ORIGINAL ARTICLE

The hierarchical stability of the seven known large size ratio triple asteroids using the empirical stability parameters

Xiaodong Liu · Hexi Baoyin · Franck Marchis

Abstract In this study, the hierarchical stability of the seven known large size ratio triple asteroids is investigated. The effect of the solar gravity and primary's J_2 are considered. The force function is expanded in terms of mass ratios based on the Hill's approximation and the large size ratio property. The empirical stability parameters are used to examine the hierarchical stability of the triple asteroids. It is found that the all the known large size ratio triple asteroid systems are hierarchically stable. This study provides useful information for future evolutions of the triple asteroids.

1 Introduction

The triple asteroids are newly found systems in the Solar System. The system (87) Sylvia is the first known triple asteroid system to be confirmed in 2004, for which the outer satellite Romulus was discovered in 2001 using Keck (Brown and Margot 2001; Margot and Brown 2001) and was also detected in HST images (Storrs et al. 2001), and the inner satellite Remus was discovered in 2004 using adaptive optics system on the Very Large Telescope (Marchis et al. 2005). Recently, Fang et al. (2012) derived the masses, orbits and primary's oblateness of (87) Sylvia system, and examined the short-term

X. Liu · H. Baoyin (✉)
School of Aerospace, Tsinghua University, Beijing 100084, China
e-mail: baoyin@tsinghua.edu.cn

F. Marchis
Carl Sagan Center, SETI Institute, 189 Bernardo Avenue,
Mountain View, CA 94043, USA

and long-term stability of the orbits of two satellites numerically. At the time of writing this paper, nine triple asteroids (including two Kuiper Belt objects) have been identified, namely, (45) Eugenia (Marchis et al. 2010; Beauvalet et al. 2011), (87) Sylvia (Marchis et al. 2005; Fang et al. 2012), (93) Minerva (Marchis et al. 2011, 2013), (216) Kleopatra (Descamps et al. 2011), (136617) 1994 CC (Fang et al. 2011; Brozović et al. 2011), (153591) 2001 SN263 (Fang et al. 2011), (136108) Haumea (Ragozzine and Brown 2009), (47171) 1999 TC$_{36}$ (Benecchi et al. 2010), and (3749) Balam (Marchis et al. 2008, 2012). The first seven are large size ratio triple asteroids, while the last two triple asteroids have their central components of a comparable size and orbiting very close to each other.

In connection with these recently discovered triple asteroids, the hierarchical stability of the large size ratio triple asteroid system including the effect of the solar gravity and primary's J_2 is analyzed in this study. The system is said to be hierarchically stable if its hierarchical configuration is not altered over a time scale that is much longer than the basic periods of the system (Milani and Nobili 1983; Walker and Roy 1983). Thus, either orbit crossing or escape of the bodies from the system is not possible. Although the four-body that consists of the triple asteroids and the Sun is non-integrable, the hierarchical stability can be examined using the Walker-Roy empirical stability criteria (Walker 1983a; Walker and Roy 1983). In this study, only the seven large size ratio triple asteroids are concerned because it is convenient to expand the force function in terms of mass ratios. The effect of the primary's J_2 is added to the traditional empirical stability parameters. In previous studies, numerical simulations were performed to investigate orbital stability of (87) Sylvia (Winter et al. 2009; Frouard and Compère 2012; Fang et al. 2012) and stability regions around (153591) 2001 SN263 (Araujo et al. 2012).

For clarity, this paper is structured as follows: in Sect. 2, the model of the large size ratio triple asteroid system involving the Sun in the four-body problem is described, and the force function is expanded in terms of mass ratios. In Sect. 3, the empirical stability parameters are used to examine the hierarchical stability of the triple asteroids. Finally, our conclusions are presented in Sect. 4.

2 Model of the large size ratio triple asteroid system involving the solar gravity and primary's J_2 in the four-body problem

The large size ratio triple asteroids involving the Sun in the four-body problem consists of a nonspherical primary with mass m_1, two small satellites (also known as moonlets) with masses m_2 and m_3, and the Sun with mass m_4. The simple four-body hierarchy instead of the double hierarchy is adopted here (Milani and Nobili 1983; Walker 1983a; Walker and Roy 1983). In other words, these four bodies are arranged that the body m_i ($i = 2, 3, 4$) is in orbit about the barycenter of all of its inner bodies, i.e., $m_1, \ldots, m_{i-1}$.

Hill's approximation is used here since the triple asteroids are close to each other, their center of mass is far away from the Sun, and the mass of the Sun dominates the four-body system (Hamilton and Krivov 1997; Scheeres 1998). Additionally, for the large size ratio triple asteroids, the mass of the primary is much larger than its moonlets, i.e. $m_1 \gg m_2, m_3$. Based on the previous study (cf. Walker 1983a, the equations before Eq. (17)) and adding the effect of J_2, the equations of motion in the expansion of the mass ratios are written as

$$\ddot{\boldsymbol{\rho}}_i = GM_i \nabla_i \left\{ \frac{1}{\rho_i} \left[1 - \frac{J_2 R_e^2 P_2(\sin\varphi_i)}{\rho_i^2} \right. \right.$$
$$\left. \left. + \sum_{k=1}^{i-1} \frac{m_k}{M_{i-1}} \alpha_{ki}^2 S'_{ki} + \sum_{l=i+1}^{n} \frac{m_l}{M_{i-1}} \alpha_{il}^3 S'_{il} \right] \right\}, \quad (1)$$

where $\boldsymbol{\rho}_i$ is the vector from M_{i-1} to m_i; G is the gravitational constant; $M_i = \sum_{j=1}^{i} m_j$, located at the barycenter of the masses $m_1, \ldots, m_i$; $\nabla_i = \frac{\partial}{\partial \boldsymbol{\rho}_i}$; $\rho_i = |\boldsymbol{\rho}_i|$; $\alpha_{ij} = \rho_i/\rho_j$; $S'_{ij} = \sum_{r=0}^{\infty} \alpha_{ij}^r P_{r+2}(C_{ij})$; $C_{ij} = \boldsymbol{\rho}_i \cdot \boldsymbol{\rho}_j / \rho_i \rho_j$; P_r is the Legendre function of degree r; J_2 is the primary's oblateness coefficient; R_e is the primary's reference radius; and φ_i is the latitude of m_i in the primary-fixed coordinate system. Taking the (216) Kleopatra system as an example, the primary is Kleopatra with the oblateness coefficient $J_2 = 0.6$, and two moonlets are S/2008 ((216)) 1 and S/2008 ((216)) 2 (Descamps et al. 2011). Therefore, the four-body system consists of the primary Kleopatra, S/2008 ((216)) 1, S/2008 ((216)) 1, and the Sun. Because the semi-major axis

of the moonlet is usually several times larger than the primary's reference radius (Richardson and Walsh 2006; Johnston 2013), the effects of higher harmonic expansions are negligible. Therefore, only J_2 term is considered in this paper.

Note that Eq. (1) is different from Eq. (17) in Walker (1983a). The reason is that m_1 dominates the whole system in Walker (1983a) while m_4 dominates our system and m_1 only dominates the large size ratio triple asteroids in the present paper.

Take m_1 as the unit of the mass, the scaled masses m_2, m_3, and m_4 are denoted as μ_2, μ_3, and μ_4, respectively. Reminding of the large size ratio property ($m_1 \gg m_2, m_3$) and the Hill's approximation ($\rho_4 \gg \rho_2, \rho_3$ and $m_4 \gg m_1, m_2, m_3$), the equations of motion take a compact form

$$\ddot{\boldsymbol{\rho}}_2 = GM_2 \nabla_2 \left[\frac{1}{\rho_2} \left(1 - \frac{J_2 R_e^2 P_2(\sin\varphi_2)}{\rho_2^2} \right. \right.$$
$$\left. \left. + \mu_3 \alpha_{23}^3 S'_{23} + \mu_4 \alpha_{24}^3 \right) \right], \quad (2)$$

$$\ddot{\boldsymbol{\rho}}_3 = GM_3 \nabla_3 \left[\frac{1}{\rho_3} \left(1 - \frac{J_2 R_e^2 P_2(\sin\varphi_3)}{\rho_3^2} \right. \right.$$
$$\left. \left. + \mu_2 \alpha_{23}^2 S'_{23} + \mu_4 \alpha_{34}^3 \right) \right], \quad (3)$$

$$\ddot{\boldsymbol{\rho}}_4 = GM_4 \nabla_4 \left[\frac{1}{\rho_4} \left(1 - \frac{J_2 R_e^2 P_2(\sin\varphi_4)}{\rho_4^2} \right. \right.$$
$$\left. \left. + \mu_2 \alpha_{24}^2 + \mu_3 \alpha_{34}^2 \right) \right]. \quad (4)$$

Note that the expressions are expanded in terms of mass ratios due to the properties of the large size ratio triple asteroid systems, which is different from Walker and Roy (1983) where the expressions were expanded in terms of the ratios of the orbital radii.

3 Hierarchical stability of the large size ratio triple asteroids

According to Walker and Roy (1983), in our problem at the collinear configuration m_1–m_2–m_3–m_4 where both the mutual perturbations of the bodies and the perturbation due to primary's J_2 are greatest. Note that this collinear configuration is only an extreme scenario in the mathematical sense. In this scenario, $C_{ij} = 1$ and $\varphi_i = 0$, so that $P_r(C_{ij}) = 1$ and $P_2(\sin\varphi_i) = -1/2 \ \forall r, i, j$; there is also the equation $S'_{ij} = 1/(1 - \alpha_{ij})$. Walker and Roy (1983) introduced the empirical stability parameters that are the sum of the disturbing terms at this configuration, when they expanded the force function in terms of the ratios of the orbital radii. In the

Table 1 Ratios of masses and orbital radii for the seven known large size ratio triple asteroids

System	μ_2	μ_3	μ_4	α_{23}	α_{34}	Primary's J_2
(45) Eugenia [a,b]	4.45×10^{-5}	4.45×10^{-5}	3.53×10^{11}	5.25×10^{-1}	2.86×10^{-6}	0.0581
(87) Sylvia [c]	4.94×10^{-5}	6.28×10^{-5}	1.34×10^{11}	5.21×10^{-1}	2.60×10^{-6}	0.09959
(93) Minerva [d]	8.60×10^{-6}	1.23×10^{-5}	5.94×10^{11}	6.01×10^{-1}	1.51×10^{-6}	0.08
(216) Kleopatra [e]	1.40×10^{-4}	3.00×10^{-4}	4.29×10^{11}	6.70×10^{-1}	1.62×10^{-6}	0.6
(136108) Haumea [f]	4.47×10^{-4}	4.50×10^{-3}	4.97×10^{8}	5.14×10^{-1}	7.74×10^{-6}	0.244
(136617) 1994 CC [g]	2.24×10^{-2}	3.50×10^{-3}	7.67×10^{18}	2.82×10^{-1}	2.50×10^{-8}	0.014
(153591) 2001 SN263 [g]	1.07×10^{-2}	2.62×10^{-2}	2.17×10^{17}	2.29×10^{-1}	5.60×10^{-8}	0.013

Note: The masses of (93) Minerva's moonlets are estimated by assuming their spherical shapes and same density with the primary

[a] Marchis et al. (2010). [b] Beauvalet et al. (2011). [c] Fang et al. (2012). [d] Marchis et al. (2013). [e] Descamps et al. (2011). [f] Ragozzine and Brown (2009). [g] Fang et al. (2011)

four-body problem, the physical significance of each empirical stability parameter Σ_i ($i = 2, 3, 4$) can be considered as the measure of the disturbance of the system's other components on the orbit of m_i relative to M_{i-1}; in order to ensure the hierarchical stability, the values of Σ_i must be less than unity and generally less than 10^{-2} (Walker and Roy 1983). Walker and Roy (1983) also pointed out that the eccentricity's effect on the Σ_i parameters is small. Thus, the eccentricity's effect is not considered in this study, just as Walker et al. (1980), Walker (1983b), and Veras and Armitage (2004) when they investigated the Hill stability of the triple systems using analytic methods. In our problem for the large size ratio triple asteroid systems, the empirical stability parameters Σ_2, Σ_3, and Σ_4 at the collinear configuration m_1–m_2–m_3–m_4 can be expressed as

$$\Sigma_2 = J_2 R_e^2 / 2\rho_2^2 + \mu_3 \alpha_{23}^3 / (1 - \alpha_{23}) + \mu_4 \alpha_{24}^3, \quad (5)$$

$$\Sigma_3 = J_2 R_e^2 / 2\rho_3^2 + \mu_2 \alpha_{23}^2 / (1 - \alpha_{23}) + \mu_4 \alpha_{34}^3, \quad (6)$$

$$\Sigma_4 = J_2 R_e^2 / 2\rho_4^2 + \mu_2 \alpha_{24}^2 + \mu_3 \alpha_{34}^2. \quad (7)$$

3.1 Without consideration of the solar gravity effect and primary's J_2

If the effect of the solar gravity and primary's J_2 is not taken into account, i.e. $\mu_4 = 0$, $\rho_4 = 0$, and $\Sigma_4 = 0$, the system only consists of three masses. Table 1 lists ratios of masses and orbital radii for the seven known large size ratio triple asteroids. Based on Eqs. (5)–(6), the values of Σ_2 and Σ_3 for the seven known large size ratio triple asteroids are shown in Fig. 1 using the parameters listed in Table 1. It is easy to see that the values of Σ_2 and Σ_3 are all much less than 10^{-2}. Therefore, these seven large size ratio triple asteroids are all hierarchically stable when without considering the solar gravity effect and primary's J_2.
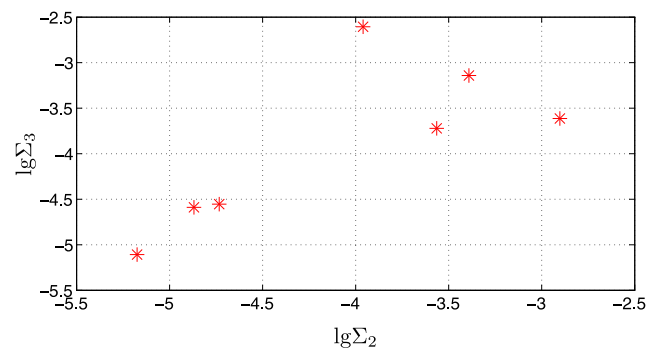

Fig. 1 The values of empirical stability parameters Σ_2 and Σ_3 for the seven known large size ratio triple asteroids without considering the solar gravity effect and primary's J_2. The actual large size ratio triple asteroids cases are marked with asterisk

3.2 With consideration of the solar gravity effect and primary's J_2

When the solar gravity effect and primary's J_2 is taken into consideration, the case is a four-body problem, which is a bit more complicated. According to Eqs. (5)–(7), the values of Σ_2, Σ_3 and Σ_4 for the seven known large size ratio triple asteroids are shown in Fig. 2. It can be seen that the values of Σ_2, Σ_3 and Σ_4 are all much less than 10^{-2} for all the known high size ratio asteroids. It means that all the known high size ratio asteroids are hierarchically stable. This result is consistent with numerical investigations of (87) Sylvia (Winter et al. 2009; Frouard and Compère 2012; Fang et al. 2012), which showed that the triple system (87) Sylvia is orbitally stable.

4 Conclusions

In the present paper, the hierarchical stability of the seven known large size ratio triple asteroids is analyzed. The solar gravity and primary's J_2 are considered. The force function
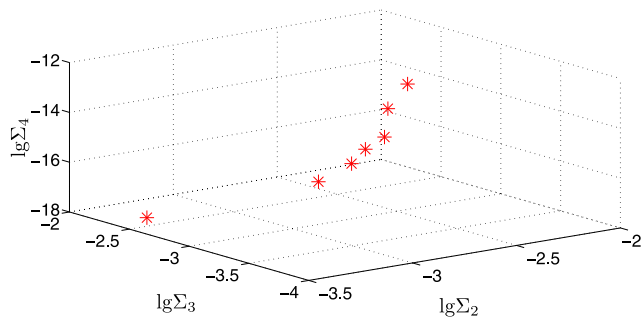
Fig. 2 The values of empirical stability parameters Σ_2, Σ_3 and Σ_4 for the seven known large size ratio triple asteroids when considering the solar gravity effect and primary's J_2. The actual large size ratio triple asteroids cases are marked with asterisk

for the four-body problem is expanded in terms of mass ratios based on the Hill's approximation and the large size ratio property. The hierarchical stability of the triple asteroids is examined using the empirical stability parameters. Without considering the solar gravity and primary's J_2, it is found that all the seven known high size ratio asteroids are hierarchically stable. While considering the solar gravity and primary's J_2, all the known high size ratio asteroids are still hierarchically stable. We concluded that the hierarchical arrangement of the system will not be altered for (45) Eugenia, (87) Sylvia, (93) Minerva, (216) Kleopatra, (136617) 1994 CC, (153591) 2001 SN263, and (136108) Haumea over a long time.

References

Araujo, R.A.N., et al.: Mon. Not. R. Astron. Soc. **423**, 3058–3073 (2012)

Beauvalet, L., Marchis, F., Lainey, V., Allain, M.: A new dynamical solution of (45) Eugenia's satellites (2011). EPSC-DPS2011-728

Benecchi, S.D., Noll, K.S., Grundy, W.M., Levison, H.F.: (47171) 1999 TC$_{36}$, a transneptunian triple. Icarus **207**, 978–991 (2010)

Brown, M.E., Margot, J.-L.: S/2001 (87) 1. IAU Circ., 7588 (2001)

Brozović, M., et al.: Radar and optical observations and physical modeling of triple near-earth asteroid (136617) 1994 CC. Icarus **216**, 241–256 (2011)

Descamps, P., et al.: Triplicity and physical characteristics of asteroid 216 Kleopatra. Icarus **211**, 1022–1033 (2011)

Fang, J., Margot, J.-L., Brozovic, M., et al.: Orbits of near-earth asteroid triples 2001 SN263 and 1994 CC: properties, origin, and evolution. Astron. J. **141**, 154–168 (2011)

Fang, J., Margot, J.-L., Rojo, P.: Orbits, masses, and evolution of main belt triple (87) Sylvia. Astron. J. **144**, 70 (2012)

Frouard, J., Compère, A.: Instability zones for satellites of asteroids: the example of the (87) Sylvia system. Icarus **220**, 149–161 (2012)

Hamilton, D.P., Krivov, A.V.: Dynamics of distant moons of asteroids. Icarus **128**, 241–249 (1997)

Johnston, W.R.: Asteroids with satellites. Last updated 13 July 2013. http://www.johnstonsarchive.net/astro/asteroidmoons.html (2013)

Marchis, F., Descamps, P., Hestroffer, D., Berthier, J.: Discovery of the triple asteroidal system 87 Sylvia. Nature **436**, 822–824 (2005)

Marchis, F., et al.: (3749) Balam. IAU Circ. 8928 (2008)

Marchis, F., et al.: A dynamical solution of the triple asteroid system (45) Eugenia. Icarus **210**, 635–643 (2010)

Marchis, F., et al.: A detailed picture of the (93) Minerva triple system. EPSC-DPS2011-653-3 (2011)

Marchis, F., et al.: Multiple asteroid systems: dimensions and thermal properties from Spitzer space telescope and ground-based observations. Icarus **221**, 1130–1161 (2012)

Marchis, F., et al.: Characteristics and large bulk density of the C-type Main-Belt triple asteroid (93) Minerva. Icarus **224**, 178–191 (2013)

Margot, J.L., Brown, M.E.: Discovery and characterization of binary asteroids 22 Kalliope and 87 Sylvia. In: AAS/Division for Planetary Sciences Meeting Abstracts, No. 33, p. 1133 (2001)

Milani, A., Nobili, A.M.: On the stability of hierarchical four-body systems. Celest. Mech. **31**, 241–291 (1983)

Ragozzine, D., Brown, M.E.: Orbits and masses of the satellites of the dwarf planet Haumea (2003 EL61). Astron. J. **137**, 4766–4776 (2009)

Richardson, D.C., Walsh, K.J.: Binary minor planets. Annu. Rev. Earth Planet. Sci. **34**, 47–81 (2006)

Scheeres, D.J.: The restricted Hill four-body problem with applications to the earth-moon-sun system. Celest. Mech. Dyn. Astron. **70**, 75–98 (1998)

Storrs, A., et al.: S/2001 (87) 1. IAU Circ., 7590 (2001)

Veras, D., Armitage, P.J.: The dynamics of two massive planets on inclined orbits. Icarus **172**, 349–371 (2004)

Walker, I.W., et al.: Stability criteria in many-body systems. I. An empirical stability criterion for co-rotational three body systems. Celest. Mech. **22**, 371–402 (1980)

Walker, I.W.: Stability criteria in many-body systems. IV. Empirical stability parameters for general hierarchical dynamical systems. Celest. Mech. **29**, 149–178 (1983a)

Walker, I.W.: On the stability of close binaries in hierarchical three-body systems. Celest. Mech. **29**, 215–228 (1983b)

Walker, I.W., Roy, A.E.: Stability criteria in many-body systems. V. On the totality of possible hierarchical general four-body systems. Celest. Mech. **29**, 267–294 (1983)

Winter, O.C., et al.: On the stability of the satellites of asteroid 87 Sylvia. Mon. Not. R. Astron. Soc. **395**, 218–227 (2009)