>) \\
|\Psi_2'> &= \frac{1}{\sqrt{\mathcal{N}}}(|N_2> -\alpha_1|N_1> -|N_2>^c +\alpha_2|N_1^c>)
\end{aligned} \tag{8}$$

with,

$$\alpha_1 = \frac{M_1 H_{12}^{(1)} + M_2 \widetilde{H^{(1)}}_{12}}{M_1^2 - M_2^2} \quad , \qquad \alpha_2 = \frac{M_1 \widetilde{H^{(1)}}_{12} + M_2 H_{12}^{(1)}}{M_1^2 - M_2^2}$$

where the normalization factor is $\mathcal{N} = 2 + |\alpha_1|^2 + |\alpha_2|^2$.

The states $|\Psi_1>$ and $|\Psi_1'>$ are eigenstates with mass eigenvalues $\pm(M_1 + H_{11}^{(1)})$. They are related by a chiral γ_5-transformation and correspond to the same physical state. The same holds for the states $|\Psi_2>$ and $|\Psi_2'>$ with eigenvalues $\pm(M_2 + H_{22}^{(1)})$. For the rest of our calculation we shall only consider $|\Psi_1>$ and $|\Psi_2>$.

The asymmetry parameter can be defined as,

$$\Delta = \sum_{i=1}^{2} \frac{\Gamma_{\Psi_i \to l} - \Gamma_{\Psi_i \to l^c}}{\Gamma_{\Psi_i \to l} + \Gamma_{\Psi_i \to l^c}} \tag{9}$$

which is a measure of the lepton asymmetry generated when the physical states $|\Psi_i>$ finally decay. This can be calculated using,

$$\begin{aligned}
\Gamma_{\Psi_1 \to l} &\propto \sum_\alpha |h_{\alpha 1} + \alpha_2 h_{\alpha 2}|^2 \\
&= \sum_\alpha \left[|h_{\alpha 1}|^2 + |\alpha_2|^2 |h_{\alpha 2}|^2 + 2\mathrm{Re}(\alpha_2 h^*_{\alpha 1} h_{\alpha 2}) \right] \\
\Gamma_{\Psi_1 \to l^c} &\propto \sum_\alpha |h^*_{\alpha 1} + \alpha_1 h^*_{\alpha 2}|^2 \\
&= \sum_\alpha \left[|h_{\alpha 1}|^2 + |\alpha_1|^2 |h_{\alpha 2}|^2 + 2\mathrm{Re}(\alpha_1 h_{\alpha 1} h^*_{\alpha 2}) \right] \\
\Gamma_{\Psi_2 \to l} &\propto \sum_\alpha |h_{\alpha 2} - \alpha_1 h_{\alpha 1}|^2 \\
&= \sum_\alpha \left[|h_{\alpha 2}|^2 + |\alpha_1|^2 |h_{\alpha 1}|^2 - 2\mathrm{Re}(\alpha_1 h_{\alpha 1} h^*_{\alpha 2}) \right] \\
\Gamma_{\Psi_2 \to l^c} &\propto \sum_\alpha |h^*_{\alpha 2} - \alpha_2 h^*_{\alpha 1}|^2 \\
&= \sum_\alpha \left[|h_{\alpha 2}|^2 + |\alpha_2|^2 |h_{\alpha 1}|^2 - 2\mathrm{Re}(\alpha_2 h^*_{\alpha 1} h_{\alpha 2}) \right]
\end{aligned} \tag{10}$$

In addition to the $CP-$violating contribution due to the mixing of the states $|N_i>$ and $|N_i^c>$, which we call δ, there is another contribution ϵ' coming from the direct $CP-$violation through the decays of $|N_i>$ and $|N_i^c>$,

$$\begin{aligned}
\Gamma_{N_i} &= \frac{1}{2}(1+\epsilon') \frac{1}{16\pi} \sum_\alpha |h_{\alpha i}|^2 M_i \\
\Gamma_{N_i^c} &= \frac{1}{2}(1-\epsilon') \frac{1}{16\pi} \sum_\alpha |h_{\alpha i}|^2 M_i
\end{aligned} \tag{11}$$

which has been discussed in the literature extensively [6, 7].

Then it is straightforward to show that the asymmetry parameter consists of the following two parts,

$$\Delta = \epsilon' + \delta \tag{12}$$

The new indirect $CP-$violation δ, which enters through the mass matrix, is given by,

$$\begin{aligned}\delta &= \mathrm{Re}\left[\sum_\alpha h^*_{\alpha 1} h_{\alpha 2} (\alpha_2 - \alpha_1^*)\right] \left(\frac{1}{\sum_\alpha |h_{\alpha 1}|^2} + \frac{1}{\sum_\alpha |h_{\alpha 2}|^2}\right) \\ &= 2\,\pi\, g^{ab} \mathcal{C} \frac{M_1 M_2}{M_2^2 - M_1^2}\end{aligned} \tag{13}$$

where,

$$\mathcal{C} = -\frac{1}{\pi} \mathrm{Im}[\sum_\alpha (h^*_{\alpha 1} h_{\alpha 2}) \sum_\beta h^*_{\beta 1} h_{\beta 2})] \left(\frac{1}{\sum_\alpha |h_{\alpha 1}|^2} + \frac{1}{\sum_\alpha |h_{\alpha 2}|^2}\right) \tag{14}$$

From this expression it is clear that this contribution becomes significant when the two mass eigenvalues are close to each other (figure 3). On the other hand the perturbation theory used for this expression is valid only for $|M_1 - M_2| \gg |H_{ij}^{(1)}|$ or $|\widetilde{H_{ij}^{(1)}}|$.

To find out the value of δ in the vicinity of $M_1 = M_2$ we now write $M_1 = M$ and $M_2 = M + \eta M$ and consider the case $M\eta \leq |H_{ij}|$ or equivalently $\eta \leq |\sum_\alpha h_{\alpha i} h^*_{\alpha j}|$. Then the total Hamiltonian,

$$\widehat{\mathcal{H}} = \begin{pmatrix} 0 & 0 & M + H_{11} & H_{12} \\ 0 & 0 & H_{12} & (M + \eta M) + H_{22} \\ M + H_{11} & H_{12} & 0 & 0 \\ H_{12} & (M + \eta M) + H_{22} & 0 & 0 \end{pmatrix} \tag{15}$$

will have the eigenvalues,

$$\begin{aligned} \Lambda^2 &\approx M^2(1+\eta) + M\,\{H_{11} + H_{22} \pm c\} \\ \text{or} \qquad \Lambda &\approx \pm\left[(M + \frac{1}{2}\eta M) + \frac{1}{2}\{H_{11} + H_{22} \pm c\}\right] \end{aligned} \tag{16}$$

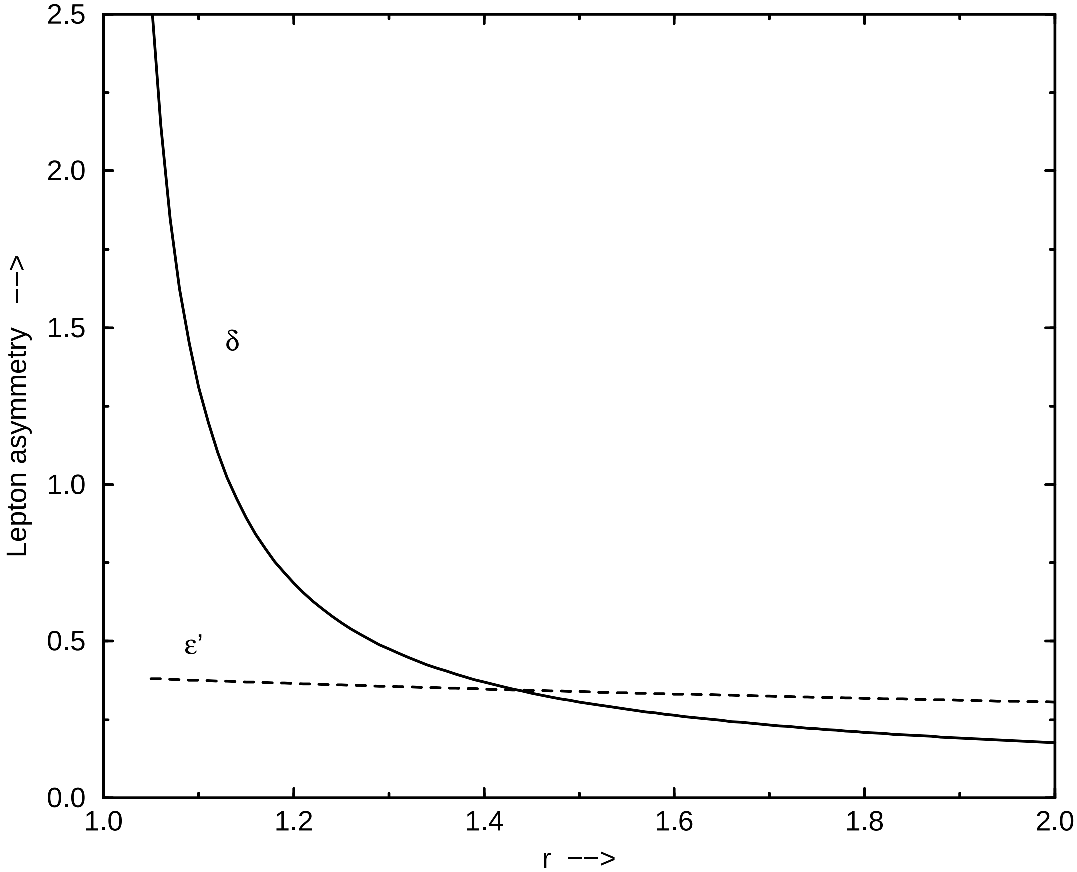

Figure 3: Comparison of the two $CP-$violating contributions to the lepton asymmetry in units of C, where $r = M_2^2/M_1^2$

with $c = \sqrt{[M\eta - (H_{11} - H_{22})]^2 + (H_{12} + \widetilde{H}_{12})^2}$, neglecting terms of order $M^2 |\sum_\alpha h_{\alpha i} h^*_{\alpha j}|^3$. The eigenvectors are now given by,

$$\begin{aligned} |\Psi_1> &= \frac{1}{\sqrt{2(1+|\alpha|^2)}}(|N_1> + \alpha|N_2> + |N_1^c> + \alpha|N_2^c>) \\ |\Psi_2> &= \frac{1}{\sqrt{2(1+|\beta|^2)}}(|N_2> + \beta|N_1> + |N_2^c> + \beta|N_1^c>) \end{aligned} \tag{17}$$

with

$$\alpha = \frac{H_{12} + \widetilde{H}_{12}}{H_{11} - H_{22} - \eta M + c} \quad , \quad \beta = \frac{H_{11} - H_{22} - \eta M - c}{H_{12} + \widetilde{H}_{12}},$$

Note that the states $|\psi_1>$ and $|\psi_2>$ are not $CP-$symmetric, because under $CP-$transformations $\alpha|N_i>$ transforms into $\alpha^*|N_i^c>$. With this we can now calculate the lepton asymmetry for the degenerate scenario defining the asymmetry parameter as before. It is now given by,

$$\delta = \mathcal{C} \frac{\pi\, g^{ab} \eta}{\eta^2 + (g^{ab})^2 \mathrm{Re}^2(\sum_\alpha h^*_{\alpha 1} h_{\alpha 2})} \tag{18}$$

This expression vanishes for $\eta \to 0$ as expected, because there is no detectable mixing between two identical particles (figure 4). The matching of the two solutions occurs at $\eta \approx \sum_\alpha h^*_{\alpha 1} h_{\alpha 2}/5$ and thus we obtained an asymmetry for small and large mass differences. This is shown in figures 3 and 4. For $\eta \approx |\sum_\alpha h_{\alpha i} h^*_{\alpha j}|$ a big enhancement of the asymmetry parameter δ, by several orders of magnitude, is achieved. The enhancement factor is roughly $1/\eta$. For large values of η the two distinct contributions ϵ' and δ become of the same order.

The dynamical evolution of the lepton number in the universe is governed by the Boltzmann equations. This is the appropriate tool to describe any deviation from thermal equilibrium. The framework and the derivation of the Boltzmann equation is reviewed in [2]; we adopt the same notation. We make the approximations of kinetic equilibrium and Maxwell-Boltzmann

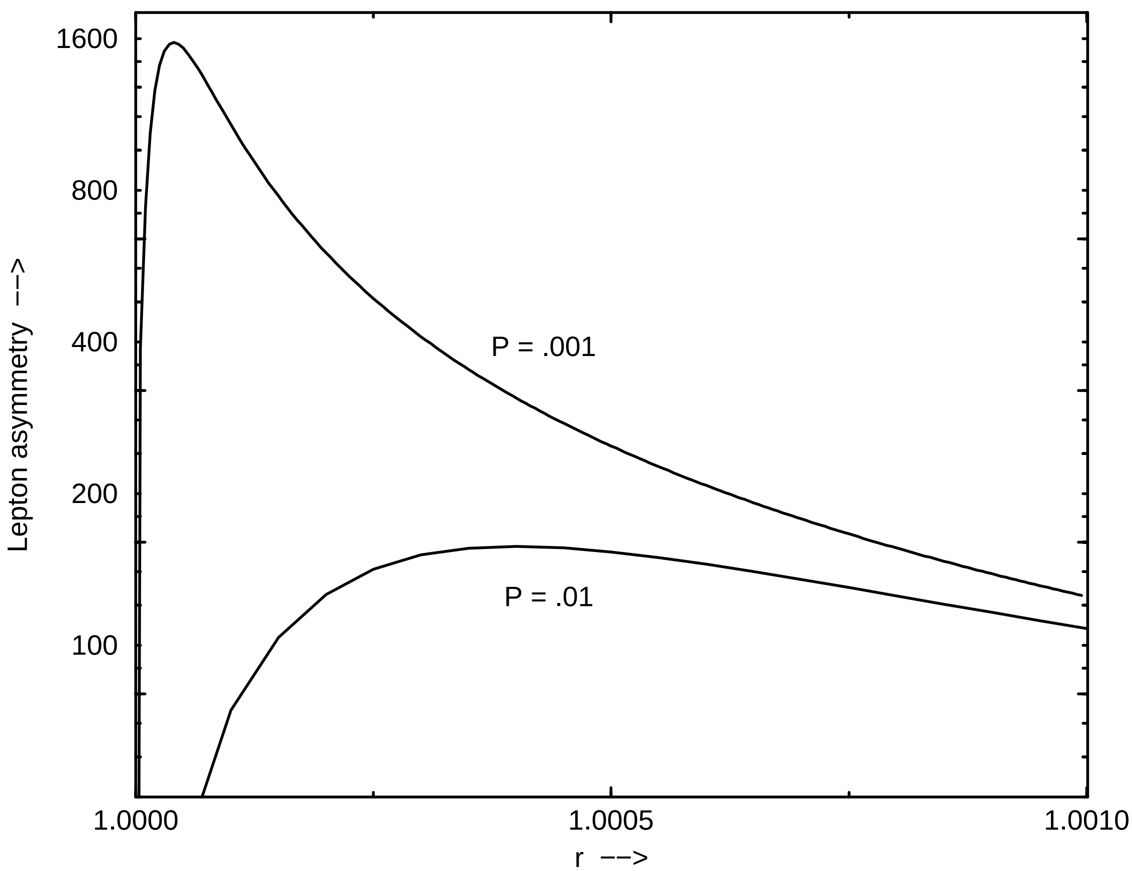

Figure 4: Lepton asymmetry in units of $\mathcal{C}$ generated for a small mass difference, where $\mathrm{P} = \sum_\alpha h^*_{\alpha 1} h_{\alpha 1}$ and $\mathrm{r} = M_2^2/M_1^2$.

statistics. The out-of-equilibrium conditions occur, when the temperature T drops below the mass scale $M_{\psi_1} = M_1 + H_{11}^{(1)}$, where the inverse decay is effectively frozen out.

The density of the lepton number asymmetry $n_L = n_l - n_{l^c}$ for the left-handed leptons can be shown to evolve in time according to the equation:

$$\frac{\mathrm{d}n_L}{\mathrm{d}t} + 3Hn_L = (\epsilon' + \delta_1)\Gamma_{\psi_1}^{th}[n_{\psi_1} - n_{\psi_1}^{eq}] - \left(\frac{n_L}{n_\gamma}\right) n_{\psi_1}^{eq}\Gamma_{\psi_1}^{th} - 2n_\gamma n_L\langle\sigma|v|\rangle \quad (19)$$

where δ_1 is the contribution proportional to $1/\sum_\alpha |h_{\alpha 1}|^2$ in the expression for δ. The second term on the left side comes from the expansion of the universe, where H is the Hubble constant. $\Gamma_{\psi 1}^{th}$ is the thermally averaged decay rate of the $|\psi_1\rangle$ state, n_γ is the usual photon density and the term $\langle\sigma|v|\rangle$ describes the thermally averaged cross-section of $l + \phi^\dagger \longleftrightarrow l^c + \phi$ scattering. We note that the first term of the right side of this equation describes the creation of lepton number and is proportional to $(\epsilon' + \delta_1)$, while the last two terms are responsible for any depletion of lepton number and are coming from the inverse decay and the scattering respectively. The density of the ψ_1 state satisfies the Boltzmann equation,

$$\frac{\mathrm{d}n_{\psi_1}}{\mathrm{d}t} + 3Hn_{\psi_1} = -\Gamma_{\psi_1}^{th}(n_{\psi_1} - n_{\psi_1}^{eq}) \quad (20)$$

In order to find a solution to this set of coupled differential equations it turns out to be useful to transform to new variables. We introduce the dimensionless variable $x = M_{\psi_1}/T$, a particle density per entropy density $Y_i = n_i/s$ and make use of the relation $t = x^2/2H(x=1)$.

In addition we define the parameter $K = \Gamma_i(x=1)/H(x=1)$ which is a measure of the deviation from equilibrium. For $K \ll 1$ at $T \approx M_{\psi_1}$ we are far from thermal equilibrium so that both inverse decays and $2 \leftrightarrow 2$ CP non-conserving scattering processes are not important and can be safely ignored. With these simplifications and the above redefinitions the Boltzmann equations effectively read :

$$\begin{aligned} \frac{\mathrm{d}Y_L}{\mathrm{d}x} &= (Y_{\psi_1} - Y_{\psi_1}^{eq})(\epsilon' + \delta_1)Kx^2 \\ \frac{\mathrm{d}Y_{\psi_1}}{\mathrm{d}x} &= -(Y_{\psi_1} - Y_{\psi_1}^{eq})Kx^2 \end{aligned} \tag{21}$$

For very large times the solution for Y_L has an asymptotic value which is approximately given by

$$Y_L = \frac{n_L}{s} = \frac{1}{g_*}(\epsilon' + \delta_1) \tag{22}$$

where g_* denotes the total number of effectively massless degrees of freedom and is of the order of $O(10^2)$ for all usual extensions of the standard model. The lepton asymmetry thus generated will then be converted to the baryon asymmetry of the universe during the electroweak phase transition [12] and is approximately given by $n_B \approx \frac{1}{3}n_L$. So we have demonstrated that the baryon asymmetry is proportional to $(\epsilon' + \delta_1)$. This new contribution was shown to be at least of the same order as the ϵ' (as shown in figure 3) and for small values of r, δ exceeds ϵ' by several orders of magnitude (figure 4).

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