An inversion approach for determining water storage changes from 3-D GPS coordinates time series in Europe

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Outline

- Inversion algorithm
- Water storage determination in Amazon basin
- Water storage determination in Danube basin
- Conclusion

Elastic responses of the Earth

Mass-loading Green's function [Farrel, 1972]

$$G_u(\theta) = \frac{a}{m_e} \sum_{n=0}^{\infty} h_n P_n(\cos\theta)$$

$$G_v(\theta) = \frac{a}{m_e} \sum_{n=1}^{\infty} l_n \frac{\partial P_n(\cos\theta)}{\partial \theta}$$

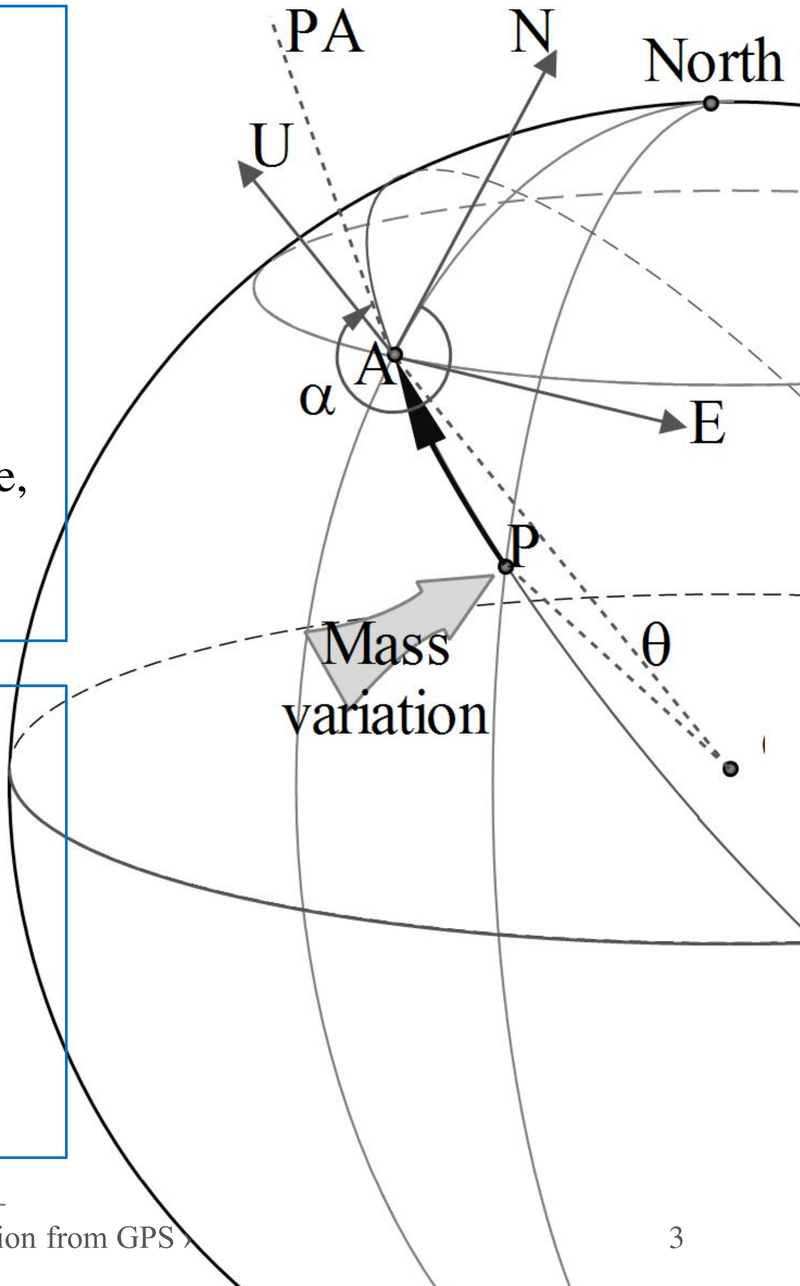
where *h* and *l* are love numbers, θ is angular distance,
P_n are the Legendre polynomials.
a and m_e are radius and total mass of the Earth.

Surface displacements (*U*, *V*):

$$\begin{bmatrix} \delta U \\ \delta V \end{bmatrix} = \begin{bmatrix} G_u \\ G_v \end{bmatrix} \delta M$$

with *M* is the mass variation

V aligns to direction P→A .



Inversion methodology

- Block load mass: homogenous equivalent water height (EWH) variations, $H_{1\ldots n}$, for predefined grids.

$$\begin{bmatrix} N_1 \\ E_1 \\ U_1 \\ \vdots \\ N_m \\ E_m \\ U_m \end{bmatrix} = \begin{bmatrix} \sum_{\theta,\lambda}^{\Omega_1^1} \rho G_v \sin\alpha \cdot dS & \sum_{\theta,\lambda}^{\Omega_2^1} \rho G_v \sin\alpha \cdot dS & \cdots & \sum_{\theta,\lambda}^{\Omega_n^1} \rho G_v \sin\alpha \cdot dS \\ \sum_{\theta,\lambda}^{\Omega_1^1} \rho G_v \cos\alpha \cdot dS & \sum_{\theta,\lambda}^{\Omega_2^1} \rho G_v \sin\alpha \cdot dS & \cdots & \sum_{\theta,\lambda}^{\Omega_n^1} \rho G_v \cos\alpha \cdot dS \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{\theta,\lambda}^{\Omega_1^m} \rho G_u \cdot dS & \sum_{\theta,\lambda}^{\Omega_2^m} \rho G_u \cdot dS & \cdots & \sum_{\theta,\lambda}^{\Omega_n^m} \rho G_u \cdot dS \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \end{bmatrix} + \boldsymbol{\varepsilon} .$$

$$\mathbf{l} \qquad \mathbf{A} \qquad \mathbf{X}$$

$$\hat{\mathbf{X}} = (\mathbf{A}^T \mathbf{W} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{W} \cdot \mathbf{l}$$

Where **W** is the weight matrix, determined from GPS error information.

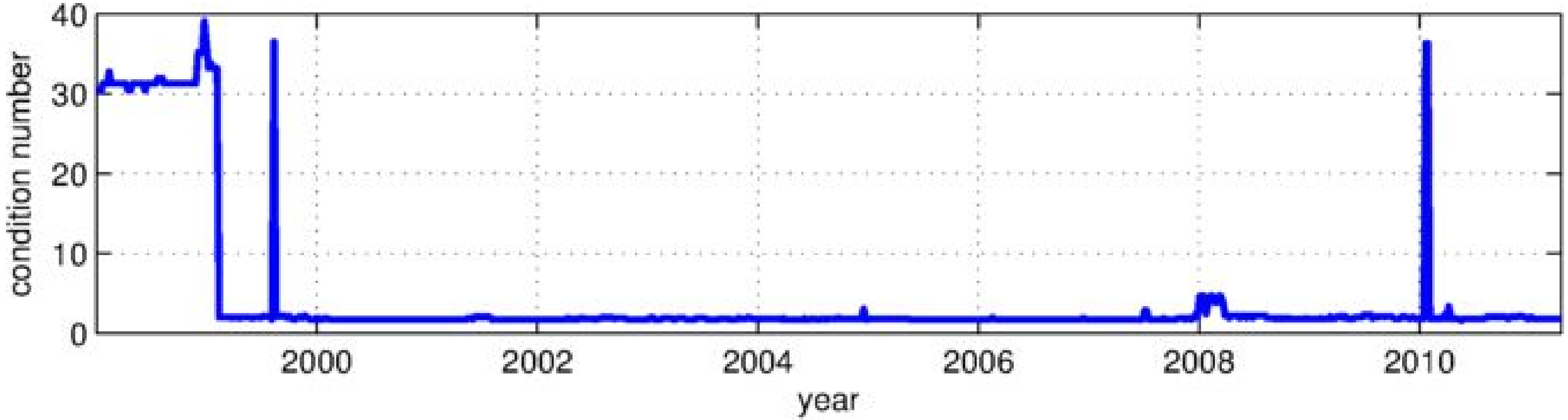
ρ = water density

θ and λ are latitude and longitude

S = surface area for block or cell

Regularization methods

- Condition number 3-block inversion case in Danube:



  - GPS avaibility
  - Distribution
  - Signal-to-noise ratio
  - Leakage effects

- Tikhonov Regularization in its most general form:

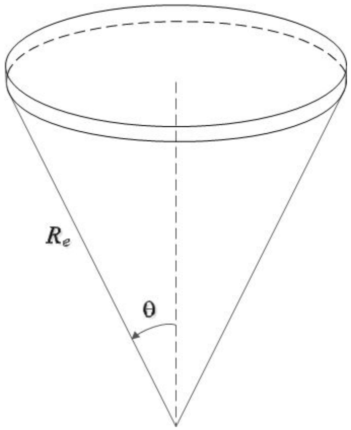
$$\min\left\{\left\|\mathbf{A}x-\varepsilon\right\|^{2}+\lambda^{2}\mathbf{I}^{2}\right\}$$

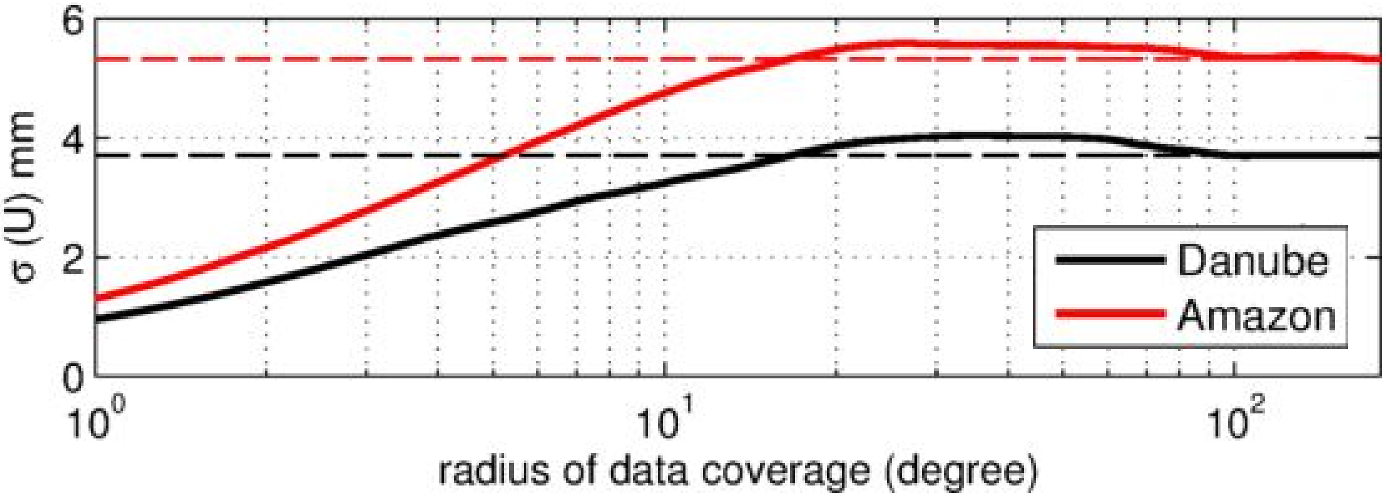
  - λ from L-curve corner determination [Hansen and O'Leary, 1993]

Data preparation

- Weekly GPS 3-D coordinate time series [Collilieux et al., 2012]
  - Linear trends are removed
  - 3-month moving average
- GRACE gravity fields
  - CSR Release 5
  - Degree-1 coefficients from Swenson [2008]
  - $C_{2,0}$ from Cheng [2005]
  - GAC de-aliasing product is restored
  - De-striping
  - No Gaussian smoothing
- Water storage from GLDAS NOAH model
  - Soil moisture
  - Snow/ice depth

Amplitude of displacements w.r.t. the spatial extent of the data





- 5-years GLDAS
- Displacements result from mass change within coverage
- 180° = global coverage

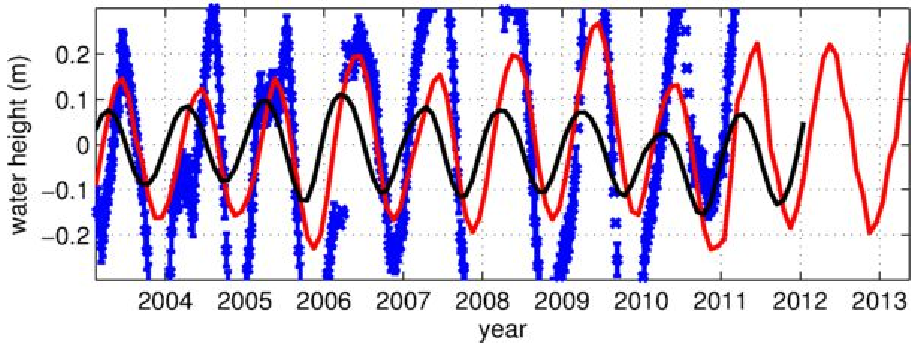
- Location dependent
- Displacements in radial direction present the loading mass up to radius of 10°-20°

- Amazon basin

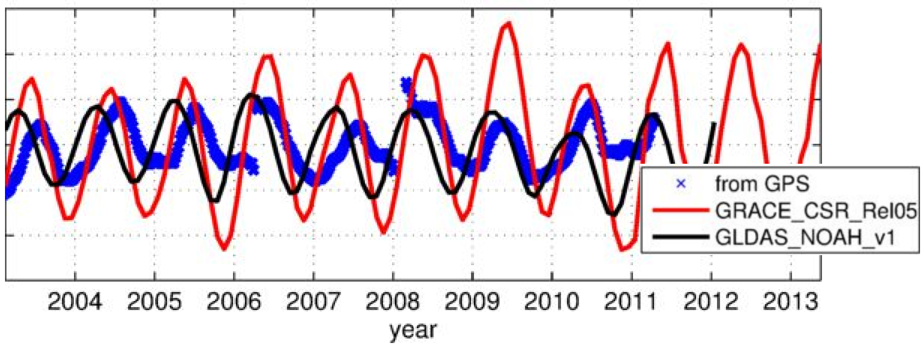
Inversion for a single disk in the Amazon Basin

- Most simple case
- Invert for mean CWS for a disk with a radius of ~12.5^{o} (same surface area as basin grid)

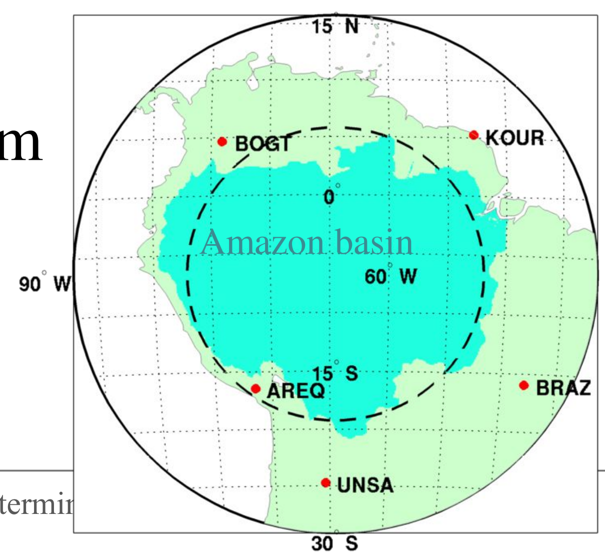
CWS from radial displacements

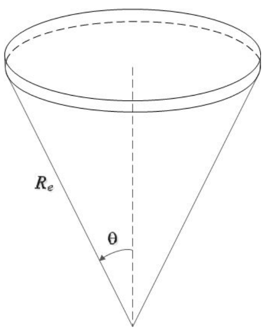


CWS from 3-D displacements



- Simple disk inversion provides reasonable results using data from only 5 GPS stations

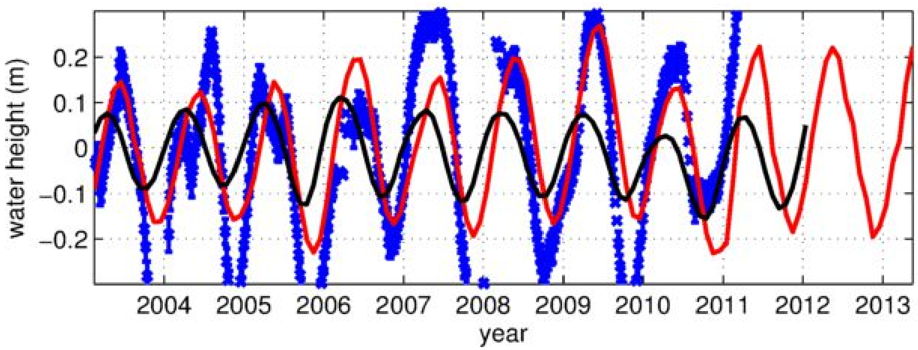




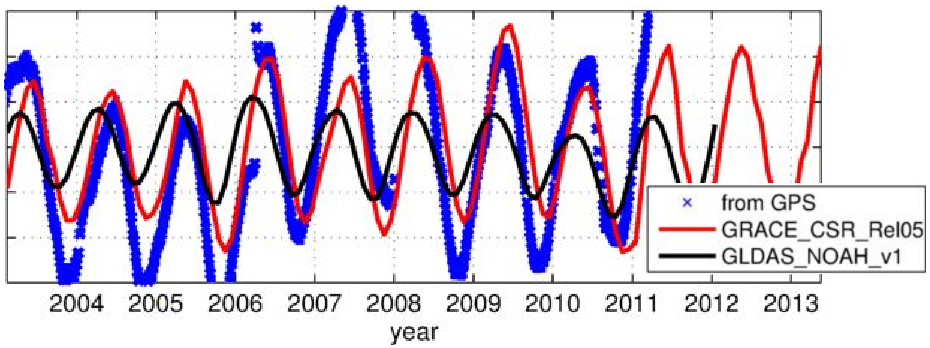
Inversion for a single CWS value in Amazon Basin

- Invert for mean CWS for the true area of the Amazon basin

CWS from radial displacements



CWS from 3-D displacements



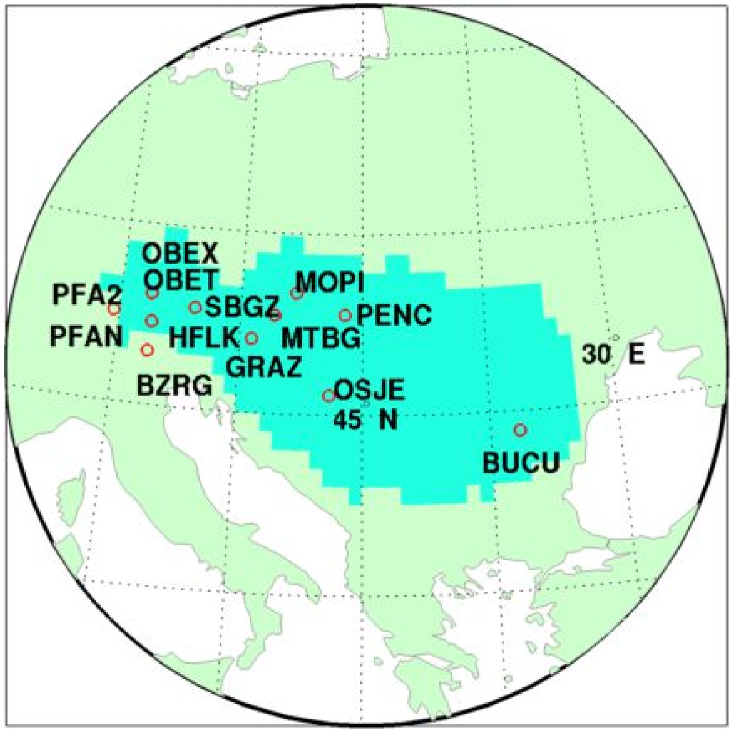
- Water storage variations determined from GPS have higher correlation to GRACE data than to GLDAS

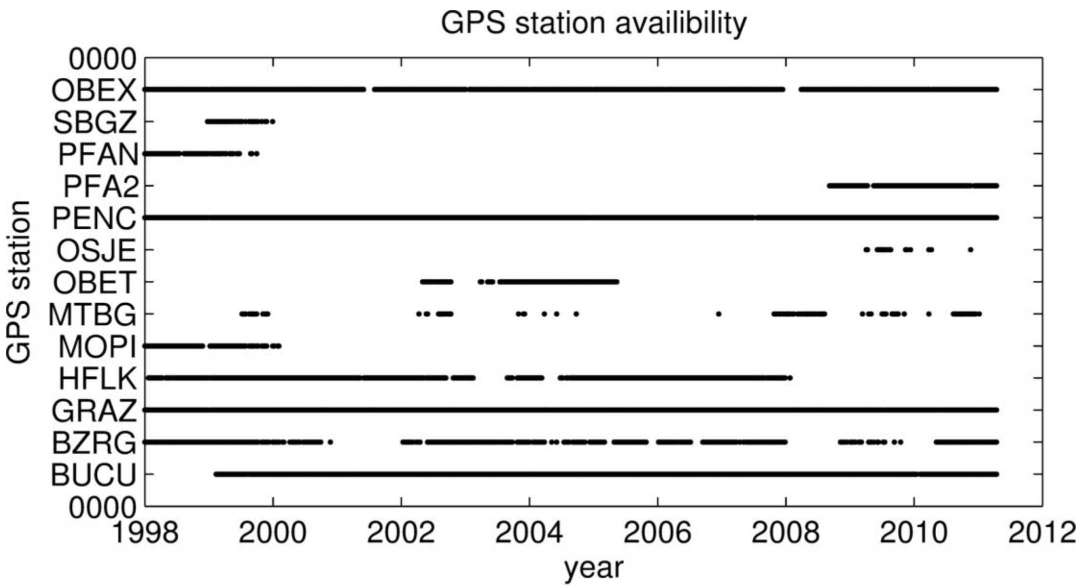
	$\rho_{\text{GRACE,GPS}}^{\text{EWH}}$	$\rho_{\text{GLDAS,GPS}}^{\text{EWH}}$
inv. Radial	0.76	0.55
inv. 3-D	0.78	0.63

- Danube basin

Danube basin

- 13 stations within/nearby the Danube basin are selected

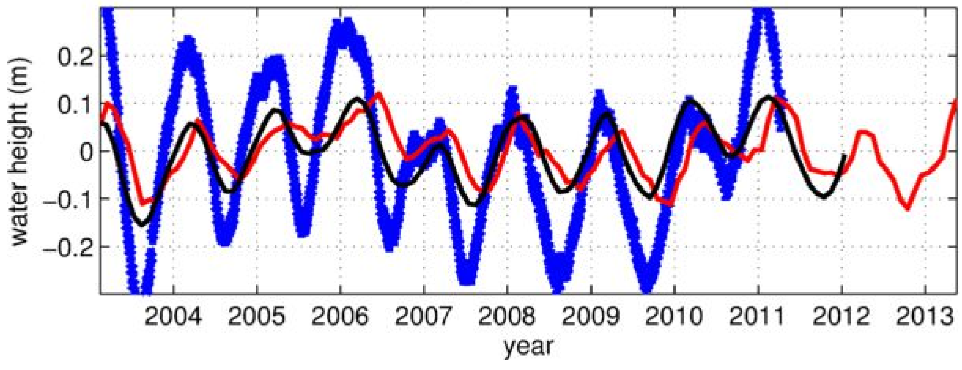




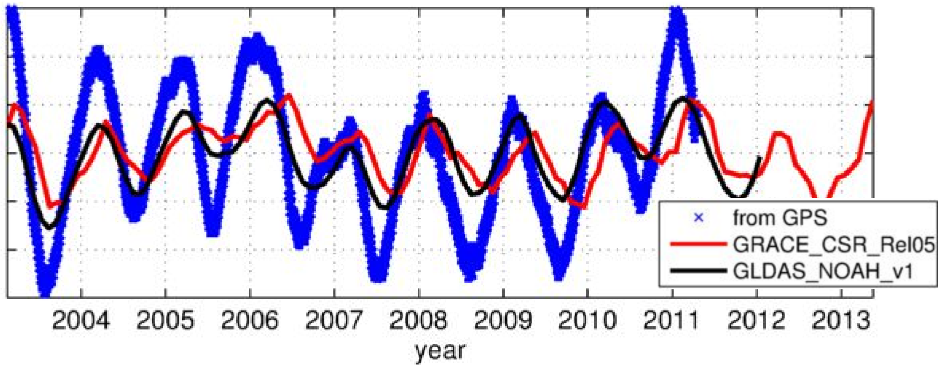
- On average, there are ~7 stations available for all epochs

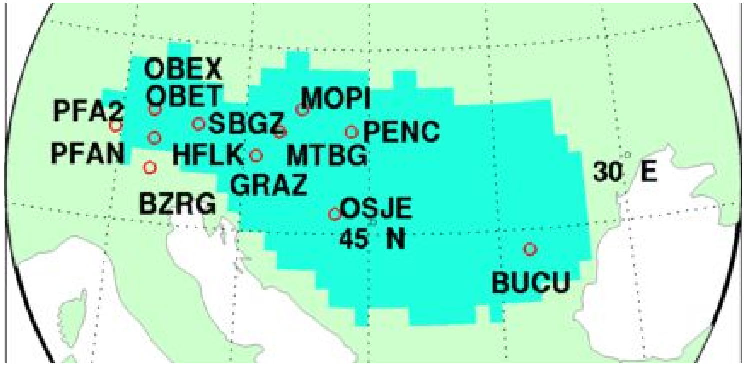
Inversion for a single block in Danube basin

CWS from radial displacements



CWS from 3-D displacements

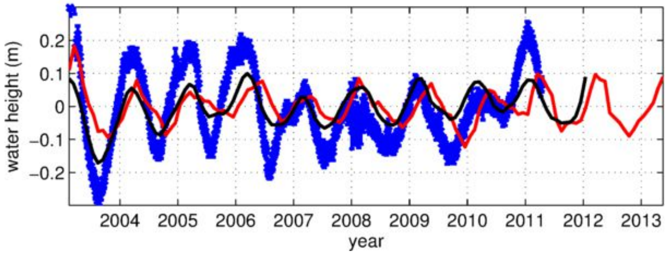




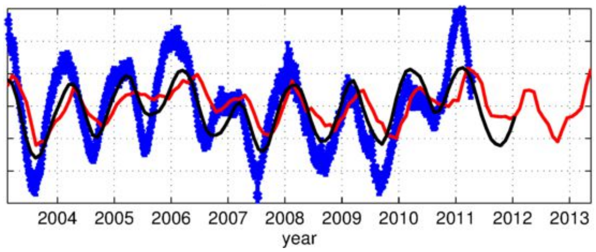
- The CWS amplitude determined from GPS is significant larger than from reference.
  - Why ?
    - Mass changes from outside of the Danube basin
    - Mass variations in smaller scale are detected by GPS

L-curve Tikhonov regularization

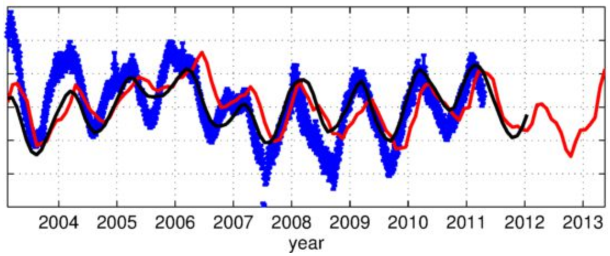
CWS from radial displacements in 1st blk

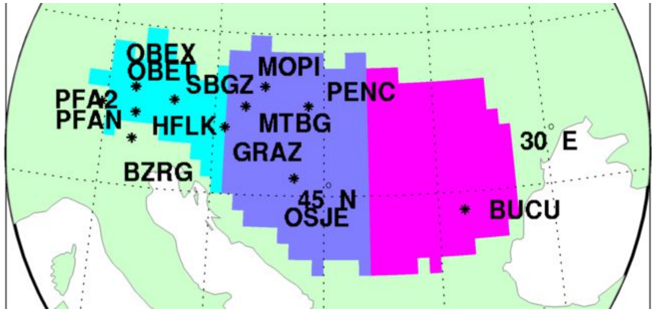


CWS from radial displacements in 2nd blk

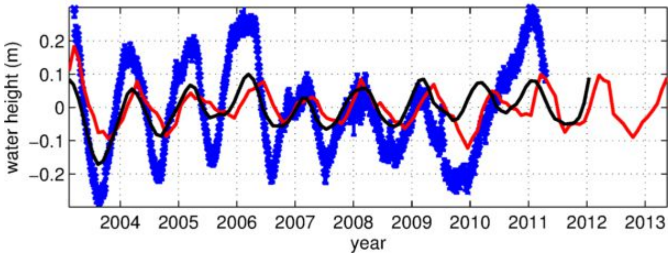


CWS from radial displacements in 3rd blk

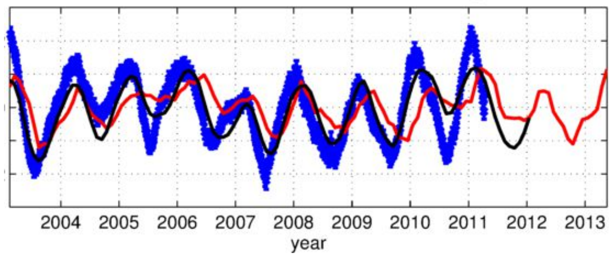




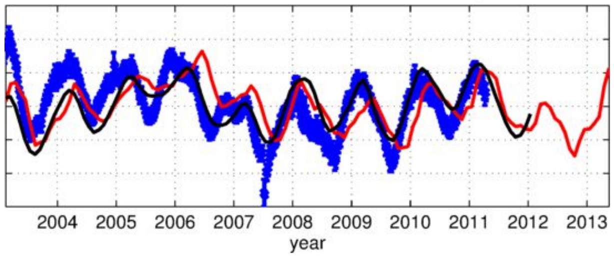
CWS from 3-D displacements in 1st blk



CWS from 3-D displacements in 2nd blk



CWS from 3-D displacements in 3rd blk



Conclusions

- GPS coordinates represent regional (10^o-20^o) mass variations.
- GPS can be used for CWS determination.
- Reasonable CWS estimates obtained for inverted GPS.
- The loading effects from outside of the study basin can not be neglected.

- To-do list:
  - Correlation between areas
  - Correlation of GPS time series
  - Introduce larger coverage
  - Validations
  - Combination of other data sources (GRACE, in-situ observations, etc.)

Bibliography

Cheng, M., and B. D. Tapley (2005), Variations in the Earth's oblateness during the past 28 years, Journal of Geophysical Research: Solid Earth, 109(B9), B09402, doi:10.1029/2004JB003028.

Collilieux, X., T. Dam, J. Ray, D. Coulot, L. Métivier, and Z. Altamimi (2012), Strategies to mitigate aliasing of loading signals while estimating GPS frame parameters, Journal of Geodesy, 86(1), 1–14, doi:10.1007/s00190-011-0487-6.

Farrell, W. E. (1972), Deformation of the Earth by surface loads, Reviews of Geophysics, 10(3), 761, doi:10.1029/RG010i003p00761.

Hansen, P., and D. O'Leary (1993), The Use of the L-Curve in the Regularization of Discrete Ill-Posed Problems, SIAM J. Sci. Comput., 14(6), 1487–1503, doi:10.1137/0914086.

Swenson, S., D. Chambers, and J. Wahr (2008), Estimating geocenter variations from a combination of GRACE and ocean model output, Journal of Geophysical Research: Solid Earth, 113(B8), B08410, doi:10.1029/2007JB005338.

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Thank you for your attention!