SLIP EFFECTS ON MIXED CONVECTIVE STAGNATION-POINT FLOW AND HEAT TRANSFER OVER A VERTICAL SURFACE

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This paper considers the steady mixed convection boundary layer flow of a viscous and incompressible fluid near the stagnation point on a vertical surface with slip effect. The temperature of the sheet and the velocity of the external flow are assumed to vary linearly with the distance from the stagnation point. The governing partial differential equations are first transformed into a system of ordinary differential equations, which is then solved numerically by a shooting method. The features of the flow and heat transfer characteristics for different values of the governing parameters are analyzed and discussed. Both assisting and opposing flows are considered. The results indicate that for the opposing flow, dual solutions exist for a certain range of the buoyancy parameter, while for the assisting flow, the solution is unique. In general, the effect of velocity slip is to reduce the wall shear stress and increase the heat transfer rate at the surface, while the thermal slip gives a vice versa behavior on the physical quantities.

Keywords: Dual solutions; Heat transfer; Mixed convection; Stagnation point; Slip.

1. INTRODUCTION

The two-dimensional stagnation point flow of an incompressible viscous fluid on a vertical sheet has attracted the attention of researchers for the past several decades because of its wide applications in industrial and practical applications. Some of the applications are cooling of electronic devices by fans, cooling of nuclear reactors during emergency shutdown, solar central receivers exposed to wind currents, and many hydrodynamic processes (Ishak et al. 2007).

Significant numbers of investigations have discovered the existence of dual solutions for the problem of a stagnation-point flow toward a vertical plate. Ramachandran et al. (1988) studied the steady laminar mixed convection in two-dimensional stagnation-point flows around heated surfaces by taking both cases of an arbitrary wall temperature and arbitrary surface heat flux variations. They found that a reversed flow developed in the buoyancy opposing flow region, and dual solutions were found to exist for a certain range of the buoyancy parameter. This problem was then extended by Devi et al. (1991) to the unsteady case, where they obtained the similar results as in Ramachandran et al. (1988). Further, Nazar et al. (2004) extended this problem to a micropolar fluid, and considered small and large values of ratio of the external velocity over the stretching velocity. Hassanien and Gorla

(1990) studied the stagnation-point flows of micropolar fluids over non-isothermal surfaces. The case of unsteady mixed convection flow of a micropolar fluid was studied by Lok et al. (2006) and they found a smooth transition from the initial unsteady-state flow to the final steady-state flow. Recently, Ishak et al. (2010) reported the existence of dual solutions for both assisting and opposing flows of an electrically conducting fluid past a vertical permeable flat plate. It may be pointed out here that less work has been done on the stagnation-point flows with slip. The similarity solutions of the Navier-Stokes equations for a stagnation-point flow towards a flat plate with slip was found by Wang (2003) where the solutions are applicable to the slip regime of rarefied gases. Later, Wang (2006) extended this problem to include the heat transfer aspect, while Ariel (2008) studied the stagnation-point flow of a viscoelastic fluid. Considering other aspect, Fang et al. (2010) have solved the problem of the slip flow over a permeable shrinking surface (without heat transfer aspect) using a second order slip flow model and presented an exact solution to the governing Navier-Stokes equations. Recently Fang and Zhang (2010) studied the heat transfer over a shrinking sheet with mass transfer. The flow is induced by a shrinking sheet with a linear velocity distribution from the slot.

In this paper, we consider the problem of a mixed convection boundary layer flow near the stagnation point on a vertical surface with slip. The effects of the buoyancy and slip parameters on the skin friction coefficient and the heat transfer rate at the surface are analyzed and discussed.

2. PROBLEM FORMULATION

Consider a steady, two-dimensional laminar boundary layer flow of a viscous and incompressible fluid near the stagnation-point on a vertical surface. It is assumed that the velocity of the free stream is $u_e(x)$, the temperature of the plate is $T_w(x)$, and the temperature of the ambient fluid is T_∞. The boundary layer equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where u and v are velocities in the x and y directions, respectively, g is the acceleration due to gravity, T the fluid temperature, β the thermal expansion coefficient and α is the thermal diffusivity. The boundary conditions are given by (Wang (2006), Andersson (2002))

$$\begin{aligned} &u=L\frac{\partial u}{\partial y},\quad v=0,\quad T=T_w+S\frac{\partial T}{\partial y}\quad \text{at}\quad y=0,\\ &u\to u_e,\quad T\to T_\infty\quad \text{as}\quad y\to\infty, \end{aligned} \tag{4}$$

where L is the slip length and S is a proportionality constant. Further, we assume that

$$u_e = a\,x, \quad T_w(x) = T_\infty + b\,x, \tag{5}$$

where a and b are constants.

To obtain similarity solution, we introduce the following similarity transformation (Wang (2006), Andersson (2002)):

$$\eta = (a/\nu)^{1/2}\,y, \quad \psi = (a\nu)^{1/2}\,x\,f(\eta), \quad \theta(\eta) = (T - T_\infty)/(T_w - T_\infty), \tag{6}$$

where η is the independent similarity variable, θ is the dimensionless temperature and ψ is the stream function defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ which identically satisfy the continuity equation (1). Substituting (6) into Eqs. (2) and (3), we get the following nonlinear ordinary differential equations:

$$f''' + f f'' - f'^2 + 1 + \lambda\,\theta = 0, \tag{7}$$

$$\frac{1}{\Pr}\theta'' + f\,\theta' - f'\,\theta = 0, \tag{8}$$

subject to the boundary conditions

$$\begin{gathered} f(0) = 0, \quad f'(0) = \delta\,f''(0), \quad \theta(0) = 1 + \gamma\theta'(0), \\ f'(\eta) \to 1, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty. \end{gathered} \tag{9}$$

Here primes denote differentiation with respect to η, $\Pr = \nu/\alpha$ is the Prandtl number, $\delta = L(a/\nu)^{1/2}$ is the velocity slip parameter, $\gamma = S(a/\nu)^{1/2}$ is the thermal slip parameter and λ $(= g\beta b/a^2)$ is the mixed convection parameter. It is worth mentioning that $\lambda > 0$ corresponds to the assisting flow, $\lambda < 0$ corresponds to the opposing flow and $\lambda = 0$ corresponds to the forced convection flow.

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are proportional to $f''(0)$ and $-\theta'(0)$, respectively.

3. RESULTS AND DISCUSSION

The nonlinear ordinary differential equations (7) and (8) subject to the boundary conditions (9) were solved numerically using the shooting method described by Zheng (2007) for some values of the velocity slip parameter δ, thermal slip parameter γ and the buoyancy or mixed convection parameter λ, while the Prandtl number Pr is fixed at 0.7 (such as air), except for comparisons with previously reported cases. Comparisons of the values of the reduced skin friction coefficient $f''(0)$ and the reduced local Nusselt number $-\theta'(0)$ with those obtained by Ramachandran et al. (1988), Devi et al. (1991), Lok et al. (2006) and Hassanien and Gorla (1990) for several values of Pr when the slip effect is absent ($\delta = 0$ and $\gamma = 0$) are listed in Table 1 and 2, respectively. It is observed that the results show a very good agreement.

Table 1. Values of $f''(0)$ for different values of Pr when $\lambda = 1, \delta = 0$ and $\gamma = 0$

Pr	Ramachandran et al. [2]	Devi et al. [3]	Lok et al. [6]	Hassanien and Gorla [5]	Present results
0.7	1.7063	1.7064	1.7064	1.70632	1.7063
1	-	-	-	-	1.6754
7	1.5179	1.5180	1.5180	-	1.5179
10	-	-	-	1.49284	1.4928
20	1.4485	1.4485	1.4486	-	1.4485
40	1.4101	-	1.4102	-	1.4101
50	-	-	-	1.40686	1.3989
60	1.3903	1.3903	1.3903	-	1.3903
80	1.3774	-	1.3773	-	1.3774
100	1.3680	1.3680	1.3677	1.38471	1.3680

The effect of velocity slip parameter δ on the reduced skin friction coefficient (wall shear stress) $f''(0)$ and the reduced local Nusselt number (heat transfer rate at the surface) $-\theta'(0)$ are shown in Figs. 1 and 2, respectively. In the meantime, the effect of thermal slip parameter γ on $f''(0)$ and $-\theta'(0)$ are shown respectively in Figs. 3 and 4. It is evident from these four figures that dual solutions exist for buoyancy opposing

Table 2. Values of $-\theta'(0)$ for different values of Pr when $\lambda = 1, \delta = 0$ and $\gamma = 0$

Pr	Ramachandran et al. [2]	Devi et al. [3]	Lok et al. [6]	Hassanien and Gorla [5]	Present results
0.7	0.7641	0.7641	0.7641	0.76406	0.7641
1	-	-	-	-	0.8708
7	1.7224	1.7223	1.7226	-	1.7224
10	-	-	-	1.94461	1.9446
20	2.4576	2.4574	2.4577	-	2.4576
40	3.1011	-	3.1023	-	3.1011
50	-	-	-	3.34882	3.3415

60	3.5514	3.5517	3.5560	-	3.5514
80	3.9095	-	3.9195	-	3.9095
100	4.2116	4.2113	4.2289	4.23372	4.2116

flow case ($\lambda < 0$). For the assisting flow $(\lambda > 0)$, the solution is unique. We identify the upper and lower branch solutions in the following discussion by how they appear in Figs. 1 to 4, i.e. the upper branch solution has a higher value of $f''(0)$ and $-\theta'(0)$ for a given δ and γ with λ, than the lower branch solution. It can be seen that for the upper branch solution, as the buoyancy parameter λ increases, both $f''(0)$ and $-\theta'(0)$ increase for the velocity and thermal slip parameters, due to the increased velocity caused by the external flow and buoyancy forces. The opposite trend can be observed for the lower branch solution. It is found that at the upper branch solution, an increase in the velocity slip parameter δ has decreased the wall shear stress, while the heat transfer rate at the surface increased gradually. Apart from that, a contrast behavior has been observed as the thermal slip parameter γ increases at the same branch. Concurrently, for the lower branch solutions, both $f''(0)$ and $-\theta'(0)$ show an increment as δ increases, while depreciation occurred as γ increases for both physical quantities. In addition, it is also observed (from Fig. 3), that in the forced convection flow ($\lambda = 0$), all values of the reduced skin friction coefficient are the same, that is $f''(0) = 0.5935$. It is worth mentioning that the heat transfer rate at the surface is always greater than zero ($-\theta'(0) > 0$), which means the heat is transferred from the surface to the fluid.

For each selected values of δ and γ, there is indeed a critical value λ_c of λ for which the solution exists. Based on our computations, we found that $\lambda_c = -2.96$, -2.9361108 and -2.96135329 for $\delta = 0.2$, 0.5 and 1.0, respectively. While, $\lambda_c = -2.96135329$, -3.489 and -4.02669366 for $\gamma = 1.0$, 1.5 and 2.0, respectively. It is worth mentioning that the computations have been performed until the point where the solution does not converge, and the calculations were terminated at that point. It may also be pointed out here that the effect of slip is to reduce the range of λ for which the solution exists.

Figs. 5 and 6 respectively present the samples of velocity $f'(\eta)$ and temperature $\theta(\eta)$ distributions for the selected values of velocity slip parameter δ when $\lambda = -2.0$ for both upper and lower branch solutions. While, the velocity and temperature profiles for the thermal slip parameter γ are depicted in Figs. 7 and 8, respectively. It is obvious that the upper branch solution display a thinner boundary layer thickness compared to the lower branch solution. It can be seen that the velocity gradient decreases as δ increases for the upper branch solution and as a result decreases the reduced skin friction coefficient $f''(0)$, while the temperature gradient increases and so does the reduced local Nusselt number $-\theta'(0)$. For the lower branch solution, both the velocity and temperature gradients is seen to increase therefore, $f''(0)$ and $-\theta'(0)$ increase as well. From Fig. 8, the effect of the thermal slip parameter γ is to diminish the fluid temperature in the boundary layer, which in turn decreases the heat transfer rate at the surface $-\theta'(0)$ for both upper and lower branch solutions. This observation is in agreement with the results presented in Fig. 4. Figs. 5 to 8 also show that the boundary conditions (9) are satisfied asymptotically, hence support the validity of the numerical results obtained, besides supporting the existence of the dual solutions presented in Figs. 1 to 4.

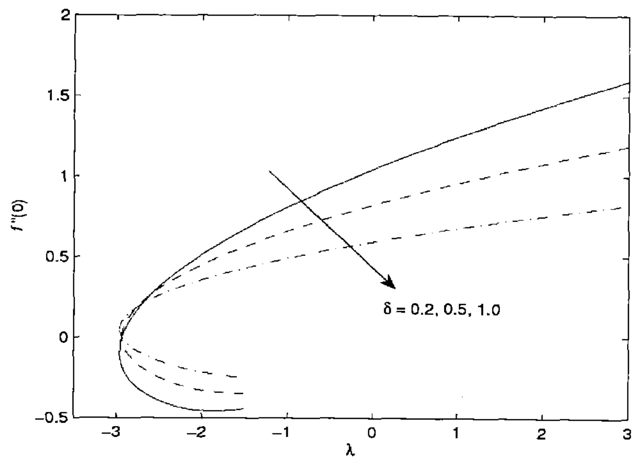

Fig. 1. Variation of skin friction coefficient $f''(0)$ with buoyancy parameter λ for various velocity slip parameter δ when $\Pr = 0.7,\ \gamma = 1$.

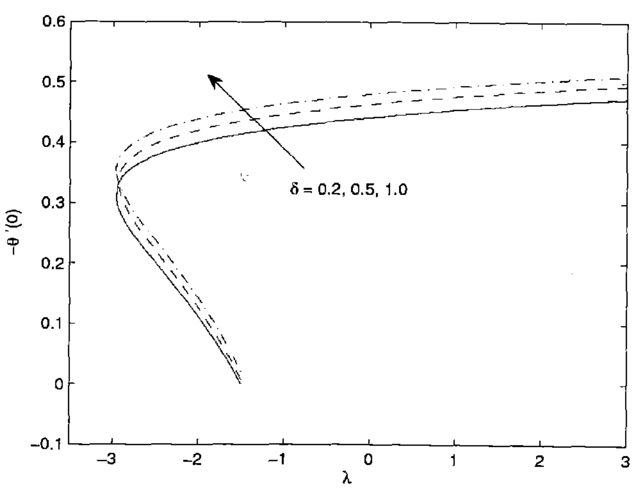

Fig. 2. Variation of local Nusselt number $-\theta'(0)$ with buoyancy parameter λ for various velocity slip parameter δ when $\Pr = 0.7,\ \gamma = 1$.

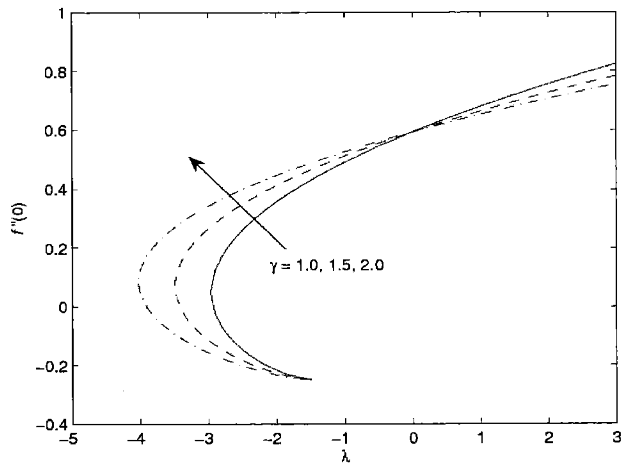

Fig. 3. Variation of skin friction coefficient $f''(0)$ with buoyancy parameter λ for various thermal slip parameter γ when $\Pr = 0.7,\ \delta = 1$.

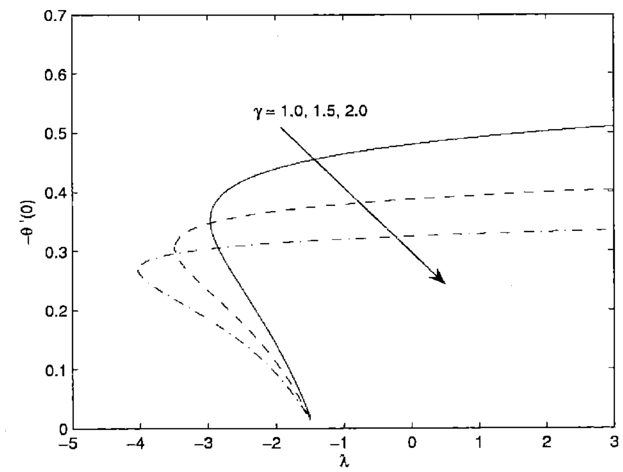

Fig. 4. Variation of local Nusselt number $-\theta'(0)$ with buoyancy parameter λ for various thermal slip parameter γ when $\Pr = 0.7,\ \delta = 1$.

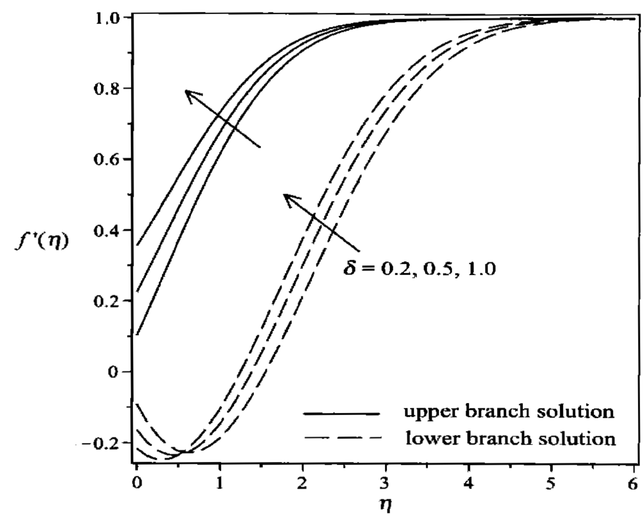

Fig. 5. Velocity profiles $f'(\eta)$ for some values of δ when $\Pr = 0.7$, $\gamma = 1$ and $\lambda = -2.0$.

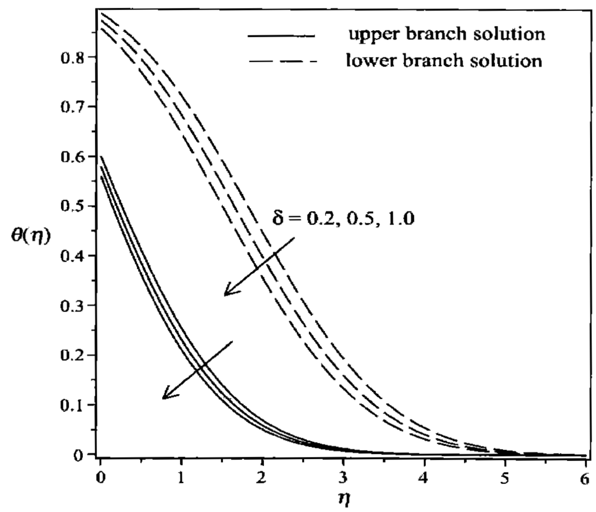

Fig. 6. Temperature profiles $\theta(\eta)$ for some values of δ when $\Pr = 0.7$, $\gamma = 1$ and $\lambda = -2.0$.

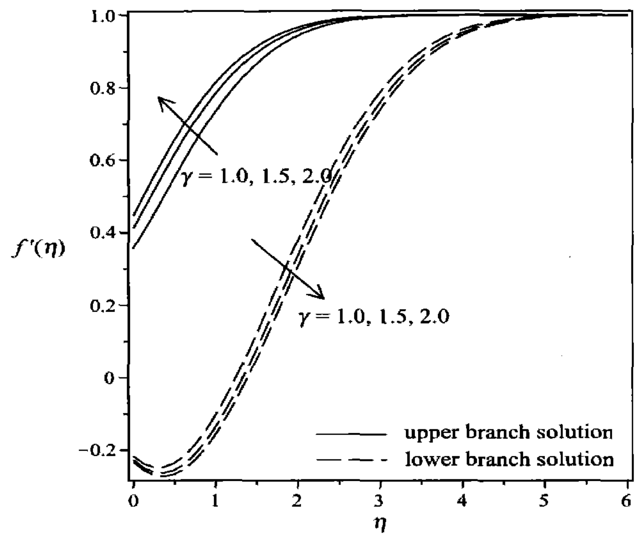

Fig. 7. Velocity profiles $f'(\eta)$ for some values of γ when $\Pr = 0.7$, $\delta = 1$ and $\lambda = -2.0$.

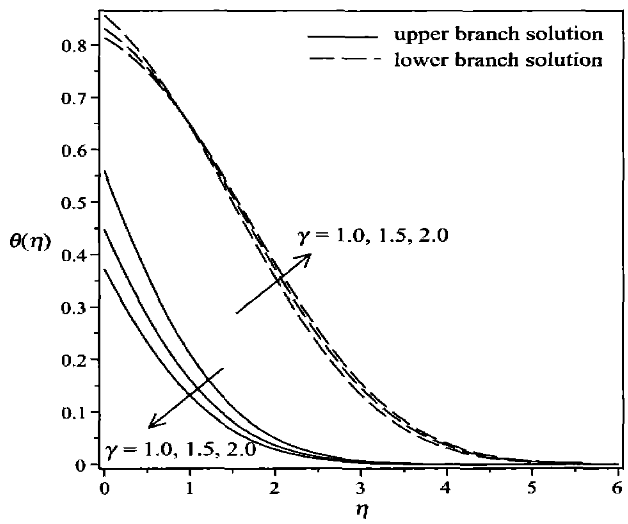

Fig. 8. Temperature profiles $\theta(\eta)$ for some values of γ when $\Pr = 0.7$, $\delta = 1$ and $\lambda = -2.0$.

4. CONCLUSION

In this paper, the flow and heat transfer characteristics near the stagnation-point on a vertical surface with slip effect were studied. The boundary layer equations governing the flow were reduced to ordinary differential equations using a similarity transformation. These equations were then solved numerically to obtain the skin friction coefficient and the local Nusselt number as well as the velocity and the temperature distributions for various values of the velocity slip parameter δ, thermal slip parameter γ and the buoyancy parameter λ, with a fixed value of the Prandtl number Pr. It was found that for the opposing flow ($\lambda < 0$), dual solutions exist for a certain range of the buoyancy parameter, whereas for the assisting flow ($\lambda > 0$), the solution is unique. Moreover, the effect of slip is to reduce the range of λ for which the solution exists.

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