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Optimal Design of Composite Structures by Advanced Mixed Integer Nonlinear Optimization

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Optimal Design of Composite Structures by Advanced Mixed Integer Nonlinear Optimization Techniques

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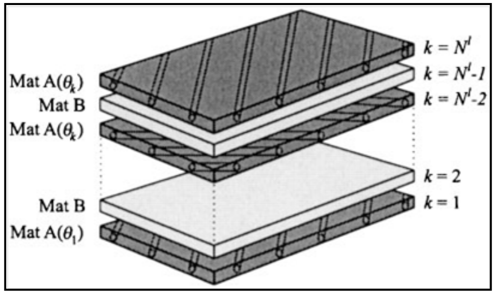
Composite Materials

❑ Composite Material



=

❑ Fibres

- Glass Fibres
- Carbon Fibres
- Cellulose Fibres
- and more …

Glass fibres

\+

❑ Matrix

- Polymers
- Metals
- Ceramics

Polyester matrix

❑ We consider **discrete multi material minimum compliance** problems.

❑ We are dealing with plies that have **constant** and **equal thicknesses** and the **number of layers** is **constant** and non varying through the entire structure.

❑ Our aim is to solve the considered problems to **global optimality** by modern special purpose **methods** and **heuristics**.

Problem Formulation – Minimum Compliance

minimize Compliance

subject to Equilibrium equations
Mass constraint
Manufacturing constraints
Single material selection per layer and element (DMO)

$$\begin{aligned}
\underset{x,u_1,\ldots,u_L}{\text{minimize}} \quad & \sum_{l=1}^{L} w_l f_l^T u_l \\
\text{subject to} \quad & K(x)u_l - f_l = 0, \qquad l = 1,\ldots,L \\
& m(x) \leq m_c, \\
& Ax \leq b, \\
& \sum_{i=1}^{N^i} x_{ijk} = 1, \qquad \forall (j,k) \\
& x_{ijk} \in \{0,1\}, \qquad \forall (i,j,k)
\end{aligned}$$

Problem Formulation – Minimum Weight

minimize Weight

subject to Equilibrium equations
Compliance Constraint
Manufacturing constraints
Single material selection per layer and element (DMO)

$$\begin{aligned}
\underset{x,u_1,\ldots,u_L}{\text{minimize}} \quad & m(x) \\
\text{subject to} \quad & K(x)u_l - f_l = 0, \qquad \forall l \\
& f_l^T u_l \leq c_l^{\max}, \qquad \forall l \\
& Ax \leq b, \\
& \sum_{i=1}^{N^i} x_{ijk} = 1, \qquad \forall (j,k) \\
& x_{ijk} \in \{0,1\}, \qquad \forall (i,j,k)
\end{aligned}$$

Problem Formulation – Minimum Compliance

minimize Compliance

subject to Equilibrium equations
Mass constraint
Manufacturing constraints
Single material selection per layer and element (DMO)

$$
\begin{aligned}
\underset{x,u_1,\ldots,u_L}{\text{minimize}} \quad & \sum_{l=1}^{L} w_l f_l^T u_l \\
\text{subject to} \quad & K(x)u_l - f_l = 0, \qquad l = 1,\ldots,L \\
& m(x) \leq m_c, \\
& Ax \leq b, \\
& \sum_{i=1}^{N^i} x_{ijk} = 1, \qquad \forall(j,k) \\
& x_{ijk} \in \{0,1\}, \qquad \forall(i,j,k)
\end{aligned}
$$

- Our models closely follow the **Discrete Material Optimization** (DMO) parameterization scheme.
- The design variables associate candidate materials from a given set to each layer and every finite element.
- All **materials** behave **linearly elastic** and the structural behavior of the laminate is described using an **equivalent single layer theory** (ESL).
- The finite element formulations are based on the **first order shear deformation theory** (FSDT).

Reformulation – Nested Analysis and Design

minimize	Compliance
subject to	Mass constraint
	Manufacturing constraints
	Single material selection per layer and element (DMO)

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \sum_{l=1}^{L} w_l f_l^T K(x)^{-1} f_l \qquad \leftarrow u_l = K(x)^{-1} f_l \\ \text{subject to} \quad & m(x) \le m_c, \\ & Ax \le b, \\ & \sum_{i=1}^{n} x_{ijk} = 1, \qquad \forall (j,k) \\ & x_{ijk} \in \{0,1\}, \qquad \forall (i,j,k) \end{aligned}$$

- (A1) The topology of the structure does not change and the stiffness matrix is **symmetric** and **positive definite**. The stiffness matrix is **linear** (or affine) in the design variables

$$K(x) = \sum_{ijk} x_{ijk} K_{ijk} = \sum_{ijk} x_{ijk} B_j^T C_{ik} B_j = \sum_j B_j^T \left(\sum_{ik} x_{ijk} C_{ik} \right) B_j$$

- (A2) The external loads $f_l \in R^{n_d} \setminus \{0\}$. Furthermore, we assume that the load vectors are **independent** of the design variables.
- (A3) The mass limit satisfies $\sum_{j=1}^{N^j} \sum_{k=1}^{N^k} t_k a_j \min_i \{\rho_i\} < m_c < \sum_{j=1}^{N^j} \sum_{k=1}^{N^k} t_k a_j \max_i \{\rho_i\}$
- (A4) Throughout the weighting factors $w_l \ge 0$ of each load case and satisfy $\sum_{l=1}^{L} w_l = 1$

Continuous Relaxation – Interior Point Method

minimize Compliance

subject to Mass constraint
Manufacturing constraints
Single material selection per layer and element (DMO)

$$
\begin{aligned}
\underset{x}{\text{minimize}} \quad & \sum_{l=1}^{L} w_l f_l^T K(x)^{-1} f_l \\
\text{subject to} \quad & m(x) \leq m_c, \\
& Ax \leq b, \\
& \sum_{i=1}^{N^i} x_{ijk} = 1, && \forall (j,k) \\
& \underline{x_{ijk} \geq 0,} && \forall (i,j,k)
\end{aligned}
$$

- By **relaxing** the **integer constraints** on the design variables we get the **continuous relaxation** of the minimum compliance problem.
- The continuous relaxation is solved by a **primal-dual interior point method**.
- The continuous problem has a **non empty feasible set**.
- The objective function is **bounded** from **below** by zero.
- There is at least one optimal solution of the continuous problem (Weierstrass theorem).
- Finding a KKT-point to the continuous problem assures **global optimality**.

Heuristic for Minimum Compliance Problems

minimize Distance to 0-1 design

subject to Mass constraint

Manufacturing constraints

Single material selection per layer and element (DMO)

$$
\begin{aligned}
\underset{x}{\text{minimize}} \quad & \|\overline{x}^0 - x\|_1 \\
\text{subject to} \quad & m(x) \leq m_c, \\
& Ax \leq b, \\
& \sum_{i=1}^{N^i} x_{ijk} = 1, \qquad \forall (j,k) \\
& x_{ijk} \in \{0,1\}, \qquad \forall (i,j,k)
\end{aligned}
$$

- Smart **rounding** of the optimal solution of the continuous relaxation.
- The method is guaranteed to find a **feasible 0-1 design**.
- There are no guarantees on the quality of the obtained design.
- The 0-1 problem has a **non-empty feasible set**.
- There exists at least one optimal solution of the 0-1 problem.

Heuristic for the Minimum Weight Problem

minimize Distance to 0-1 design

subject to Compliance constraint

Manufacturing constraints

Single material selection per layer and element (DMO)

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \|x - \overline{x}^{m-1}\|_1 \\ \text{subject to} \quad & c_l(\overline{x}^n) + \nabla c_l(\overline{x}^n)(x - \overline{x}^n) \leq c_l^{\max}, \quad \forall n = 0, \dots, m-1, \quad \forall l \\ & (\overline{x}^n - \hat{x}^n)^T (x - \overline{x}^n) \geq 0, \quad \forall n = 1, \dots, m-1 \\ & Ax \leq b, \\ & \sum_{i=1}^{N^i} x_{ijk} = 1, \quad \forall (j,k) \\ & x_{ijk} \in \{0,1\}, \quad \forall (i,j,k) \end{aligned}$$

minimize Distance to 0-1 design

subject to Compliance constraint

Manufacturing constraints

Single material selection per layer and element (DMO)

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \|x - \hat{x}^m\|_2^2 \\ \text{subject to} \quad & c_l(x) \leq c_l^{\max}, \quad \forall l \\ & Ax \leq b, \\ & \sum_{i=1}^{N_i} x_{ijk} = 1, \quad \forall (j,k) \\ & x_{ijk} \geq 0, \quad \forall (i,j,k) \end{aligned}$$

Global Optimization – Outer Approximation

minimize	Compliance
subject to	Compliance constraint
	Mass constraint
	Manufacturing constraints
	Single material selection per layer and element (DMO)

$$
\begin{aligned}
\underset{\eta, x}{\text{minimize}} \quad & \eta \\
\text{subject to} \quad & c(x^p) + (\nabla c(x^p))^T (x - x^p) - \eta \leq 0, \qquad p = 1, \ldots, P \\
& m(x) \leq m_c, \\
& Ax \leq b, \\
& \sum_{i=1}^{N^i} x_{ijk} = 1, \qquad \forall (j,k) \\
& x_{ijk} \in \{0,1\}, \qquad \forall (i,j,k) \\
& \eta \geq 0
\end{aligned}
$$

- **Approximate** the non-linear objective and constraint functions **with linear functions**.
- Solve a **sequence** of **linear mixed 0-1 problems**. Guaranteed to converge to **global minimizer**.
- Since the function $c(x)$ is convex the **linearization constraints** represent **supporting hyperplanes**.
- The **feasible set** of the 0-1 problem is **non-empty**.

Global Optimization – Outer Approximation

Algorithm: Outer Approximation for solving the minimum compliance problem.

Solve the continuous relaxation. Denote the optimal solution $\overline{x}^0$

Generate a compliance inequality on $\overline{x}^0$, i.e.

$$c(\overline{x}^0) + (\nabla c(\overline{x}^0))^T (x - \overline{x}^0) - \eta \leq 0$$

Solve the rounding heuristic problem. Denote the optimal solution $\hat{x}$

Set the lower bound $lb = z_R$

Compute the displacement vectors $u_l = K(\hat{x})^{-1} f_l$. Set the upper bound

$$ub = \sum_l w_l f_l^T u_l$$

Set $p = 1$

while $(ub - lb)/ub > \epsilon_0$ **do**

Solve the linear mixed 0-1 problem. Denote the optimal design $\hat{x}^p$

Generate compliance inequalities for all solutions of the master problem.

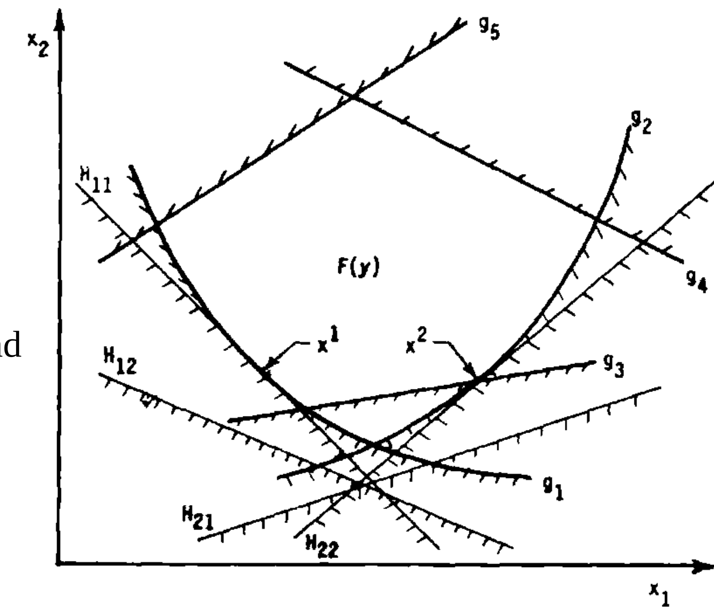
Update the lower bound $lb \longleftarrow \max\{lb, z_{OA}\}$

For all solutions of the master problem, compute the corresponding displacement vector.

Update the upper bound $ub \longleftarrow \min\{ub, \sum_{l=1}^{L} w_l f_l^T u_l\}$

Set $p \longleftarrow p + 1$

end



Gap improvement Heuristic/Method – Feasibility Pump

minimize Distance to 0-1 design

subject to Compliance constraint

Mass Constraint

Manufacturing constraints

Single material selection per layer and element (DMO)

$$
\begin{aligned}
\underset{x}{\text{minimize}} \quad & \|x - \overline{x}^{m-1}\|_1 \\
\text{subject to} \quad & c(\overline{x}^n) + (\nabla c(\overline{x}^n))^T (x - \overline{x}^n) \leq c^{\max}, && \forall n = 0, \ldots, m-1 \\
& (\overline{x}^n - \hat{x}^n)^T (x - \overline{x}^n) \geq 0, && \forall n = 1, \ldots, m-1 \\
& m(x) \leq m_c, \\
& Ax \leq b, \\
& \sum_{i=1}^{N^i} x_{ijk} = 1, && \forall (j,k) \\
& x_{ijk} \in \{0,1\}, && \forall (i,j,k)
\end{aligned}
$$

minimize Distance to 0-1 design

subject to Compliance constraint

Mass Constraint

Manufacturing constraints

Single material selection per layer and element (DMO)

$$
\begin{aligned}
\underset{x}{\text{minimize}} \quad & \|x - \hat{x}^m\|_2^2 \\
\text{subject to} \quad & c(x) \leq c^{\max}, \\
& m(x) \leq m_c, \\
& Ax \leq b, \\
& \sum_{i=1}^{N^i} x_{ijk} = 1, \quad \forall (j,k) \\
& x_{ijk} \geq 0, \quad \forall (i,j,k)
\end{aligned}
$$

Gap improvement Heuristic/Method

Algorithm: Gap improvement Heuristic/Method for the minimum compliance problem.

Solve the continuous relaxation. Denote the optimal solution $\overline{x}^0$

Generate a compliance inequality on $\overline{x}^0$, i.e. $c(\overline{x}^0) + (\nabla c(\overline{x}^0))^T(x - \overline{x}^0) - c^{\max} \leq 0$

Solve the rounding heuristic problem. Denote the optimal solution $\hat{x}$

Set the lower bound $lb = z_R$

Compute the displacement vectors $u_l = K(\hat{x})^{-1} f_l$. Set the upper bound $ub = \sum_l w_l f_l^T u_l$

Set the target value $c^{\max} = (ub + lb)/2$

while $(ub - lb)/ub > \epsilon_0$ **do**

Set $p = 1$

while $c(\hat{x}^p) > c^{\max}$ **do**

Attempt to solve the 0-1 problem.

If *it is infeasible* **then**

Update the lower bound $lb \leftarrow c^{\max}$

Update the target value $c^{\max} \longleftarrow (ub + lb)/2$

else

Denote the optimal solution $\hat{x}^p$

Generate compliance inequalities on the solutions from the 0-1 problem.

Solve the nonlinear relaxation. Denote the optimal solution $\overline{x}^p$

Generate a compliance inequality on $\overline{x}^p$, i.e. $c(\overline{x}^p) + (\nabla c(\overline{x}^p))^T(x - \overline{x}^p) - c^{\max} \leq 0$

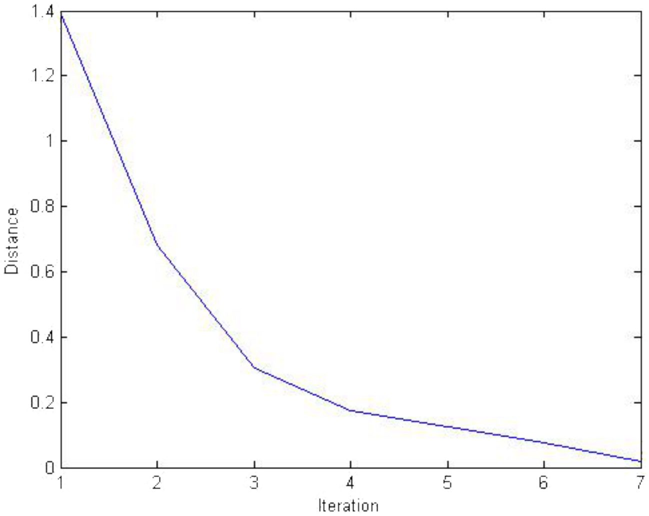
Set $p \longleftarrow p + 1$

end

end

Update the upper bound $ub \leftarrow c(\hat{x}^p)$. Update the target value $c^{\max} \longleftarrow (ub + lb)/2$

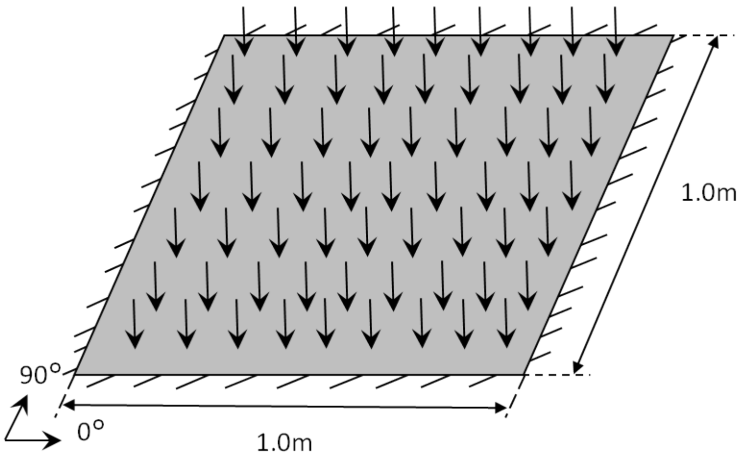
end



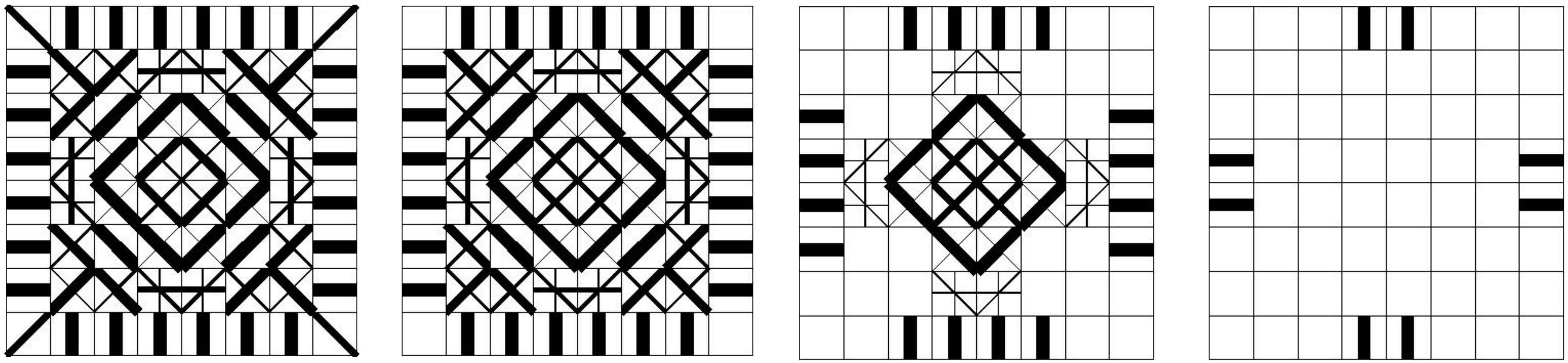
Numerical Example – Layered Clamped Plate

Problem	Elements	DOF	Variables		Foam	Orthotropic
P1G1	4x4	405	640	**Ex [GPa]**	0.065	34.0
P1G2	8x8	1445	2560	**Gxy [GPa]**	-	9.0
P1H1	16x16	5445	10240	**Major Poisson's ratio**	0.47	0.29
P1H2	32x32	21125	40960	**Density [kg/m³]**	200.0	1910.0

- Distributed Out of Plane (Static) Loading
- Clamped plate 1.0m x 1.0m x 0.005m
- 8 layers of equal thickness
- 5 candidate materials
    - Glass fiber reinforced epoxy { -45º, 0º, 45º, 90º }
    - Isotropic polymeric foam (allowable mass is 49.4[kg]
- Q9 Plate Elements

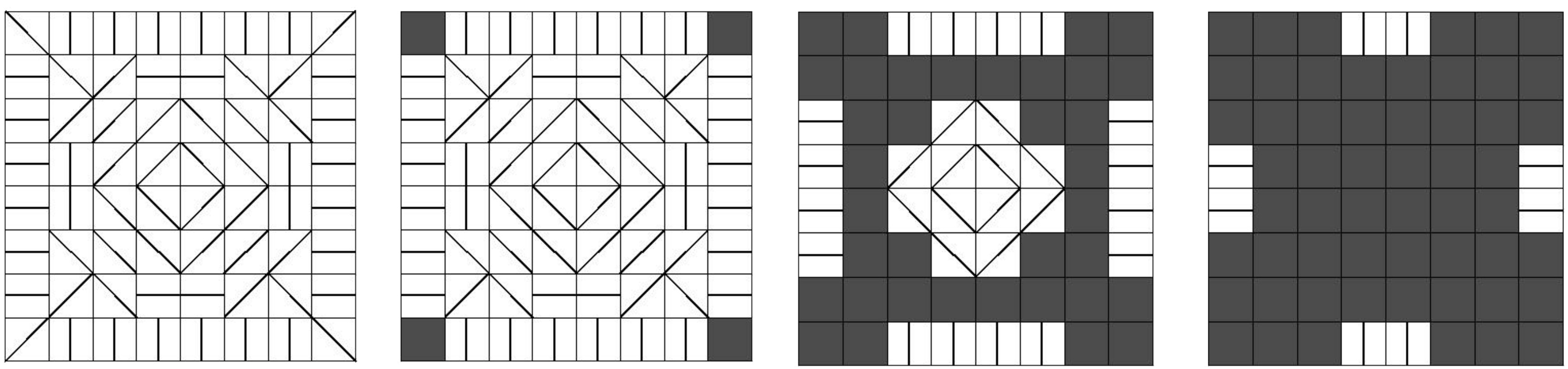


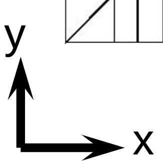
Continuous Relaxation



Layer 1 *Layer 2* *Layer 3* *Layer 4*

Rounding Heuristic – Relative Optimality Gap of 2.8%





Layer 1 *Layer 2* *Layer 3* *Layer 4*

Numerical Example – Layered Clamped Plate

Numerical Results with the Heuristics

Problem	Objective	Elements	DOF	Variables	Itns I.P.	Itns Heuristic	Time [h:m:s]		Bounds		Gap (%)
							I.P.	Heuristic	Lower	Upper	
P1H1	Compliance	16x16	5445	10240	20	1	00:01:54	00:00:02	1.797	1.933	7.0
P1H1	Weight	16x16	5445	10240	21	17	00:02:03	00:22:43	42.560	43.428	2.0
P1H2	Compliance	32x32	21125	40960	22	1	00:15:07	00:00:15	1.787	1.923	7.1
P1H2	Weight	32x32	21125	40960	20	10	00:24:07	00:58:46	37.803	38.920	2.9

- The obtained results showcase the **excellent convergence properties** and the ability of the primal-dual interior point method to react swiftly to changes of scale of our problems.

- The method managed to converge in 20 to 22 iterations in all the examined cases.

Numerical Example – Layered Clamped Plate

Numerical Results with the gap improvement method

Problem	Objective	Elements	DOF	Variables	Time [h:m:s]	Itns	Bounds		Gap (%)
							Lower	Upper	
P1H1	Compliance	16x16	5445	10240	00:10:12	2	1.797	1.810	<1%
P1H1	Weight	16x16	5445	10240	0:21:51	1	42.560	42.994	<1%
P1H2	Compliance	32x32	21125	40960	15:32:12	3	1.787	1.804	<1%
P1H2	Weight	32x32	21125	40960	12:25:45	2	37.803	38.082	<1%

Numerical Results with Outer Approximation

Problem	Objective	Elements	DOF	Variables	Time [h:m:s]	O.A.		Bounds		Gap (%)	
						Itns	Cuts	Lower	Upper	Heuristic	O.A.
P1G1	Compliance	4x4	405	640	00:00:12	1	4	1.651	1.654	2.8	<1%
P1G1	Weight	4x4	405	640						<1%	
P1G2	Compliance	8x8	1445	2560	00:10:00	50	94	1.817	1.835	49.1	<1%
P1G2	Weight	8x8	1445	2560	00:23:48	111	352	37.484	37.484	2.9	<1%

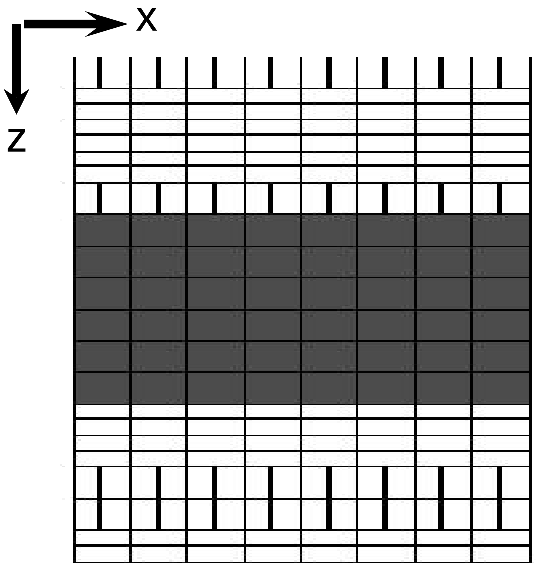
- The gap improvement algorithm was used as a **global optimization method** and was able to solve all the considered problem instances to **global optimality**.

- **Outer approximation** is able to solve the considered problem only for **small scale** instances.

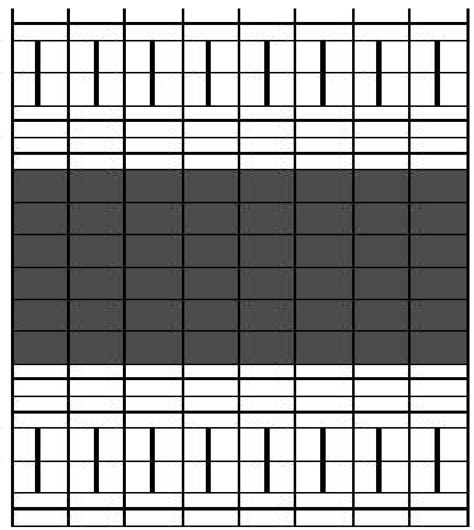
Ongoing Work – Manufacturing Constraints

- Articles covering optimal design of composite structures under manufacturing constraints are scarce in the literature and generally only (small) part of a structure and small-scale problems are considered.

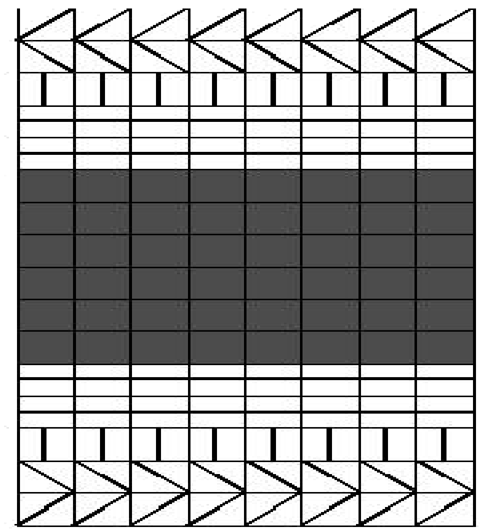
- It is common practice to divide the structure into panels that may be designed independently, and consider manufacturing constraints **in plane** and **through the thickness** of the composite.



Layer Wise Constant Fiber Orientation



Contiguity Constraints



Damage Tolerance Requirements

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THANK YOU FOR YOUR ATTENTION